Analysis of optimization techniques using multi-objective genetic algorithms applied in the preliminary single stage axial flow turbine design.

Osmar Francisco Reis Silva, osmarfr@gmail.com
Jesuíno Takachi Tomita., jtakachi@ita.br

Instituto Tecnológico de Aeronáutica
Praça marechal Eduardo Gomes, 50- Vila das Acácias
Cep: 12228-900, São José dos Campos- SP- Brasil

Abstract. *The turbomachinery design process consists of several steps to obtain the final machine geometry. The first step is the preliminary design based on the design-point operation condition as determined from a thermodynamic cycle calculation. At this point the turbomachinery designer must be capable of handling many geometrical parameters as space-chord-ratio, aspect ratio, hub-to-tip ratio and so on. The use of optimization techniques can supply important information on the behavior of these parameter and its influence on the turbomachinery performance. This work deals with a preliminary design process of a single stage axial flow turbine using loss models based on entropy generation correlations, developed by Denton and the use of optimization techniques that uses Genetic Algorithms to support the designer in decisions of different turbine geometrical parameters. The comparison between different optimization techniques results are presented and discussed. The preliminary design is calculated from an in house computational program and the optimization tool is the modeFRONTIER commercial package.*

Keywords*: optimization, axial turbine, gas turbine.*

1. INTRODUCTION

The search for optimal aerodynamic design of an axial turbine has several conflicting factors leading the designer to a trade-off between high efficiency, high performance, low levels of losses, lower weight, lower cost, without adversely affecting the required parameters. This paper presents a methodology for optimizing the efficiency of a single stage axial turbine at design-point using artificial intelligence. In a search fields as vast as the area of turbomachinery using genetic algorithms has produced excellent results.

This work perform a comparison between two optimization models using genetic algorithms applied in the design of an axial turbine preliminary design using the loss model developed by Denton. This model is based on entropy generation to predict the losses that influence isentropic efficiency of the turbomachinery. In the prediction model developed by Denton losses evaluation differ from the methodologies developed by other authors as Ainley and Mathieson, Dunham and Came and Kacker and Okapuu, which are based on tuning curves obtained from several different tests over turbines years of testing.

The optimization tool is couple the turbine preliminary design program. The implementation of this technique presents a set of optimal solutions known as Pareto front. The techniques involving multi-objective optimizers have the advantage of searching a global optimum of the search space while in single-objective methods seeks a local optimum, not guaranteeing that this point is also reached the global optimum, this set of optimal solutions allows the designer to choose the geometrical parameters most appropriate to the design requirements.

The methods used are multi-objctive genetic algorithm (MOGA II) and non-dominated sorting genetic algorithm (NSGA II), using the same number of generations and mutation rate in order to compare not only results in better efficiency, but computational time and also Pareto-optimal generated by the algorithms. For the optimization process the commercial SOFTWARE MODEFRONTIER was used.

2. THE LOSS MODEL DEVELOPED BY DENTON

This loss model is based on the entropy increase due to the viscous friction effects in either boundary layer or free shear layers, heat transfer across temperature difference, non equilibrium processes such as those that occur in very rapid expansions (Denton, 1999).

The total loss is determined by the sum of the profile loss (ζ_p), the trailing edge loss (ζ_{te},), boundary-layer effects (ζ_{pp}), tip clearance (ζ_{tip}) and shock wave (ζ_{chock}) losses.

- Profile loss:

  The profile loss is due to the boundary layer on the blade. It is calculated by:

$$x = \frac{T_2 \Delta s}{m {V_2^2}/{2}} \quad , \tag{1}$$

where Δs is given for:

$$\Delta s = \sum_s^p h_p c_d \int_0^{cx} c_d \rho V^3 d\left(\frac{x}{c_s}\right) \quad , \tag{2}$$

where h_p is blade height, C_s is blade surface length; c_x is the blade chord; Cd is the dissipation coefficient; V is the relative velocity; T the static temperature; ρ is the density; m is the mass and x is the position along the blade.

- Trailing edge loss:
  At the trailing edge the loss is due to the limitation of blade thickness and has influence in the flow angle on the pressure, and in the boundary layer. The trailing edge loss is described as:

$$\varsigma_{Te} = \frac{C_{pb} t_e}{s \cos \alpha_2} + \frac{2\theta}{s \cos \alpha_2} + \frac{\delta t_e}{s \cos \alpha_2} \tag{3}$$

Where C_{pb} is:

$$C_{pb} = \frac{P - P_{ref}}{0{,}5 \rho V_{ref}^2} \tag{4}$$

Since the terms P_{ref} is the reference pressure and V_{ref} is the reference speed on the surface of the blade just before the trailing edge. Where t_e is trailing edge thickness, s is blade spacing, θ is the momentum boundary layer thickness, δ is the and α_2 the flow angle (Schlichiting and Gersten, 2000). According to (Denton, 1987) a satisfactory value for c_p is $0.1 < C_{pb} < 0.2$.

- Tip clearance loss:

The loss in the tip clearance is due to the flow leakage between rotor tip and casing. A turbine can be shrouded or unshrouded, where for the first type of loss coefficient is described by the angles of entry and exit of the flow, besides the flux that passes through the top and also the main channel of the blade. In the case of unshrouded blades the loss is due to flow leakage, blade chord and height, velocity distribution on the pressure and suction sides of the blade, relative output speed and pitch. Shrouded turbine the loss is given by:

$$\varsigma = \frac{T\Delta s}{0{,}5 V_2^2} = 2\frac{m_L}{m_m} = \left(1 - \frac{\tan \beta_1}{\tan \beta 2} \sin^2 \beta_2\right) \tag{5}$$

With m_L is the flow from the top, m_m main flow and β_1 and β_2 the blades angles input and output respectively. The unshrouded model is calculated by:

$$\varsigma = \frac{2 c_d k' c}{h_p s \cos \beta_2} \int_0^{cx} \left(\frac{V_p}{V_s}\right)^3 \left(1 - \frac{V_p}{V_s}\right) \sqrt{\left(1 - \left(\frac{V_p}{V_s}\right)^2\right)} d\left(\frac{z}{c}\right) \tag{6}$$

Where k´ is the effective value of tip clearance and c is the chord.

- Boundary layer loss:
  The boundary layer loss is calculated by the sum of the blade to blade and blade streamwise losses.

- Streamwise losses:
  The main factor in this loss source is the entropy generation due to the temperature variation:

$$\Delta s = 0.25 \int_0^{cx} \frac{c_d \left(V_s^4 - V_p^4\right)}{T\left(V_s - V_p\right)} \rho t_l dx \tag{7}$$

Where V_s and V_p are velocities at blade suction and pressure sides, respectively and t_l the blade local thickness. Thus,

$$\varsigma_{canal} = \frac{T_2 \Delta s}{m \frac{V_2^2}{2}} \tag{8}$$

The losses in boundary layer on the wall between grids are calculated by:

$$\varsigma_{grid} = \frac{T_2 \Delta s}{m \frac{C_2^2}{2}} \tag{9}$$

Where C_2 is the absolute velocity and Δs is calculated by:

$$\Delta s = \int_0^{Aw} \frac{c_d \rho C_2^2}{T_2} dA \tag{10}$$

Thus, the loss in the boundary layer is represented by the sum:

$$\varsigma_{pp} = \varsigma_{canal} + \varsigma_{grid} \tag{11}$$

- Shock wave loss:
  The shock wave formation causes high losses inside the turbine blade rows. It can be calculated with:

$$\varsigma_{chock} = \frac{T_2 \Delta s}{m \frac{V_2^2}{2}} \tag{12}$$

Where of Δs is given by

$$\Delta s = C_\upsilon \frac{2\gamma(\gamma - 1)}{3(\gamma - 1)^2} \left(M_2^2 - 1\right)^3 \tag{13}$$

Where C_v is the specific heat, γ is ratio of specific heats and M_2 the outlet Mach number.

Finally the total loss can be determined with the following expression:

$$\varsigma_{total} = \varsigma_p + \varsigma_{te} + \varsigma_{tip} + \varsigma_{pp} + \varsigma_{chock} \tag{14}$$

3. OPTIMIZATION TECHNIQUE APPLIED ON GAS TURBINE COMPONENTS:

The turbomachinery designers should have a high expertise to decide and to choose the key parameters to start the preliminary design. Generally, geometrical parameters can be changed to improve efficiency and enhance the design characteristics (Keskin *et al*, 2006). Optimization tools can support designer decisions.

The use of genetic algorithms in the turbine design, have been widely used and has shown good results in projects that work with multiple objectives. Through this method it is possible to work with different objectives at the same time and get a very effective design, the multi-objective optimization presents a set of optimal solutions, allowing the designer to choose the best configuration according to the designers requirements. The optimization methods used in this work are MOGA II and NSGA II.

3.1 MOGA II

The MOGA II (Multi-Objctive Genetic Algorithm) is an evolution of MOGA and introduced itself as an efficient multi-objective genetic algorithm that uses an intelligent multi search elitism (Öksüz and Akmandor, 2010). This new elitism operator is able to preserve some excellent solutions without causing premature convergence to local optima.

For simplicity, MOGA II requires only fewer user-provided parameters, several other parameters are internally settled in order to provide robustness and efficiency to the optimizer.

The algorithm to attempt a number of evaluations equal to the size of the loaded Design of Experiments, the initial population for MOGA II algorithm, multiplied by the number of generation defined in the process which can be chosen according to the computational resources available.

The size of the run is usually defined by the computing resources available. A rule of thumb would suggest possibly accumulate an initial DOE of at least 16 design configurations and a possibly more than twice the number of variable multiplied by the number of objectives.

The functions Cross-over operation defines the efficiency of the algorithm at the same time decreasing its strength, in general the extremes 0 and 1 should be avoided as for nonlinear problems increase the chance of the algorithm to converge to a local optimum, a value of 0.5 is normally used (modeFRONTIER, 2000). The probability of selection sets the chance that there are no changes in the project settings during the process of development while maintaining the diversity between the points.

There are three types of MOGA algorithms:

Stable evolution: uses all the computed configurations as soon as they are available in a "first in - first out" mode.
Generational Evolution: works on a set of design configurations that are periodically updated when one generation is completed.

Adaptive Evolution: the choice of the genetic algorithm operators is done dynamically during the search. The Probability of Directional Cross-Over and the Probability of Mutation are taken as initial conditions. During the search the adaptive algorithm changes their ratio according to the evaluation results. The adaptive evolution can be used when there is no clear idea about the probabilities of operators.

3.2 NSGA II

The method NSGAII (non-dominated sorting genetic algorithm) as MOGA II is a fast and elitist multi-objective algorithm. According with (MODEFRONTIER™, 2000) the main features are: A fast non-dominated sorting produce is implemented. Sorting of the individuals given of a given population according to the level of non-domination is a complex task: due to non-dominated sorting algorithms are in general computationally expensive for a large population size is adopted a solution performs an intelligent sorting strategy.

Is implemented a system of elitism for multi-objective search, using an elitism-preserving approach. Elitism is introduced storing all non-dominated solutions discovered so far, beginning from the population. Elitism enhances the convergence properties towards the true Pareto-optimal set.

The constraint handling method does not make use of penalty parameters. The algorithm implements a modified definition of dominance in order to solve constrained multi-objective problems efficiently.
The NSGA II allows both continuous, real- coded, and discrete, binary- coded, design variables. The original feature is the application of a genetic algorithm in the field of continuous variables.

A parameter-less diversity preservation mechanism is adopted. Diversity and spread of solutions is guaranteed without use of sharing parameters, since NSGA II adopts a suitable parameter-less niching approach. It uses the crowding distance, which estimates the density of solutions in the objective space, and the crowded comparison operator, which guides the selection process towards a uniformly spread Pareto frontier.

4. PRELIMINARY AXIAL TURBINE DESIGN.

The turbine design is presented by (Saravanamuttoo, 2001) for a turbojet and is based on the mean diameter condition and use the theory of free vortex, the rotational and tangential speed are defined according to the speed of the compressor which is ever more critical due to deceleration of the fluid. The design parameters of axial turbine are given by table 1.

Table 1: Parameters of original design

Mass flow	$\dot{m}$	20	kg/s
Isentropic efficiency	η_t	0.9	-
Inlet temperature	T_{01}	1100	K
Temperature drop	T_{01}- T_{03}	145	-
Pressure ratio	p_{01}/ p_{03}	1.873	-
Inlet pressure	p_{01}	4	bar
Rotational speed	N	250	rev/s
Mean blade speed	U	340	m/s

Based on the design considerations and loss model adopted, geometrical data and the efficiency obtained are presented in table 2.

Table 2: Parameters of the original design of the axial turbine.

blade row	nozzle	rotor
s/c	0.86	0.83
h	0.536	0.0691
h/c	3	3
c	0.0175	0.023
s	0.01506	0.0191
n	90	71
η		88%

The parameters used in the optimization process are: s/c (NGV and rotor); h/c and consequently the number of blades at each blade row. These parameters have influence not only in the efficiency but, in the manufacturing complexity.
During optimization process the same was parameter choose for the two algorithms as shown in table 3.

Table 3: Parameters of the genetic algorithms

Initial population	100
Generations	50
Rate cross-over	0.5

The workflow is shown in Fig. 1, three design variables were defined, the objective function was defined with the original design variables and the outputs variables are defined by the designer. In this work the output variables are: efficiency, quantification of losses and number of nozzle and rotor blades, finally the goal is determined for each output variable, maximize or minimize. The optimization process was implemented using commercial software modeFRONTIER. Table 2 shows the parameters of the original design where n is the blade number and η the efficiency of the turbine.

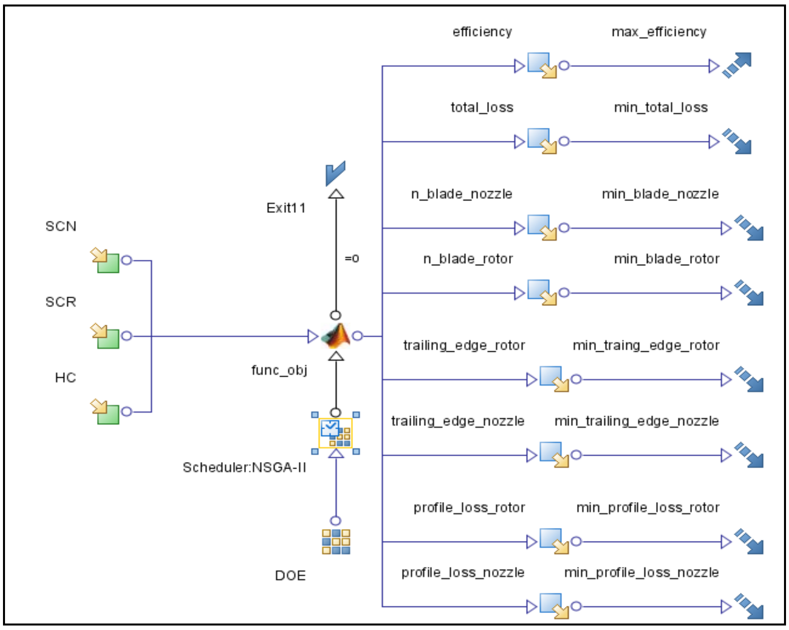


Figure 1: Schematic of the optimization process in the environment modeFRONTIER.

Other parameters will be assessed in future stages of the project as stress and vibration problems among others relevant points. These parameters can have a limit value and be included an design constrains.

5. RESULTS

5.1 RESULTS USING MOGA II

The results achieved through the application of MOGA II showed an increase in efficiency, reduction in the number of blades in nozzle and rotor rows and low computational cost. The Pareto Front show the efficiency of the turbine as a function of parameter s/c in the nozzle (SCN) and rotor (SCR) shown in Fig.2.

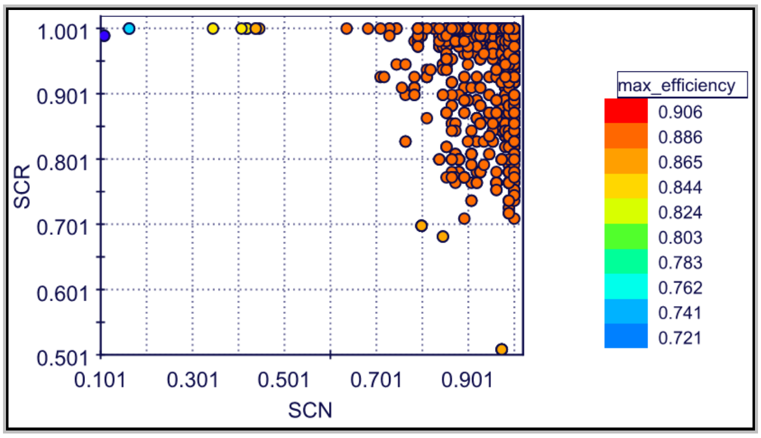


Figure 2: Front Pareto showing the various viable projects based on the ratio s/c in the nozzle and rotor.

Figure 3 show the combination of several designers achieved for blades numbers and efficiency. Each line corresponds to a viable design giving the designer a range of choices. This shows that there are a range of configurations to the designer in which he can choose the number of blades considering efficiency, alignment of flow and loading issues blade.

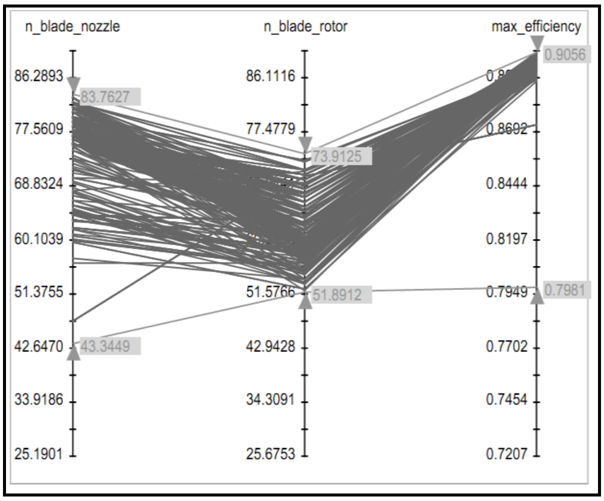


Figure 3: Influence of the choice of the number of blades in turbine efficiency.

Figure 4 show the various design settings and its efficiency as a function of designs variables, where HC is the ratio h/c, SCN is the s/c on nozzle and SCR is the s/c on rotor, for values of HC within the range of security according Saravanamuttoo a value between 3 and 4 is indicated. There are several configurations possible with SCN and SCR and more critical relationship happens in the SCN, where the number of possibilities is smaller.

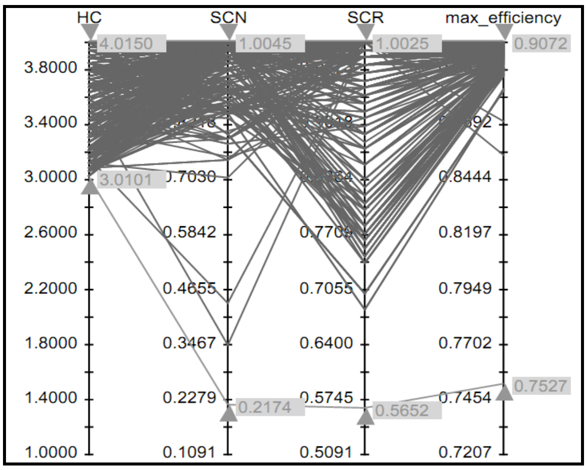
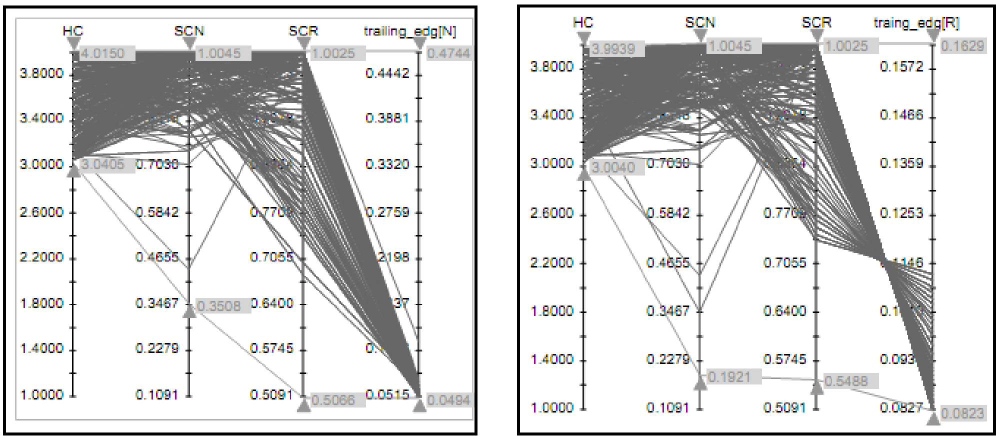


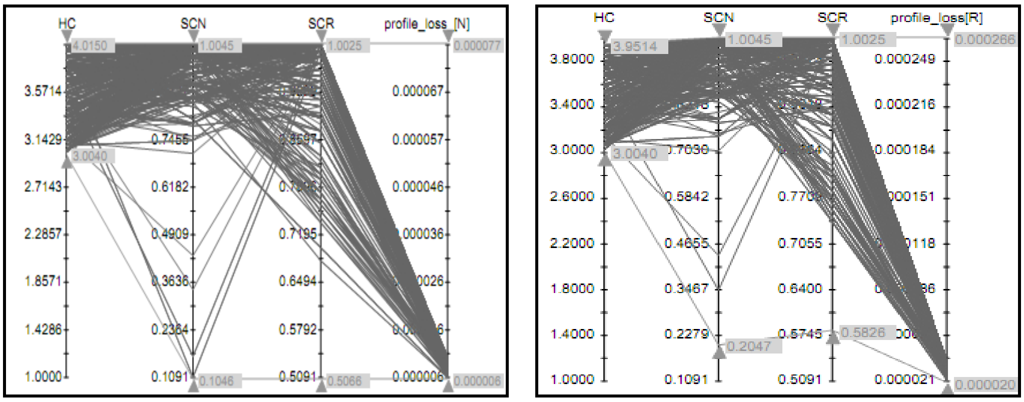
Figure 4: Influence of variation of geometrical variables in efficiency.

The higher loss occurs at the trailing edge and on the profile, so the behaviors of these losses were analyzed in the nozzle and rotor with the variation of design variables as shown in the Fig.5 and Fig.6. (a) and (b).



(a) (b)

Figure 5: Variation of trailing edge loss for each blade row: (a) nozzle; (b) rotor.



(a) (b)

Figure 6: Variation of profile loss for each blade row: (a) nozzle; (b) rotor.

5.2 RESULTS NSGA II

The Fig. 7 show that this algorithm as well as the MOGA II resulted in significant gains in efficiency compared to original design. Therefore the group of Pareto Optimal achieved by this algorithm was fewer. On the other hand the value of maximum efficiency comparable which MOGA II.

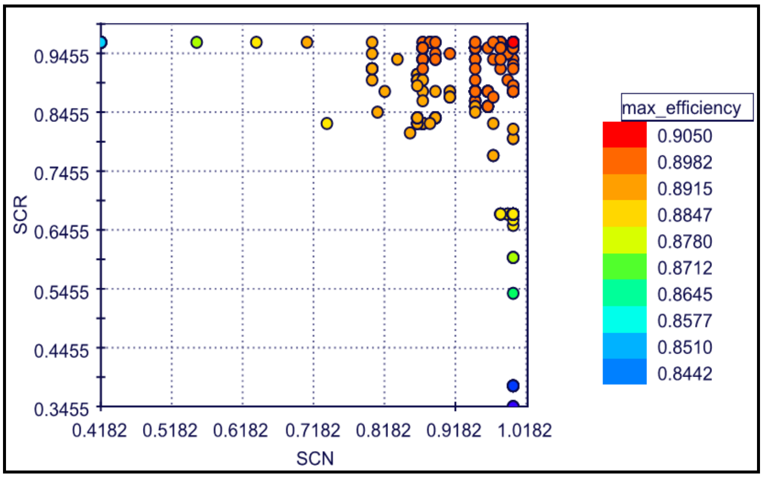


Figure 7: Front Pareto showing the various possible and viable projects based on the ratio s/c in the nozzle and rotor.

The influence of number of blades in the efficiency is shown in Fig. 8, where the red line represented the best configuration for a given number blades as a function of efficiency, as so the optimization using MOGA II shown in Fig.3 occurred a reduction in the number of stator and rotor blades row, although it is computational cost is slightly higher than the MOGA II.

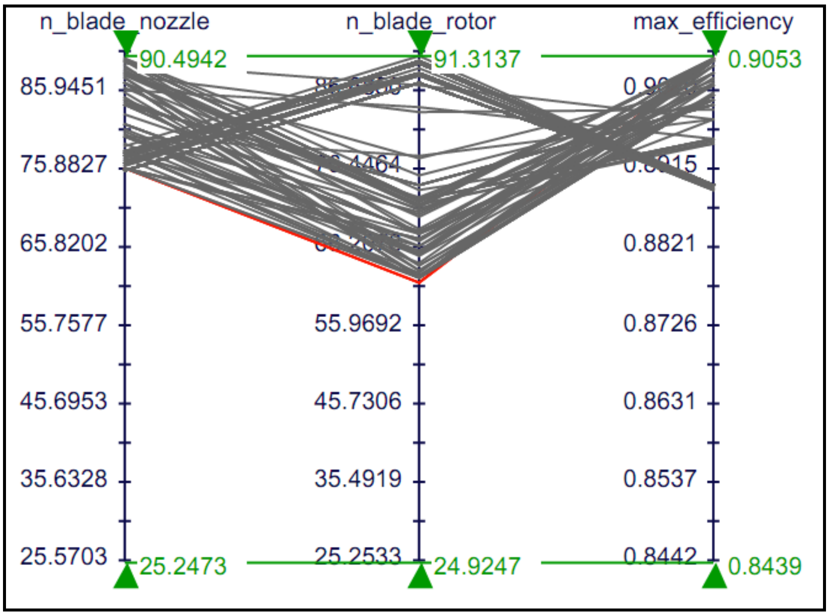


Fig. 8: Possible design configuration varying the number of blades.

The Fig. 9 show several combinations for designs variable presented in the Pareto set and the efficiency obtained. Again the red line represents the best parameters of the design variable for maximum efficiency. The choice of ratio h/c, (HC), considers security issues for the project to avoid vibrations and stress problems (Saravanamuttoo, 2001).

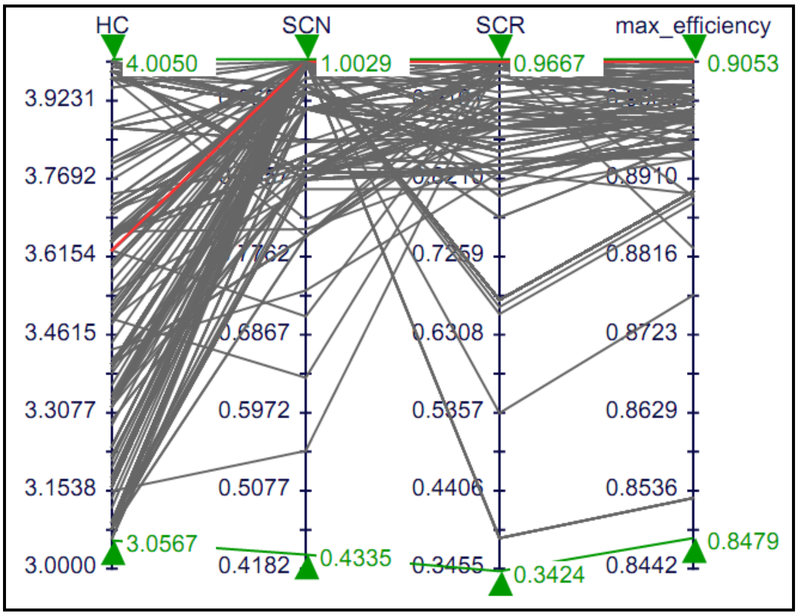
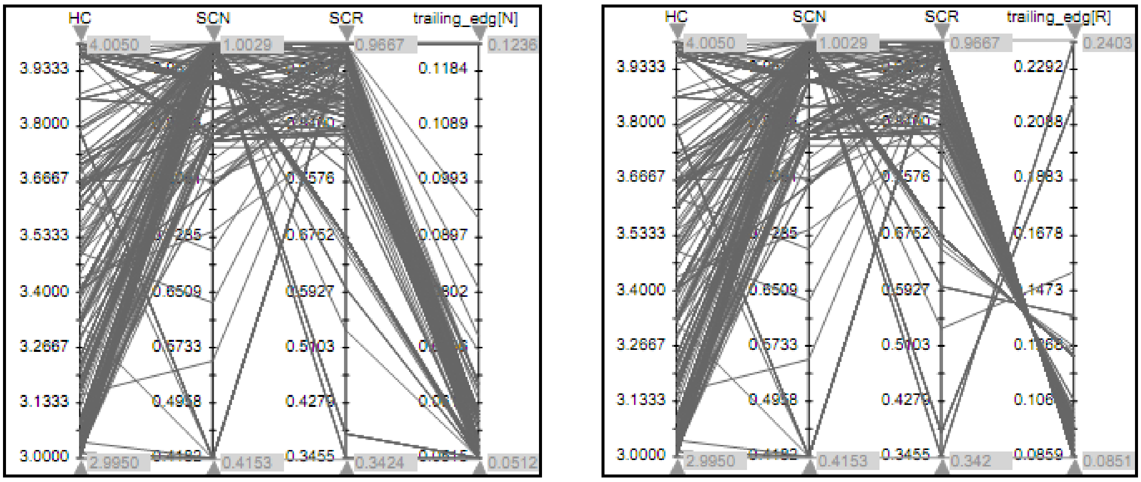


Figure 9: Influence of variation of input variables in efficiency.

On the other hand the Fig. 10 presents the influence of these parameters in the trailing edge loss in the nozzle and rotor. Was obtained a considerable reduction in losses with optimization compared to original design that has 0.0595 in nozzle and 0.1109 in rotor.



(a) (b)

Figure 10: Influence of variation of input variables in loss trailing edge (a) nozzle; (b) rotor.

The Figure 11 shows the set of designs possibilities with variations of the design variable a function of loss profile. The best results presented were for values about 1.0 for SCN and SCR. Only the variations of HC does not present significative changes in loss and efficiency, but should be considered a safety issue due to problems of stress and vibration, on the other hand the variations of all combinations parameters implies significative results.

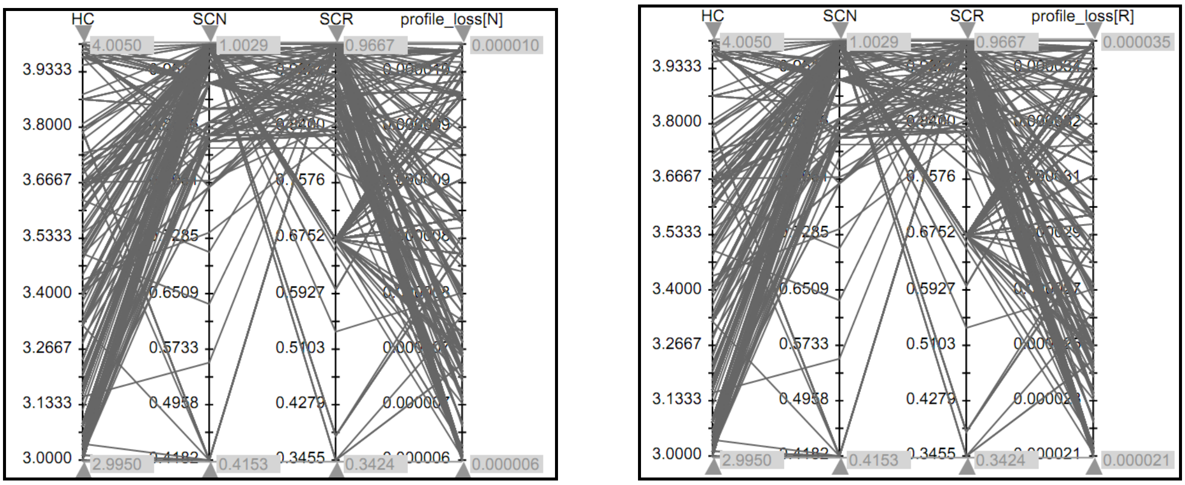


Figure 11: Influence of variation of input variables in profile loss (a) nozzle; (b) rotor.

The table 3 presents a comparison between the original design and optimized using MOGAII and NSGAII. According to (Saravanamuttooo, 2001) it is advisable to choose the even number of blades pairs in the rotor and add in the nozzle to avoid vibration problems and stress. The table 3 below show the best configuration presented by each algorithm.

Table 3: comparison between original and optimized designs

Parameters	Original	MOGA II	NSGA II
NGV blade number	91	79	77
Rotor blade number	70	60	62
s/c (nozzle)	0.86	0.96	0.98
s/c (rotor)	0.83	1	0.96
h/c	3.0	3.27	3.05
NGV profile loss	8.1×10^{-5}	1.024×10^{-5}	9.93×10^{-7}
Rotor profile loss	2.2×10^{-5}	3.53×10^{-7}	3.49×10^{-7}
NGV trainling edge loss	0.0595	0.053463	0.05152
Rotor trainling edge loss	0.1109	0.082732	0.08585
η	0.88	0.906	0.904

6. CONCLUSION.

Most real-world engineering problems involve simultaneously optimizing multi-objectives where considerations of trade-off between high efficiency, high performance, low levels of losses and lowers cost.

The loss model development by Denton is based in entropy variation and presented better results when compared to others methods (Hess, 2006).

For the axial turbine evaluated in this work, the most significant loss sources were from profile and trailing edge. Using MOGA II and NSGA II coupled with the preliminary design tool developed in this work, was possible to obtain better values of the geometrical parameters supplied by the designer to start the design procedure.

The results from optimization process achieved good values of blades numbers for each row (NGV and rotor) and improve the profile and trailing edge losses due to the variation in s/c (NGV and rotor) and h/c. For future purpose, it is possible to vary the blade angles and turbine length. In some applications as rocket turbine, the weight is an important parameter to optimize due to the high cost of the mission. Thus, the number of blades and turbine size strong influence.

Both algorithms have proven successful in differentiating itself Pareto-optimal set size and computational time, while MOGA II showed a slightly smaller computational time and bigger set of Pareto compared with NSGA II.

The use of multi-objective optimization has the advantage of searching a global optimum of the search space while in single-objective methods to optimize seek a local. Optimization is an indispensable tool in the design of turbomachineries.

7. ACKNOWLEDGEMENTS

The author would like to thank VSE- Vale Soluções em Energia for providing funding for this work allowing this publication. Discuss with Dr. Marcio Mendonça Teixeira that are very helpful.

8. REFERENCES

Abdullah Konak, David W. Coit, Alice E. Smith, 2006, “The Multi-objective optimization using genetic algorithms: A tutorial”, Reability Engineering and system Safety 91(2006) 992-1007. ELSEVIER.

Denton, J. D.,1999, “Loss mechanisms in turbomachines”, Part I:Entropy creation in fluid flows, von Karman Institute for Fluid Dynamics, Lecture Series 1999-02, “ Loss generation in turbomachines”, pp. 1- 26.

Denton, J. D. and Cumpsty, N. A. Loss mechanisms in turbomachines. I Mech E Paper C 260/1987.

Hess, Ricardo., “ Estudo comparativo de métodos de analise de desempenho de turbinas axiais”, Dissertação de mestrado, Instituto Tecnológico de Aeronáutica, 2006.

Keskin, Akim., Dutta, A. K. and Bestle Dieter., 2006, “Modernal compressor using aerodynamic blading process using multi-objective optimization” ASME, Turbo Expo 2006: Power for Land, Sea and Air GT2006-90206.

MODEFRONTIER™ 4 User Manual, 2000.

Öksüz Özhan and Akmandor S. I., 2010, “Multi-objective Aerodynamic Optimization of Axial Turbine blades Using a Novel Multilevel genetic algorithm”. ASME, Journal of turbomachinery, vol. 132, 041009.

Saravanamuttoo, H.I.H., Cohen, H., and Rogers, G.F.C., 2001, “Gas turbine Theory”, Prentice hall, 5 ed, England, pp., 305-356.

Schilichting, H., Gersten, K., “Bonudary layer Theory”, 2001, 8 eth revised and Enlarged Edition, Springer, pp.30-49.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.