DIFFUSION IN SOLIDS OF REVOLUTION VIA GALERKIN-BASED INTEGRAL METHOD

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Abstract. *Drying is a process that involves heat and mass transfer, which by liquid evaporation at the surface causes the removal of the moisture contained within the product. In order to describe the drying process more adequately, it is important to consider the body shape factor in the development of mathematical model. Thus, this study aims to develop a mathematical model to predict drying of solids of revolution, using the liquid diffusion theory. The analytical solutions of the diffusion equations via Galerkin-based integral method is obtained by considering constant thermo-physical properties and equilibrium boundary condition at the surface of the solid. Results of the moisture content distribution within the solid of revolution and drying kinetic are presented and analyzed for differents solids.*

Keywords*: Drying, Method GBI, Simulation, Solid of Revolution*

1. INTRODUCTION

Drying of solids is one of the oldest processes used in various industries, such as agricultural, ceramics, chemical, food, pharmaceutical, paper, mineral and polymer ones. Because it is a process that involves simultaneous transfer of heat, mass and momentum in the solid, it generates the need for effective models for simulating the process (Menon and Mujumdar, 1987).

During the process of drying, the moisture transport from the interior to the surface of the material may occur as liquid and/or vapor, depending on the type of product and the percentage of moisture in the product. In the case of agricultural products, removal of moisture promotes conservation of the given products for much longer, concentrating their flavor and nutritional values, as well as making transportation, handling, preparation and storage easier. By comparison with the other separation techniques, drying differs due to the removal of molecules which, in this case is obtained by a movement of liquid caused by a partial difference on the vapor pressure between the surface of the product to be dried and the air that surrounds it. In the case of food, the removal of water from the wet material is accomplished up to a level where the deterioration caused by microorganisms can be diminished (Lima et al., 2004).

The process of drying should occur in a controlled and uniform way in order to avoid high moisture and temperature gradients inside the material which can cause loss of product quality. Dehydration alter the physical and chemical properties and as a result, affect the process of heat and mass transfer, so it is crucial to identify its effects and how to control them.

In an attempt to further elucidate the drying process, several theories about the drying of solids have been proposed and several mathematical models were developed, with the majority based on the theory of liquid diffusion. Models based on the liquid diffusion theory are used to describe the thin layer drying of various types of products (Lima et al., 2004; Doymaz, 2005; Amendola and Queiroz, 2007; Hacihafizoglu et al., 2008).

When considering simple geometries of one and two dimensions to describe the drying processes of solids with arbitrary shape, some discrepancies are found in the drying kinetics and moisture content distribution as well as the temperature inside the solid, when compared to experimental results. Such a fact occurs because the mathematical models do not faithfully represent the geometry of the body. In this context it is important to consider the body shape factor in the development of mathematical models in order to describe the physical phenomenon in a more authentic way and therefore, to increase the level of assurance of the proposed model. When considering the influence of this parameter, it is expected that the deviations between the theoretical and experimental results are minimized.

Analytical solutions of the diffusion equation by using constant or convective boundary conditions and constant or variable diffusive coefficients, for diverse geometries as parallelepiped, cylinder and sphere, can be found in the literature, such as Carslaw and Jaeger (1959), Luikov and Mikhailov (1965), Luikov (1968), Skelland (1974), Crank (1992), and Gebhart (1993). For bodies of elliptical shape, can be cited the following works: Haji-Sheikh and Sparrow (1966), Haji-Sheikh and Sparrow (1967), Payne et al. (1986), Lima (1999),Oliveira (2001), Lima et al. (2004), Silva et al. (2010) and Santos et al. (2010). However, hardly any researches are related to other more complex geometries.

In this sence this paper aims to theoretically study drying of solids of revolution via integral method based on Galerkin, assuming the equilibrium boundary condition at the surface (Dirichlet condition) and constant diffusion coefficient within the solid.

2. MATHEMATICAL MODELLING

Based on the theory of liquid diffusion, which states that the movement of liquid inside a porous solid has as main agent the existence of a moisture concentration gradient, the second law of Fick has been used as mathematical model in describing such phenomenon (Brooker et al., 1992) as follows:

$$\frac{\partial M}{\partial t} = \nabla \cdot (D \nabla M) \tag{1}$$

where M is the moisture content of the solid which varies along time and D is the diffusion coefficient.

In order to enable the solution of the physical issue, the following considerations were taken:

(a) The solid is homogeneous and isotropic;
(b) The moisture distribution inside the solid is uniform at the beginning of the process;
(c) Thermophysical properties are constant throughout the process;
(d) The solid consists of dry substance and water in liquid phase;
(e) The phenomenon of drying occurs by diffusion of liquid water inside the solid and by evaporation of water at the surface.

Hence, the solution of Eq. (1) can be written as follows (Payne et al., 1986):

$$M = \sum_{n=1}^{N} C_n \Psi_n e^{-\gamma_n t} + M_e \tag{2}$$

where M_e is the equilibrium moisture content.

When substituting Eq. (2) into Eq. (1), considering C_n, γ_n, M_e and D are constant and Ψ_n constant with the time, we can meet:

$$\left[\gamma_n \psi_n + \nabla \cdot \left(D \nabla \Psi_n\right)\right] = 0 \tag{3}$$

to each value of n.

In Eq. (3), Ψ_n is a specific function, obtained by linear combination of a set of basic functions properly selected, where f_j, also known as Galerkin function, is an element of the set of basic functions and the d_{nj} are constants which must be determined. This function is given by:

$$\Psi_n = \sum_{j=1}^{N} d_{nj} f_j \tag{4}$$

The Ψ_n function is selected so that the homogeneous boundary condition (balance condition) is satisfied. The function f_j with j ranging from 1 to N compose a set of basic functions (Farias, 2002).

When substituting Eq. (4) into Eq. (3), we have:

$$\sum_{j=1}^{N} d_{nj} \left[\gamma_n f_j + \nabla \cdot \left(D \nabla f_j\right)\right] = 0 \tag{5}$$

Furthermore, using the Galerkin procedure, which involves multiplying Eq. (5) by $f_i dV$ and integrate it over the volume (Kantorovich and Krylov, 1960), the following result can be obtained:

$$\int_v \sum_{j=1}^{N} d_{nj} \left[\gamma_n f_j + \nabla \cdot \left(D \nabla f_j\right)\right] f_i dV = 0 \tag{6}$$

Writing Eq. (6) in matrix form, we have:

$$\left(\bar{A} + \gamma_n \bar{B}\right) \bar{d}_n = 0 \tag{7}$$

where $\bar{A}$ and $\bar{B}$ are square matrices of N x N elements, whose elements are calculated by the following equations:

$$a_{ij} = \int_v f_i \nabla \cdot \left(D \nabla f_j \right) dV \tag{8.a}$$

$$b_{ij} = \int_v f_i f_j dV \tag{8.b}$$

Thus, after determining the matrices $\bar{A}$ and $\bar{B}$, the values of γ_n and $\bar{d}_n$ are determined.
In order to apply the boundary conditions to the problem the following identity is used:

$$\int_v f_i \nabla \cdot \left(D \nabla f_j \right) dV = \int_v \nabla \cdot \left(D f_i \nabla f_j \right) dV - \int_v D \nabla f_i \nabla f_j dV \tag{9}$$

Considering D constant, the following can be written:

$$\int_v f_i \nabla \cdot \left(D \nabla f_j \right) dV = \int_s D f_i \left(\frac{\partial f_j}{\partial \vec{n}} \right) dS - \int_v D \nabla f_i \nabla f_j dV \tag{10}$$

When dealing with the homogeneous conditions of first kind (prescribed M) it is determined that $f_j = 0$. Thus, the first term of the second member of the Eq. (10) igual to zero and the second term is always symmetric.

In order to obtain the coefficients C_n of Eq. (2), we use the initial condition ($t = 0$). In this case, $M = M_0$, so the following equation can be obtained:

$$M_0 = \sum_{n=1}^{N} C_n \Psi_n + M_e \tag{11}$$

Using the Galerkin procedure once again, multiplying Eq. (11) by $f_i dV$ and integrating it over the volume (Kantorovich and Krylov, 1960), we obtain:

$$\int_v f_i \left(M_0 - M_e \right) dV = \int_v f_i \sum_{n=1}^{N} C_n \Psi_n dV \tag{12}$$

The solution of Eq. (12) is a set of N linear algebraic equations that will allow finding the C_n and consequently complete the solution to the problem.

The average moisture content of the solid is given by:

$$\bar{M} = \frac{1}{V} \int_v M dV \tag{13}$$

where V is the volume of the solid studied.

As an application, the methodology will be used to describe the drying of various solids of revolution (fig. 1) with the same aspect ratio (b/a = 2.0). The contour of the solids is being defined here as a basic function of the following kind:

$$\emptyset_1 = 1 - {r^m}/{a^m} - {z^2}/{b^2} \tag{14}$$

where m is a number that defines the geometry of the solid being studied.

In this study, we simulated the drying of five solids of revolution considering m = 0.5, 1.0, 2.0, 3.0 and 4.0, in Eq. (14).

Therefore, considering m = 2.0 in Eq. (14), the solid will be a prolate spheroid, like show the fig. 1.

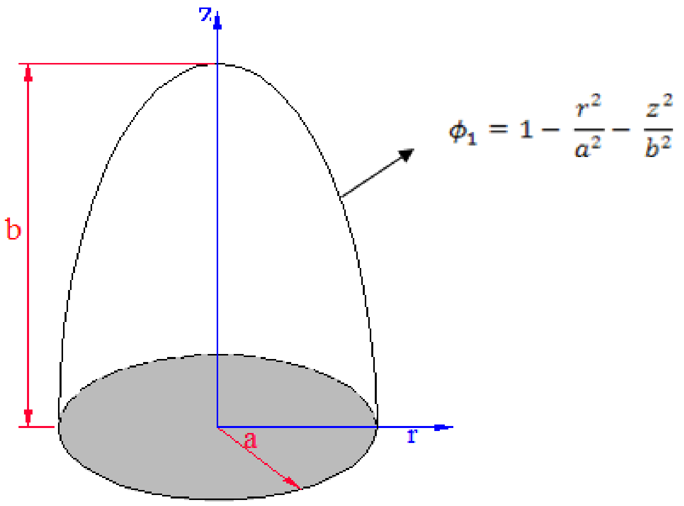


Figure 1. Prolate spheroid and its characteristics.

In this case, the volume of the solid shown in fig. 1 is given by:

$$V = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^2/b^2}} r\,dz\,dr\,d\theta \tag{15.a}$$

or,

$$V = \frac{2}{3}\pi a^2 b \tag{15.b}$$

Therefore, using Eqs. (8.a) and (8.b), one can reach:

$$a_{ij} = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^2/b^2}} f_i \nabla\cdot(D\nabla f_j) r\,dz\,dr\,d\theta \tag{16.a}$$

and

$$b_{ij} = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^2/b^2}} f_i f_j r\,dz\,dr\,d\theta \tag{16.b}$$

where the values of C_n to the prolate spheroid are calculated using the following equation:

$$\int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^2/b^2}} f_i (M_0 - M_e) r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^2/b^2}} f_i \sum_{n=1}^{N} C_n \Psi_n r\,dz\,dr\,d\theta \tag{17}$$

The basic functions of first kind are determined from the first basic function, which is given by:

$$f_1 = \emptyset_1 \emptyset_2 \emptyset_{3\ldots} \emptyset_N \tag{18}$$

where $\emptyset = 0$ is an equation that describes one the surfaces of the body, and N is the number of surfaces that define the body to be studied.

Therefore, a set of basic functions of first kind is defined, for instance, as follows:

$$f_j^1 = f_1 r^{m_j} z^{n_j} \tag{19}$$

where, $j = 0, 1, 2, 3, \ldots$, $m_j = 0, 2, 4, 6, \ldots$ and $n_j = 0, 2, 4, 6 \ldots$, and $f_1 = \emptyset_1$ (for prolate spheroid).

In order to simulate the drying of solids of revolution presented in this work, we developed a computer code in the Mathematica® language (version 7.0) (Wolfram, 2009). All work was carried out at the Computational Laboratory of Thermal and Fluids, LCTF, of the Department of Mechanical Engineering Center for Science and Technology, Federal University of Campina Grande, Brazil, by using a computer Pentium IV, Intel Xenon Dual with 3.06 GHz, 2048 Mb ram and 80 Gb of HD.

3. RESULTS AND DISCUSSIONS

Figures 2 and 3 represent the drying kinetics of some solids of revolution of aspect ratio b/a = 2.0. All the curves representing the average moisture content of spheroids are described in terms of Fourier number ($Fo = {}^{Dt}/_{a^2}$). It can be observed in all figures which the dimensionless average moisture of the solids considered, decreases rapidly at the beginning of the process due to high concentration gradients within the solid in this instant.

By examining fig. 2, one can observe that for m = 0.5, the dimensionless average moisture within the spheroid decreases much faster than for m = 4.0. In other words, for the same aspect ratio, the higher the value of m in Eq. (14) the lower the area/volume ratio will be formed. Since the area/volume relationships is directly proportional to the drying velocity, it is noticed that for m = 4.0, the drying process of this solid will be slower when compared to other spheroids studied in this work. High rates of drying (high concentration gradients) induce elevated thermal, hydraulic and mechanical stress within the solid, which could result in the manifestation of cracks or distortions in the product, which may come to compromise its final quality.

It is also clear from fig. 2 that for the same Fourier number there are different moisture levels, so factor m in Eq. (14) defines how the body influences the drying kinetics. Since the spheroid characterized by m = 0.5 has a higher area/volume ratio, one can state that the higher the area/volume ratio the greater the drying rate. This observation is consistent with existing works in literature (Lima, 1999; Carmo, 2000; Farias, 2002).

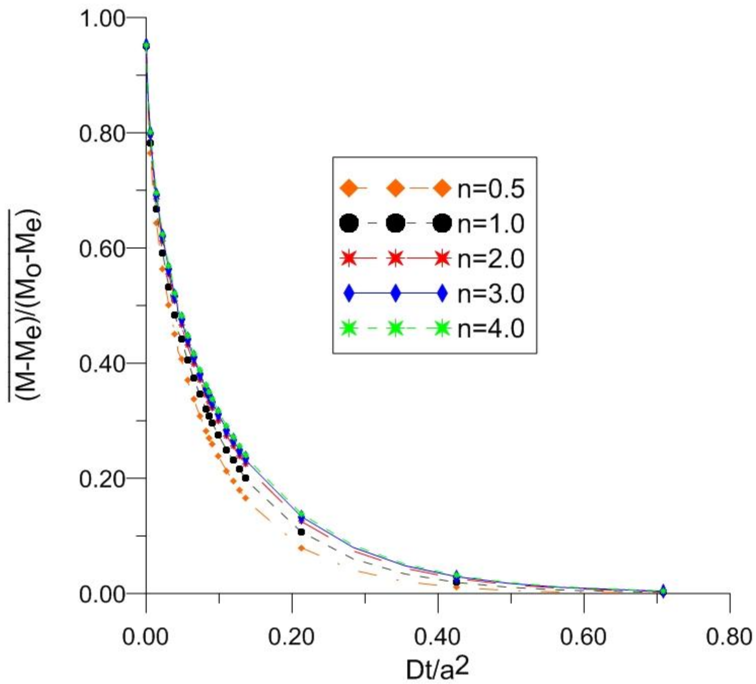


Figure 2. Average moisture content within five solids of revolution with the same aspect ratio as a function of Fourier number.

Figure 3 was used to validate the methodology and results obtained in this work with literature data (Lima et al., 2004). This figure shows the drying kinetics of a prolate spheroid of aspect ratio b/a = 2.0. Analyzing fig. 3, one can find good agreement between the results thereby proving that this work presents consistent results.

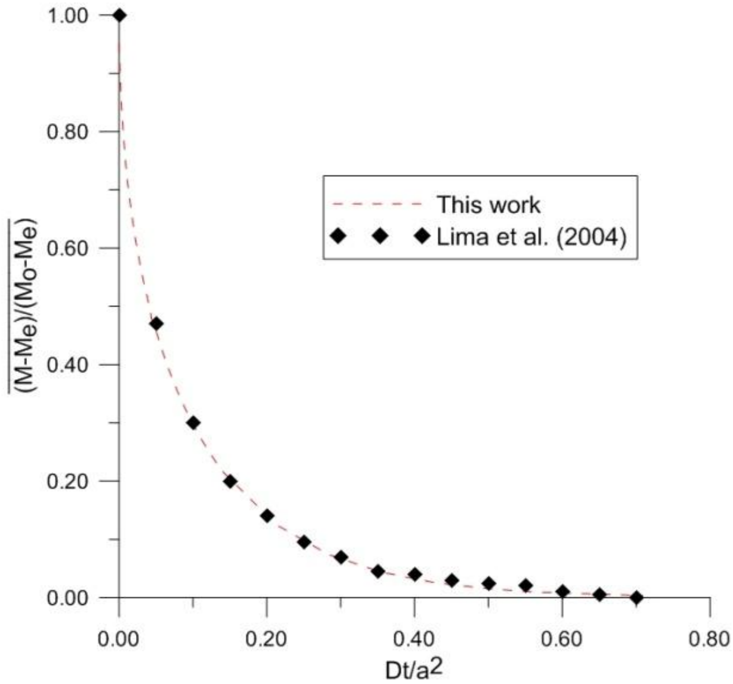


Figure 3. Comparison between the results of moisture content in the center of a spheroid with m = 2.0 and b/a = 2.0, obtained in this work and reported in the literature.

Figures 4 to 8, show the distribution of the dimensionless moisture content inside the solids of revolution with aspect ratio b/a = 2.0, in three dimensionless time (Fourier number): 0.0708617, 0.0992063 and 0.141723.

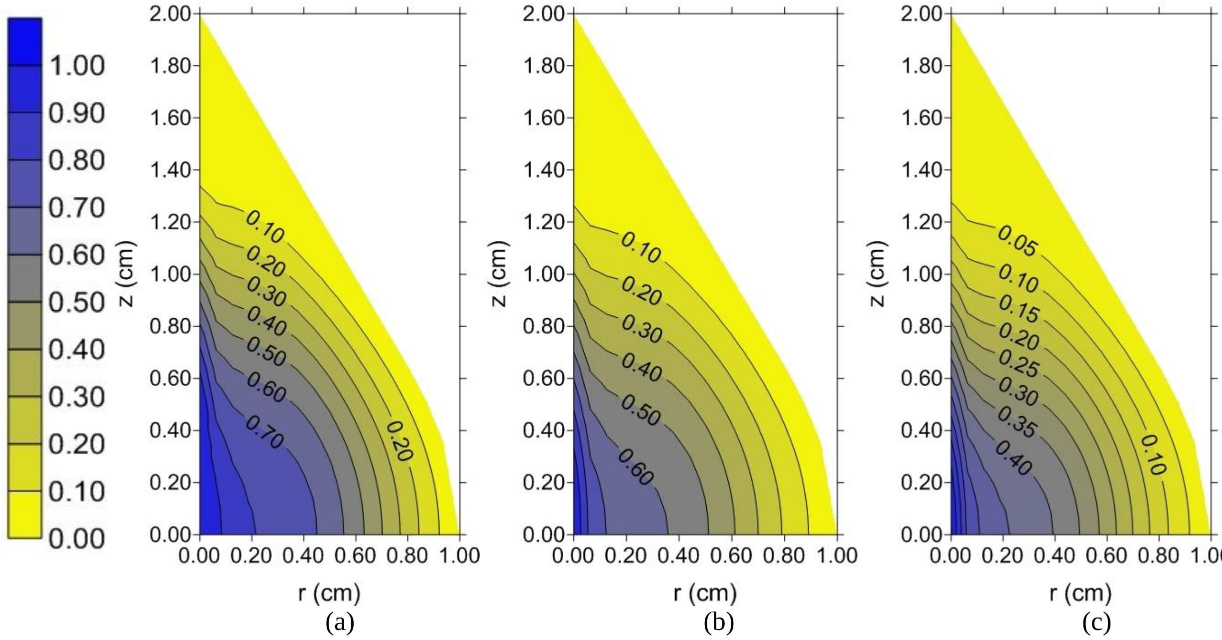
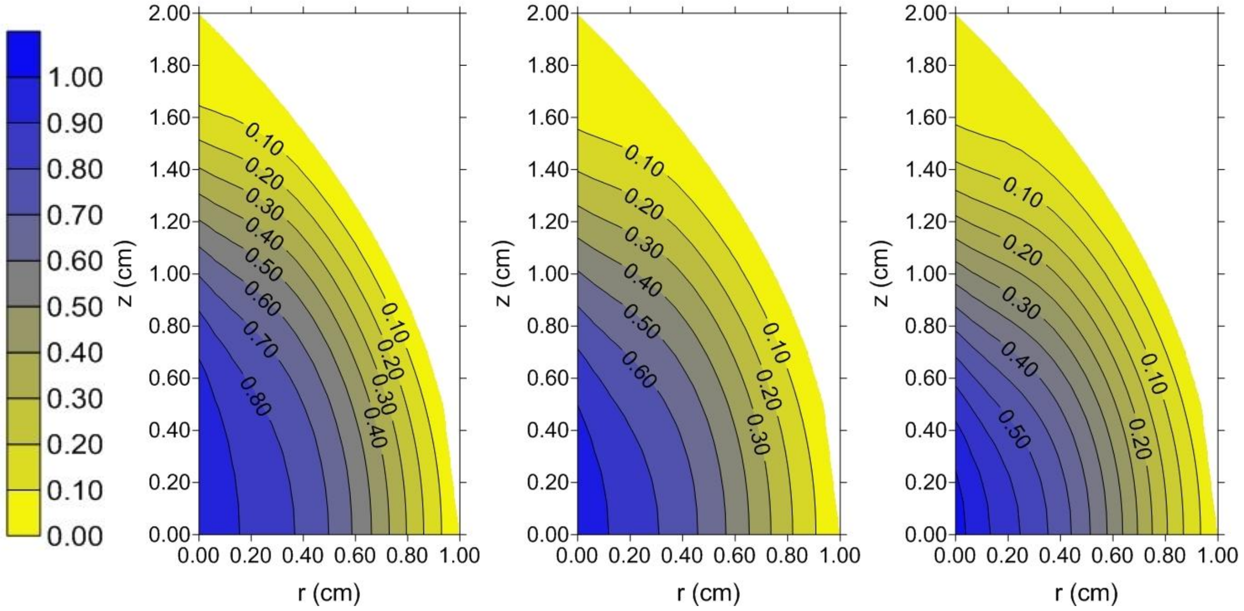
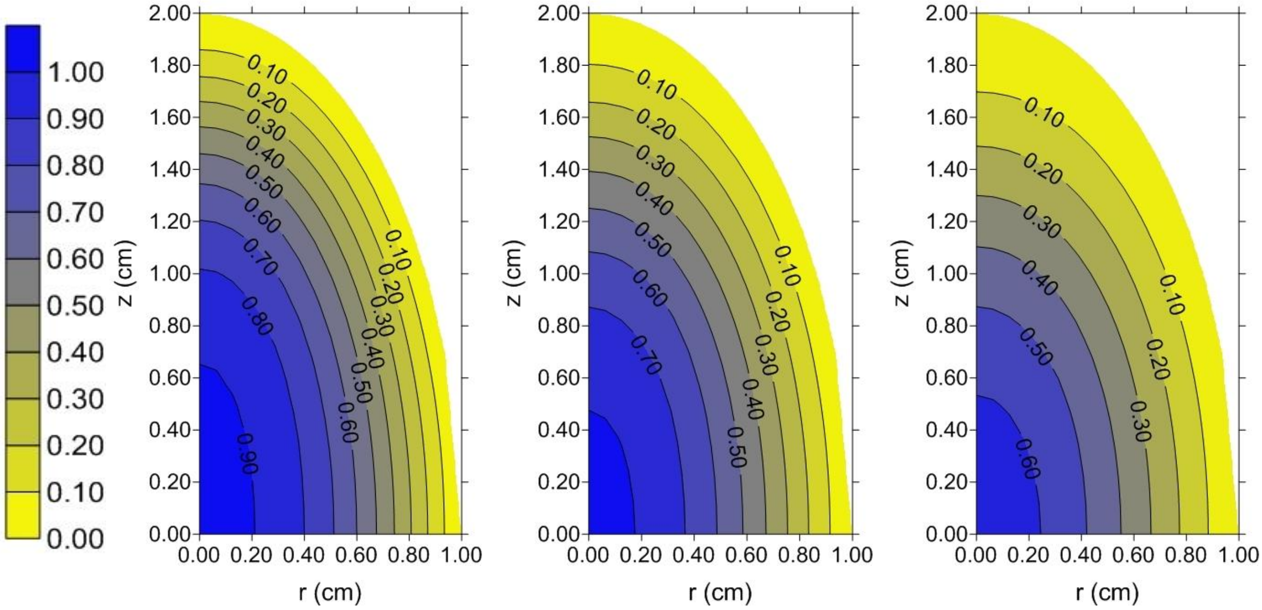


Figure 4. Distribution of dimensionless moisture content within a solid of revolution formed with m = 0.5 and b/a = 2.0 for the following dimensionless times: a) Fo=0.0708617, b) Fo=0.0992063 and c) Fo=0.141723.



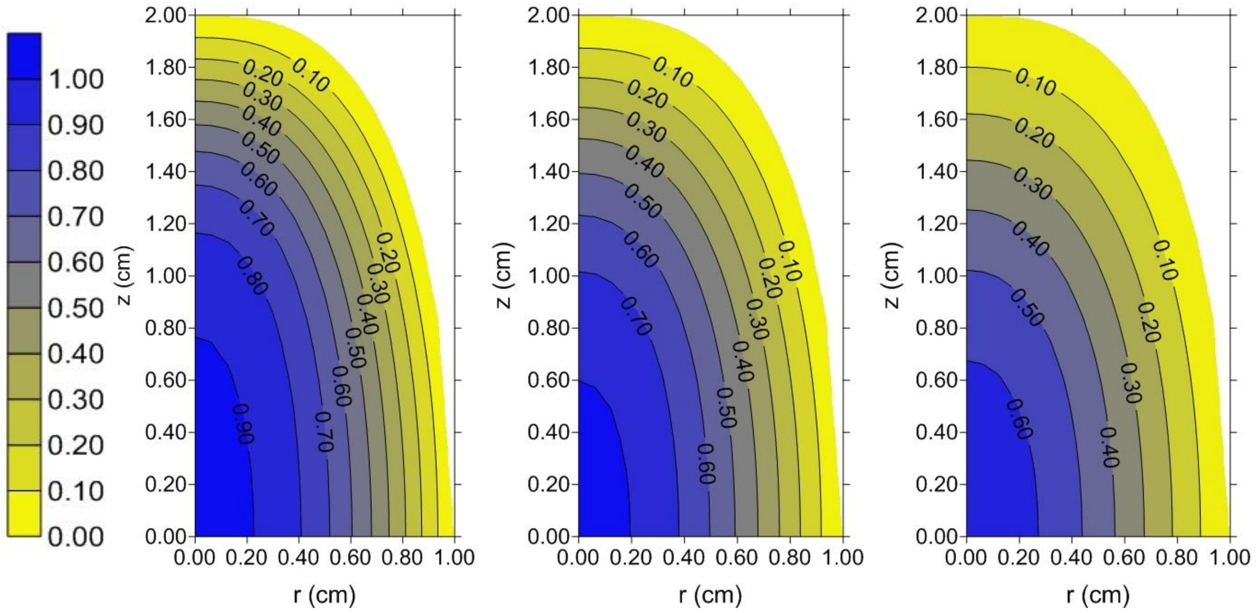
(a) (b) (c)

Figure 5. Distribution of dimensionless moisture content within a solid of revolution formed with m = 1.0 and b/a = 2.0 for the following dimensionless times: a) Fo=0.0708617, b) Fo=0.0992063 and c) Fo=0.141723.



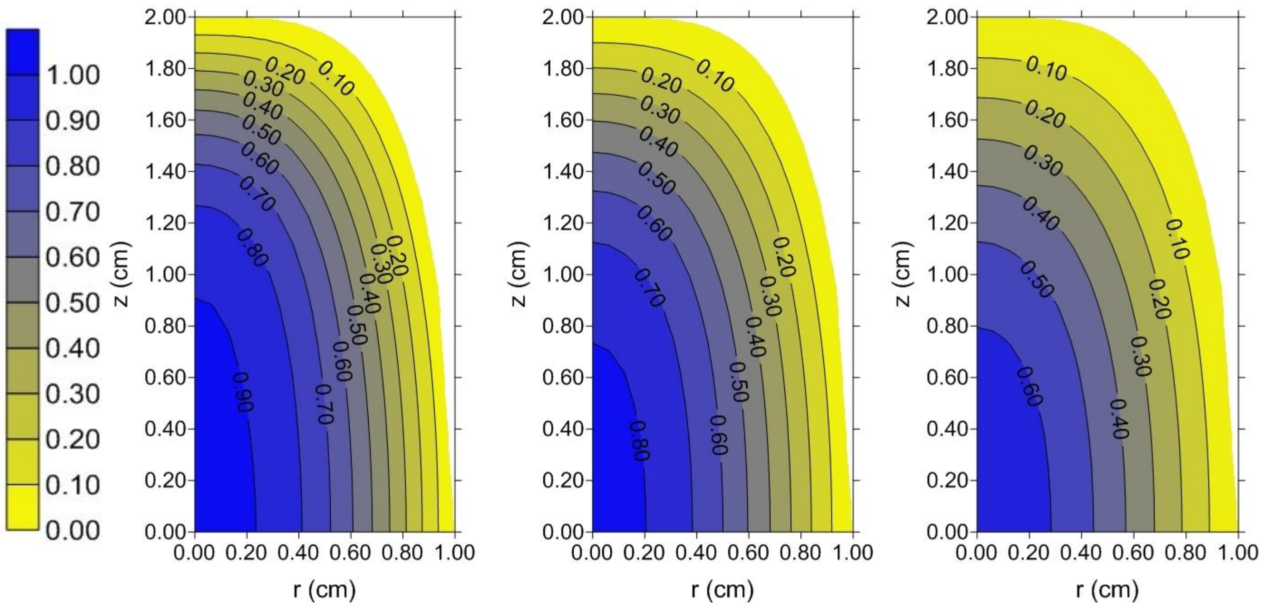
(a) (b) (c)

Figure 6. Distribution of dimensionless moisture content within a solid of revolution formed with m = 2.0 and b/a = 2.0 for the following dimensionless times: a) Fo=0.0708617, b) Fo=0.0992063 and c) Fo=0.141723.



(a) (b) (c)

Figure 7. Distribution of dimensionless moisture content within a solid of revolution formed with m = 3.0 and b/a = 2.0 for the following dimensionless times: a) Fo=0.0708617, b) Fo=0.0992063 and c) Fo=0.141723.



(a) (b) (c)

Figure 8. Distribution of dimensionless moisture content within a solid of revolution formed with m = 4.0 and b/a = 2.0 for the following dimensionless times: a) Fo=0.0708617, b) Fo=0.0992063 and c) Fo=0.141723.

From figs. 4 to 8, one can verify the existence of concentration gradients during the drying process and that they are higher in early times for each of the five solids. However it is observed that these moisture gradients are higher in solids that have a lower "m" value, which represent the shape factor of the body, being consistent with fig. 2. Since higher moisture gradients will result in greater movement of fluid within the solid, it to dry more quickly.

It is also worth pointing out that for all cases presented, this phenomenon is related with the fact that the highest concentration gradients that occur at the edge of the solid close to z = 2.0, being lower when the solid tend to a sphere (z = 1.0). It is observed that the region z = 2.0 upholds the lowest moisture concentrations for all times in any solids. Therefore, if by any chance the drying rate is such that it will cause a fracture or a deformation in the solid, it is very likely that such effects occur in the edge region. For this reason, several authors mention this edge effect (Oliveira, 2001; Nascimento, 2002).

From figs. 4, 5, 6, 7 and 8, one can observe the presence of iso-concentration lines, which have the same form of the solid and also that the moisture concentrations are higher within the solid thus showing that the movement of liquid occurs from the interior to the surface of the solid of revolution, as expected.

4. CONCLUSIONS

Upon completion of this work and data analysis, the following conclusions were reached: the mathematical model used to obtain the analytical solution of the problem was appropriate, thus it is also suitable for obtaining solution to transient problems such as cooling, heating or wetting; the body shape has a direct influence on its drying kinetics; the highest moisture gradients occur on the edge of the solid of revolution, being this area the most affected by thermo-hydro-mechanical stresses and being more susceptible to the appearance of defects such as cracks and deformations, and the relation area/volume is a major factor that influencing directly at the drying rate of a solid which is observed in figs. 4 to 8.

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