Fault detection of sensors and actuators in smart structures

Daniel Augusto Pereira, danielaugusto@fem.unicamp.br
Alberto Luiz Serpa, serpa@fem.unicamp.br
Department of Computational Mechanics, Faculty of Mechanical Engineering, University of Campinas - UNICAMP

Abstract. *Smart structures can be used for Active Vibration Control (AVC) and also for Structural Health Monitoring (SHM). In such cases it is desirable that the instrumentation involved is reliable and any faults must be detected and isolated as soon as possible. In this work, some Fault Detection and Isolation (FDI) approaches applied to SHM and AVC systems were evaluated. The structure used for tests was a cantilever beam instrumented with two sensors and two actuators, all piezoelectric elements. Output observers and filters were used to conduct fault detection. They provide estimates of the measured outputs. The subtraction between measured and estimated quantities provides a signal called residue, which should be small in the absence and large in the presence of fault. These output observers/filters can be designed in various ways and the following approaches were considered: state observer, Kalman filter and H-infinity filter. In addition to know if a fault occurred, it is also necessary to determine in which element of the system the fault is present and under which severity. For this, the strategy adopted was to use a bank of observers where each component is an output observer designed taking into account a system with only a specific fault. This scheme can be interpreted as a matrix, where each line refers to a certain severity and each column to one of the monitored instruments. Thus, the plant output is compared with the estimated output for each of the observers, giving rise to various signs of residue. Within each column of the bank, the smallest amount of residue indicates the degree of degradation of the instrument. So the faults are isolated. The severity of these faults can be determined by some indicators. Among a myriad of possibilities, three indicators were used: Root Mean Square Difference, Sum of the Modulus of the Error and Modal Assurance Criterion. These indicators were applied to the residue supplied by each component of bank of observers. Computational tests were performed on a beam modeled by finite element method, initially without controller (SHM system) and after in the presence of an H-infinity controller (AVC system). Disturbances, measurement noises, modeling uncertainties and simultaneous faults were considered.*

Keywords: *structural health monitoring, fault detection and isolation, output estimators, indicators, smart structures*

1. INTRODUCTION

Structural Health Monitoring (SHM) in flexible structures can be accomplished by piezoelectric sensors and actuators (Sohn *et al.*, 2004; Park and Inman, 2007). Active Vibration Control (AVC) can also be performed by this type of instrumentation (Chopra, 2002). However, these instruments can fail and some sorts of techniques to monitor them must be applied.

The sensor monitoring has received attention in Structural Health Monitoring (SHM) community in the last years, especially under the subject Sensor Validation (Friswell and Inman, 1999; Kerschen *et al.*, 2005; Abdelghani and Friswell, 2007). The actuator fault case has also been treated (Koh *et al.*, 2005). In control and chemical engineering community the monitoring of fault has been approached under the subject Fault Detection and Isolation (FDI) (Ding, 2008; Hwang *et al.*, 2010; Venkatasubramanian *et al.*, 2003), where the applications always involve closed loop systems.

An alternative to achieve sensor validation is the use of model based techniques that provide predictions of the measured outputs, with a post-processing step in which an indicator, or index, is applied (Friswell and Inman, 1999; Abdelghani and Friswell, 2007). The sensor validation can also be performed by data driven methods, without the need of a structural model of the system. In this case, Principal Component Analysis (PCA) is a usual possibility (Kerschen *et al.*, 2005).

There are few works with Fault Detection and Isolation techniques applied to structural health monitoring (Liberatore *et al.*, 2006; Mechbal *et al.*, 2006). The adopted methods are model-based, based on Fault Detection Filters (Liberatore *et al.*, 2006) and H-infinity theory (Mechbal *et al.*, 2006). These methods are only two possibilities among many reported in the literature (Ding, 2008; Hwang *et al.*, 2010; Venkatasubramanian *et al.*, 2003). The primitive formulation of the fault detection filters is based on a simple Luenberger output observer (Chen and Patton, 1999; Ding, 2008), from which started the application of more elaborated residual generation schemes, with the estimation provided, for example, by Kalman and H-infinity filters.

The schemes for Sensor Validation or Fault Detection and Isolation are based on an estimation module followed by an evaluation one where an indicator is applied to support the decision about the occurrence and localization of faults. The aim of this work is an evaluation of some output estimation techniques (output observer, Kalman filter and H-infinity filter) and also of some indicators (Root Mean Square Difference, Sum of the Modulus of the Error and Modal Assurance Criterion).

2. FAULT DETECTION AND ISOLATION

The strategy adopted for fault detection is the usage of output estimators in order to compare the estimated signals with the measured ones, subtracting them and generating error signals which are the residue. An illustration of this idea can be seen in Fig. 1. For fault isolation, a bank of observers/filters is taken into account.

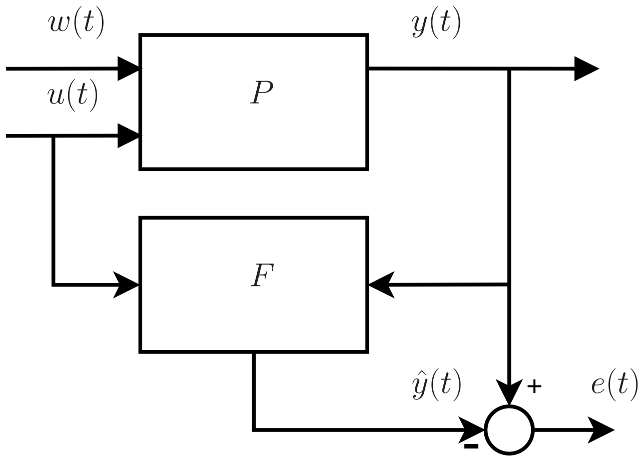


Figure 1. Residual generation scheme. The signals are: unknown inputs $w(t)$, known inputs $u(t)$, measured output $y(t)$, estimated output $\hat{y}(t)$, estimation error $e(t)$ or residue. P is the plant and F the output estimator.

2.1 Fault detection by output estimation

2.1.1 Output observer

The output observer takes into account the nominal model of the plant, the input $u(t)$ and the measured output $y(t)$ to produce the estimate, internally incorporating a feedback. This observer is described by (Astrom and Murray, 2008):

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \quad (1)$$
$$\hat{y}(t) = C\hat{x}(t) + Du(t) \quad (2)$$

where L is a matrix chosen by a pole placement procedure to ensure that the estimation error converges asymptotically to zero with a desired rate.

2.1.2 Kalman filter

The Kalman filter incorporates knowledge about the disturbances acting and interfering in dynamic behavior of the system (Astrom and Murray, 2008; Franklin *et al.*, 1998). The plant model considered in the design is

$$\dot{x}(t) = Ax(t) + Bu(t) + Gd(t) \quad (3)$$
$$y(t) = Cx(t) + Du(t) + Hd(t) + n(t) \quad (4)$$

where $d(t)$ and $n(t)$ are white noise disturbances such that $E[d(t)] = E[n(t)] = 0$, $E[d(t)d(t)^T] = Q$, $E[n(t)n(t)^T] = R$ and $E[d(t)n(t)^T] = 0$. The Kalman filter is designed in order to minimize the covariance of the estimation error in steady state, given by

$$P = \lim_{t\to\infty} E[(x(t) - \hat{x}(t))(x(t) - \hat{x}(t))^T]$$

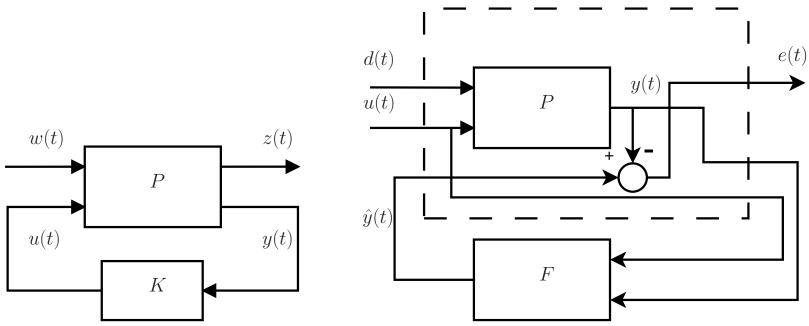
so that the optimal solution requires the Kalman filter given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \quad (5)$$
$$\hat{y}(t) = C\hat{x}(t) + Du(t). \quad (6)$$

The matrix of gains L is determined according to methodology described in (Astrom and Murray, 2008).

2.1.3 H-infinity filter

Another alternative design of an output estimator is the H-infinity filter. In this case, it is formulated an optimization problem where the goal is minimize the interference of disturbances on the estimation error. A standard scheme of the H-infinity design problem is shown in Fig. 2(a), where the controller K is the one that minimizes the influence of disturbances w in the performance z . Figure 1 shows de original residual generation problem, where the filter F is used to generate the output estimative z. Then, an estimation error is generated. The puprose of the H-infinity design is to minimize the influence of the exogenus inputs w in the performance output z (Skogestad and Postlethwaite, 1996; Zhou *et al.*, 1996). The scheme of Fig. 1 can be converted to the standard H-infinity scheme of Fig. 2(a), as shown in Fig. 2(b). Weighting functions (Zhou *et al.*, 1996) can be used to enhance the performance.



(a) Standard H-infinity. The signals are: exogenous inputs $w(t)$, performance output $z(t)$, control input $u(t)$, measured output or controller input $y(t)$. P is the plant and K is the controller or filter.

(b) FDI in standard H-infinity form. The signals are: disturbance input $d(t)$, known input $u(t)$, measured output $y(t)$, estimated output $\hat{y}(t)$, estimate error $e(t)$ or residue.

Figure 2. H-infinity problem.

There are several ways to solve the H-infinity problem: direct search, where the optimization problem is solved; solution of the corresponding Riccati equations; employment of techniques based on linear matrix inequalities (LMI). The development of the theory involved in these solutions is quite extensive and will be omitted in this text because escape the scope of the work. It should be emphasized that the solution of the problem can be obtained from several computer packages, highlighting the Robust Control Toolbox of MATLAB, which was used in this work.

2.2 Bank of observers

Besides indicating whether there is a fault, it is also necessary to determine in which element of the system the fault is present and under which severity. For this, one strategy is to design a bank of observers (or filters) where each component of the bank is designed taking into account a system with only a specific sensor or actuator fault (Chen and Patton, 1999; Ding, 2008). The output of the plant must be compared with the estimated output provided by each observer/filter. The estimated output closer to the measured output, with less residue value, will be that from the observer designed taking into consideration the current fault. So it is possible to isolate the fault, i.e., indicate in which element of the system it occurred and also the severity.

A possible structure of this scheme of fault isolation can be seen in Fig. 3. This scheme is presented as a matrix, where each row gives a certain fault severity and each column is related to a fault in a specific sensor or actuator. For example, imagine that the levels of failure can vary from 0 to 100%, with a step of 10% from one level to another, pretending to monitor five sensors. In this case, the bank will have eleven rows and five columns. In the second line the observers will be designed considering 10% faults and in the third column the observers will be designed considering fault in the third sensor. Thus, each of 55 observers of the bank will produce an estimate of the output and one that is closest to the measured output indicates which sensor has failed and severity of crashes.

2.3 Indicators

The element of the bank of observers which produces the estimated output closer to the measured output will allow the isolation of the fault. For this, one must adopt a metric that allows the comparison between the measured output and each of the estimates to determine the closer one. Some well known metrics are applied as indicators.

2.3.1 Root Mean Square (RMS) Difference - RMSD

For an arbitrary n-dimensional data array $x = \{x_1, \ldots, x_n\}$, the RMS value of x is given by:

$$x_{RMS} = \sqrt{\frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n}}.$$

The indicator RMSD is given by the difference between the RMS value of the measured signal y and the RMS value

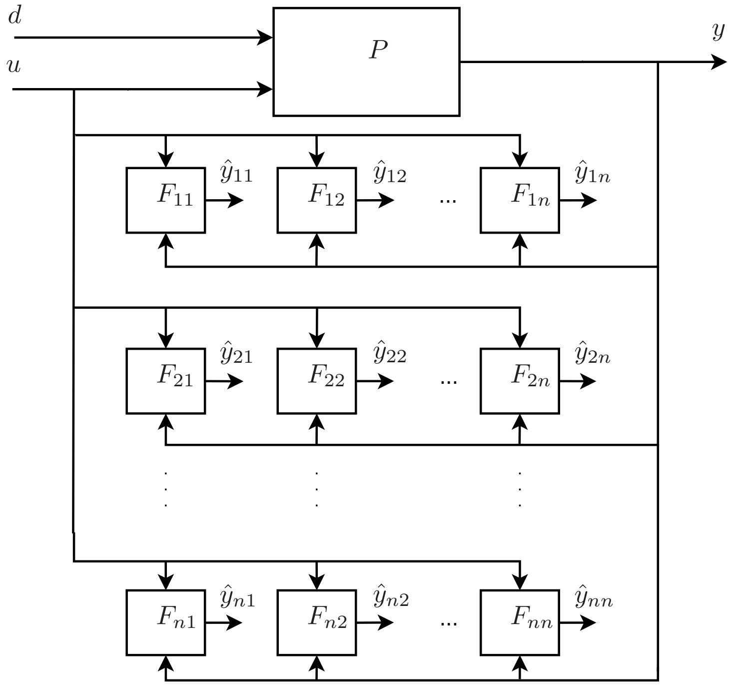


Figure 3. Bank of observers.

of the estimated signal $\hat{y}$:

$$DRMS = y_{RMS} - \hat{y}_{RMS}. \tag{7}$$

2.3.2 Sum of the Modulus of the Error - SME

The estimation error is $e = y - \hat{y}$. The indicator SME is given by:

$$SME = \sum_{i=1}^{n} |e_i| \tag{8}$$

where e is n-dimensional.

2.3.3 Modal Assurance Criterion - MAC

The MAC is given by (Allemang, 2003):

$$MAC = \frac{(\hat{y}^T y)^2}{(y^T y)(\hat{y}^T \hat{y})}. \tag{9}$$

3. THE CANTILEVER BEAM

The evaluation of the estimators and indicators will be accomplished through simulations in a cantilever beam, which is shown in Fig. 4. This structure was modeled using the Finite Element Method (FEM), considering a discretization with 10 elements and 10 nodes. The degrees of freedom (DOF) are displacement towards $\vec{z}$ and rotation around $\vec{y}$. Properties, dimensions and the obtained natural frequencies of the beam can be seen in Tab. 1. The model adopted in this work is a truncated version considering just the first five modes, i.e., tenth order state-space model.

The system inputs are moments and the outputs are angular displacements (rotations) around $\vec{y}$, which are both applied by piezoelectric (PZT) patches working as actuator and sensor, respectively. There are two actuators, each one attached to a finite element and producing at each node identical moments of opposite signs; and two sensors, each one also attached to a finite element and producing a measurement signal that gives the difference between the rotations at the element nodes. There is still a PZT patch to apply a disturbance signal that impose moments in the same way as the actuators.

The placement of the sensors, actuators and disturbance are defined in Tab. 2. So, using this relationship between inputs and outputs, it was designed an H-infinity controller in order to reduce the performance criterion represented by the transverse displacement at the free end. Taking the H-infinity formulation represented by Fig. 2(a) and considering a truncated fourth order plant for the design, a fourth order controller was designed focusing the attenuation of the first mode. It produced the singular value diagram shown in Fig. 5. The MATLAB function *hinfsyn.m* was used in the design.

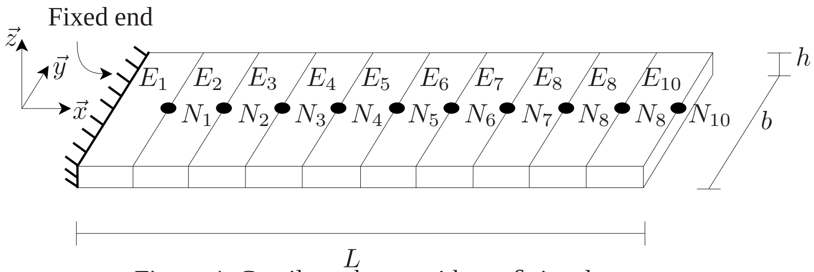


Figure 4. Cantilever beam with ten finite elements.

Table 1. Properties, dimensions and first six natural frequencies of the beam.

(a) Properties and dimensions.

Quantity	Value
Elastic modulus(E)	6.9×10^{10} $[Pa]$
Base (b)	3.2×10^{-2} $[m]$
High (h)	3.0×10^{-3} $[m]$
Cross-sectional area (A)	9.6×10^{-5} $[m^2]$
Length (L)	1.0 $[m]$
Density (ρ)	2.7×10^3 $[kg/m^3]$

(b) Natural frequencys.

Frequencys [rad/s]
1.5393×10^1
9.6469×10^1
2.7017×10^2
5.2980×10^2
8.7718×10^2
1.3141×10^3

4. RESULTS

The faults considered in the simulations were percentage degradations in the signals provided by sensors and actuators. For example, a 10% fault of sensor 1 means that the available signal has just 90% amplitude of the original one. The same is valid for the actuator case. Considering this type of faults, simulations were performed for the case without control and with control. In open loop, the excitation signals are sine sweeps generated by the MATLAB function *chirp.m* with amplitude 5.0×10^{-3} and frequencies between 10 and 20 $[rad/s]$. It is also considered a disturbance signal, with amplitude 1.0×10^{-5} in the same frequency range of the actuator signals, and white noise of amplitude $1,0 \times 10^{-7}$ corrupting the measurement signals. Model uncertainties are also taken into account: the available beam model is tenth order but a truncated version with just the first three modes is used in the design of the estimators producing sixth order models (except for the case of the H-infinity filter). Then, the simulations are performed with the original tenth order model.

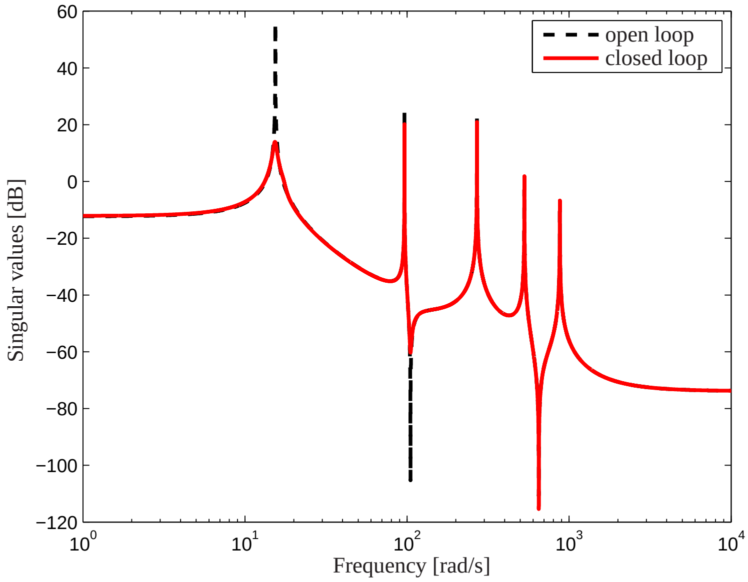
The output observer design It was designed in such a way that the matrix L provides an estimation error with the same dynamics of the original plant. As each estimator inside the bank of observers is based in a different plant, various different matrices L were computed using the MATLAB function *place.m* and the presentation of these results is not necessary.

The Kalman filter design The Kalman filters were designed with $Q = 1,0 \times 10^3$ and $R = I_{ny}$, where I is an identity matrix and ny is the number of measured outputs, which is 2. The design was performed with the MATLAB function *kalman.m*.

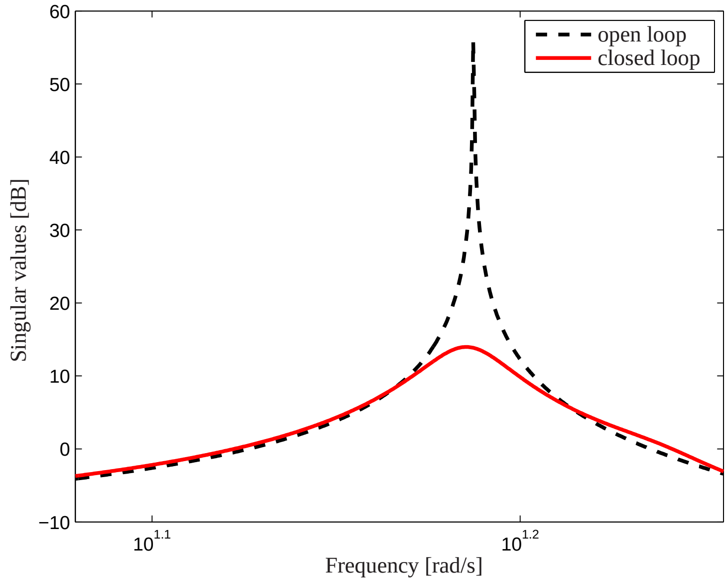
The H-infinity filter design The H-infinity filter design was performed by the MATLAB function *hinfsyn.m* and the weighting functions shown in Tab. 3 were used. Because of this weighting functions, the H-infinity filter is of eighth order.

Table 2. Localization of actuators, sensors and disturbance.

PZT	finite element
sensor 1	E_4
sensor 2	E_6
actuator 1	E_2
actuator 2	E_8
disturbance	E_5



(a) Open loop × closed loop



(b) First mode detail shown the attenuation provided by the controller.

Figure 5. Singular values diagram.

The bank of observers It is designed with 10 levels, the first one designed without considering faults and the next levels with a step of 10% from one to another. This way, 10 levels were produced, from 0% to 90%.

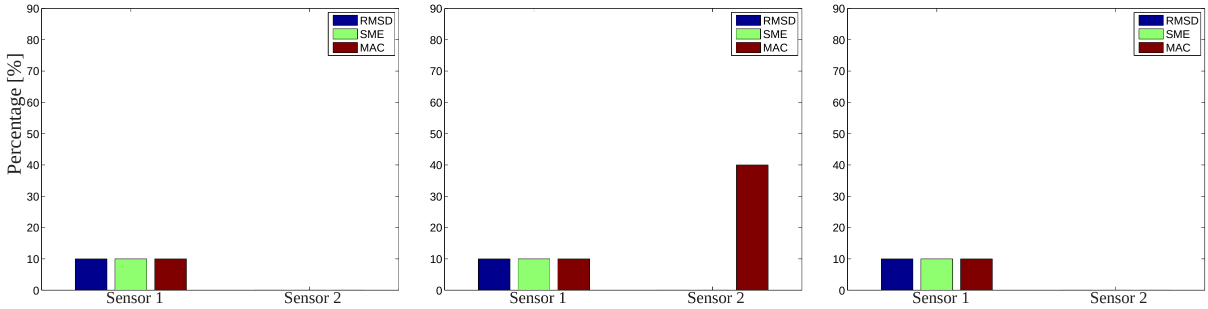
The first result is shown in Fig. 6, where a 10% fault is present in sensor 1. The estimators and indicators are all applied. When there is a controller closing the loop, the simulation with the same disturbances and noises produced the results that are presented in Fig. 7. When the controller is not present, the three estimators allowed the correct detection and isolation of the 10% fault in sensor 1, but the MAC indicator was not successful with the Kalman filter. For this same fault, but with the controller, the Kalman filter is not adequate, so does the MAC indicator under any estimator.

A 30% fault in sensor 2 has also been simulated both for cases with and without controller and the results are shown in Fig. 8 and in Fig. 9. When the controller is not present, it can be seen that all the estimators correctly isolated the fault, but the MAC indicator provided wrong results for sensor 1 when the state observer and the Kalman filter were used. In closed loop, just the SME indicator was successful with all the estimators, whereas MAC fails under any circumstance and RMSD is adequate in isolation just with the state observer despite detecting with the others.

Simultaneous faults of 30% in sensor 1 and 40% in sensor 2 were evaluated and shown in Fig. 10, just in the case

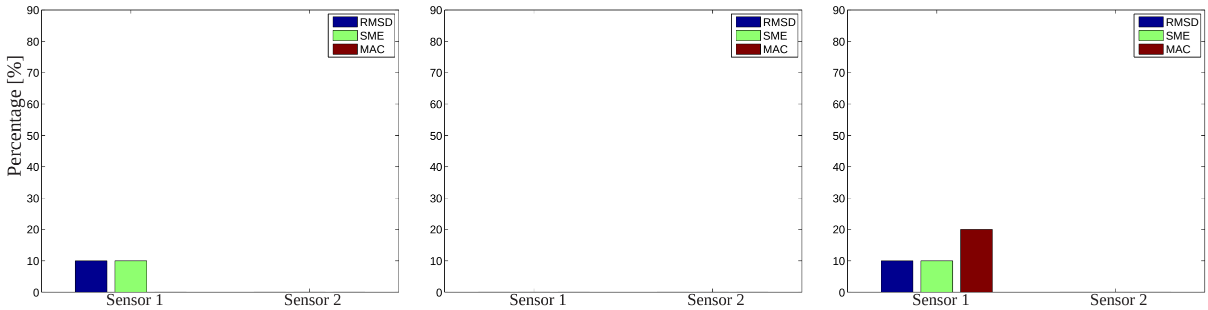
Table 3. Weigthing functions (filters) used in the H-infinity design of output estimators. g is the gain in the rejection band, G the gain in the passband and ω_c the cutoff frequency of the filter/function. The superscripts s and a are related to the sensor and actuator monitoring design cases respectively.

Filter	g	G	ω_c	order	type
$W^s_{z_1}$	$1,0 \times 10^{-6}$	$1,0 \times 10^{0}$	$1,0 \times 10^{2}$	1	low-pass
$W^s_{z_2}$	$1,0 \times 10^{-6}$	$1,0 \times 10^{0}$	$1,0 \times 10^{2}$	1	low-pass
$W^a_{z_1}$	$1,0 \times 10^{-3}$	$1,0 \times 10^{0}$	$1,0 \times 10^{2}$	1	low-pass
$W^a_{z_2}$	$1,0 \times 10^{-3}$	$1,0 \times 10^{0}$	$1,0 \times 10^{2}$	1	low-pass



(a) State observer. (b) Kalman filter. (c) H-infinity filter.

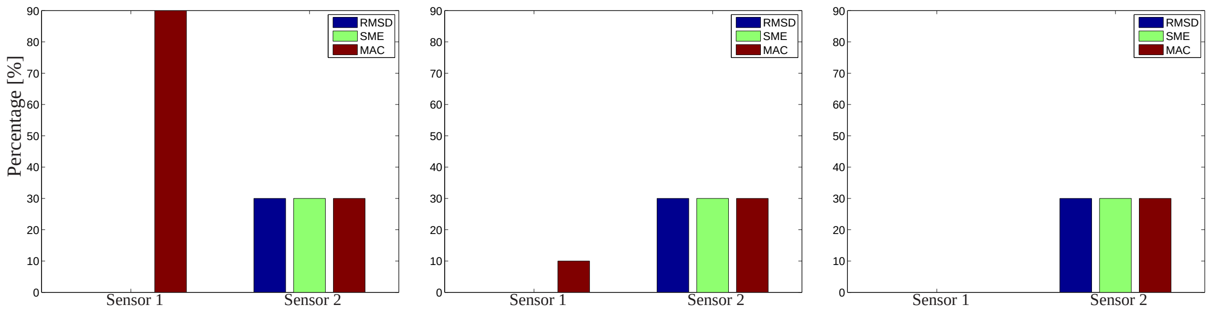
Figure 6. Fault of 10% in sensor 1. Without control.



(a) State observer. (b) Kalman filter. (c) H-infinity filter.

Figure 7. Fault of 10% in sensor 1. With control.

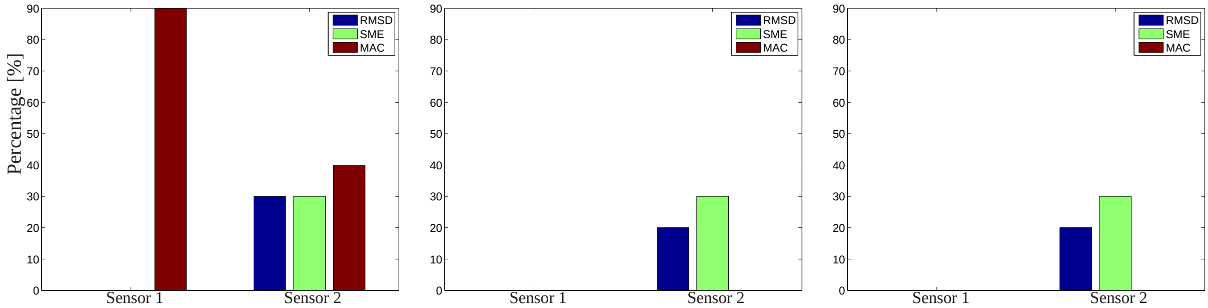
without controller. All the methods are unsuitable for detection and isolation of simultaneous faults. Results with control are also improper and are omitted.



(a) State observer. (b) Kalman filter. (c) H-infinity filter.

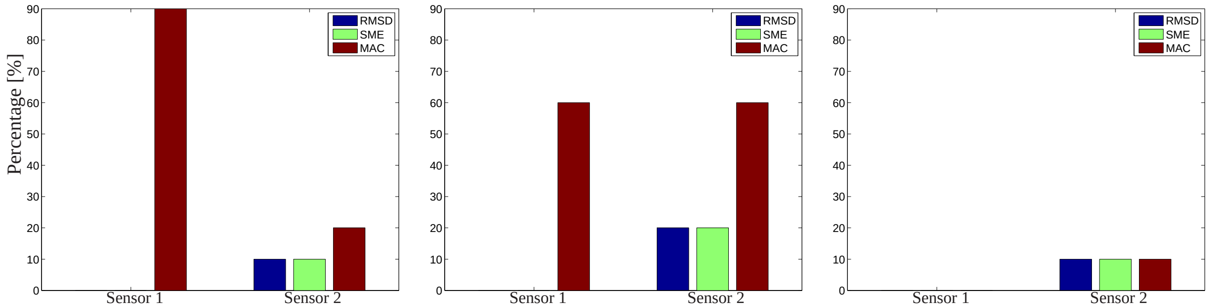
Figure 8. Fault of 30% in sensor 2. Without control.

A simulation with a fault of 60% in actuator 1 is shown in Fig. 11. It shows that the methods considered in this work were not adequate for actuator fault detection and isolation, neither in the case without control nor with control, which results are omitted.



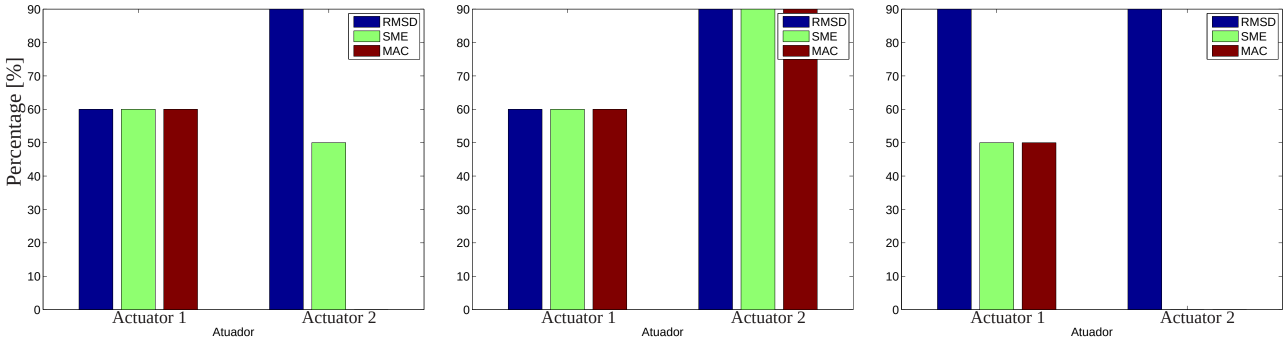
(a) State observer. (b) Kalman filter. (c) H-infinity filter.

Figure 9. Fault of 30% in sensor 2. With control.



(a) State observer. (b) Kalman filter. (c) H-infinity filter.

Figure 10. Fault of 30% in sensor 1 and 40% in sensor 2. Without control.



(a) State observer. (b) Kalman filter. (c) H-infinity filter.

Figure 11. Fault of 60% in actuator 1. Without control.

These results show that the fault detection and isolation strategy based on the output estimators in conjunction with the bank of observers is useful just for single sensor faults, not for simultaneous faults. Indeed, the methodologies are not applicable for actuator fault detection and isolation. For open loop, all the estimators provided reasonable results, but the MAC indicator has some deficiencies. In closed loop, MAC is even worse and RMSD presented some problems. The H-infinity estimator provided slightly better results.

It is important to notice that the methodologies presented are not useful for simultaneous monitoring of sensor and actuator faults.

5. CONCLUSION

This paper has presented different methods, all inside the same perspective, for the monitoring of sensors and actuators faults. The simulations has shown that the simple output observer was capable to isolate sensor faults and results with the H-infinity technique were a little better. In the case of controlled systems, some of the indicators stop working. It denotes that improvements and new developments must be done to perform better monitoring in closed loop situations.

It is also important to note that a large number of simulations could be done to statistically show which estimators and indicators are more suitable to each situation. Also, progress is required for actuator fault monitoring.

6. ACKNOWLEDGEMENTS

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