A NEW NUMERIC FORMULATION TO OBTAIN THE EQUIVALENT TO THE SUPERSONIC INTENSITY IN ACOUSTIC SOURCES WITH ARBITRARY GEOMETRIES.

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Abstract. *This work presents a new numeric technique for the computation of the numerical equivalent to the supersonic acoustic intensity, which provides the identification of the regions of a noise source with arbitrary geometry that effectively contribute to the sound power radiated to the far field by filtering recirculating and evanescent sound waves. The proposed technique is entirely formulated on the vibrating surface. The acoustic power radiated is obtained through a numerical operator (a matrix) that relates it with the distribution of superficial normal velocity, which is obtained through the boundary element method. Such matrix, called power operator, possesses the property of being Hermitian. The advantage of this characteristic is that their eigenvectors form an orthonormal set for the velocity field. It is applied to the power operator the decomposition in eigenvalues and eigenvectors, becoming possible, due the orthonormality of the eigenvectors, to get the numerical equivalent to the supersonic intensity. An example of a noise source with geometry in the shape of a finite cylinder was implemented and the results are discussed in this article.*

Keywords: *Supersonic intensity, Boundary element method, Acoustic power radiated.*

1. INTRODUCTION

The study of the phenomenon of acoustic radiation of vibrating surfaces have, as it is well known, many applications. Among the many aspects of this phenomenon, one that received special attention from researchers in recent years is the reconstruction of sound sources.

With the competition increasing, result of globalization, industries have sought tirelessly to improve its products in various aspects such as safety, comfort and durability. This way, industries that work with products whose noise is a factor of considerable importance for their quality, have invested great resources in finding solutions for problems generated within that field of engineering. In many industries, including automotive, aerospace, and appliances, where the noise is, except in some rare cases, such as speakers, an undesired effect, there is an investment in technologies that improve the acoustic quality of their products, since this is an important aspect for consumers at the time of purchase.

Williams (1995) proposed the concept of supersonic acoustic intensity, expressing the pressure and normal velocity to the source as an inverse spatial Fourier transform. However, instead of performing the integration across the whole field of wave number, this is cut at the extremes of the region that corresponds to propagating waves, not evanescent. The sound intensity calculated, received then the name of *supersonic intensity* (SI). Its main applicability is to distinguish, on the source surface, the regions that effectively contribute to the sound power radiated into the far field. Williams also concerns about the difference between his new approach and the simple use of the acoustic intensity, showing that in the near field, there is a recirculation of energy, which puts it at a disadvantage in relation to the supersonic intensity as an identifier parameter of areas of high contribution to the far field. Williams's work, however, is restricted to separable geometries, where the spatial Fourier transform can be applied directly.

Magalhães (2002) presents a methodology for obtaining a numerical equivalent to SI to arbitrary geometries. Strongly based on the singular values decomposition (SVD), the author presents simulated tests where the supersonic intensity was able to clearly identify the edge modes at a cylindrical source with flat caps (non-separable geometry), which eliminated regions where the acoustic intensity indicates, erroneously, that there is injection of energy to the far field. It was tested numerically, too, a source with geometry close to the envelopment of an inner combustion engine, where the supersonic intensity also proved capable to revealing the regions that have significant contribution to the sound power, as opposed to the acoustic intensity, which indicates high values where there is only recirculation of energy. Although efficient, the method presented by Magalhães requires the creation of a grid of points in the far field (a hologram) so that one can obtain a numerical equivalent to the supersonic intensity, generating a computational cost higher than the methodology that will be outlined here.

In this paper, we present a new numerical methodology to calculate the equivalent to the SI. Unlike the existing numerical technique, this one is formulated entirely on the vibrating surface. An example for a cylinder with caps was implemented and the results are discussed. This article represents new step in the research work in acoustic radiation. This paper therefore seeks to validate a numerical technique to solve the problem of acoustic radiation and the corresponding calculation of the supersonic intensity for geometries that do not have analytical solutions.

2. MATHEMATICAL DEVELOPMENT

2.1 Supersonic Intensity

The SI is a tool coming from the Fourier approach with huge value for the localization of sound sources. The basic idea behind SI is to extract an intensity value which excludes the contribution of the evanescent components (subsonic), remaining the portion corresponding to the propagating components (supersonic), so that it can highlight regions that effectively transfer energy to the far field.

Consider a vibrating surface on the xy plane ($z = 0$). The distribution of pressure and normal velocity in the plane can be given, respectively, by the two-dimensional spatial inverse Fourier transform as:

$$\hat{p}(x, y, 0, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{p}(k_x, k_y, 0, \omega) e^{i(k_x x + k_y y)} \, dk_x dk_y, \tag{1}$$

$$\hat{v}_n(x, y, 0, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{v}_n(k_x, k_y, 0, \omega) e^{i(k_x x + k_y y)} \, dk_x dk_y., \tag{2}$$

where k_x and k_y are the wavenumbers in the plane directions, $\tilde{p}$ and $\tilde{v}_n$ are, respectively, the angular spectrum of the pressure and normal velocity, ω is the angular frequency and i is the imaginary unit, as usual.

The supersonic pressure, $\hat{p}^{(\mathbf{s})}$, and the normal supersonic velocity, $\hat{v}_n^{(\mathbf{s})}$, are written as

$$\hat{p}^{(\mathbf{s})}(x, y, 0, \omega) = \frac{1}{4\pi^2} \int \int_{Cr} \tilde{p}(k_x, k_y, 0, \omega) e^{i(k_x x + k_y y)} \, dk_x dk_y, \tag{3}$$

$$\hat{v}_n^{(\mathbf{s})}(x, y, 0, \omega) = \frac{1}{4\pi^2} \int \int_{Cr} \tilde{v}_n(k_x, k_y, 0, \omega) e^{i(k_x x + k_y y)} \, dk_x dk_y, \tag{4}$$

where Cr is the region inside the radiation circle (see Williams (1995)) and Fig.1.

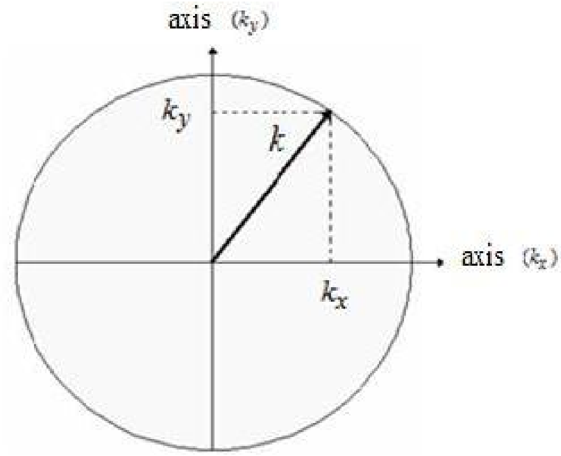

Figure 1. Radiation circle.

This way, $\hat{p}^{(\mathbf{s})}$ and $\hat{v}_n^{(\mathbf{s})}$ are reconstructed by filtering the evanescent waves (subsonic) from the components of the plane wave, keeping only the supersonic part. Then, the SI is defined as

$$\hat{I}^{(\mathbf{s})}(x, y, 0, \omega) = \frac{1}{2} \Re \big[\hat{p}^{(\mathbf{s})}(x, y, 0, \omega) \hat{v}_n^{(\mathbf{s})}(x, y, 0, \omega)^* \big], \tag{5}$$

where "$*$" means conjugate and $\Re$ stands for real part.

An important result shown by Williams (1999) is that the sound power, Π, calculated with the use of acoustic intensity (AI) is the same as that calculated with the SI. In other words,

$$\Pi = \int_S \hat{I}(x, y, 0, \omega) \, dS = \int_S \hat{I}^{(\mathbf{s})}(x, y, 0, \omega) \, dS. \tag{6}$$

2.2 Numerical Approach

Aiming to create a computational tool capable of recognizing the regions of a vibrating surface that are actually contributing to the radiated sound power, we will introduce a numerical modeling of the radiation problem obtaining an approximation for the calculation of SI. The normal velocity field of the vibrating surface is obtained through the finite element method (FEM).

To obtain the pressure field, we use the boundary element method (BEM). The method is based on the use of Green's theorem to calculate the fundamental solution of the Helmholtz equation to obtain an integral contour of the domain, called Kirchoff-Helmholtz integral theorem, given by

$$c\hat{p}(X,\omega) = \int_\Gamma \left(i\omega\rho_0 \hat{v}_n(X_s,\omega) G(X_s|X) - \hat{p}(X_s,\omega)\frac{\partial G(X_s|X)}{\partial n_s} \right) d\Gamma, \tag{7}$$

with

$$c = \begin{cases} 0, X \in \Omega_i \\ 1, X \in \Omega \\ \frac{\alpha}{4\pi}, X \in \Gamma \end{cases}$$

where $X = (x, y, z)$, $X_s = (x, y, 0)$, Γ is the surface contour, Ω is the area in question, n_s is the normal to the surface (see Fig. 2), ρ is the density of the fluid, β is the solid angle formed by the existence of some irregular part on the contour Γ, and $G(X_s|X)$ is the free field Green function of that is given by (see Stakgold (1979)),

$$G(X_s|X) = \frac{e^{ik|X_s - X|}}{4\pi|X_s - X|}. \tag{8}$$

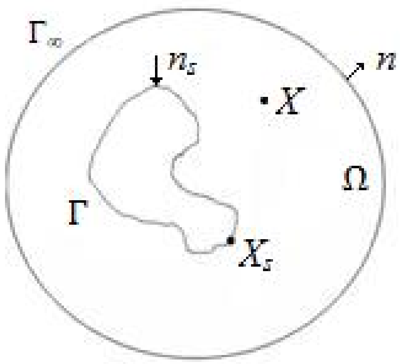

Figure 2. Geometry for the BEM.

The relation between pressure and velocity at the surface is given by

$$H\hat{p} = G\hat{v}_n, \tag{9}$$

with H and G obtained from the BEM. For details about H and G, see Holmstrom (2001).

The Eq. (9) can also be written as

$$\hat{p} = \mathbf{R}\hat{v}_n, \tag{10}$$

where $\mathbf{R} = H^{-1}G$ is called *surface operator*.

Knowing $\hat{v}_n$ and $\hat{p}$, then it can be obtained the normal component of acoustic intensity (AI), as

$$\hat{I} = \frac{1}{2}\Re[\hat{p}\,\hat{v}_n^*]. \tag{11}$$

Xu and Hang (2010) demonstrated that the sound power Π can be obtained through an operator that relates the normal velocity distribution on the vibrating surface. To obtain such operator, the authors used BEM, finding the Eq. (12)

$$\Pi = \hat{v}_n^H \bar{Q}\hat{v}_n, \tag{12}$$

where the superscript H indicates conjugate transpose. $\bar{Q}$ is, in this work, is called the *power operator*. This operator will be used to obtain the numeric equivalent to the SI, and it is Hermitian. In other words, $\bar{Q} = \bar{Q}^H$, which guarantees that all eigenvalues are real and its eigenvectors form an orthonormal basis for the velocity surface (Anton and Rorres (2001)). This is a crucial fact to the attainment of supersonic velocity, and subsequent computating of the supersonic intensity.

To proceed with the model formulation to calculate the supersonic intensity in sound sources with arbitrary geometries, it is necessary to write the *power operator* in terms of its eigenvalues and eigenvectors; such decomposition is called ED (Eigen-Decomposition).

$$\bar{Q} = VDV^{-1}, \tag{13}$$

where V is a matrix where its columns are the eigenvectors of $\bar{Q}$, V^{-1} is the inverse of V and D is a diagonal matrix containing the eigenvalues of $\bar{Q}$.

Using the fact that the matrix V is unitary, as $V^H V = VV^H = I_d$, where I_d is the identity matrix, we can write the Eq. (13) as

$$\bar{Q} = VDV^H, \tag{14}$$

and then, the sound power can be written as

$$\Pi = \hat{v}_n^H VDV^H \hat{v}_n, \tag{15}$$

or, alternatively, as

$$\Pi = \sum_{i=1}^{r} \lambda_i \langle \hat{v}_n^H, V_i \rangle \langle V_i^H, \hat{v}_n \rangle, \tag{16}$$

where r is the total number of eigenvalues, λ_i are the eigenvalues of $\bar{Q}$, called *eigenvalues of velocity*, V_i^H are the lines of V^H e V_i are the columns of V and $\langle\,\rangle$ stands for internal product, as usual. In other words, the eigenvectors of $\bar{Q}$, are called *own standard of velocity*, because they form a kind of set of modes for the velocity distribution. The λ_i are arranged in descending order, $|\lambda_1| > |\lambda_2| > \cdots > |\lambda_r|$ to be possible to apply the truncation criterion given in Eq. (21).

Watching the series (16), it is worth noting that discarding eigenvalues of negligible magnitude, there is still an excellent approximation to the sound power, showing that the subsequent eigenvalues constitute an insignificant contribution to the radiated sound power. As it is the case for sources with separable geometry, the sound pressure calculated from only supersonic modes is practically the same as calculated using all modes, including subsonic, that means,

$$\sum_{i=1}^{r} \lambda_i \langle \hat{v}_n^H, V_i \rangle \langle V_i^H, \hat{v}_n \rangle \approx \sum_{i=1}^{r_c} \lambda_i \langle \hat{v}_n^H, V_i \rangle \langle V_i^H, \hat{v}_n \rangle, \tag{17}$$

with $r_c \leq r$, where r_c is a sufficient amount of retained eigenvalues.

To determine the supersonic intensity magnitude, first the normal supersonic velocity $\hat{v}_n^{(s)}$ must be obtained. For this we use the orthonormally of the eigenvectors of $\bar{Q}$, in other words, the columns of V. The supersonic velocity is written using only the eigenvalues retained in the series (16),

$$\hat{v}_n^{(\mathbf{s})} = \sum_{i=1}^{r_c} \langle V_i^H, \hat{v}_n \rangle V_i, \tag{18}$$

where V_i^H are the lines of V^H e V_i are the columns of V.

The supersonic sound pressure can be obtained by inserting $\hat{v}_n^{(\mathbf{s})}$, obtained in Eq. (18), in Eq. (10),

$$\hat{p}^{(\mathbf{s})} = \mathbf{R}\hat{v}_n^{(\mathbf{s})}. \tag{19}$$

Knowing the normal supersonic velocity and the supersonic pressure, we can then obtain the normal component of supersonic intensity,

$$\hat{I}^{(\mathbf{s})} = \frac{1}{2}\Re[\hat{p}^{(\mathbf{s})} \hat{v}_n^{(\mathbf{s})*}]. \tag{20}$$

3. RESULTS

This section presents a numerical test in which a cylindrical surface with flat caps, with base radius of 0.5 m and height of 5 m, is excited in mode (6.1) at a frequency of 96 Hz. The frequency of coincidence is 240 Hz. The discretization of the whole surface has 1727 elements. It must be remembered that the vibration modes represent only the configuration of the structure when it vibrates with a certain frequency, so the absolute value of the components comprising the vector mode of vibration has no significance, being important only the relation between them. As a result, it is usual to represent the vibration modes through a determined standard to facilitate its interpretation and comparison. It will be used, in the following results, the a normalization where it is considered the highest value of the components of the velocity vector equal to the unity in millimeters (or 10^{-3}m).

An important parameter is the *dimensionless frequency* ν, defined as the ratio between the excitation frequency and the coincidence frequency.

Although the main interest of this work is the study of sound radiation, no matter at this time details of the elasticity of the vibrating structure, it is appropriate to mention that the material of the cylindrical surface is aluminum, which has elastic modulus of 7×10^{10} Pa, Poison coefficient of 0.35 and density of 2730 kg/m^3. To obtain the distribution of the surface velocity, it was used the FEM through the FEMLAB software.

In order to validate the efficiency of this technique that uses the ED of the power operator, it was performed a numerical test to calculate the supersonic intensity from the sound source coming from the cylinder mentioned earlier, comparing the results with those obtained using the SVD of the radiation operator developed by Magalhães (2002).

The Fig. 3 shows the normal velocity and the acoustic intensity, for $\nu = 0.4$.

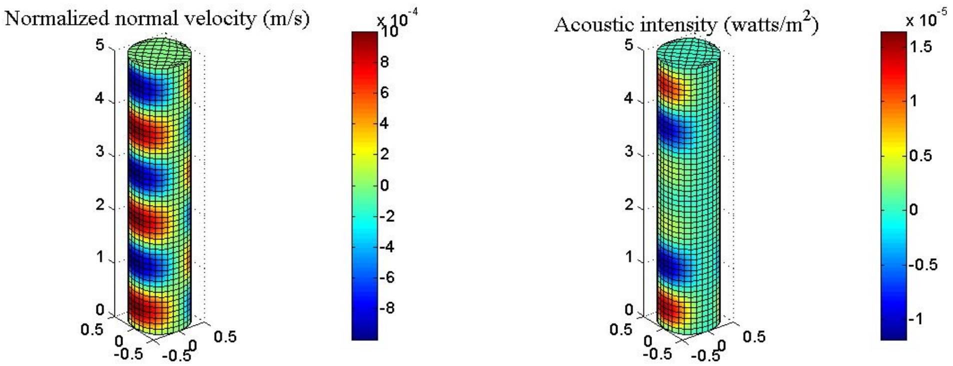

Figure 3. Normalized normal velocity and acoustic intensity, for $\nu = 0.4$.

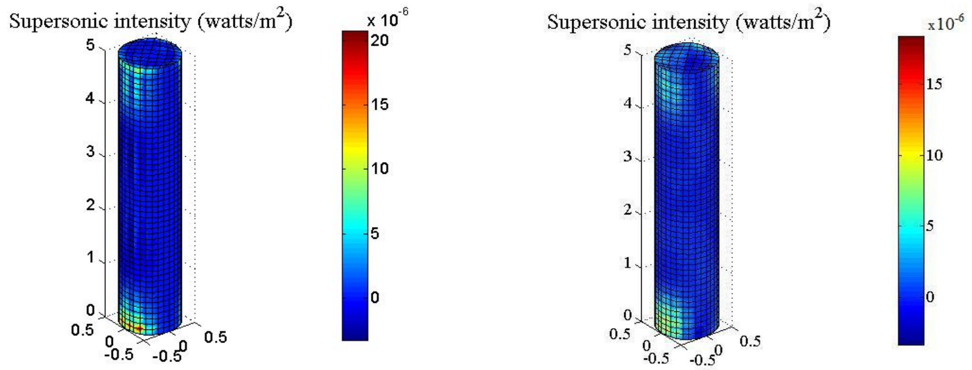

Figure 4. Supersonic intensity via ED and SVD, respectively, for $\nu = 0.4$.

Observing Fig. 4, it can be noticed that the numerical techniques showed close results in the identification of efficiently radiating areas. Comparing Figs. 4 and 3, it can be observed that the acoustic intensity indicates, incorrectly, regions that contribute with the generation of sound power to be radiated, while using the SI these regions are filtered out, leaving only those who really contribute to the generation of sound power (edge mode).

To obtain the supersonic intensity, it was used the following automatic criteria to choose the value of r_c

$$\left| \frac{\sum_{i=1}^{r} \lambda_i \langle \hat{\mathbf{v}}^H, V_i \rangle \langle V_i^H, \hat{\mathbf{v}} \rangle - \sum_{i=1}^{r_c} \lambda_i \langle \hat{\mathbf{v}}^H, V_i \rangle \langle V_i^H, \hat{\mathbf{v}} \rangle}{\sum_{i=1}^{r} \lambda_i \langle \hat{\mathbf{v}}^H, V_i \rangle \langle V_i^H, \hat{\mathbf{v}} \rangle} \right| < \delta, \tag{21}$$

where δ is a small value. It was adopted here $\delta = 10^{-3}$.

3.1 Convergence in relation to the number of eigenvalues kept

Figure 5 illustrate the convergence of the sound power with the eigenvalues in the series of Eq. (16) and also the distribution curve of such values, where from about 45 eigenvalues a good approximation is obtained.

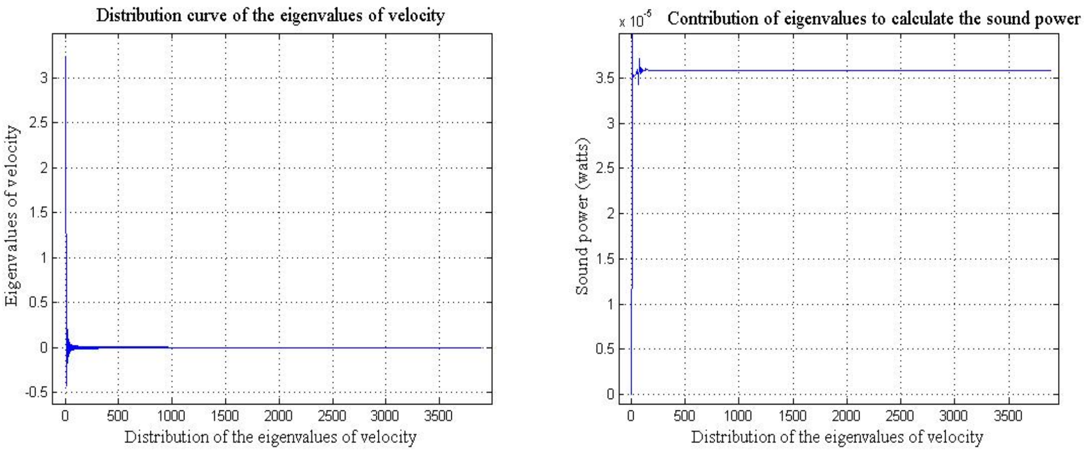

Figure 5. Distribution curve of the eigenvalues of velocity and contribution of such values to calculate the sound power.

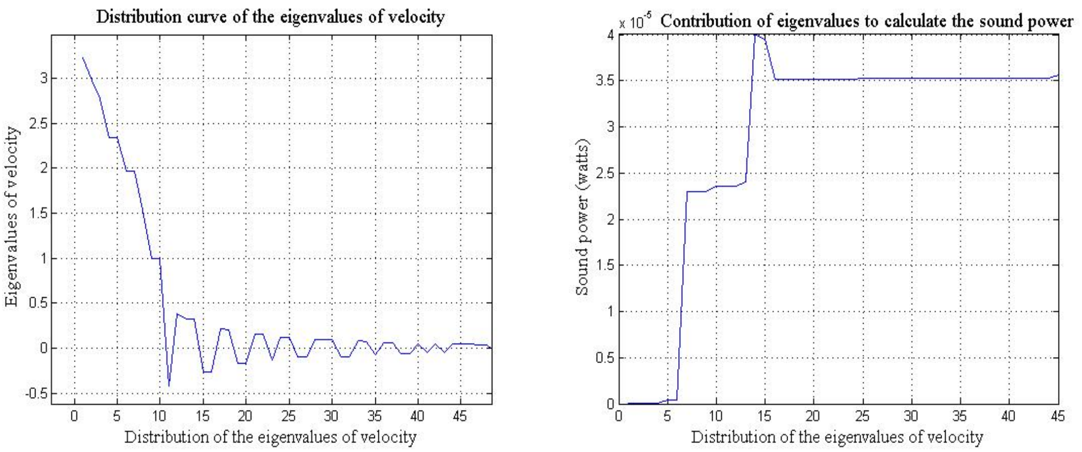

Figure 6. Distribution curve of the eigenvalues of velocity and contribution of such values to calculate the sound power, in detail.

As it can be observed in the Figs. 5 and 6, there are oscillations in the sign of the terms of the series that calculates the sound power, a fact probably due to the recirculation of energy presented in the calculation of the acoustic intensity, whereas when it is not indicated recirculation of energy in its calculation, such oscillations do not occur, as seen in Fig. 13.

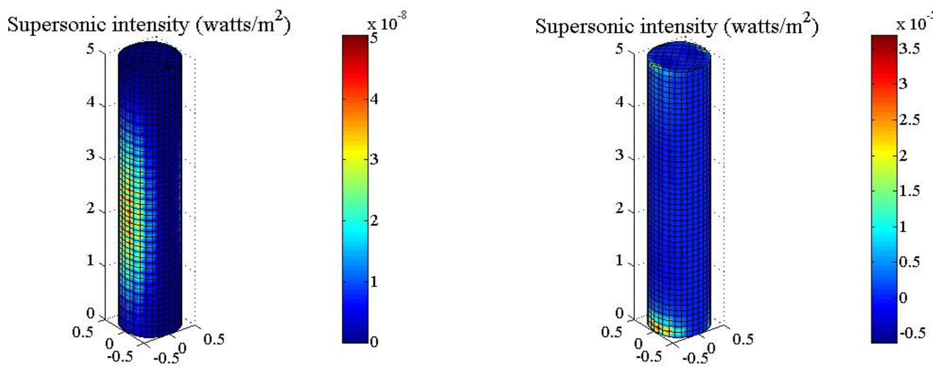

Figure 7. Supersonic intensity by ED, $r_c = 5$ and $r_c = 15$, respectively, for $\nu = 0.4$.

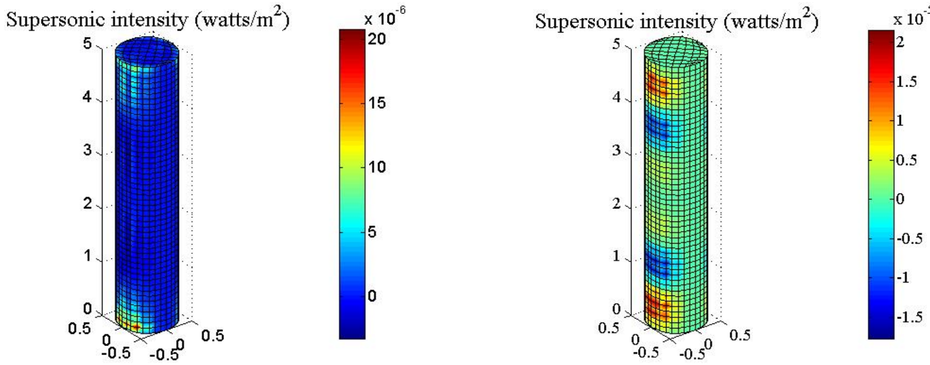

Figure 8. Supersonic intensity by ED, $r_c = 45$ and $r_c = 405$, respectively, for $\nu = 0.4$.

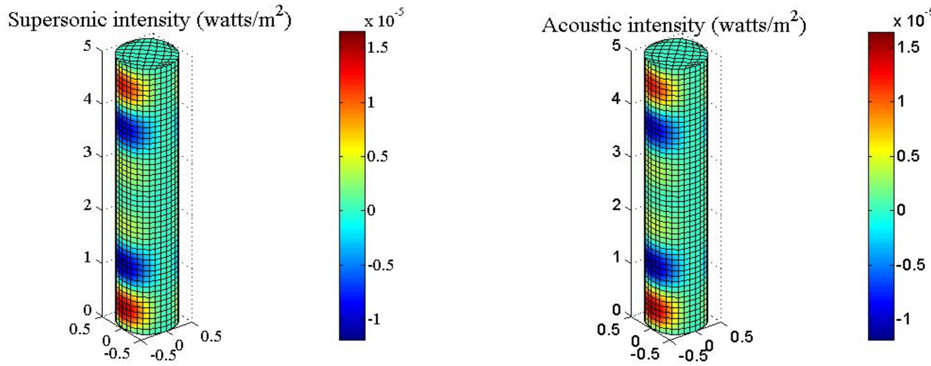

Figure 9. Supersonic intensity by ED, $r_c = 3645$ and acoustic intensity, respectively, for $\nu = 0.4$.

Observing Figs. 7, 8 and 9, one can observe the fact that the supersonic intensity converges to the acoustic intensity as the number of eigenvalues in the series (16) is increased, as expected.

3.2 Convergence with respect to the dimensionless frequency

In the literature, it can be found that the use of supersonic intensity to identify radiating regions is only advantageous when the excitation frequency is less than the coincidence frequency of the sound source (Wallace (1972)). When the excitation frequency approaches the coincidence frequency the supersonic intensity approach the acoustic intensity. Aiming to illustrate this convergence, we performed a numerical test with the same cylindrical sound source mentioned before, for different excitation frequencies.

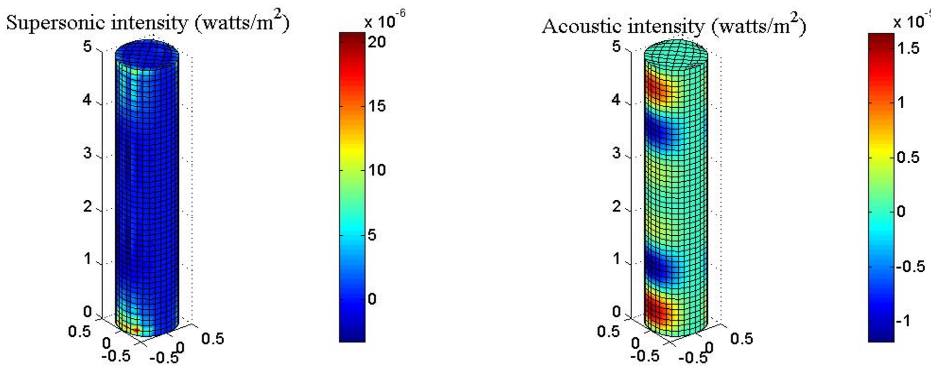

Figure 10. Supersonic Intensity by ED and acoustic intensity, for $\nu = 0.4$.

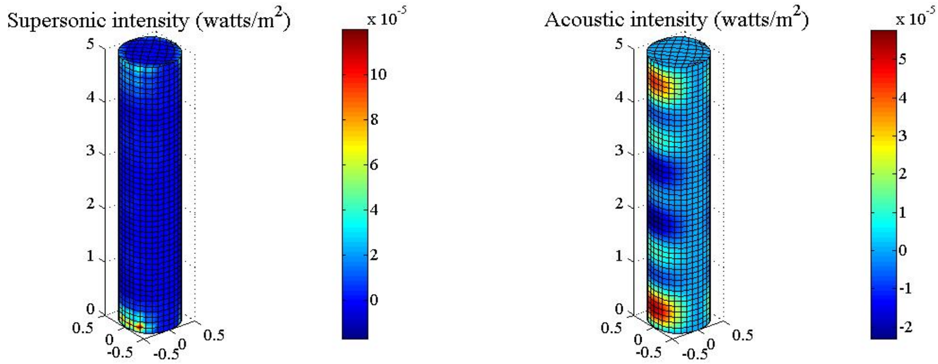

Figure 11. Supersonic Intensity by ED and acoustic intensity, for $\nu = 0.6$.

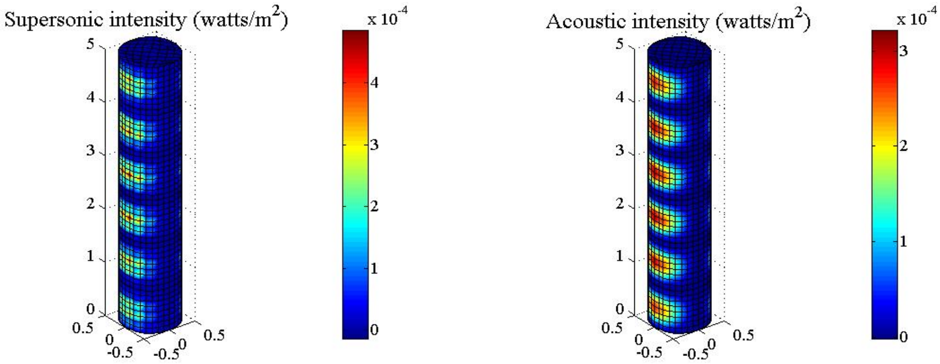

Figure 12. Supersonic Intensity by ED and acoustic intensity, for $\nu = 1$.

The results presented in Figs. 10, 11 and 12 demonstrates the convergence of supersonic intensity to the acoustic intensity as the excitation frequency approaches the coincidence frequency.

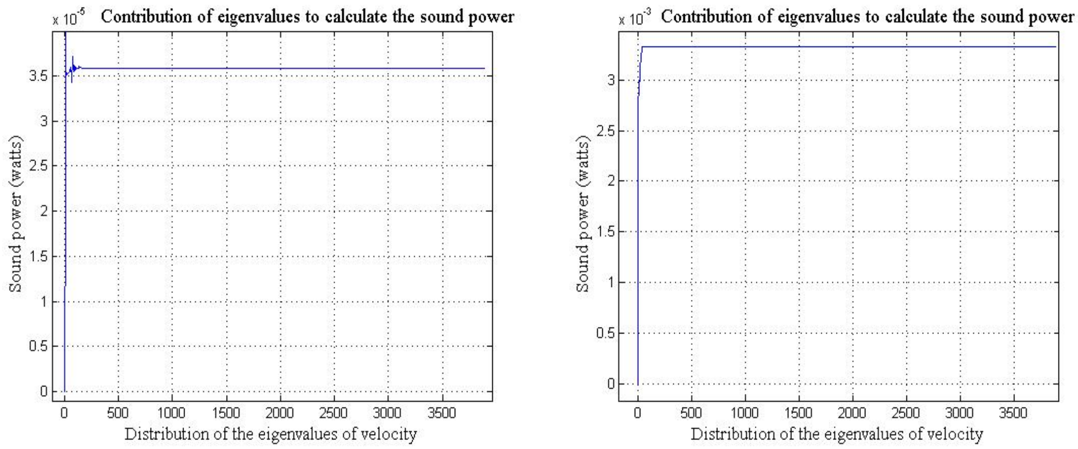

Figure 13. Contribution of eigenvalues to calculate the sound power, for $\nu = 0.4$ and $\nu = 1$, respectively.

As it can be observed in the Fig. 13, when $\nu = 0.4$ there are oscillations in the sign of the terms of the series that calculates the sound power, and when $\nu = 1$ there aren't such oscillations, fact probably due to the recirculation of energy presented in the calculation of the acoustic intensity for $\nu = 0.4$, and not presented in the calculation of the acoustic intensity for $\nu = 1$.

3.3 Computational Cost

In this section, it will be presented some data related to the computational cost of the numerical techniques used to compute the supersonic intensity. It was considered the worst case, where all values (eigenvalues or singular values) are used. The tests were done on a computer with an Intel Core 2 Duo T6500, 4 gigabytes of memory, and using the software Matlab 7.6.

The Tab. 1 and the Fig. 13 indicate the time, in seconds, spent by each technique, using different element meshes, thus demonstrating the advantage of the technique that uses the ED of power operator developed in this work.

Table 1. Computational Cost.

Number of elements	Technique: ED	Technique: SVD	Advantage%
100	4.84 s	7.96 s	39.13
225	21.04 s	35.66 s	41.00
400	60.49 s	111.46 s	45.73
625	153.81 s	283.85 s	45.81
900	332.66 s	619.16 s	46.27
1225	639.44 s	1215.99 s	47.41
1600	1135.47 s	2211.10 s	48.64
2025	2001.13 s	3896.60 s	48.64
2500	3277.21 s	6437.28 s	49.10
3025	5062.35 s	11405.26 s	55.61
3600	7305.53 s	16520.88 s	55.78
4025	11207.93 s	25588.88 s	56.20

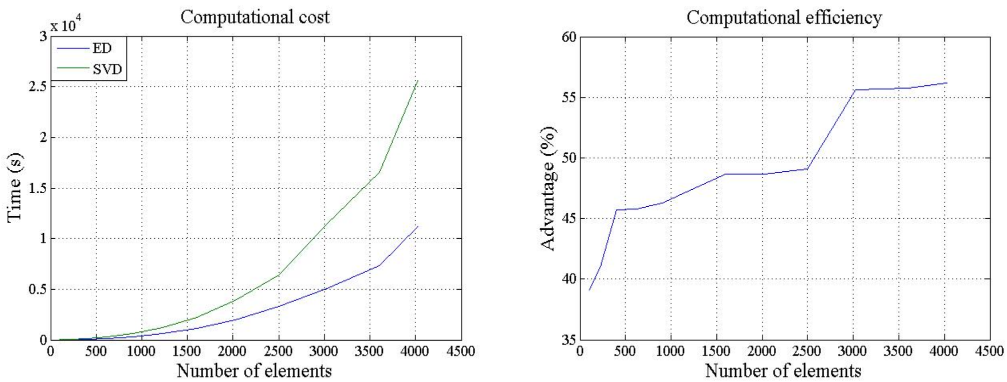

Figure 14. Computational cost and efficiency

As indicated in Tab. 1 and in Fig. 14, the technique that uses the ED of the power operator is advantageous in the computational point of view and as the number of elements used in discretization of the geometry is increased, it increases the computational advantage, compared to the SVD technique that uses the radiation operator. Remember, the number of elements to be used on the discretization will depend on the geometry of the sound source. For example, a small plate will need few elements, while the envelope of an airplane will need several elements. In general, the accuracy of the solution is increased with increasing the number of elements in the discretization.

4. CONCLUSIONS AND FUTURE WORKS

This paper presents a new numerical technique to the problem of acoustic radiation of a sound source with arbitrary geometry, with which one can get the equivalent to the supersonic intensity, allowing then to identify the regions of a sound source that effectively contribute to the generation of power that will be radiated to the the far field.

The numerical technique that uses SVD in the radiation operator is very satisfactory for the identification of radiating regions, however requires that it has to be created a grid of points located at a distance from the source, so you can extract the supersonic velocity using the series developed by Borgiotti (1990).

The numerical technique presented in this work, which uses the ED of the power operator, was also very satisfactory for the identification of radiating regions, with the advantage of beind done in points that belong to the sound source surface, bringing as benefit a significant reduction of the computational time spent.

In the presented results it was possible to observe the convergence of supersonic intensity to the acoustic intensity as eigenvalues are added to the series (16), or as the excitation frequency approaches the coincidence frequency. These facts are predicted to occur according to the literature, giving credibility to the numerical model.

For future works, we propose to study the ideal truncation of the eigenvalues for the technique that uses the ED, and apply the numerical technique developed in other arbitrary geometry surface, such as the envelope of an alternative compressor or a muffler.

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