# MINIMUM MASS IN THERMOELASTIC SOLID STRUCTURES

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***Abstract****. This work proposes a formulation for optimization of tridimensional (3D) structure layouts submitted to mechanic and thermal shipments. The main goal of the formulation is to minimize the structure mass submitted to an effective state of stress of von Mises, with stability and lateral constraint variants. A criterion of global measurement was used for intents a parametric condition of stress fields. To avoid singularity problems was considerate a release on the stress constraint. On the optimization was used a material approach where the homogenized constitutive equation was function of the material relative density. On the solution of the optimization problem, was applied the Augmented Lagrangian Method, that consists on minimum problem sequence solution with box-type constraints, resolved by a second order projection method which uses the method of the quasi-Newton without memory, during the problem process solution. The topology optimization problem when considerate some stress criterion, can generate feasible topology results like solutions of realistic problems of engineering, but this causes ill-conditioning of the optimization problem.*

***Keywords****: topology, termoelasticity, optimization, global criterion, relaxation.*

## 1. INTRODUCTION

The objective of this work is to develop a computational procedure for the determination of the optimum topology of structures and components subjected to mechanical and thermal loads. The reduction of the cost of manufacturing a given component or product may be obtained by applying some optimization tool. In the particular case of components or products obtained by an injection process (plastic or metal powder), the possibility to consider complex geometry allow us to explore the flexibility of the process by designing optimized molds with an optimum topology for the domain of the component.

One of the most difficult decisions in the designing phase is to specify the layout of the geometry of the component. Once the layout or topology of the component is defined we may concentrate in the definition of the optimum shape of the domain, sizing of some additional geometric parameters used to define the model and some material properties, (Suzuki and Kikuchi, 1990; Suzuki and Kikuchi, 1991; Bendsoe and Kikuchi, 1998; Bendsoe, 1995).

In general, the appropriate choice of the layout is strongly dependent of the designer, what implies in the necessity of a designer with a large practical experience. The decision process associated with the definition of the optimum layout of component may be done automatically by employing a topology optimization software, (Bendsoe and Sigmund, 2003; Bendsoe and Rodrigues, 1991; Bendsoe *et al*. 1993).

In this work, the layout optimization is done by considering a Solid Isotropic Microstructure with Penalization (SIMP). The material density function $\rho$ are the design parameters and varies continuously from 0 to 1, taking the value of 1.0 for a solid material and 0.0 for a void material, Costa Jr. and Alves (2003). To avoid numerical singularity, the lower bound of material, $\rho_{\min}$, is introduced as

$$0 < \rho_{\min} \le \rho \le 1. \tag{1}$$

## 2. FORMULATION OF THE PROBLEM

### 2.1. Determination of the Thermo Mechanical Problem

The thermal problem considered in this work is illustrated in Fig. 1.

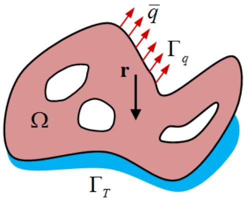


Figure 1: Definition of the Thermal Problem.

Here, the body domain is defined by $\Omega$ that is a tridimensional solid structure, with $\partial\Omega = \Gamma_T \cup \Gamma_q$; $\Gamma_T \cap \Gamma_q = \varnothing$. Denoting by $\Gamma_T$ and $\Gamma_q$ the part of the boundary where the temperature and the heat flux are prescribed respectively. At this point, we define the set of admissible temperatures, $W_T$, and the set of the temperature variations, $Var_T$, to be given as: $W_T = \left\{ T \middle| T \in H^1(\Omega) :\ T = \bar{T} \text{ at } \Gamma_T \right\}$ and $Var_T = \left\{ \hat{T} \middle| \hat{T} \in H^1(\Omega) :\ \hat{T} = 0 \text{ at } \Gamma_T \right\}$, where $\bar{T}$ is a prescribed temperature imposed over the temperature boundary $\Gamma_T$. We consider the source/sink to be given by a convection heat transfer from the body to a fluid, *i. e.*, $r = -h(T - T_\infty)$ where $T_\infty$ denotes the temperature of the fluid and $h$ the convection heat transfer coefficient. Notice that, if $T > T_\infty$ then heat is removed from the body and if $T < T_\infty$ heat is given to the body, (Alves and Alves, 1999; Silva, 2007).

The weak formulation of the thermal problem may be stated as: Let $T = T^* + T_p$, where $T_p \in W_T$ is a known field. The problem consists in the determination of $T^* \in Var_T$ such that,

$$a_T(T^*, \hat{T}) = l_T(\hat{T}), \quad \forall \hat{T} \in Var_T \tag{2}$$

where

$$a_T(T^*, \hat{T}) = \int_\Omega \mathbf{K}^H \nabla T^* \cdot \nabla \hat{T}\, d\Omega + \int_\Omega h T^* \hat{T}\, d\Omega \tag{3}$$

and

$$l_T(\hat{T}) = \int_\Omega h(T_\infty - T_p)\hat{T} d\Omega + \int_{\Gamma_q} \bar{q}\, \hat{T} d\Gamma - \int_\Omega \mathbf{K}^H \nabla T_p \cdot \nabla T d\Omega \tag{4}$$

Here, the conductivity matrix $\mathbf{K}^H = k^H \mathbf{I}$, where $k^H$ is the homogenized thermal conductivity of the material that is porous material dependent, such that

$$k^H = \rho^\eta k \tag{5}$$

where $k$ is the conductivity parameter for the fully density material, $\rho$ the relative density material and $\eta$ is the SIMP penalty parameter, (Cho and Choi, 2005; Rodrigues and Fernandes, 1993).

The mechanical problem considered in this work is illustrated in Figure 2.

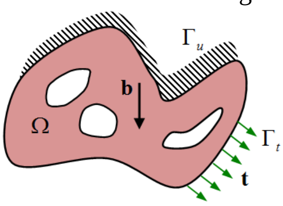


Figure 2: Definition of the Mechanical Problem.

Here $\partial\Omega = \Gamma_u \cup \Gamma_t$; $\Gamma_u \cap \Gamma_t = \varnothing$ . Here, $\Gamma_u$ and $\Gamma_t$ represent the part of the boundary where the displacement and the traction are prescribed respectively. At this point we define the set of admissible displacements, $W_u$, and the set of the displacement variations, $Var_u$, to be given as: $W_u = \left\{ \mathbf{u} \middle| \mathbf{u} \in [H^1(\Omega)]^3 :\ \mathbf{u} = \bar{\mathbf{u}} \text{ at } \Gamma_u \right\}$ and $Var_u = \left\{ \mathbf{v} \middle| \mathbf{v} \in [H^1(\Omega)]^3 :\ \mathbf{v} = 0 \text{ at } \Gamma_u \right\}$. Here, for simplicity we consider $\bar{\mathbf{u}} = 0$. As a result, $W_u = Var_u$.

The weak formulation of the mechanical problem may be stated as: Let $T \in W_T$ be the solution of the thermal problem. Then, the problem consists in the determination of $\mathbf{u} \in W_u$ such that

$$a_u(\mathbf{u},\mathbf{v}) = l_u(\mathbf{v}), \quad \forall \mathbf{v} \in Var_u . \tag{6}$$

Where

$$a_u(\mathbf{u},\mathbf{v}) = \int_\Omega \boldsymbol{\sigma}(\mathbf{u}) \cdot \boldsymbol{\varepsilon}(\mathbf{v}) d\Omega , \tag{7}$$

$$l_u(\mathbf{v}) = \int_\Omega \mathbf{b} \cdot \mathbf{v}\, d\Omega + \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{v}\, d\Gamma , \tag{8}$$

and been $T_0$ the reference temperature of the body

$$\boldsymbol{\sigma}(\mathbf{u}) = \mathbf{D}^H \boldsymbol{\varepsilon}(\mathbf{u}) - (T - T_0)\mathbf{B}^H \tag{9}$$

Now, since we consider the material (matrix) to be isotropic, we have:

$$B_{ij} = B_0 \delta_{ij}, \qquad B_0 = \frac{E\alpha}{1-2\nu}. \tag{10}$$

Here, α is the linear thermal expansion coefficient, $(T - T_0) B_{ij}$ is the thermal stress tensor and $\mathbf{D}$ is the generalized Hooke's law for a linear elastic body, (Brahim-Otsmane *et al.*, 1989; Francfort, 1983). Moreover,

$$D_{ijrs} = \lambda \delta_{ij} \delta_{rs} + \mu(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}) \tag{11}$$

with

$$\lambda = \frac{\nu E}{(1-\nu)(1-2\nu)}, \qquad \mu = G = \frac{E}{2(1-\nu)} . \tag{12}$$

Where $\lambda$ and $\mu$ are the Lame´s constants, $\nu$ is the Poisson´s ratio and $E$ is the Young modulus.

The constitutive matrix $\mathbf{D}^H$ adapted to intermediary density material, proposed by Cho and Choi (2005) is given

$$\mathbf{D}^H_{ijkl} = \rho^\eta \mathbf{D}_{ijkl} . \tag{13}$$

### 2.2. Formulation of the Problem

The objective of this work is to determine the optimum layout of the structure obtained as solution to an optimization problem. The optimization problem consists in the minimization of the mass of the structure subjected to an effective von Mises equivalent stress and side constraints. The design variable is the relative density of the material, represented by $\rho$, for dealing with the problem of stress criteria in mass minimization was used SIMP exponential penalty system to describe the constitutive relation of the material, which is used $\eta = 3$, with this choice, proposed by Sigmund and Petersson (1998), we get a description of feasible microstructure material.

The problem may than be formulated as:

$$\min_{\rho(x)} \int_\Omega \rho\, d\Omega , \tag{14}$$

such that

$$\sigma^*_{eq}\left(\rho(\mathbf{x}), \mathbf{u}\left(\rho(\mathbf{x}), \mathbf{x}, T\right), T\right) - \sigma_y \le 0 \tag{15}$$

$$\rho_{\inf} - \rho \le 0 \tag{16}$$

$$\rho - \rho_{\sup} \le 0 , \qquad \forall\, \mathbf{x} \in \Omega \tag{17}$$

where $\mathbf{u}$ and $T$ are obtained as a solution to the problem:

$$a_T(T^*, \hat{T}) = l_T(\hat{T}) , \qquad \forall\, \hat{T} \in Var_T \tag{18}$$

$$a_u(\mathbf{u},\mathbf{v}) = l_u(\mathbf{v}) , \qquad \forall\, \mathbf{v} \in Var_u \tag{19}$$

and $T = T^* + T_p$, for a given $T_p \in W_T$. The effective von Mises stress, for this microstructure is considered to be given as:

$$\sigma^*_{eq} = \frac{\sigma_{eq}}{\rho^\eta} . \tag{20}$$

The Karush-Kuhn-Tucker necessary optimality conditions associated with this problem is given by: Let $L$ denote the lagrangian functional associated with the problem, *i. e.*,

$$L(\rho, \mathbf{u}, T^*, \lambda_\sigma, \lambda_i, \lambda_s) = \int_\Omega \rho\, d\Omega + \int_\Omega \lambda_\sigma \left(\sigma^*_{eq}(\rho, \mathbf{u}(\rho, \mathbf{x}, T^*), T^*) - \sigma_y\right) d\Omega + \int_\Omega \lambda_i (\rho_{\inf} - \rho)\, d\Omega + \int_\Omega \lambda_s (\rho - \rho_{\sup})\, d\Omega , \tag{21}$$

where $\lambda_\sigma$, $\lambda_i$, and $\lambda_s$ are the Lagrange multipliers associated with the inequality constraints, Costa Jr. and Alves (2003). Then, the optimality conditions are given by:

$$\lambda_\sigma \geq 0, \qquad \lambda_i \geq 0, \qquad \lambda_s \geq 0, \tag{22}$$

$$\lambda_\sigma (\sigma_{eq} - \sigma_y) = 0, \quad \lambda_i (\rho_{\inf} - \rho) = 0, \quad \lambda_s (\rho - \rho_{\sup}) = 0, \tag{23}$$

$$\sigma_{eq} - \sigma_y \leq 0, \quad \rho_{\inf} - \rho \leq 0, \quad \rho - \rho_{\sup} \leq 0 \quad \text{and} \tag{24}$$

$$1 + \lambda_\sigma \frac{\partial \sigma^*_{eq}}{\partial \rho} - \lambda_i + \lambda_s = 0, \qquad \forall\, \mathbf{x} \in \Omega . \tag{25}$$

### 2.3. Stress Singularity Problem

In order to open the degenerated parts of the design space with the possibility of creating or removing holes without violating the effective stress constraint we apply the $\kappa$ - relaxation technique (Duysinx, 1998; Duysinx and Sigmund, 1998; Duysinx and Bendsoe, 1998). In this work, we implement an automatic and systematic strategy to reduce the initial perturbation parameter $\kappa$. The stress relaxation parameter is decremented as we get closer to the solution. Now, let $\rho_{\sup} = 1$ be the relative density associated with the full material condition. Then, knowing that $\mathbf{u} = \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T)$, the relaxed admensionalized effective stress constraint may be written as:

$$g(\rho(\mathbf{x}), \mathbf{u}, T) = \rho(\mathbf{x}) \left( \frac{\sigma^*_{eq}(\rho(\mathbf{x}), \mathbf{u}, T)}{\sigma_y} - 1 \right) + \kappa (\rho_{\sup} - \rho(\mathbf{x})) \leq 0 . \tag{26}$$

From this consideration, the relaxed minimization problem may be formatted as:

$$\min_\rho \int_\Omega \rho \, d\Omega \tag{27}$$

such that

$$\rho \left( \frac{\sigma^*_{eq}}{\sigma_y} - 1 \right) + \kappa (\rho_{\sup} - \rho) \leq 0 \tag{28}$$

$$\rho_{\inf} - \rho \leq 0 \tag{29}$$

$$\rho - \rho_{\sup} \leq 0, \qquad \forall\, \mathbf{x} \in \Omega \tag{30}$$

## 3. DISCRETIZATION OF THE PROBLEM

In order to solve the thermo-mechanical problem we apply the Galerkin Finite Element Method. Moreover, we consider the material density associated with each finite element linearly distributed. Consequently, the material properties related to given element are characterized by a single microstructure. Thus, for each element we have a design variable "a" which represents the size of the void of the microstructure that fully represents the given finite element material properties. From this consideration, the number of design variables is given by the number of finite elements in the mesh.

Furthermore, we make use of the slope-constrained conditions proposed by Petersson and Sigmund (1998). These conditions are employed in order to ensure the existence of a solution to the layout optimization problem and to eliminate the well-known checkerboard instability problem (Bendsoe, 1995; Sigmund and Petersson, 1998), that occur in the Galerkin finite element discretization, when using a low order interpolation base function, in the approximation space. Thus,

$$\left( \frac{\partial \rho}{\partial x} \right)^2 \leq C_x^2 , \quad \left( \frac{\partial \rho}{\partial y} \right)^2 \leq C_y^2 \quad \text{and} \quad \left( \frac{\partial \rho}{\partial z} \right)^2 \leq C_z^2 \tag{31}$$

Here, the constants $C_x$, $C_y$ and $C_z$ define the bounds for the components of the gradient of the relative density. These bounds are imposed component wise with the objective of properly imposing a symmetry condition, which may be used in some particular cases.

The discretized problem may be formulated as:

$$\min_\rho \int_\Omega \rho \, d\Omega \tag{32}$$

such that

$$\frac{\sigma^*_{eq}(\rho(\mathbf{x}), \mathbf{u}(\rho(\mathbf{x}), \mathbf{x}, T), T)}{\sigma_y} + \kappa \left( \frac{1}{\rho_0} - \frac{1}{\rho} \right) - 1 \leq 0, \quad \forall\, \mathbf{x} \in \Omega \tag{33}$$

and

$$\rho_{\inf} - \rho_i \leq 0 , \tag{34}$$

$\rho_i - \rho_{\sup} \leq 0$,  $i$=1,...,$n$ ($n$ is the number of nodes in the mesh) (35)

### 3.1 Global Stress Condition

Notice that, the effective stress constraint is a parametric constraint that must be satisfied for $\forall \mathbf{x} \in \Omega$. In order to handle this parametric constraint we relax the pointwise criteria and consider a global integrated constraint. This can be done by replacing a parametric constraint of the type

$$g\left(\rho(\mathbf{x}),\mathbf{u}\left(\rho(\mathbf{x}),\mathbf{x},T\right),T^*(\rho)\right) \leq 0, \quad \forall \mathbf{x} \in \Omega \tag{36}$$

by the following associated global constraint:

$$\bar{g}\left(\rho(\mathbf{x}),\mathbf{u}\left(\rho(\mathbf{x}),\mathbf{x},T\right),T^*(\rho)\right) = \left\{\frac{1}{\Omega}\int_\Omega \left\langle \rho(\mathbf{x}),\mathbf{u}\left(\rho(\mathbf{x}),\mathbf{x},T\right),T^*(\rho)\right\rangle^p d\Omega\right\}^{1/p} = 0 . \tag{37}$$

Here, in order to enforce the point wise constraint we must consider $p \to +\infty$. However, for practical purposes we consider $p = 2$ where $\langle g(\bullet)\rangle$ denotes the positive part of the function g(•), *i. e.*, $\langle g(\bullet)\rangle = \max\{0, g(\bullet)\}$, Silva (2007).

### 3.2 Formulation of the Discretized Problem

In order to discretize the problem is applied the Galerkin Finite Element Method. It is employed a four nodes tetrahedron finite element that interpolates not only the displacement components but also the relative density $\rho$. Consequently, the discretization formulation of the problem way be stated as:

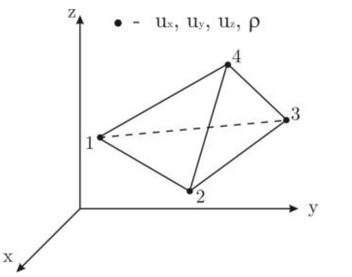


Figure 3: Displacement and density fields.

$$\min \int_\Omega \rho d\Omega \tag{38}$$

Subject to:

(i) Stress Constraint:

$$\rho(\mathbf{x})\left(\frac{\sigma^*_{eq}\left(\rho(\mathbf{x}),\mathbf{u}\left(\rho(\mathbf{x}),\mathbf{x}\right)\right)}{\sigma_y} - 1\right) + \rho(\mathbf{x})\kappa\left(\rho_{\sup} - \rho(\mathbf{x})\right) \leq 0 \tag{39}$$

at this work was proposed the follow global criteria:

$$\bar{g}(\rho,\mathbf{u}(\rho)) = \left\{\frac{1}{\Omega}\int_\Omega \left\langle \rho\left(\frac{\sigma^*_{eq}}{\sigma_y} - 1\right) + \rho\kappa\left(\rho_{\sup} - \rho\right)\right\rangle^p d\Omega\right\}^{1/p} \leq 0 \tag{40}$$

where, $\langle f(\mathbf{x})\rangle = \max\{0, f(\mathbf{x})\}$, for all positive part of $f(\mathbf{x})$.

$$\max_{\mathbf{x}\in\Omega}\left|\left\langle \rho\left(\frac{\sigma^*_{eq}(\rho)}{\sigma_y} - 1\right) - \kappa(1-\rho)\right\rangle\right| \leq 0. \tag{41}$$

(ii) Side Constraint:

$$\rho_{\inf} - \rho_i \leq 0 \quad \text{and} \quad \rho_i - \rho_{\sup} \leq 0; \quad i=1,...,n \ (n \text{ is the number of nodes in the mesh}) \tag{42}$$

(iii) Stability Constraint

$$\bar{h}_j(\rho) = \left\{\frac{1}{\Omega}\int_\Omega \left\langle h_{ej}(x_e)\right\rangle^p d\Omega\right\}^{1/p}; \quad j = x, y \text{ and } z \tag{43}$$

with

$$h_{ej}(\rho) = \frac{\sqrt{\left(\frac{\partial \rho}{\partial j}\right)^2}}{c_j} - 1 \;\; ; \;\; j = x, y, z \quad \text{and} \quad e = 1, \ldots, n_e \tag{44}$$

The constants $c_x$, $c_y$ and $c_z$ impose a superior limit to the components of the relative density gradient.

Consider a generic element, according to Fig. 3, with $\mathbf{x}_n = (x_i, y_i, z_i)$; $i = 1,...,4$ vertices coordinates and $\mathbf{x}_m = (x_m, y_m, z_m)$ as barycenter coordinates of tetrahedral element.

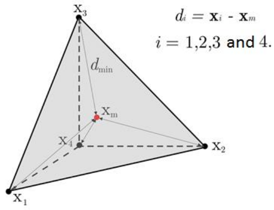


Figure 4: Four nodes tetrahedral coordinates.

$$d_{\min} = \min_j \left\| \mathbf{d}_j \right\| \;\; \text{p/} \;\; i = 1,...,4 \tag{45}$$

$$c_x^e = c_y^e = c_z^e = \frac{1}{d_{\min}}. \tag{46}$$

### 3.3 Augmented Lagrangian Method

Being $\boldsymbol{\rho} \in \mathbf{X}$ with $\mathbf{X} = \left\{ \boldsymbol{\rho} \in \mathbb{R}^n \,\middle|\, \rho_{\inf} \le \rho_i \le \rho_{\sup}, i = 1, \ldots, n \right\}$ and $n$ is the number of nodes in the mesh.

Step – 1. Initial Conditions: $k = 0$, $\lambda^k = 0$, $\mu^k = \mathbf{0}$, $erro = 1,0$, $\zeta$, $\omega^k$ and $tol$.

Step – 2. While $erro > tol$, to do:

(i) Solution of the minimization problem with side constraint:

$$\min \Pi(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{\mu}; \zeta, \boldsymbol{\omega}), \quad \forall\, \boldsymbol{\rho} \in \mathbf{X} \tag{47}$$

where,

$$\Pi(\rho, \lambda, \mu; \zeta, \omega) = f(\rho) + \frac{1}{\zeta} \sum_{e=1}^{n_e} \Lambda_e(g_e, \zeta\lambda_e) + \sum_{j=1}^{3} \left[ \frac{1}{\omega_j} \sum_{e=1}^{n_e} \Psi_e^j\left(h_e^j, \omega_j \mu_{ej}\right) \right] \tag{48}$$

with

$$\Lambda_e(g_e, \zeta\lambda_e) = \begin{cases} g_e(g_e + \zeta\lambda_e) & , \text{ if } g_e \ge -\dfrac{\zeta\lambda_e}{2} \\ -\left(\dfrac{\zeta\lambda_e}{2}\right)^2 & , \text{ if } g_e < -\dfrac{\zeta\lambda_e}{2} \end{cases}, \tag{49}$$

and

$$\Psi_e^j\left(h_e^j, \omega_j \mu_{ej}\right) = \begin{cases} h_e^j\left(h_e^j + \omega_j \mu_{ej}\right) & , \text{ if } h_e^j \ge -\dfrac{\omega_j \mu_{ej}}{2} \\ -\left(\dfrac{\omega_j \mu_{ej}}{2}\right)^2 & , \text{ if } h_e^j < -\dfrac{\omega_j \mu_{ej}}{2} \end{cases} \;\; ; \;\; j = 1,...,3. \tag{50}$$

(ii) Update of Lagrange multipliers

$$\lambda_e^{k+1} = \max\left\{0, \lambda_e^k + \frac{2}{\zeta} g_e\left(\mathbf{x}^k\right)\right\} \tag{51}$$

and

$$\mu_{ej}^{k+1} = \max\left\{0, \mu_{ej}^k + \frac{2}{\omega_j} h_e\left(\mathbf{x}^k\right)\right\} \;\; ; \;\; j = 1,...,3. \tag{52}$$

(iii) Update of penalty parameters

$$\zeta^{k+1} = \begin{cases} \gamma_1 \zeta^k & \text{with } \gamma_1 \in (0,1), \text{ if } \gamma_1 \zeta^k > \zeta^{crit} \\ \zeta^{crit} \end{cases} \tag{53}$$

and

$$\omega_j^{k+1} = \begin{cases} \beta_j \omega_j^k & \text{with } \beta_j \in (0,1), \text{ if } \beta_j \omega_j^k > \omega_j^{crit} \\ \omega_j^{crit} \end{cases} \quad ; \quad j = 1,\ldots,3. \tag{54}$$

(iv) Error

$$a = \max_e \left|\lambda_e^{k+1} - \lambda_e\right|, \; b = \max_e \left|\mu_{e1}^{k+1} - \mu_{e1}\right|, \; c = \max_e \left|\mu_{e2}^{k+1} - \mu_{e2}\right| \text{ and } d = \max_e \left|\mu_{e3}^{k+1} - \mu_{e3}\right| \tag{55}$$

so, $erro = \max\{a,b,c,d\}$.

Step – 3. End

The problem can be formulated as: $\lambda, \mu_1, \mu_2, \mu_3 \in \mathbb{R}^{n_e}$, $n_e$ is the number of elements in the mesh and $\zeta, \omega_1, \omega_2, \omega_3 \in \mathbb{R}$, determinate $\boldsymbol{\rho}^* \in \mathbb{R}^n$, such that:

$$\boldsymbol{\rho}^* = \arg\min \Pi(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{\mu}; \zeta, \boldsymbol{\omega}) \;, \quad \forall\, \boldsymbol{\rho} \in \mathbf{X}.$$

## 4. ALGORITHM

Here was used a solver of bound constrained Truncated-Newton method. The Truncated-Newton method is preconditioned by a limited-memory Quasi-Newton method with a further diagonal scaling. Similar results were obtained with the TANGO algorithm of Andreani *et al.* (2004), Andreani *et al.* (2005) and Birgin and Martinez (2002).

## 5. NUMERICAL APPLICATIONS

### 5.1 Problem Case (1)

Here we consider a cube with 0.05m of edge according to illustrated in Figure 5, which the material properties (stainless steel AISI 304) to be given as: Young Modulus $E = 193 GPa$, Poisson's ratio $\nu = 0.29$. The distributed load (on the top), $\mathrm{t} = 207{,}000\, kN/m$. The reference (initial) temperature of the body is $T_R = 20°C$, the temperature of the fluid is $T_f = 25°C$ and the prescribed temperature at the clamped edge (on the bottom) is $T_p = 100°C$. The yield stress, $S_y = 207\, MPa$. The heat conductivity of the material, $k = 16.6\, W/m°C$. The coefficient $\beta = 17 \times 10^{-6}\; m/m°C$. The convection heat transfer coefficient, $h = 5\, W/m^2\, °C$.

Here, it was analyzed the ¼ of block structure with 2504 elements and 594 nodes.

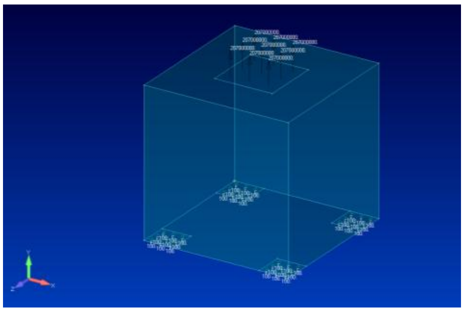

Figure 5: Definition of the Problem Case (1).

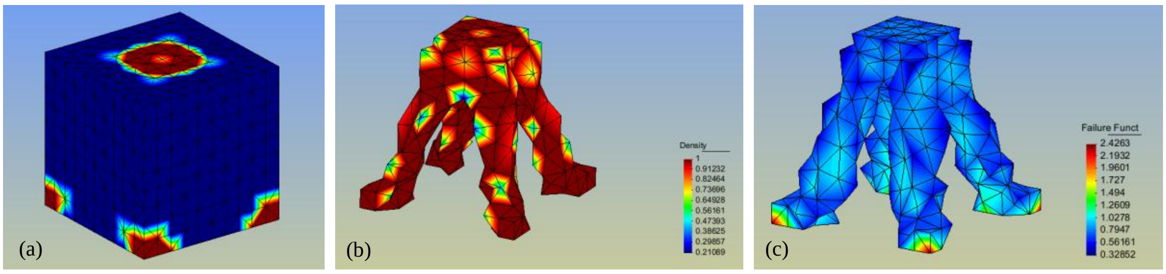


Figure 6: (a) Full Problem with optimum mass distribution; (b) Mass distribution topology of the problem and (c) failure function distribution.

The same problem analyzing ¼ of block structure with 3629 elements and 850 nodes.

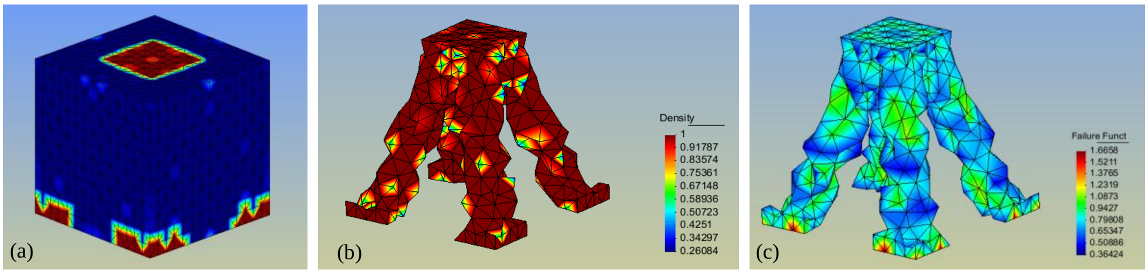

Figure 7: (a) Full Problem with optimum mass distribution; (b) Mass distribution topology of the problem and (c) failure function distribution.

Now, a mesh with 1034 nodes and 4557 elements.

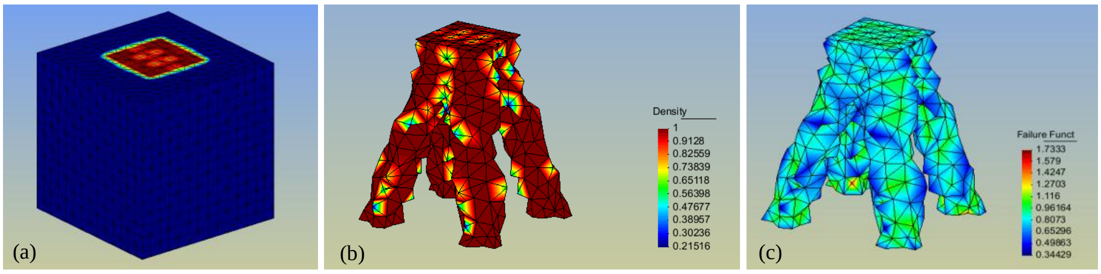

Figure 8: (a) Full Problem with optimum mass distribution; (b) Mass distribution topology of the problem and (c) failure function distribution.

## 5.2 Problem Case (2)

Consider a problem according to Figure 9. The case consists in one block of dimensions: $a = 0.1m$, $b = 0.04m$ and $c = 0.01m$. The properties material are: Young's modulus, $E = 193GPa$, Poisson's ratio $\nu = 0.29$. The distributed load, $t = 207\,kN/m$. The reference (initial) temperature of the body is $T_R = 20^o C$, the temperature of the fluid is $T_f = 25^o C$ and the prescribed temperature at the clamped edge (on the bottom) is $T_P = 100^o C$. The yield stress, $S_y = 207\,MPa$. The heat conductivity of the material, $k = 16.6W/m^o C$. The coefficient $\beta = 17 \times 10^{-6} m/m^o C$. The convection heat transfer coefficient, $h = 5W/m^{2o} C$.

Here, it was analyzed ½ of block structure with 1302 elements and 354 nodes.

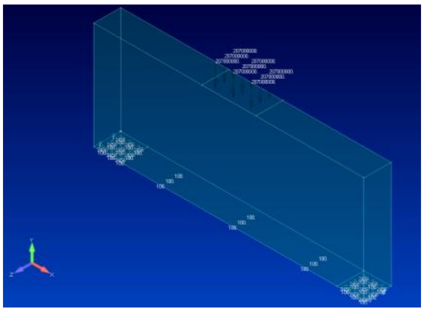
Figure 9: Definition of the Problem Case (2).

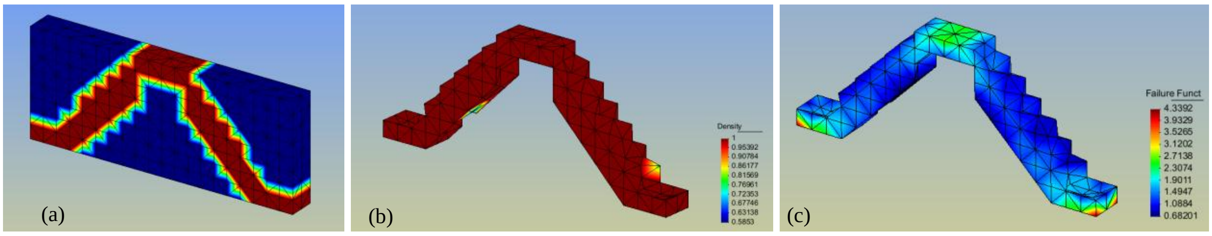

Figure 10: (a) Full Problem with optimum mass distribution; (b) Mass distribution topology of the problem and (c) failure function distribution

The same problem analyzing ½ block structure with 5976 elements and 1358 nodes.

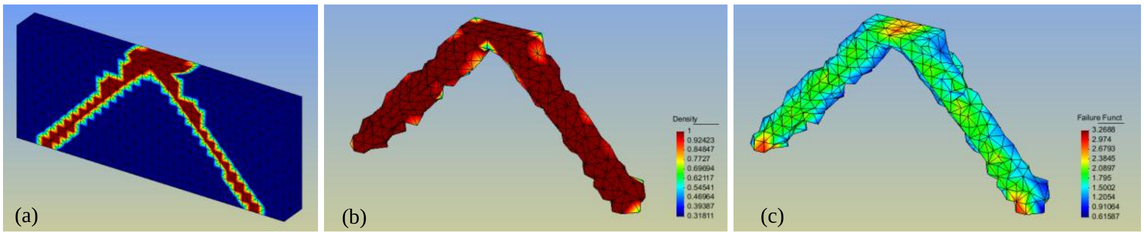


Figure 11: (a) Full Problem with optimum mass distribution; (b) Mass distribution topology of the problem and (c) failure function distribution.

## 6. CONCLUSION

The usage of a multigrid approach or remeshing procedure is important to increase the rate of convergence to the optimum layout of the problem. With this approach, we are able to handle problem with a large number of design variables.

The results were promising, given the stress constraint and tested in meshes with few elements, but for a sharp optimum layout require a very refined mesh, suggesting high computational cost. This formulation shows be promising for the implementation of h-adaptive process, that is, the implementation of an intelligent process of refinement of the mesh with information of the topology gotten in the original mesh.

One of the disadvantages of the adopted approach is that we need to determine the element matrices and vectors what increase the computational cost when compared with the pixel type of strategy employed by many Authors. However, the pixel approach requires a refined mesh in order to describe the material boundary with some precision.

## 7. REFERENCES

Alves, M.K. and Alves, B.K., 1999, “Topology Optimization of Elastic Structures”, IV Congresso Iberoamericano de Engenharia Mecânica, CIDIM 99, Vol.2 - Mecânica dos Sólidos, Santiago, Chile.

Andreani, R., Birgin, E.G., Martínez, J.M. and Schuverdt, M.L., 2004 "Augmented Lagrangian methods under the Constant Positive Linear Dependence constraint qualification", Mathematical Programming.

Andreani, R., Birgin, E.G., Martínez, J.M. and Schuverdt, M.L., 2005 "On Augmented Lagrangian methods with general lower-level constraints", Technical Report MCDO-050304, Department of Applied Mathematics, UNICAMP, Brazil.

Bendsoe M.P. and Kikuchi N., 1998, “Generating Optimal Topologies in Structural Design Using a Homogenization Method”. Comp. Meth. Appl. Mechs. Engrg., 71, 197-224.

Bendsoe, M.P. and Sigmund, O., 2003, “Topology Optimization: Theory, Methods and Applications”, Springer, New York, 2nd ed.

Bendsoe, M.P. and Rodrigues, H.C., 1991, “Integrated Topology and Boundary Shape Optimization of 2-D solids”, Comp. Meth. Appl. Mech. Engng., 87, 15-34.

Bendsoe, M.P.; Díaz, A.R. and Kikuchi, N., 1993, “Topology and Generalized Layout Optimization of Elastic Structures”, Loc. Cit. Bendsoe and Mota Soares, 159-206.

Bendsoe, M.P., 1995, “Optimization of Structural Topology, Shape, and Material”, Springer-Verlag.

Birgin, E.G. and Martínez, J.M., 2002 "Large-scale active-set box-constrained optimization method with spectral projected gradients", Computational Optimization and Applications 23, pp. 101-125.

Brahim-Otsmane, S., Francfort, G.A. and Murat F., 1989, “Homogenization in Thermoelasticity”, Publications du Laboratoire d´Ánalyse Numérique, Universite Pierre et Marie Curie, Centre National de la Recherche Scientifique, R89011.

Cho, S. and Choi J.Y., 2005, “Efficient Topology Optimization of Thermo-Elasticity Problems Using Coupled Field Adjoint Sensitivity Analysis Method”, Finite Elements in Analysis and Design, 41, 1481–1495.

Costa Jr., J.C.A. and Alves, M.K., 2003, “Layout Optimization with *h*-adaptivity of Structures”, Int. J. Numer. Meth. Engng; John Wiley & Sons, 58: 83–102.

Duysinx, P., 1998, “Topology Optimization with Different Stress Limits in Tension and Compression”, International report: Robotics and Automation, Institute of Mechanics, University of Liège, Liège, Belgium.

Duysinx, P. and Sigmund, O., 1998, “New Developmets in Handling Stress Constraints in Optimal Material Distribution”, 7th AIAA/USAF/NASA/ISSMO symposium on Multidisciplinary Design Optimization, Saint Louis, Missouri, USA.

Duysinx, P. and Bendsoe, M.P., 1998, “Topology Optimization of Continuum Structures with Local Stress Constraints”, Int. Jnl of Num. Meth. In Eng., vol 43, pp.1453-1478.

Francfort, G.A., 1983, “Homogenization and Linear Thermoelasticity”, SIAM J. Math. Anal., 14, 696-708.

Petersson, J. and Sigmund, O., 1998, "Slope constrained topology optimization", Int. J. Numer. Meth. Engng., 41, 1417-1434.

Rodrigues, H.C. and Fernandes, P., 1993, "Topology Optimization of Linear Elastic Structures Subjected to Thermal Loads", Bc. Cit. Bendsoe and Mota Soares, 437-450.

Sigmund, O. and Petersson, J., 1998, "Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh dependencies and local minima". Structural Optimization, vol. 16. Springer-Verlag: Berlin, 68–75.

Silva, P.S.R., 2007, "Estruturas Termoelásticas sob Otimização Topológica e *H*-adaptatividade", MSc. Dissertation, Post-graduate Program of Mechanical Engineering, UFRN, Natal-RN, Brazil.

Suzuki, K. and Kikuchi, N., 1990, "Generalized Layout Optimization of Shape and Topology in Three Dimensional Shell Structures", kept. No.90-05, Dept. Mech. Engrg. and Appl. Mech., Comp. Mech. Lab., University of Michigan, USA.

Suzuki, K. and Kikuchi, N., 1991, "Shape and Topology Optimization for Generalized Layout Problems Using the Homogenization Method". Comp. Meth. Appl. Mechs, Engng., 93, 291-318.

## 8. RESPONSIBILITY NOTICE

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