THE INFLUENCE OF SHORT CRACKS ON THE FATIGUE LIMIT

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Abstract. *Most structural components are designed against fatigue crack initiation, by procedures which do not recognize cracks. Large cracks may be easily detected, but small cracks may pass unnoticed if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural design for very long fatigue lives should avoid fatigue crack initiation* ***and*** *be tolerant to undetectable short cracks. However, this self-evident requirement is still not used in fatigue design routines, which just intend to maintain the loading at the structural component critical point below its fatigue limit. Nevertheless, most long-life designs work just fine, which means that they are somehow tolerant to undetectable short cracks. But the question "how much tolerant" cannot be answered by SN procedures alone. This important problem can only be solved by adding a proper short crack fatigue growth threshold requirement to the "infinite" life design criterion. This paper evaluates the tolerance to short 1D and 2D cracks, and proposes a design criterion for infinite fatigue life which explicitly considers it.*

Keywords*: short cracks, non-propagating cracks, fatigue life prediction, fatigue limit.*

1. INTRODUCTION

The notch sensitivity $0 \le q \le 1$ is widely used in mechanical design to correlate the linear elastic (LE) stress concentration factor (SCF) $K_t = \sigma_{max}/\sigma_n$, to $K_f = 1 + q\cdot(K_t - 1) = S_L/S_{Lntc}$, its corresponding fatigue SCF, which is used to quantify the actual notch effect on the fatigue strength of structural components. S_L and S_{Lntc} are the fatigue limits measured on standard (smooth and polished) and on notched test specimens (TS), usually under fully alternated loads. However, these limits can be defied for any $R = \sigma_{min}/\sigma_{max}$ ratio, $S_L(R)$ and $S_{Lntc}(R)$. σ_{max} and σ_{min} are the maximum and minimum LE stress at the notch root caused by σ_n, the nominal stress that would act at that point if the notch did not affect the stress field around the notch. It is well known that q can be associated with the relatively fast generation of tiny non-propagating fatigue cracks at notch roots when $S_L/K_t < \sigma_n < S_L/K_f$. Thus, the notch sensitivity can be predicted from the fatigue behavior of short cracks emanating from notch tips, using relatively simple but sound mechanical principles, which do not require heuristic arguments, or arbitrary fitting parameters (Castro and Meggiolaro, 2009).

The stress field gradients around notch roots controls the fatigue crack propagation (FCP) behavior of short cracks emanating from them. For any given material, q depends not only on the notch tip radius ρ, but also on its depth b, meaning that shallow and elongated notches of same radius ρ may have quite different sensitivities q. Note that "short crack" here means "mechanical" not "microstructural" short crack, since material isotropy is assumed in their modeling, a simplified hypothesis experimentally corroborated (Meggiolaro et al, 2007, Wu et al, 2010).

The short cracks FCP threshold must be smaller than the long crack threshold $\Delta K_{th}(R)$, otherwise the stress range $\Delta\sigma$ required to propagate them would be higher than the material fatigue limit $\Delta S_L(R)$. Indeed, assuming that the FCP process is primarily controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{(\pi a)}$, if short cracks with $a \to 0$ had the same $\Delta K_{th}(R)$ threshold of long cracks, then their propagation by fatigue would require $\Delta\sigma \to \infty$, a physical non-sense (Lawson et al, 1999). The FCP threshold of short fatigue cracks under pulsating loads $\Delta K_{th}(a, R = 0)$ can be modeled using El Haddad-Topper-Smith (1979) or ETS characteristic size a_0, which can be estimated from $\Delta S_0 = \Delta S_L(R = 0)$ and $\Delta K_0 = \Delta K_{th}(R = 0)$. This clever trick reproduces the Kitagawa-Takahashi (1976) plot trend, using a modified SIF range $\Delta K'$ to describe the fatigue propagation of any crack, short or long,

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}\text{ , where } a_0 = (1/\pi)(\Delta K_0/\Delta S_0)^2 \tag{1}$$

Using this a_0 trick, it is indeed possible to reproduce the expected limits $\Delta K_{th}(a \to \infty) = \Delta K_0$ and $\Delta\sigma(a \to 0) = \Delta S_0$, see Fig. 1. Knowing that steels typically have $6 < \Delta K_0 < 12MPa\sqrt{m}$, ultimate tensile strength $400 < S_U < 2000MPa$, and fatigue limit $200 < S_L < 1000MPa$ (since very clean high-strength steels tend to maintain the $S_L/S_U \cong 0.5$ trend of lower strength steels under fully alternated loads, with $R = -1$); and estimating by Goodman the pulsating ($R = 0$) fatigue limit as $\Delta S_0 = 2S_U S_L/(S_U + S_L) \Rightarrow 260 < \Delta S_0 < 1300MPa$; it can then be expected that the maximum range of the ETS short crack characteristic size a_0 for steels should be contained in the range

$$(1/\pi)(\Delta K_{0_{min}}/\Delta S_{0_{max}})^2 \cong 7 < a_0 < 700\mu m \cong (1/\pi)(\Delta K_{0_{max}}/\Delta S_{0_{min}})^2 \tag{2}$$

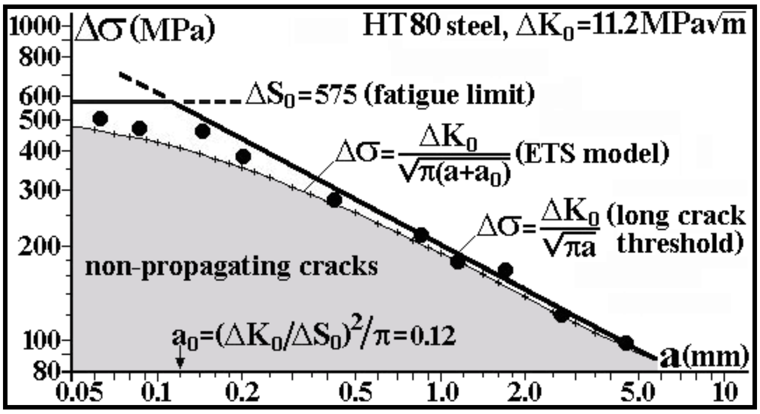

Figure 1: Kitagawa-Takahashi plot describing the fatigue propagation of short and long cracks under $R = 0$ in a HT80 steel with $\Delta K_0 = 11.2MPa\sqrt{m}$ and $\Delta S_0 = 575MPa$: long cracks with $a >> a_0$ stop when $\Delta\sigma \leq \Delta K_0/\sqrt{(\pi a)}$, very short cracks with $a << a_0$ stop when $\Delta\sigma \leq \Delta S_0$, and the ETS curve predicts crack stop when $\Delta\sigma \leq \Delta K_0/\sqrt{\pi(a + a_0)}$.

This a_0 range may be overestimated, since the minimum threshold ΔK_{0min} is not necessarily associated with the maximum fatigue crack initiation limit ΔS_{0max}, neither is ΔK_{0max} always associated with ΔS_{0min}. But it nevertheless justifies the “short crack” denomination used for cracks of a similar small size, and highlights the short crack dependence on the FCP threshold and on the fatigue limit of the material. In other words, it can be expected that cracks up to a few millimeters may still behave as short cracks in some steels, meaning they may have a smaller propagation threshold than that measured with long crack, which have $a >> a_0$.

Since the strengths of typical aluminum alloys are $70 < S_U < 600MPa$, $30 < S_L < 230MPa$, $40 < \Delta S_0 < 330MPa$, and $1.2 < \Delta K_0 < 5MPa\sqrt{m}$, their maximum a_0 (over)estimated range, and thus their short crack influence scale, is wider than the steels range, $\sim 1\mu m < a_0 < \sim 5mm$.

As ETS $\Delta K'$ has been deduced using Griffith’s plate SIF, $\Delta K = \Delta\sigma\sqrt{(\pi a)}$, Yu *et al* (1988) used the non-dimensional geometry factor $g(a/w)$ from the SIF expression $\Delta K = \Delta\sigma\sqrt{(\pi a)}\cdot g(a/w)$ to deal with other geometries, re-defining

$$\Delta K' = g(a/w)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,, \quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(g(a/w)\cdot\Delta S_0)\right]^2 \tag{3}$$

But the tolerable stress range $\Delta\sigma$ under pulsating loads tends to the fatigue limit ΔS_0 when $a \to 0$ only if $\Delta\sigma$ is the range at the notch root, instead of the nominal stress range. However, the geometry factors $g(a/w)$ listed in SIF tables usually include the notch SCF, using $\Delta\sigma$ instead of $\Delta\sigma_n$ as the nominal stress. A clearer way to define a_0 when the short crack departs from a notch root is to explicitly recognize this practice, separating the geometry factor $g(a/w)$ into two parts: $g(a/w) = \eta\cdot\varphi(a)$, where $\varphi(a)$ depends on the stress gradient ahead of the notch tip, which departs from the notch SCF as the crack length $a \to 0$, whereas η encompasses all the remaining terms, such as the free surface correction:

$$\Delta K' = \eta\cdot\varphi(a)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,, \quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(\eta\cdot\Delta S_0)\right]^2 \tag{4}$$

Alternatively, from an operational point of view, the short crack problem can be more clearly modeled by letting the SIF range ΔK retain its original equation, while the FCP threshold expression (under pulsating loads) is modified to become a function of the crack length a, namely $\Delta K_0(a)$, resulting in

$$\Delta K_0(a) = \Delta K_0\cdot\sqrt{a/(a+a_0)} \tag{5}$$

The El Haddad-Topper-Smith’s equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant’s (1977) reasoning, a more general equation can be used introducing an adjustable parameter γ to fit experimental data

$$\Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{6}$$

Note that equations (1-5) result from equation (6) if $\gamma = 2.0$. The bi-linear limit, $\Delta\sigma(a \leq a_0) = \Delta S_0$ for short cracks, and $\Delta K_0(a \geq a_0) = \Delta K_0$ for long ones, is obtained when $g(a/w) = \eta\cdot\varphi(a) = 1$ and $\gamma \to \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \leq \gamma \leq 8$, but $\gamma = 6$ better reproduces classical Peterson q-plots based on fatigue data obtained by testing TS with semi-circular notches, as discussed in Castro and Meggiolaro (2009). Using (6) as the FCP threshold, then any crack departing from a free smooth or notched surface under pulsating loads should propagate if

$$\Delta K = \eta \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \quad (7)$$

where $\eta = 1.12$ is the free surface correction□. As fatigue depends on two driving forces, the stress range $\Delta\sigma$ and its peak σ_{max}, (7) can be extended to consider σ_{max} (indirectly modeled by the R-ratio) influence in short crack behavior. First, the short crack characteristic size should be defined using the FCP threshold for long cracks $\Delta K_R = \Delta K_{th}(a >> a_R, R)$, and the fatigue limit ΔS_R, both measured or properly estimated at the desired R-ratio, then

$$a_R = (1/\pi)\left[\Delta K_R/(1.12 \cdot \Delta S_R)\right]^2 \quad (8)$$

Likewise, the corresponding short crack FCP threshold should be re-written as

$$\Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \quad (9)$$

All these details are important when such models are used to make predictions in real life situations, as they do influence the calculation results. In particular, neglecting the σmax effect on fatigue can lead to severe non-conservative life estimations, a potentially dangerous practice unacceptable for design or analysis purposes.

2. BEHAVIOR OF SHORT CRACKS WHICH DEPART FROM ELONGATED NOTCHES

It is worth to mechanically justify why a crack starting from a sharp notch root can propagate for a while before stopping and becoming non-propagating (under fixed loading conditions.) A reasonable estimate for the SIF of a small crack a departing from the elliptical notch tip in an Inglis plate, with semi-axes $b >> a$ and c, and root radius $\rho = c^2/b$, is $K_I(a) \cong \sigma_n \cdot \sqrt{(\pi a)} \cdot f_1(a, b, c) \cdot f_2(free\ surface)$, where the $2b$ axis is centered at the x co-ordinate origin, σ_n is the nominal stress (perpendicular to a and b); $f_1(a, b, c) \cong \sigma_y(x)/\sigma_n$; $\sigma_y(x)$ is the stress at $(x = b + a, y = 0)$ ahead of the notch tip when there is no crack; and $f_2 = 1.12$. $\sigma_y(x = b + a, y = 0)$ is given by (Schijve, 2001):

$$f_1 = \frac{\sigma_y(x, y=0)}{\sigma_n} = 1 + \frac{(b^2 - 2bc)(x - \sqrt{x^2 - b^2 + c^2})(x^2 - b^2 + c^2) + bc^2(b-c)x}{(b-c)^2(x^2 - b^2 + c^2)\sqrt{x^2 - b^2 + c^2}} \quad (10)$$

The slender the elliptical notch is, meaning the smaller their semi-axes c/b and tip radius to depth ρ/b ratios are, the higher is its SCF. But high K_t imply in steeper stress gradients $\partial\sigma_y(x, y = 0)/\partial x$ around notch tips, since LE stress concentration induced by any elliptical hole drops from $K_t = 1 + 2b/c = 1 + 2\sqrt{(b/\rho)} = \sigma_y(1)/\sigma_n \geq 3$ at its tip border to about $1.82 < K_{1.2} = \sigma_y(1.2)/\sigma_n < 2.11$ (for $b \geq c$) at a point just $b/5$ ahead of it, meaning their Saint Venant's controlling distance is associated with their depth b, not with their tip radii ρ . This is the cause for the peculiar growth of short cracks which depart from elongated notch roots. Their SIF, which should tend to increase with their length $a = x - b$, may instead decrease after they grow for a short while because the SCF effect in $K_I \cong 1.12 \cdot \sigma_n \sqrt{(\pi a)} \cdot f_1$ may diminish sharply due the high stress drop close to the notch tip, overcompensating the crack growth effect. This $K_I(a)$ estimate can be used to evaluate non-propagating fatigue cracks tolerable at notch roots, using the short crack FCP behavior. E.g., if a large steel plate with $S_U = 600MPa$, $S_L = 200MPa$ and $\Delta K_0 = 9MPa\sqrt{m}$ works under $\Delta\sigma_n = 100MPa$ at $R = -1$, verify if it is possible to change a circular $d = 20mm$ central hole by an elliptical one with axis $2b = 20mm$ (perpendicular to σ_n) and $2c = 2mm$, without inducing the plate to fail by fatigue.

Neglecting the buckling problem (important in thin plates), the circular hole has safety factor against fatigue crack initiation $\phi_F = S_L/K_f \cdot \sigma_n = 200/150 \cong 1.33$, as this large hole has $K_f \cong K_t = 3$. But the sharp elliptical hole would not be admissible by traditional SN design routines, since it has $\rho = c^2/b = 0.1mm$, thus a very high $K_t = 1 + 2b/c = 21$. Its notch sensitivity estimated from usual q plots (Juvinall 1967) would be $q \cong 0.32 \Rightarrow K_f = 1 + q \cdot (K_t - 1) = 7.33$, thus it would induce $K_f \cdot \sigma_n = 376MPa > S_L$. However, as this K_f value is sensibly higher than typical values reported in the literature, it is worth to re-study this problem considering the short crack FCP behavior. Supposing $\Delta K_{th}(R < 0) \cong \Delta K_0$ as usual, $\Delta K_0(a) = \Delta K_0/[1+(a_0/a)]^{-0.5}$ (by ETS), $S'_L = 0.5S_U$ (the material fatigue limit, as FCP modeling does not need modifying factors required to estimate S_L), $\Delta S_0 = S_U/1.5$ (by Goodman) and $a_0 = (1/\pi)(1.5\Delta K_0/1.12 \cdot S_U)^2 \cong 0.13mm$, the SIF ranges $\Delta K_I(a)$ for the two holes are compared to the FCP threshold $\Delta K_0(a)$ in Fig. 2. The SIF for cracks departing from the circular notch remains below the $\Delta K_0(a)$ FCP threshold curve (which considers the short crack behavior) up to

$a \cong 1.54mm$. Thus, if a small surface scratch locally augments the stress range and initiates a tiny crack at that hole border, it would not propagate under this fixed $\Delta\sigma_n = 100MPa$ and $R = -1$ load, confirming its "safe" prediction made by traditional *SN* procedures. Only if a crack with $a > 1.54mm$ is introduced at this circular hole border by any other means, it would propagate by fatigue under those otherwise safe loading conditions.

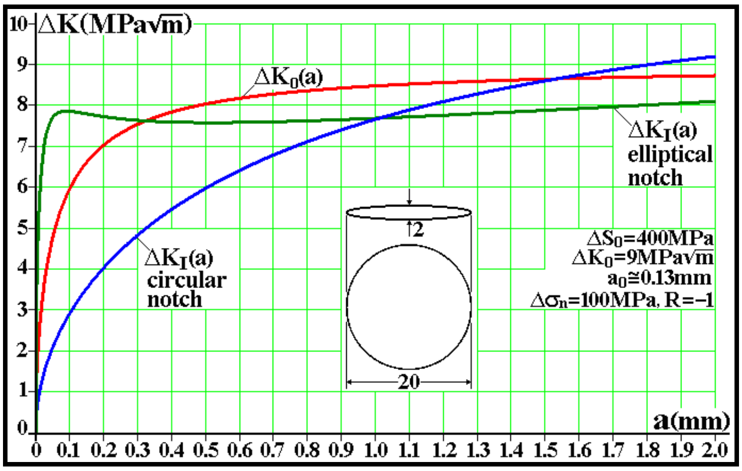

Figure 2: Using equation (9), it is estimated that cracks should not initiate at the circular hole border, which tolerates cracks $a < 1.54mm$, while the crack which initiates at the elliptical notch tip stops after reaching $a \cong 0.33mm$.

Under these same loading conditions, the $\Delta K_I(a)$ curve for the elliptical hole starts above $\Delta K_0(a)$, thus a crack should initiate at its border, as expected from its high K_t. But as this tiny crack propagates through the high stress gradient ahead of the notch root, it sees rapidly diminishing stresses around its tip during its early growth, which overcompensate the increasing crack size effect on $\Delta K_I(a)$. This crack SIF becomes smaller than $\Delta K_0(a)$ at $a \cong 0.33mm$, when it stops and becomes non-propagating (if $\Delta\sigma_n$ and R remain fixed), see Fig. 1. As fatigue failures include crack initiation and growth up to fracture, both notches could be considered safe for this service loading. But the non-propagating crack at the elliptical notch tip, a clear evidence of fatigue damage, renders it much less robust than the circular one, as discussed in Castro and Meggiolaro (2009).

For analysis purposes, the SIF range of a single crack with length a emanating from a semi-elliptical notch with semi-axes b (co-linear to a) and c at the edge of a very large plate loaded in mode I can be written as

$$\Delta K_I = \eta \cdot F\left(a/b, c/b\right) \cdot \Delta\sigma\sqrt{\pi a} \tag{11}$$

where $\eta = 1.12$, and $F(a/b, c/b)$ can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and of the notch SCF, given by

$$K_t = \left[1 + 2(b/c)\right] \cdot \left\{1 + \left[0.12 \big/ (1 + c/b)^{2.5}\right]\right\} \tag{12}$$

To obtain expressions for F, extensive finite element calculations were performed for several cracked semi-elliptical notches. The numerical results, which agreed well with standard solutions (Tada et al, 1985), were fitted within 3% using empirical equations (Meggiolaro et al, 2007, Wu et al, 2010)

$$F\left(a/b, c/b\right) \equiv f\left(K_t, s\right) = K_t\sqrt{\left[1 - exp\left(-sK_t^2\right)\right] \big/ sK_t^2}\ ,\ c \le b \text{ and } s = a/(b + a) \tag{13}$$

$$F'\left(a/b, c/b\right) \equiv f'\left(K_t, s\right) = K_t\left[1 - exp\left(-K_t^2\right)\right]^{-s/2}\sqrt{\left[1 - exp\left(-sK_t^2\right)\right] \big/ sK_t^2}\ ,\ c \ge b \tag{14}$$

The SIF expressions include the semi-elliptical notch effect through F or F'. Indeed, as $s \to 0$ when $a \to 0$, the maximum stress at its tip $\sigma_{max} \to F(0, c/b) \cdot \sigma_n = K_t \cdot \sigma_n$. Thus, the η-factor, but not the $F(a/b,c/b)$ part of K_I, should be considered in the short surface crack characteristic size a_0, as done in equation (3). Note also that the semi-elliptical K_t includes a term $[1 + 0.12/(1 + c/b)^{2.5}]$ which can be interpreted as a free surface correction (FSC), since as $c/b \to 0$ and the semi-elliptical notch tends to a crack, its $K_t \to 1.12 \cdot 2\sqrt{(b/\rho)}$. Such 1.12 factor is the notch FSC, not the crack FSC

η. Indeed, when $c/b \to 0$, this 1.12 factor disappears from the F expression, which gives $F(a/b, 0) = 1/\sqrt{s}$, and therefore $\Delta K_I = \eta \cdot F \cdot \Delta\sigma \cdot [\pi \cdot a]^{0.5} = \eta \cdot \Delta\sigma \cdot [\pi \cdot (a + b)]^{0.5}$, as expected, since the resulting crack for $c \to 0$ would have length $a + b$.

Traditional q estimates (Peterson 1974), based on the fitting of questionable semi-empirical equations to very few experimental data points, assume it depends only on the notch root ρ and on the material ultimate strength S_U. Thus, similar materials with the same S_U but different ΔK_0 should have identical notch sensitivities. The same should occur with shallow and deep or elongated notches of identical tip radii. However, whereas well established empirical relations relate the fatigue limit ΔS_0 to the tensile strength S_U of many materials, there are no such relations between their FCP threshold ΔK_0 and S_U. Moreover, it is also important to point out that the q estimation for elongated notches by the traditional procedures can generate unrealistic K_f values, as exemplified above. In conclusion, such traditional estimates should not be taken for granted.

The proposed model, on the other hand, is based on the FCP mechanics of short cracks which depart from elliptical notch roots, recognizing that their q values are associated with their tolerance to non-propagating cracks. It shows that their notch sensitivities, besides depending on ρ, ΔS_0, ΔK_0 and γ, are also strongly dependent on their shape, given by their c/b ratio. Their corresponding Peterson's curve is well approximated by the semi-circular $c/b = 1$ notch, but this curve is ***not*** applicable for much different c/b ratios. Therefore, the proposed predictions indicate that these traditional notch sensitivity estimates should ***not*** be used for elongated notches (Castro and Meggiolaro, 2009, Meggiolaro et al, 2007), a forecast experimentally verified (Wu et al, 2010), as discussed in the following section.

3. EXPERIMENTAL VERIFICATION OF ELONGATED NOTCH SENSITIVITY PREDICTIONS

Fatigue tests were carried out on modified SE(T) specimens of thickness $t = 8mm$ and width $w = 80mm$, to find the number of cycles required to re-initiate the crack after drilling a stop-hole of radius ρ centered at its tip, generating an elongated slit with $b = 27.5mm$. The original objective of those tests was to study life improvements obtained by the stop-hole repair technique, but these tests can also be used to support the validity of the model proposed to describe the short crack FCP behavior. Due to space limitations, such tests are very briefly described here, but details are available in Wu et al (2010). The TS were made from an Al alloy 6082 T6, with $S_Y = 280MPa$, $S_U = 327MPa$, and Young's modulus $E = 68GPa$. The particularly careful tests were made at 30Hz under fixed load range at $R = 0.57$, to avoid any crack closure influence on their FCP behavior. The TS were first pre-cracked until reaching the required crack size. Then they were removed to introduce the stop-holes in a milling machine, using a slight under-size drill precisely centered at their crack tips. Finally, the holes were enlarged to reach their final diameter using a reamer. The stop-hole sizes were large enough to remove the previous plastic zones.

After the stop-hole repair, the fatigue crack re-initiation lives at the tip of the resulting elongated notch can be modeled by εN procedures using (i) the alloy parameters $\sigma'_f = 485$MPa, $b = -0.0695$, $\varepsilon'_f = 0.733$ and $c = -0.827$, and Ramberg-Osgood's coefficient and exponent of the cyclic stress-strain curve, $H = 443$MPa and $h = 0.064$ (Borrego et al, 2003); (ii) the nominal stress range and R-ratio; and finally (iii) the stress concentration factor of the notches generated after repairing the cracks by a stop-hole at their tips, which can be calculated by FE ($K_t = 11.8$, 8.1, and 7.6 for the 3 stop-hole radii, $\rho = 1$, 2.5, and 3 mm.)

The repaired crack can be modeled by calculating the stress and strain maxima and ranges at the stop-hole root border by Neuber's rule, and by using them to calculate the crack re-initiation lives by an appropriate $\Delta\varepsilon \times N$ rule, considering the influence of the mean loads. Neglecting this effect could lead to severely non-conservative predictions, as the R-ratio used in the tests was high (Coffin-Manson predictions are highly non-conservative, thus useless in this case).

The lives predicted for the two larger holes reproduced reasonably well the tests results, but the predictions for the smaller $\rho = 1mm$ hole turn out to be too conservative in comparison to the measured data. But note that better-than-predicted fatigue lives do not mean that the smaller hole is more efficient than the larger ones. Indeed, as it could be expected, the larger stop-holes are associated with longer fatigue crack re-initiation lives for a given load (the apparent SIF range ΔK_{app} applied after the repaired specimens were remounted on the fatigue testing machine, were calculated treating the resulting $a = 27.5mm$ slits as if they were cracks). From a modeling point of view, the main result obtained from the analysis is that εN life predictions made using traditional procedures based on K_t, Neuber, and Morrow or SWT were satisfactory for the larger stop-holes, but severely underestimated the re-initiation lives for the smaller one.

There are few mechanical reasons which can explain the better than expected fatigue lives obtained from the specimens with the smaller stop-holes. One of them would be the presence of significant compressive residual stresses at the $\rho = 1mm$ stop-hole tips. But all the stop-holes were drilled and reamed following identical procedures. Thus, it is reasonable to assume that drilling and reaming processes left the same sub-surface residual stress state around all hole borders, since all hole sizes were large enough to remove the previous crack tip plastic zones, leaving only virgin material ahead of their roots. Moreover, as the bigger stop-hole lives were quite well predicted supposing $\sigma_{res} = 0$, it is difficult to justify why high compressive residual stresses would be present only at the $\rho = 1mm$ stop-hole roots. The same can be said about the surface finish of the stop-holes. However, the smaller stop-holes generate elongated notches with a larger K_t than the bigger holes, thus with a much steeper stress gradient near their roots. As discussed above, this effect can significantly affect the growth of short cracks and, consequently, the stop-hole fatigue notch sensitivity, possibly providing a sound mechanical explanation for the measured behavior.

Indeed, when using K_f instead of K_t with the traditional εN procedures, calculating the elongated notch sensitivity q by the method proposed here, all the estimated fatigue crack re-initiation lives reproduce quite well the measured results. The Al 6082 T6 fatigue limit and fatigue crack propagation threshold under pulsating loads required to calculate K_f are estimated as $\Delta K_0 = 4.8\ MPa\sqrt{m}$ and $\Delta S_0 = 110MPa$, following traditional structural design practices. The γ exponent was chosen as $\gamma = 6$, as recommended by Castro and Meggiolaro (2009). Figure 3 presents the lives predicted by the elastic of Morrow's equation (which is an extension of the Goodman line) and by the Smith-Watson-Topper (SWT), which are similar in this case, and reproduce well the measured data for the $\rho = 1mm$ hole. Note that the term "prediction" can in fact be used here, since the curves result from re-initiation life estimations calculated using material properties, without considering any of the measured data points. Thus they are indeed predicted, not data-fitted curves.

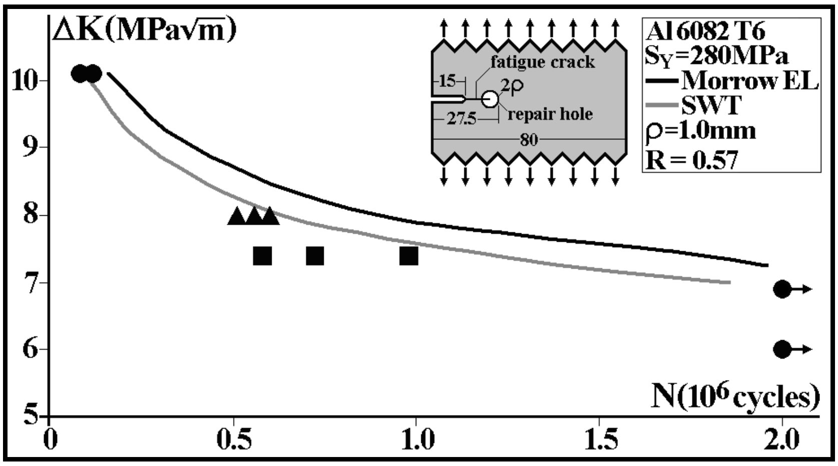

Figure 3: Predicted and measured crack re-initiation lives after introducing stop-holes with radii $\rho = 1.0mm$ at the tip of the previous crack, using the properly calculated K_f of the resulting elongated slit (instead of its K_t) and appropriate εN procedures.

4. A CRITERION TO DEFINE FUNCTIONALLY ADMISIBLE SHORT CRACKS

Based on the encouraging life estimations for these fatigue crack re-initiation data, the reverse path can be followed, assuming the methodology presented here can be used to generate an unambiguous acceptance criterion for small cracks, a potentially much useful tool for practical applications. Most structural components are designed against fatigue crack initiation, using εN or SN procedures which do not recognize cracks. Hence, their "infinite life" predictions may become unreliable when such cracks are introduced by any means, and not quickly detected and properly removed. Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections, if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural components designed for very long fatigue lives should be designed to be tolerant to short cracks.

However, this self-evident requirement is still not usually included in fatigue design routines, as most long-life designs just intend to maintain the stress range at critical points below their fatigue limits, guaranteeing that $\Delta\sigma < S_R/\phi_F$, where ϕ_F is a suitable safety factor. Nevertheless, most long-life designs work well, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN or εN procedures alone. Such problem can be avoided by adding equations (7-9) to the "infinite" life design criterion to tolerate a crack of size a. In its simplest version, this criterion should then be written as

$$\Delta\sigma < \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot g(a/w) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\}, \text{ where } a_R = (1/\pi) \cdot [\Delta K_R / \eta \Delta S_R]^2 \quad (15)$$

As fatigue limits ΔS_R considers microstructural defects inherent to the material, (15) complements it considering the component tolerance to cracks. A simple case study can clarify how useful this concept can be, as discussed next.

Due to a rare manufacturing problem, a batch of an important component was marketed with small surface cracks, causing some unexpected annoying failures. The task was to estimate how such cracks affect the stresses those steel components could tolerate under uniaxial fatigue loads, knowing that their rectangular cross section has $2mm$ by $3.4mm$; that their measured fatigue limit under $R = -1$ is $S_L = 246MPa$; and that $S_U = 990MPa$. Note that as $S_L \cong S_U/4$, it should include surface roughness effects which should not affect the cracks. But, in the absence of reliable information, the only safe option is to use the measured S_L value to estimate S_R and a_R. Therefore, by Goodman

$$S_R = \left[S_L S_U(1-R) \right] / \left[S_U(1-R) + S_L(1+R) \right] \tag{16}$$

The mode I stress range $\Delta\sigma$ tolerable by this component when it has a uniaxial surface crack of depth a is then

$$\Delta\sigma < \frac{\Delta K_R / \varphi_F}{\sqrt{\pi a}\,[\,0.752 + 2.02\frac{a}{w} + 0.37(1-\sin\frac{\pi a}{2w})^3\,]\,\sec\frac{\pi a}{2w}\sqrt{\frac{2w}{\pi a}\tan\frac{\pi a}{2w}}\cdot\left[1+\left(\frac{a_R}{a}\right)^{\gamma/2}\right]^{1/\gamma}} \tag{17}$$

where $w = 3.4mm$ was the component width, and its $g(a/w)$ geometry factor was obtained from Tada et al (1985). Figures 4-6 plot the maximum tolerable stress ranges (assuming $\phi_F = 1$) for several R-ratios. As the FCP threshold of this component was not available, it had to be estimated. The typical threshold range for steels is $6 < \Delta K_0 < 12MPa\sqrt{m}$. It is usual to assume $\Delta K_R \cong \Delta K_0$ for $R < 0$ loads (except if the load history contains severe underloads). Lower limit estimations for positive R are $\Delta K_{th}(0 < R \leq 0.17) = 6MPa\sqrt{m}$, and $\Delta K_{th}(R > 0.17) = 7\cdot(1 - 0.85R)$ (Castro and Meggiolaro, 2009). Using $\eta = 1.12$ and $\Delta K_0 = 6MPa\sqrt{m}$, the short crack characteristic value is estimated as $a_0 = 59\mu m$. Figure 4 shows that if this component works e.g. under $\Delta\sigma = 286MPa$ and $R = -0.12$, it tolerates cracks up to $a \cong 105\mu m$, and if it works under $\Delta\sigma = 176MPa$ and $R = 0.44$, it can sustain cracks up to $a \cong 150\mu m$. Figure 5 present the same curve, but using semi-log coordinates to enhance this component small tolerance to short cracks.

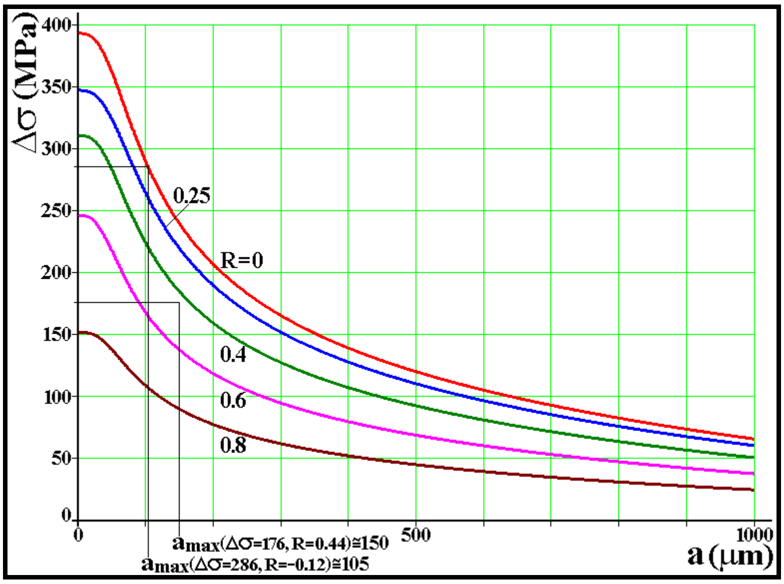

Figure 4: Surface crack of size a effect in the largest stress range $\Delta\sigma_R(a)$ tolerable by a strip of width $w = 3.4mm$ loaded in mode I, for various R-ratios (supposing $\Delta K_0 = 6MPa\sqrt{m}$ and $\gamma = 6$, thus $a_0 = 59$ and $a_{0.8} = 55\mu m$).

Therefore, this simple (but sensible) model indicates that this component is not too tolerant to 1D surface cracks. However, as this conclusion is based on estimated properties, it is worth to study its sensibility to the assumed values. Figure 6 shows the prediction range associated with the typical interval expected for the estimated properties, enhancing how important it is to measure them. Note that equation (15) assumes that the short crack is unidimensional and grows without changing its original plane. Note also that this model only describes the behavior of macroscopically short cracks, as it uses macroscopic material properties. Thus it can only be applied to short cracks which are large in relation to the characteristic size of the intrinsic material anisotropy (e.g. its grain size). Smaller cracks grow inside an anisotropic and usually inhomogeneous scale, thus their FCP is also affected by microstructural barriers, such as second phase particles or grain boundaries. However, as grains cannot be mapped in most practical applications, such problems, in spite of their academic interest, are not really a major problem from the fatigue design point of view.

However, this model has another limitation which may be more important for practical applications: it assumes that the short crack can be completely characterized by its depth a. But most short cracks are surface or corner cracks, which tend to grow by fatigue at least in two directions, maintaining their original plane when they are loaded under pure mode I conditions. In these cases, they can be modeled as bidimensional (2D) cracks which grow both in depth and width. In reality, both long and short cracks (these meaning cracks not much larger than a_R) only behave as 1D cracks after having cut all the component width to become a through crack, with a more or less straight front which propagates in an approximately uniform way. Thus, equation (17) must be adapted to consider the influence of 2D short cracks in the fatigue limit.

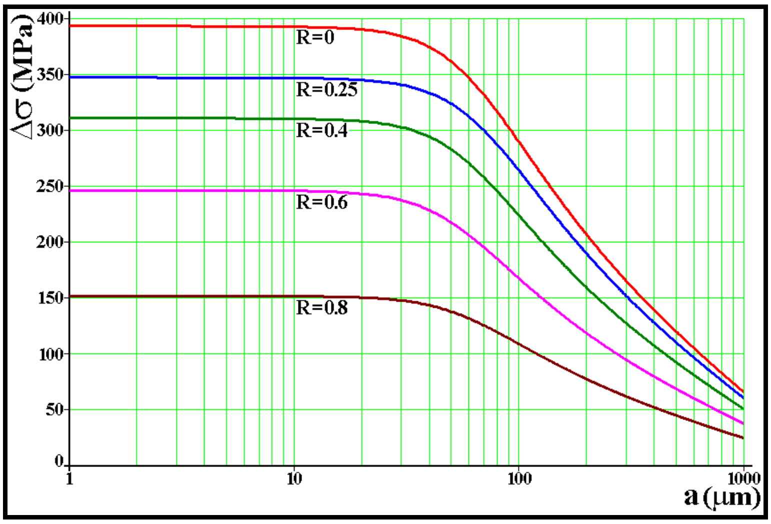

Figure 5: Similar to Fig. 4, but with semi-log scale to enhance the short crack tolerance. Small cracks with $a < 30\mu m$ have practically no effect in its fatigue resistance.

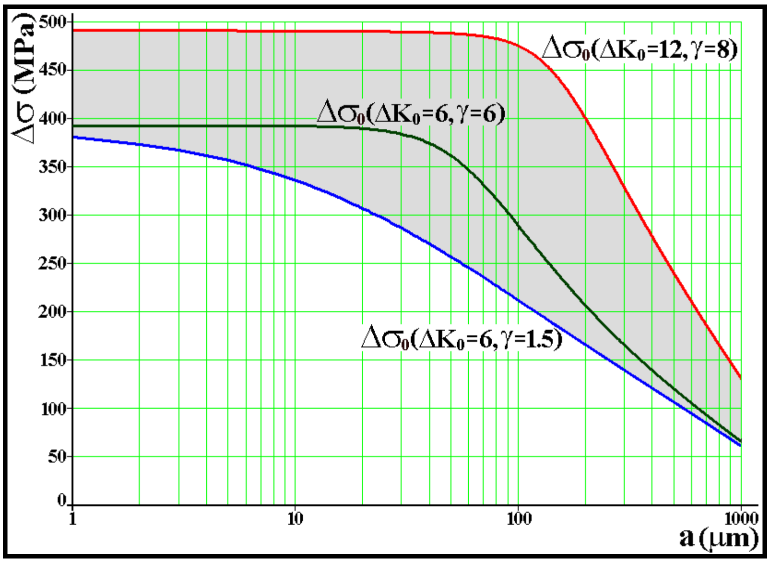

Figure 5: Typical steel threshold $6 < \Delta K_0 < 12MPa\sqrt{m}$ and γ exponent $1.5 < \gamma < 8$ ranges influence in the largest mode I stress ranges $\Delta\sigma_0$ tolerated by the $w = 3.4mm$ strip, as a function of the 1D superficial crack size a.

This can be done by assuming that: (i) the cracks are loaded in pure mode I, under quasi-constant $\Delta\sigma$ and R conditions, with no major overloads; (ii) material properties measured (or estimated) testing 1D specimens may be used to simulate the FCP behavior of 2D cracks; and (iii) 2D surface or corner cracks can be well modeled as having an approximately elliptical front, thus their SIF can be described by the classical Newman-Raju equations (1984). In this case, it can be expected that the component tolerance to cracks be given by:

$$\Delta\sigma < \begin{cases} \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot \Phi_a(a,c,w,t) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\} \\ \Delta K_R \Big/ \left\{ \sqrt{\pi c} \cdot \Phi_c(a,c,w,t) \cdot \left[1 + (a_R/c)^{\gamma/2} \right]^{1/\gamma} \right\} \end{cases} \quad (18)$$

For semi-elliptical or quart-elliptical surface cracks in a plate of thickness t, the SIF in the semi-axis directions, or in the depth a and width c directions, $K_{I,a} = \sigma\sqrt{(\pi a)}\cdot\Phi_a$ and $K_{I,c} = \sigma\sqrt{(\pi c)}\cdot\Phi_c$, are given by complicated functions, which enhance the operational advantage of treating the FCP threshold as a function of the crack size, $\Delta K_{th}(a)$, as claimed above. For structural calculations and design purposes, it is indeed relatively simple to use either equation (15) or (18) to evaluate the influence of surface cracks on the component fatigue strength. Moreover, it is not too difficult to adapt the 2D equations to include notch effects. Φa and Φc expressions are reproduced in Castro and Meggiolaro (2009).

5. CONCLUSIONS

A generalized El Haddad-Topper-Smith's parameter was used to model the threshold stress intensity range for short cracks dependence on the crack size, as well as the behavior of non-propagating fatigue cracks. This dependence was used to estimate the notch sensitivity factor q of semi-elliptical notches, from studying the propagation behavior of short non-propagating cracks that may initiate from their tips. The predicted notch sensitivities reproduced well the classical Peterson's q estimates for circular holes or approximately semi-circular notches, but it was found that the notch sensitivity of elongated slits has a very strong dependence on the notch aspect ratio, defined by the ratio c/b of the semi-elliptical notch that approximates the slit shape having the same tip radius. These predictions were confirmed by experimental measurements of the re-initiation life of long fatigue cracks repaired by introducing a stop-hole at their tips, using their calculated K_f and appropriate εN procedures. Based on this promising performance, a criterion to evaluate the influence of small or large surface cracks in the fatigue resistance was proposed.

6. ACKNOWLEDGEMENTS

CNPq has provided research scholarships for the authors.

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