MODELING OF THE DYNAMIC BEHAVIOR OF STRUCTURES PROVIDED WITH TUNED MASS DAMPERS UNDER MOVING LOADS: A BOND GRAPHS APPROACH

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Abstract. *Classic models used for analysis and simulation of structures with Tuned Mass Dampers (TMD), although show good results, are not modular. The present work takes advantage of bond graphs modularity to easily model the system. The literature presents methodologies of modeling the structure by Finite Elements Method (FEM), with dampers modeled by oscillators located of convenient form in order to reduce vibrations. In the present work, the bond graphs methodology is employed to model separately the subsystems, the final system model being achieved by union of the subsystems bond graphs in a modular way. The bridge subsystem is modeled via bond graphs with the aid of FEM's theory, and subject to the vehicle and TMD subsystems dynamic loads. Comparison between the dynamic response of the structures with and without TMD, considered for different vehicle speeds, shows the vibration attenuation provided by the damper.*

Keywords*: Bond Graphs, Moving Loads, Tuned Mass Dampers, Structures, Modular*

1. INTRODUCTION

The moving load problem has had increasing interest of several engineering areas, since it was observed that structures subject to this kind of load show bigger deflection than that subjected to correspondent static loads (Warburton, 1976). Initially, the problem was approached through simple models which considered constant or harmonic loading with the structure represented by a Bernoulli beam (Biggs, 1964). Efforts had been undertaken in the direction of refining the model in study in order to allow the consideration of structures submitted to more complex loading (Bessa 2000).

The Bond Graphs technique has shown itself as a good alternative to deal with systems composed by distinct nature subsystems (Karnopp., et al, 1990). Due its modular characteristic, the technique allows the development of each subsystem separately, with the complex system model being reached through the assembly of the simpler subsystems models.

The present work shows the application of the bond graphs to the problem of a moving load over a bridge with TMD. The bridge structure, the vehicle and the TMD are modeled separately, and then joined together to achieve the global model, taking advantage of bond graphs modularity.

Results of numerical simulation are presented, proving the efficiency of the vibrations absorber device, as well as the versatility of the generalized Bond Graphs technique .

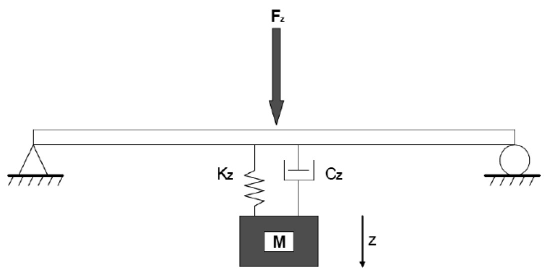

Figure 1. TMD installed in a simple beam

2. PASSIVE VIBRATIONS ABSORBERS (TUNED MASS DAMPERS)

TMD's are one of the most widely used passive control mechanisms for structures. They consist of a mass-spring-damper system attached to the main structure with the aim of reduce its vibration through energy dissipation. They reduce the energy dissipation demand in primary structural members under action of external load when the absorber frequency is tuned to a particular structural frequency. When this frequency is excited, it creates a phase lag between the damper and the structure, dissipating great amount of energy in the TMD.

The expressions for attainment of recommendable parameters of mass, stiffness and damping of the TMD as a function of the structure's properties and imposed loading, are available in literature (Lara, 2007).

The scheme of a TMD connected to a structure is illustrated in the Fig (1).

3. FIRST HYPOTHESYS: BEAM WITHOUT TMD UNDER MOVING LOAD FROM ONE DEGREE OF FREEEDOM QUARTER CAR

Aiming for analyze the influence of a TMD applied to the structure, here is first considered a structure without TMD subject to moving load. The structure is modeled as a Bernoulli beam bi-supported divided into four beam elements according to the finite element method. The moving load is modeled as a one degree of freedom quarter car which crosses the structure at a constant speed *v*, as illustrated in Fig. 2.

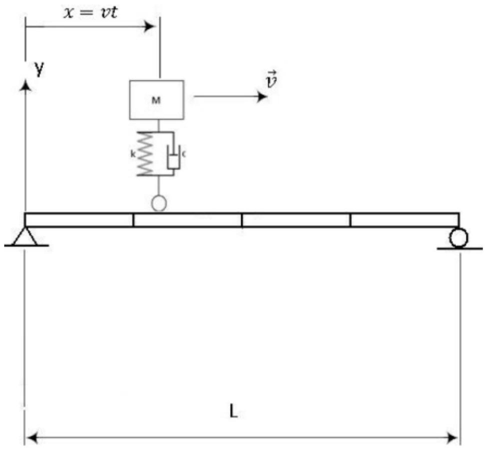

Figure 2. Structure without TMD under moving load

The vertical displacement of the beam three central nodes are considered to analyze the beam behavior under the moving load considered, for three values of the car speed of travel.

3.1 System Modeling

The development of the mathematical model shown in Fig.3 follows the same steps of that one considered by Da Silva (1994), which is applicable to any structure that can be represented in terms of its mass **M**, flexibility $\mathbf{K}^{-1}$, and damping **B** matrices. The structural systems representation considered there, associates these matrices from FEM to the inertial $\mathbb{I}$, capacitive $\mathbb{C}$, and dissipative $\mathbb{R}$ fields of the bond graphs technique, respectively. Due to the load mobility, its application point over the structure is time dependent as well as its modulus, which arises from the dynamic subsystem. Thus, this load effect along each structural element is defined by the multiport transformer MTF, modulated by the vector **Φ,** which dimension is equal to the number of structure's global degrees of freedom (FERREIRA, 2006. The vector **Φ** is a function of the load position x_e over the each element, and is given by Eq. (1), where v is the constant vehicle velocity, n_e is the number of the element under the load, and L_e is the length of such element.

$$x_e = vt - (n_e - 1)L_e \tag{1}$$

The modulated multiport transformer MTF delivers the correspondent load (force or moment) resulting of the vehicle interaction to each structure degree of freedom through the FEM's interpolation functions.

The correspondent system's bond graph is given in Fig. (3), which is attained from the union of the vehicle and structure subsystems bond graph.

The general form of the state space model for the system is given by Eq. (2), where $\boldsymbol{X}$ is the states vector; $\boldsymbol{A}$**,** the state matrix; $\boldsymbol{B}$**,** the entries matrix, and $\boldsymbol{U}$ the entries vector. Writing the state equations for this particular problem, the specific state model given in Eq. (3) is achieved.

The scalars p_2 and q_5 are the momentum and vertical displacement associated to the vehicle; and $\boldsymbol{p_{10}}$ and $\boldsymbol{q_{12}}$ are the vectors momentum and generalized displacement (vertical displacements and rotations) associated to structure's global degrees of freedom.

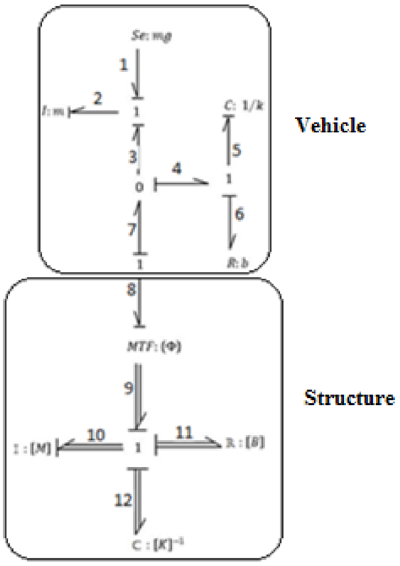

Figure 3. Structure-vehicle system Bond Graph, without TMD

$$\dot{\boldsymbol{X}} = \boldsymbol{AX} + \boldsymbol{BU} \tag{2}$$

$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_5 \\ \dot{\boldsymbol{p}}_{\boldsymbol{10}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{12}} \end{bmatrix} = \begin{bmatrix} -\frac{b}{m} & k & -b\Phi[M]^{-1} & [0] \\ -\frac{1}{m} & 0 & -\Phi[M]^{-1} & [0] \\ -\Phi^T\frac{b}{m} & \Phi^T k & -(\Phi^T b\Phi - [B])[M]^{-1} & -[K] \\ [0] & [0] & [M]^{-1} & [0] \end{bmatrix} . \begin{bmatrix} p_2 \\ q_5 \\ \boldsymbol{p_{10}} \\ \boldsymbol{q_{12}} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ [0] \\ [0] \end{bmatrix} . e_1 \tag{3}$$

3.2 System Simulation

Through a developed computational code in MATLAB 7.0, the dynamic behavior of the structure under moving load was simulated for three distinct values of constant speed (10 km/h, 70 km/h and 100 km/h). The parameters for the structure and the mechanical subsystems are given in Tab. 1:

Table 1. Beam and the vehicle parameters

Beam	Cross section area (m^2)	0.08
	Total length (m)	40
	Element length (m)	10
	Inertia relative to X axis (m^4)	1.07x10^{-3}
	Young modulus (N/m^2)	210x10^9
	Density (kg/m^3)	7850
	Structural damping (%)	5
Vehicle	1/4 vehicle mass (kg)	250
	Spring constant (N/m)	13000
	Damping coefficient (N.s/m)	600

3.3 System's Response

The vertical displacements at beam nodes 2, 3 and 4 due to the moving load, for the three speed values referred above are illustrated in Figs. (4), (5), and (6), respectively.

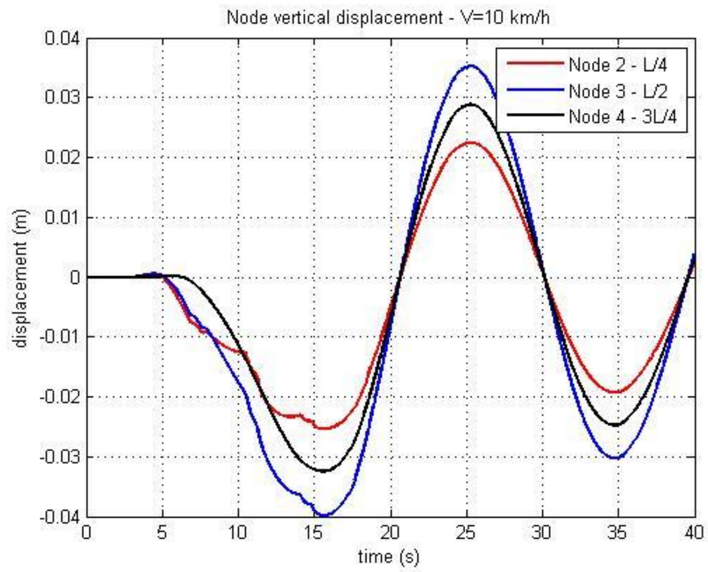

Figure 4. Node displacement (speed 10 km/h)

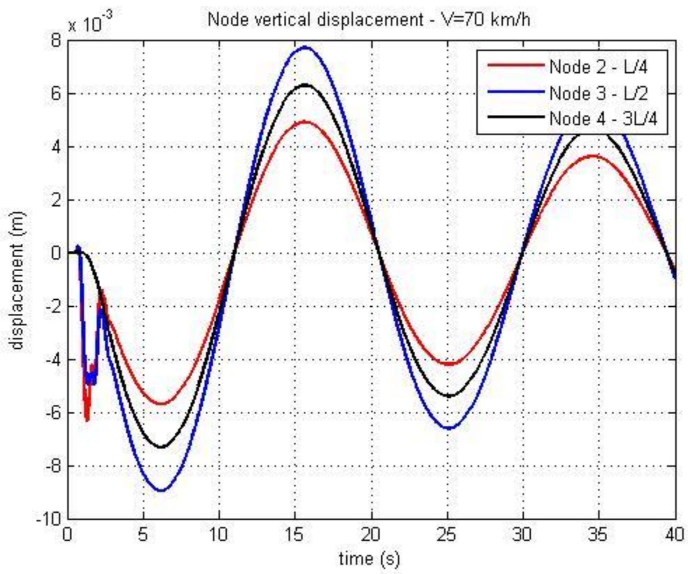

Figure 5. Node displacement (speed 70 km/h)

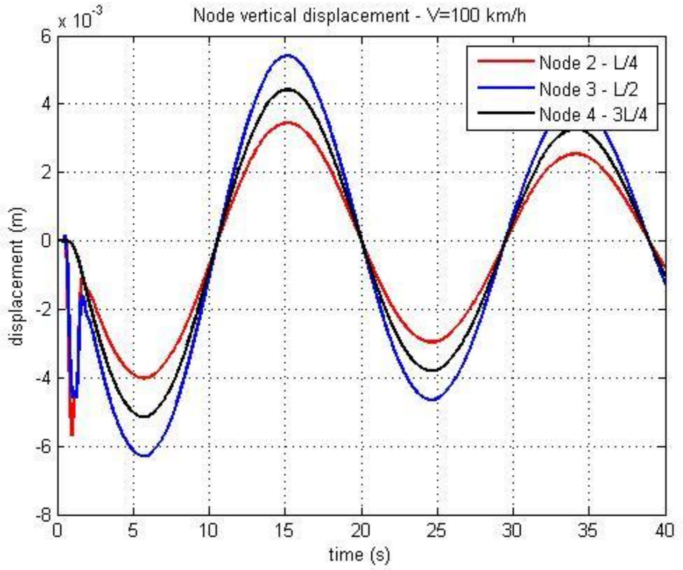

Figure 6. Node displacement (speed 100 km/h)

Table 2. Midspan node maximum displacement

Midspan node maximum displacement	
Velocity (km/h)	**Displacement (m)**
10	-39.8×10^{-3}
70	-8.95×10^{-3}
100	-6.3×10^{-3}

It can be noticed from Fig. (4), (5) and (6) that the midspan node 3 shows the biggest displacement followed by the node 4 and node 2, respectively. Although nodes 2 and 4 are symmetrically positioned from the structure central node 3, they show different displacement due to the load asymmetry.

The time values for which the vehicle enters and leaves the structure varies with vehicle velocity. Once here it was considered an initial longitudinal displacement of 10 m before the vehicle enters the bridge, and a bridge length of 40 m; the time for entering and leaving the bridge are respectively 3.6 s and 18 s, for 10 km/h velocity - Fig. (4); 0.51s and 2.57 s, for a 70 km/h velocity -Fig. (5); and 0.36 s and 1.8 s, for a 100 km/h velocity – Fig (6).

The midspan node maximum displacement is shown in Table 2 for the vehicle velocities of travel considered.

From the simulation results shown above, it can be noticed that midspan node maximum displacement raises inversely with the speed of travel. It can be also seen that maximum displacement happens with the vehicle still over the structure for the lowest speed, and with the vehicle out of the structure for the other velocities.

4. SECOND HYPOTHESYS: BEAM WITH TMD UNDER MOVING LOAD FROM ONE DEGREE OF FREEDOM QUARTER CAR.

Once the first model above was achieved and simulated as shown in section 3, the structure with TMD under moving load, as schematically shown in Fig. (7), is modeled in the present section.

4.1 System Modeling

The bond graph of the subsystem structure-absorber is built and connected to the vehicle subsystem bond graph to build the overall system model, as shown in Fig. (8). It is considered that the vibration absorber (TMD) is installed at the beam midspan, according to Fig. (7).

The state space model of the system structure-vehicle-TMD set is attained following the same steps as in subsection 3.1, resulting in Eq. (4). The scalars p_2 and q_6 are the momentum and vertical displacement associated to the vehicle; $\boldsymbol{p_9}$ and $\boldsymbol{q_{10}}$ are the vectors momentum and generalized displacement (vertical displacements and rotations) associated to structure's global degrees of freedom; and the scalars p_{14} and q_{12} are the momentum and vertical displacement associated to the TMD.

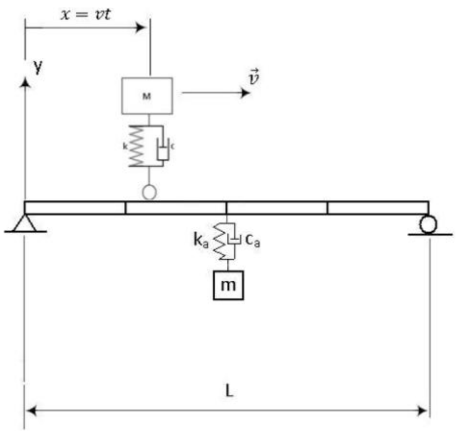

Figure 7. Structure with TMD under moving load

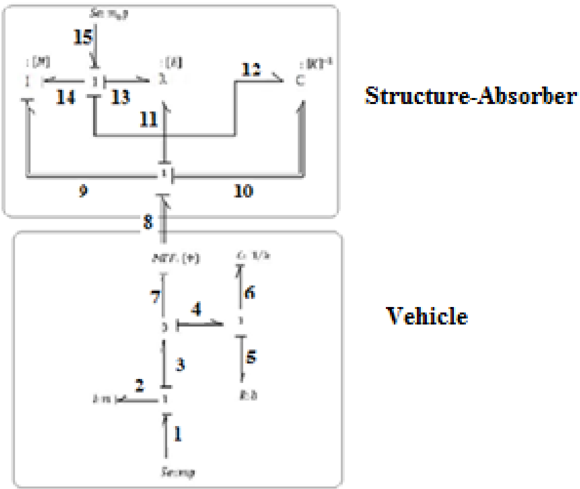

Figure 8. Bond Graph for structure-vehicle system with TMD

$$\begin{bmatrix} \dot{p}_2 \\ \dot{q}_6 \\ \dot{\boldsymbol{p}}_{\boldsymbol{9}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{10}} \\ \dot{q}_{12} \\ \dot{p}_{14} \end{bmatrix} = \begin{bmatrix} -\frac{b}{m} & -k & b\Phi D^{-1} I_{11}^{-1} & [0] & 0 & -b\Phi D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} \\ \frac{b}{m} & 0 & -\Phi D^{-1} I_{11}^{-1} & [0] & 0 & \Phi D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} \\ \Phi^T \frac{b}{m} & \Phi^T k & -\Phi^T b \Phi D^{-1} I_{11}^{-1} - R_{11} D^{-1^{-1}} I_{11}^{-1} + R_{12} F^{-1} I_{22}^{-1} I_{21} I_{11}^{-1} & -C_{11} & -C_{12} & -\Phi^T b\Phi D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} + R_{11} D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} - R_{12} F^{-1} I_{22}^{-1} \\ 0 & 0 & D^{-1} I_{11}^{-1} & [0] & [0] & D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} \\ 0 & 0 & -F^{-1} I_{22}^{-1} I_{21} I_{11}^{-1} & [0] & 0 & F^{-1} I_{22}^{-1} \\ 0 & 0 & R_{21} D^{-1} I_{11}^{-1} + R_{22} F^{-1} I_{22}^{-1} I_{21} I_{11}^{-1} & -C_{21} & -C_{22} & R_{21} D^{-1} I_{11}^{-1} I_{12} I_{22}^{-1} - R_{22} F^{-1} I_{22}^{-1} \end{bmatrix}.$$

$$\begin{bmatrix} p_2 \\ q_6 \\ \boldsymbol{p_9} \\ \boldsymbol{q_{10}} \\ q_{12} \\ p_{14} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ [0] & [0] \\ [0] & [0] \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_{15} \end{bmatrix} \tag{4}$$

4.2 System Simulation

The TMD parameters used in the simulation are given in Tab. 3.

Table 3. TMD parameters

TMD	Mass (kg)	80
	Spring constant (N/m)	54000
	Damping coefficient (N.s/m)	800

4.3 System's Response

Simulation results shown in Fig. 9, 10 and 11, highlight the time elapsed from the beginning of the vehicle travel up to the moment it leaves the structure. As the velocities considered change, so do the final simulation time in order to capture the vehicle travel over the bridge. From subsection 3.3, final times are respectively 18 s, for 10 km/h velocity; 2.57 s, for a 70 km/h velocity; and 1.8 s, for a 100 km/h velocity. Midspan node vertical displacement for the structure with and without TMD are shown in Fig (9), (10) and (11) for vehicle speed of 10, 70 and 100 km/h, respectively.

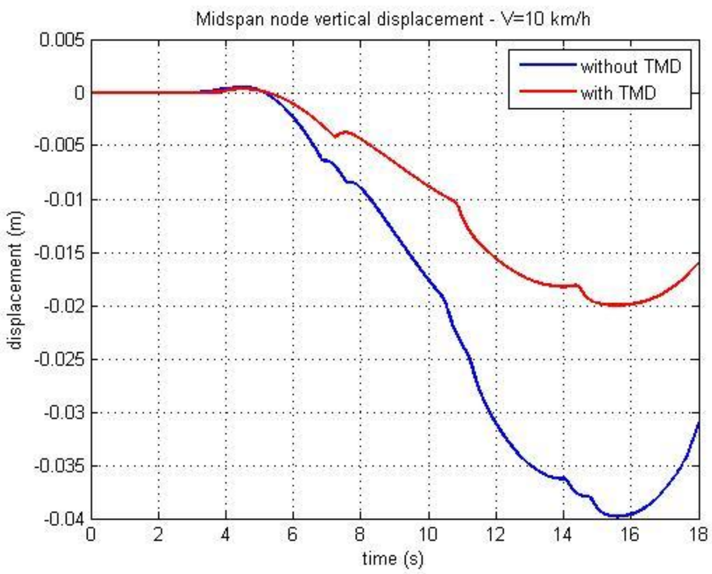

Figure 9. Central node displacement (speed 10 km/h)

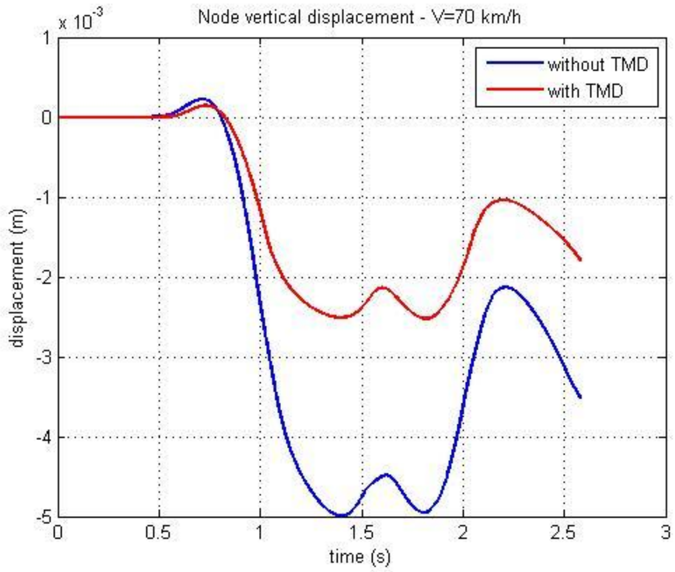

Figure 10. Central node displacement (speed 70 km/h)

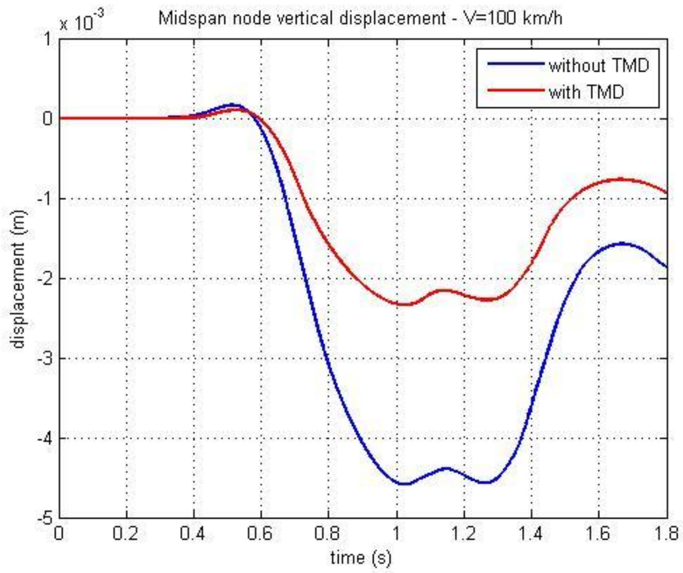

Figure 11. Central node displacement (speed 100 km/h)

Table 4. Maximum central node displacement with the vehicle over the bridge

	Central node displacement		
Velocity	10 km/h	70 km/h	100 km/h
Without TMD	-39.8×10^{-3} m	-5.0×10^{-3} m	-4.6×10^{-3} m
With TMD	-20.0×10^{-3} m	-2.5×10^{-3} m	-2.34×10^{-3} m

It can be noticed the reduction of maximum vertical displacement at midspan node as the vehicle velocity increases, according to Fig (9), (10) and (11); and a mean reduction of 50 per cent in the node displacement in the structures with TMD, according to Tab. 4.

5. FINAL CONSIDERATIONS

The present work shows the application of bond graphs technique to the problem of a moving load applied to a structure endowed with a TMD. The mathematical model of a structure (without TMD) subject to a moving load was developed first, and then the final model of the structure with TMD was achieved simply by the union of the TMD's bond graph model to the first system bond graph model, taking advantage of bond graph's modularity.

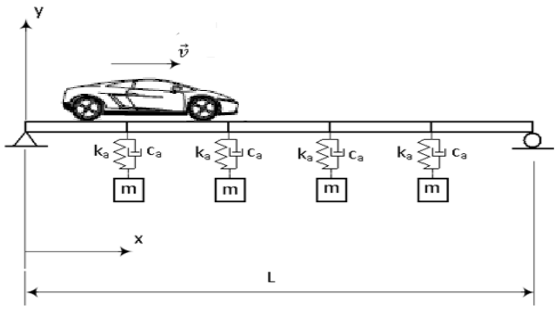

Figure 12. Structure with multiple TMD subject to moving load from 4-DOF vehicle

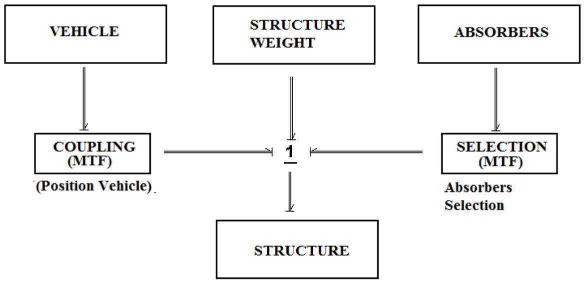

Fig 13. Coupling scheme for multiple TMD absorbers with 2-DOF vehicle (structure weight as a subsystem)

Simulation results of the structure without TMD shows that midspan node maximum displacement raises inversely with the speed; and that it happens with the vehicle still over the structure for the lowest speed, and with the vehicle out of the structure for the other velocities.

Simulation results of the structure with TMD shows a reduction of maximum vertical displacement node at midspan as the velocity increases, just like in the situation before; and a reduction of 50 per cent in the structure displacement if compared with the structure without TMD. The results therefore highlight TMD's efficiency.

The following step will be the application of the Bond Graphs technique to a structure connected to multiple TMD subject to moving loads arising from interaction with mechanical 4 and 7 degrees of freedom vehicle subsystems, as illustrated in Fig. (12) and (13).

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