ON NONLINEAR DYNAMICS BEHAVIOR OF A BUILDING STRUCTURE, EXCITED BY A NON-IDEAL MOTOR

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Abstract. *In this paper, we investigated the vibrations of a non-ideal structure (NIS) (simple portal frame under the excitation of an unbalanced DCl motor), by means of a nonlinear sub-structure with properties of nonlinear energy sink (NES) and relatively small mass.*

Keywords*: Non-ideal structure, nonlinear energy sink, energy pumping*

1. INTRODUCTION

In practical situations, the dynamics of the forcing system cannot be considered as given a priori, and it must be taken as also a consequence of the dynamics of the whole system. In other words, the forcing system has a limited energy source, as that provided by an electric motor for example, and thus its own dynamics is influenced by that of the oscillating system being forced as in (Balthazar et al, 2003).This increases the number of degrees of freedom, and is called a non-ideal problem(NIS). The study of non-ideal vibrating systems, that is, when the excitation is influenced by the response of the system, has been considered a major challenge in theoretical and practical engineering research.

In this paper, we aim to analyze a possible practical application. An unbalanced non-ideal DC motor shear building type foundation structure that suffers the Sommerfeld Effect of getting stuck at resonance (energy imparted to the motor being used to excite large amplitude motions of the supporting structure).

We remarked that the term energy pumping refers to the rapid and irreversible transfer of energy from a vibrating mechanical system to an attached nonlinear energy sink (NES). The occurrence of energy pumping depends on the essential nonlinearity of the sink stiffness. The concept of extracting energy away from a system in a simple fashion, so as to reduce its amplitude of vibration, is novel phenomenon and it forms the basis of concept of energy pumping(See as an example: Vakakis et al., 2008)). They have been shown that properly designed; essentially nonlinear local attachments may passively absorb energy from transiently loaded linear subsystems, acting in essence as (NES).

Here, we investigated the bifurcations of parameters and the dynamic interaction of the non-ideal structure attachment coupled to nonlinear essentially oscillator (NES) for vibration attenuation.

2. MATHEMATICAL MODEL AND DERIVATION OF GOVERNING EQUATIONS

Here, we deal with a modified model of the system studied by (Felix et al. 2009). A schematic of this coupled dynamical system is shown in Fig. 1

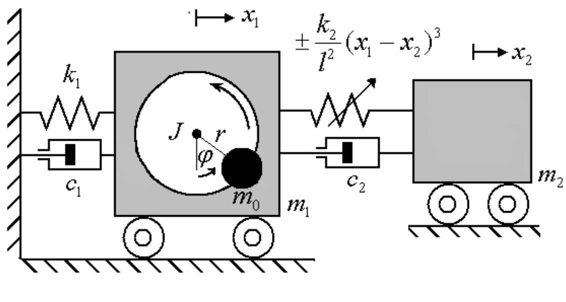


Figure 1. Schematic of a non-ideal oscillator (NIS) attachment coupled to nonlinear essentially oscillator (NES)

where (m_0, m_1, k_1, c_1, $x_1(t)$, $\varphi(t)$, J, r) denote the unbalanced mass, mass, linear stiffness, linear damping, displacement, angular displacement, inertia moment, eccentricity of the non-ideal structure and (m_2, k_2, c_2, $x_2(t)$)

denote the mass, non-linear stiffness, linear damping, displacement of the essentially nonlinear oscillator. (Taking into account that $m_1 = m_0 + M$ and $I = J + m_0 r^2$. The governing equations of motion for this system are:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1 x_1 + \frac{k_2}{l^2}(x_1 - x_2)^3 = m_0 r\dot{\varphi}^2 \sin\varphi - m_0 r\ddot{\varphi}\cos\varphi,$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + \frac{k_2}{l^2}(x_2 - x_1)^3 = 0, I\ddot{\varphi} = L(\dot{\varphi}) - H(\dot{\varphi}) - m_0 r\ddot{x}_1\cos\varphi, \quad L(\dot{\varphi}) - H(\dot{\varphi}) = u_1 - u_2\dot{\varphi} \quad (1)$$

where are u_1 related to voltage applied across the armature of the DC motor, that is, a possible control parameter of the problem, and u_2 is the constant for each model of DC motor considered. Defining the following non-dimensional parameters:

$$\tau = \omega_1 t \ (\dot{x}_1 = \frac{dx_1}{dt} = \frac{dx_1}{d\tau}\frac{d\tau}{dt} = \omega_1 x_1'),\ u = \frac{x_1}{r},\ v = \frac{x_2}{r},\ \alpha_1 = \frac{c_1}{m_1\omega_1},\ \varepsilon = \frac{m_2}{m_1},\ \eta_1 = \frac{m_0}{m_1},\ \eta_2 = \frac{m_0 r^2}{I},$$

$$\alpha_2 = \frac{c_2}{m_2\omega_1},\ \alpha_3 = \frac{k_2}{m_1}\left(\frac{r}{l}\right)^2,\ a = \frac{u_1}{I\omega_1^2},\ b = \frac{u_2}{I\omega_1},\ d = \frac{u_3}{I},\ \omega_1 = \sqrt{\frac{k_1}{m_1}}, \Gamma(\varphi') = \frac{L(\omega_1\varphi') - H(\omega_1\varphi')}{I\omega_1^2} = a - b\varphi', \quad (2)$$

the dimensionless resulting equations of motion are:

$$u'' + \alpha_1 u' + \alpha_2(u' - v') + u + \alpha_3(u - v)^3 = \eta_1(\varphi'^2 \sin\varphi - \varphi''\cos\varphi),$$
$$\varphi'' = \Gamma(\varphi') - \eta_2 u''\cos\varphi, \varepsilon v'' - \alpha_2(u' - v') - \alpha_3(u - v)^3 = 0. \quad (3)$$

and we will obtain, by considering the following change of variables:

$$u + \varepsilon v = x, u - v = y \quad (4)$$

$$x'' + \frac{1}{1+\varepsilon}x = -\frac{\alpha_1}{1+\varepsilon}(x' + \varepsilon y') - \frac{\varepsilon}{1+\varepsilon}y + \eta_1(\varphi'^2\sin\varphi - \varphi''\cos\varphi), \varphi'' = \Gamma(\varphi') - \frac{\eta_2}{1+\varepsilon}(x'' + \varepsilon y'')\cos\varphi,$$
$$y'' + \frac{\varepsilon}{1+\varepsilon}y = -\frac{\alpha_1}{1+\varepsilon}(x' + \varepsilon y') - \alpha_2\frac{(1+\varepsilon)}{\varepsilon}y' - \frac{1}{1+\varepsilon}x - \alpha_3\frac{(1+\varepsilon)}{\varepsilon}y^3 + \eta_1(\varphi'^2\sin\varphi - \varphi''\cos\varphi). \quad (5)$$

It is a dynamical system with variable y that denotes the relative displacement between the (NIS) and the (NES), whereas variable x denotes the oscillation of the center of mass of the entire dynamical system. The important advantage of Eq. (8) of that, there are the terms $\frac{1}{1+\varepsilon}x$ and $\frac{\varepsilon}{1+\varepsilon}y$, permits the application of standard perturbation technique such as, here, averaging. For small oscillations in the x, y and φ motion is assumed $\hat{\varepsilon}$ <1 as a small parameter value, the equations of motion for this system (7) can be written as:

$$x'' + \delta_1^2 x = \hat{\varepsilon}\left[-\hat{\alpha}_1 x' - \hat{\alpha}_2 y' - \hat{\alpha}_3 y + \hat{\eta}_1\varphi'^2\sin\varphi\right],\ y'' + \delta_2^2 y = \hat{\varepsilon}\left[-\hat{\alpha}_1 x' - \hat{\alpha}_4 y' - \hat{\alpha}_5 x - \hat{\alpha}_6 y^3 + \hat{\eta}_1\varphi'^2\sin\varphi\right]$$
$$\varphi'' = \hat{\varepsilon}\left[\hat{\Gamma}(\varphi') + \hat{\eta}_2 x\cos\varphi + \hat{\eta}_3 y\cos\varphi\right], \quad (6)$$

Where

$$\delta_1^2 = \frac{1}{1+\varepsilon},\ \delta_2^2 = \frac{\varepsilon}{1+\varepsilon},\ \hat{\varepsilon}\hat{\alpha}_1 = \frac{\alpha_1}{1+\varepsilon},\ \hat{\varepsilon}\hat{\alpha}_2 = \frac{\varepsilon\alpha_1}{1+\varepsilon},\ \hat{\varepsilon}\hat{\alpha}_3 = \frac{\varepsilon}{1+\varepsilon},\ \hat{\varepsilon}\hat{\eta}_1 = \eta_1,\ \hat{\varepsilon}\hat{\Gamma}(\varphi') = \Gamma(\varphi'),$$
$$\hat{\varepsilon}\hat{\eta}_2 = \frac{\eta_2}{(1+\varepsilon)^2},\ \hat{\varepsilon}\hat{\eta}_3 = \frac{\varepsilon^2\eta_2}{(1+\varepsilon)^2},\ \hat{\varepsilon}\hat{\alpha}_4 = \frac{\varepsilon\alpha_1}{1+\varepsilon} + \alpha_2\frac{(1+\varepsilon)}{\varepsilon}, \hat{\varepsilon}\hat{\alpha}_5 = \frac{1}{1+\varepsilon},\ \hat{\varepsilon}\hat{\alpha}_6 = \alpha_3\frac{(1+\varepsilon)}{\varepsilon}. \quad (7)$$

3. NUMERICAL RESULTS

In the following simulations unless otherwise noted we choose the parameters values: α_1 = 0.01, α_2 =0.2, α_3=0.5, ε =0.10, η_1=0.1, η_2 =0.3, b =1.5. The initial conditions are nulls. Figure 2 represents the resonance curve, showing the dynamics of the NIS during the passage through resonance region ($\varphi' \approx 1$). We plotted the amplitudes of oscillation of the angular velocity and foundation displacement without NES (triangle line) and with NES (point line) versus the voltage (control parameter) in the range $1.2 \leq a \leq 3.0$, considering an increment $\Delta a = 0.01$ and over the dimensionless time range $0 \leq \tau \leq 3000$. This graph is estimated by numerical simulation, defining the amplitude for angular velocity of the motor shaft as maximum and minimum value of the oscillations, and the amplitude for the foundation oscillation as (maximum value-minimum value)/2 on the stationary state motion from Eq. (7).

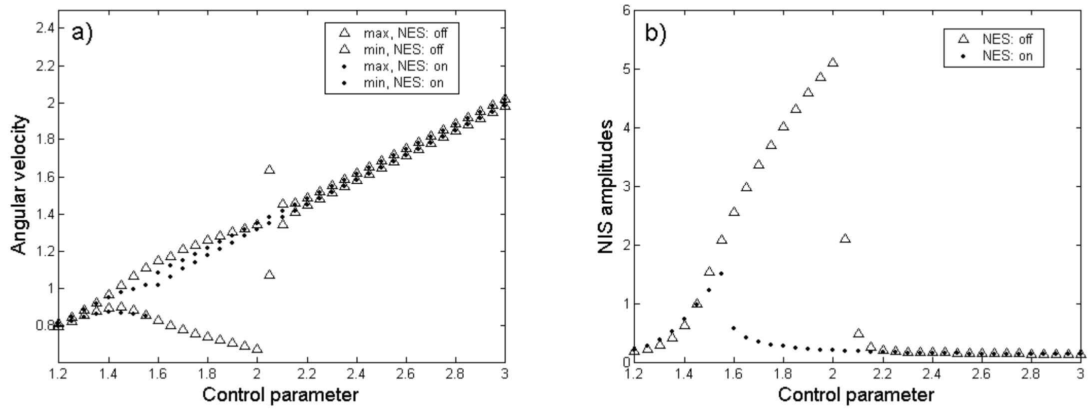
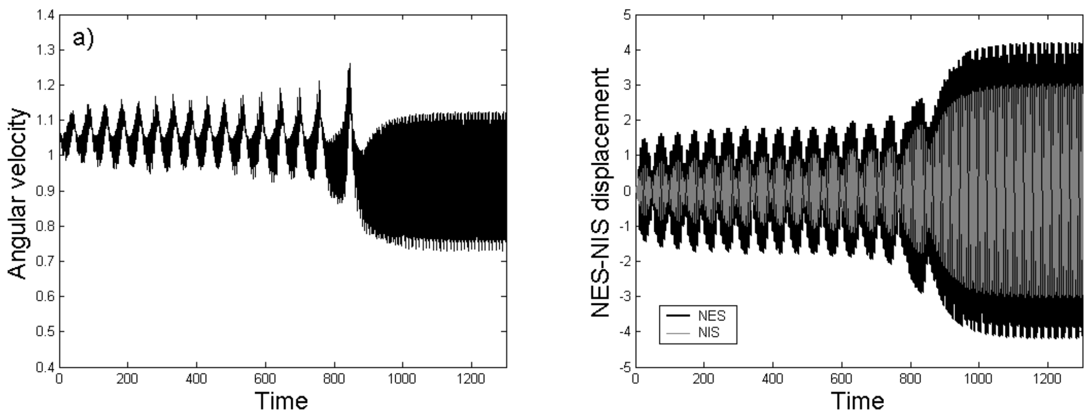


Figure 2. Amplitudes versus control parameter (triangle line of NES off and point line of NES on): a) angular velocity and b) NIS.

The Fig. 2(a), shows a predominant interaction between the motor and foundation. Here, the oscillation amplitudes are large of the angular velocity (NES off) in the range $1.5 \leq a < 2.0$ and without sensible change of the motor frequency due the resonance capture phenomenon, initiating the jump phenomenon in the vicinity $a = 2.1$, while generating large vibration amplitude of the foundation, as it is shown in Fig. 2(b) (Sommerfeld effect).

Considering the NES on, the Sommerfeld effect is attenuated, as it is observed in Fig 2(a), point line, while the vibration amplitude of the foundation are reduced drastically as is shown in Fig 2(b), point line. The influence of the damping in the system NES-NIS was observed through of a set of numerical test set with the angular velocity next of the resonance region with $a = 1.6$, the resulting is shown in Fig. 3.

Taking the other parameters fixed, we have: for $\alpha_2 \in [0, 0.013]$, the dynamic behavior is of instable motion combined of quasi-periodic and periodic regime due the phenomenon of the resonance capture, in this case, we choose $\alpha_2 = 0.013$, as is shown in Fig 3a. For $\alpha_2 \in [0.014, 0.06]$, the dynamic behavior is of stable quasi-periodic motion (beat phenomenon), in this case, we choose $\alpha_2 = 0.06$, as is shown in Fig 3b. Finally, for $\alpha_2 \in [0.07, 0.2]$, the dynamic behavior is of stable periodic motion, in this case we choose $\alpha_2 = 0.2$, as is shown in Fig 3c.In the right column, Figs 3a and b) the trace of gray line is of NIS and of black line is of NES, while the Fig. 3c, the trace of black line is NIS and of gray line is of NES.



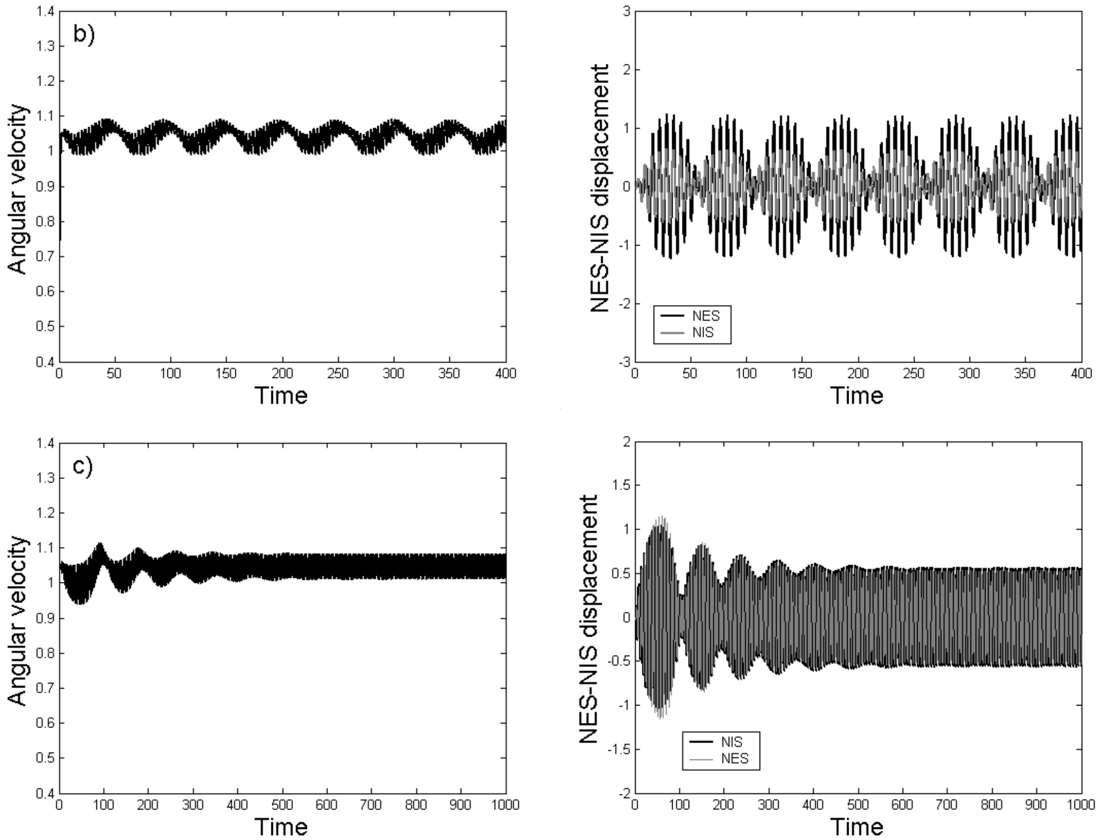
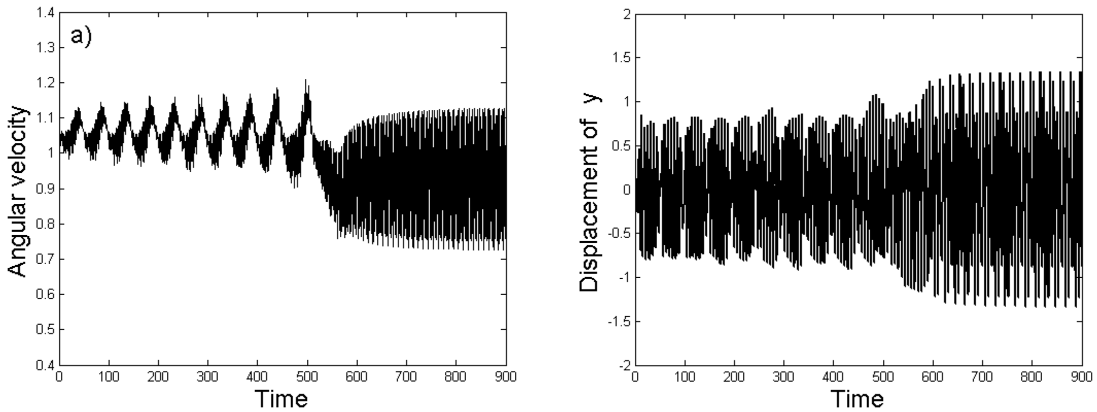


Figure 3. Time histories of the angular velocity (left column) and NES and NIS displacement (right column) for a =1.6: a) α_2 =0.013, b) α_2 =0.06, c) α_2 =0.2.

We consider now the system of Eq. (8), from the time histories of the angular velocity φ' and relative displacement between NIS and NES, y, we can to obtain the same dynamic behavior as was obtained in Fig. 3, and these results are shown in Fig. 4.



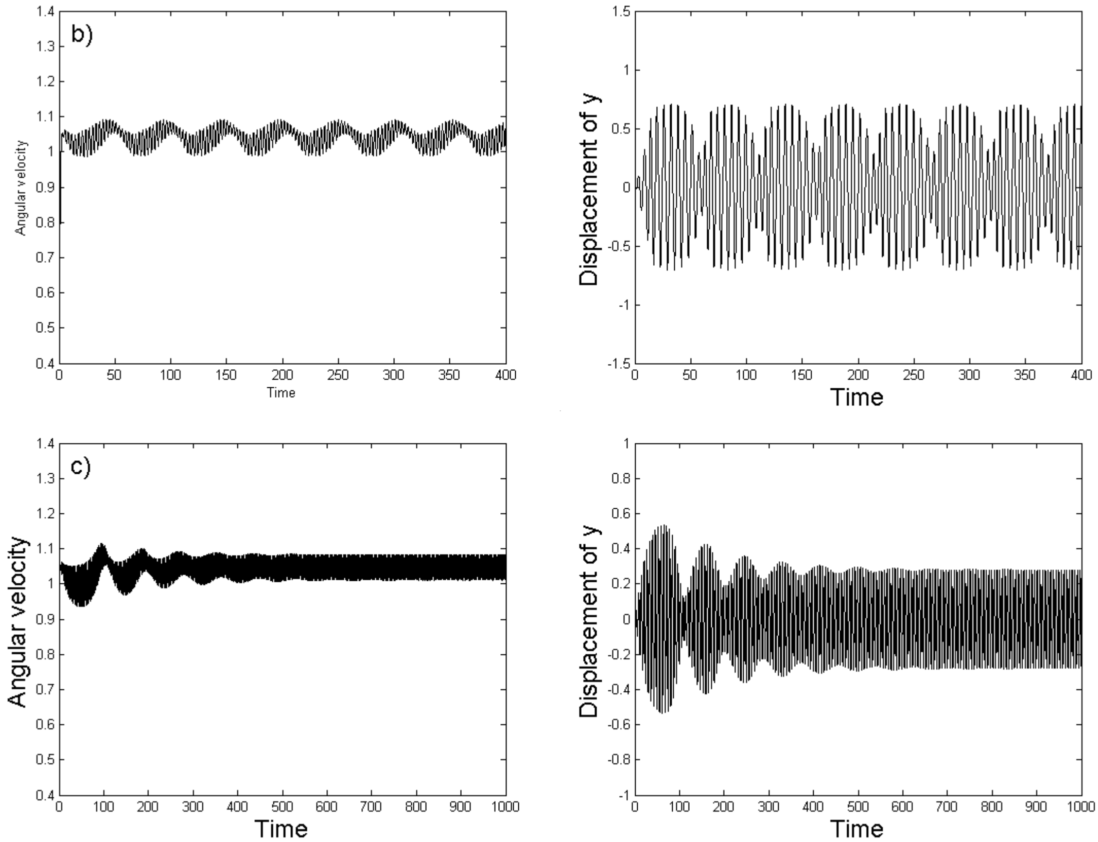
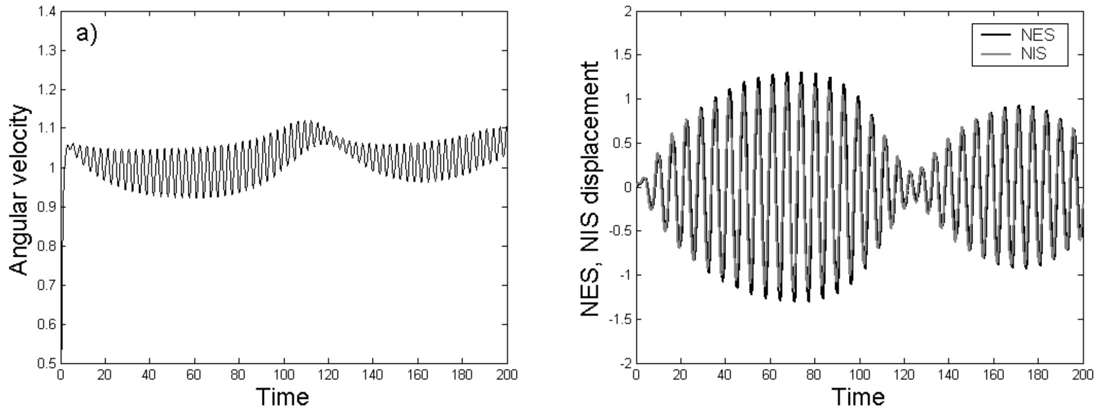


Figure 4. Time histories of the angular velocity (left column) and y displacement (right column) for a =1.6: a) α_2 =0.013, b) α_2 =0.06, c) α_2 =0.2.

With an impulse applied to the foundation of NIS, in this case will be the initial condition $u' \neq 0$ and the others are nulls of Eq. (6), the effect of the initial condition on the transient motion were studied in the numerical simulations depicted in Fig. 5.

The results, Fig. 5, show near of resonance region a motion of synchronization in phase between the NES and NIS foundation response where the black line is the NES response and gray line is the NIS foundation response as is shown in the right column, while the left column correspond to angular velocity. Fig. 5a shows the beat motion and stable for u' = 0.10 ∈ [0.0, 0.14], Fig. 5b show the periodic motion and instable due the angular velocity is captured in resonance region for u' = 0.15 ∈ [0.15, 2.0], and Fig. 5c show the periodic motion with characteristic of response of an underdamped system to a non-ideal excitation for u' = 3.0 ∈ [2.1, 5.0].



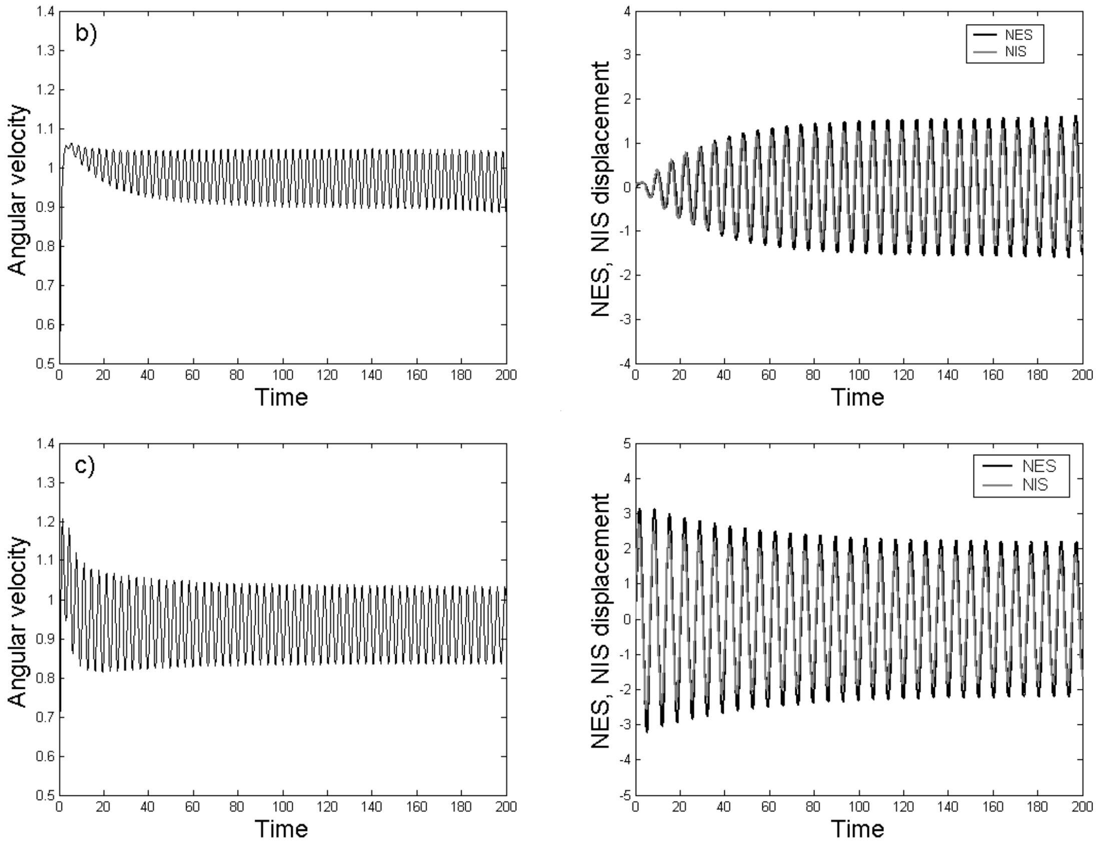


Figure 5. Time histories of the angular velocity (left column) and NES and NIS displacement (right column) for a =1.6: a) u' =0.10, b) u' =0.15, c) u' =3.0.

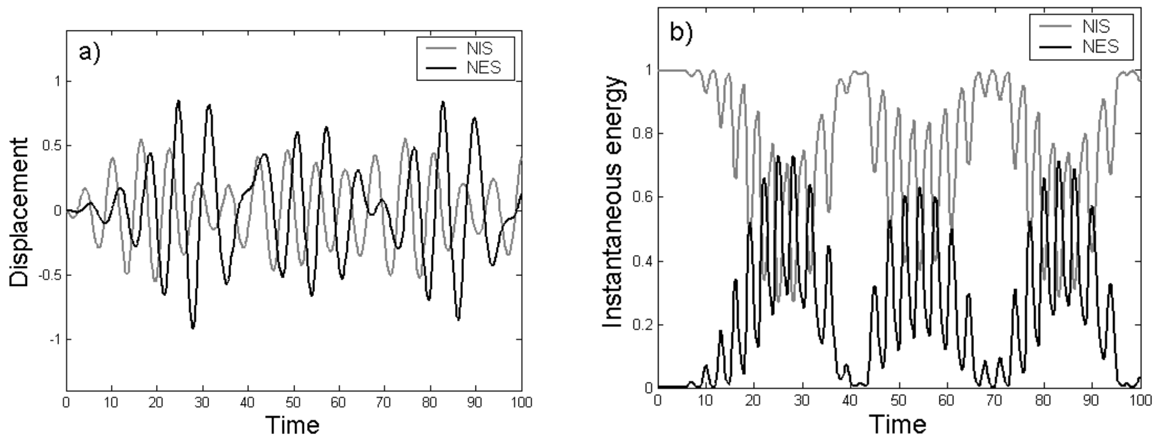
In the following numerical results were investigated the influence of the parameters of hardening cubic stiffness and damping of NES, and the size of the NIS mass, shown in Fig. 6, in this, correspond for the values new of the parameters: α_2 =0.05, α_3 =1.0, ε =0.2 and the others are fixed. We observe that, Fig. 6b, near of resonance region, the reduction of the amplitude of oscillation of NIS foundation in predominant in comparison with the results of Fig. 2. While, the reduction of the Sommerfeld is predominant in the angular velocity of the DC motor, as is shown in Fig. 6a.

The Fig. 7, show the comparison of amplitudes of oscillation of the NES and the NIS, in this case, is present the energy transfer during the passage of resonance. The Fig. 8, above, shows beat phenomenon and transfer energy is transferred from the NIS to the NES during each cycle of the beat for η =1, α =5.0, a =1.6 (angular velocity near of resonance capture) from the numerical results of the Eqs. (6), (13) and (14). To 0< τ <20, the instantaneous energy of the NES increases while of the NIS decreases,

Fig. 8(b). In the motion of synchronization of system NIS-NES, the amplitude of NIS is decreasing while of NES is increasing in same time interval.

The Fig. 9 shows the influence of the coefficients of damping and stiffness of the attachment on the energy transfer between the NIS and NES. We consider α_2 =0.05 and α_3 =0.2 and the others parameters are fixed.

We can be to observe that, in the time interval 20< τ <30, the amplitude of NIS decreasing while of NIS increasing, Fig. 10(a), due the predominant energy transfer of the NIS for the NES in the same time interval, as is shown in Fig. 10(b). Here, we show a characteristic of tuning of frequencies, internal resonance, between the NIS and NES.



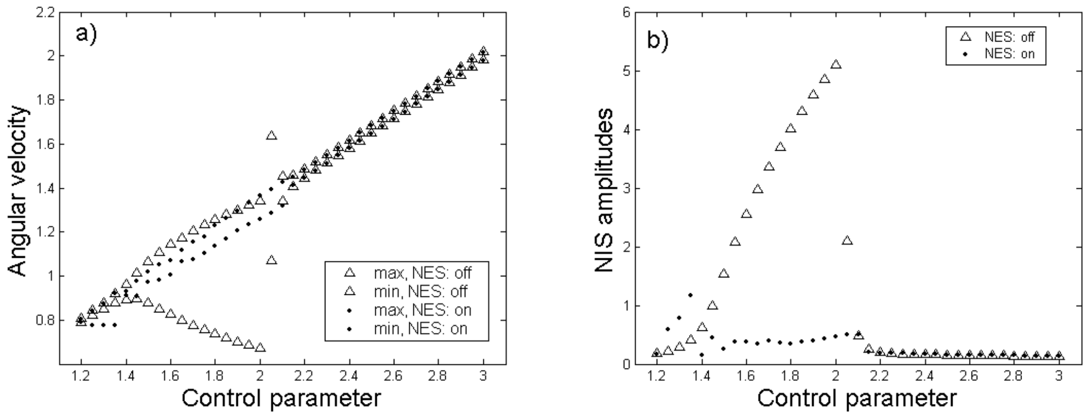


Figure 6. Amplitudes versus control parameter (triangle line of NES off and point line of NES on): a) angular velocity and b) NIS.

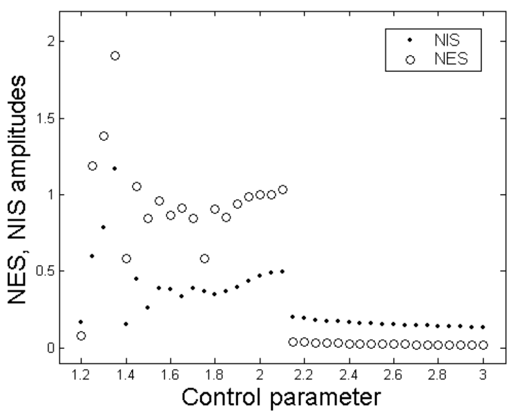


Figure 7. Transfer of energy between the response of NES (circle line) and NIS (point line).

4. ENERGY PUMPING AND BEAT PHENOMENA

In order to demonstrate the energy transfer (energy pumping) and beat phenomenon in the passage of resonance, we present the instantaneous energy with respect to total energy of the system carried by the NIS and NES, respectively. The quantity $E_{NIS} = E_{NIS}(t)$ can be interpreted as the energy in the NIS, here is included the kinetic energy of the motor $I\frac{\dot{\varphi}^2}{2}$, while $E_{NES} = E_{NES}(t)$ then represents the energy associated with the attachment of the NES. The Eqs. (8) and (9) are reduced in dimensionless form:

$$E_{NIS,inst} = \frac{E_{NIS}}{E_{NIS}+E_{NES}} = \frac{m_1\frac{\dot{x}_1^2}{2}+k_1\frac{x_1^2}{2}+I\frac{\dot{\varphi}^2}{2}}{m_1\frac{\dot{x}_1^2}{2}+k_1\frac{x_1^2}{2}+I\frac{\dot{\varphi}^2}{2}+m_2\frac{\dot{x}_2^2}{2}+k_2\frac{(x_2-x_2)^4}{4}}, \tag{8}$$

$$E_{NES,inst} = \frac{E_{NES}}{E_{NIS}+E_{NES}} = \frac{m_2\frac{\dot{x}_2^2}{2}+k_2\frac{(x_2-x_2)^4}{4}}{m_1\frac{\dot{x}_1^2}{2}+k_1\frac{x_1^2}{2}+I\frac{\dot{\varphi}^2}{2}+m_2\frac{\dot{x}_2^2}{2}+k_2\frac{(x_2-x_2)^4}{4}}. \tag{9}$$

The quantity $E_{NIS} = E_{NIS}(t)$ can be interpreted as the energy in the NIS, here is included the kinetic energy of the motor $I\frac{\dot{\varphi}^2}{2}$, while $E_{NES} = E_{NES}(t)$ then represents the energy associated with the attachment of the NES. The Eqs. (8) and (9) are reduced in dimensionless form, in the variables u and v _ Eq(4)

$$\tilde{E}_{NIS,inst} = \frac{\frac{u'^2}{2}+\frac{u^2}{2}+\eta\frac{\varphi'^2}{2}}{\frac{u'^2}{2}+\frac{u^2}{2}+\eta\frac{\varphi'^2}{2}+\frac{v'^2}{2}+\alpha\frac{(u-v)^4}{4}}, \quad \tilde{E}_{NES,inst} = \frac{\frac{v'^2}{2}+\alpha\frac{(u-v)^4}{4}}{\frac{u'^2}{2}+\frac{u^2}{2}+\eta\frac{\varphi'^2}{2}+\frac{v'^2}{2}+\alpha\frac{(u-v)^4}{4}}, \tag{10}$$

$$\tilde{E}_{NIS,inst} = \frac{E_{NIS,inst}}{k_1 r^2}, \quad \tilde{E}_{NES,inst} = \frac{E_{NES,inst}}{\varepsilon k_1 r^2}, \quad \eta = \frac{I}{m_1 r^2}, \quad \alpha = \frac{k_2}{\varepsilon k_1}\frac{r^2}{l^2}. \tag{10a}$$

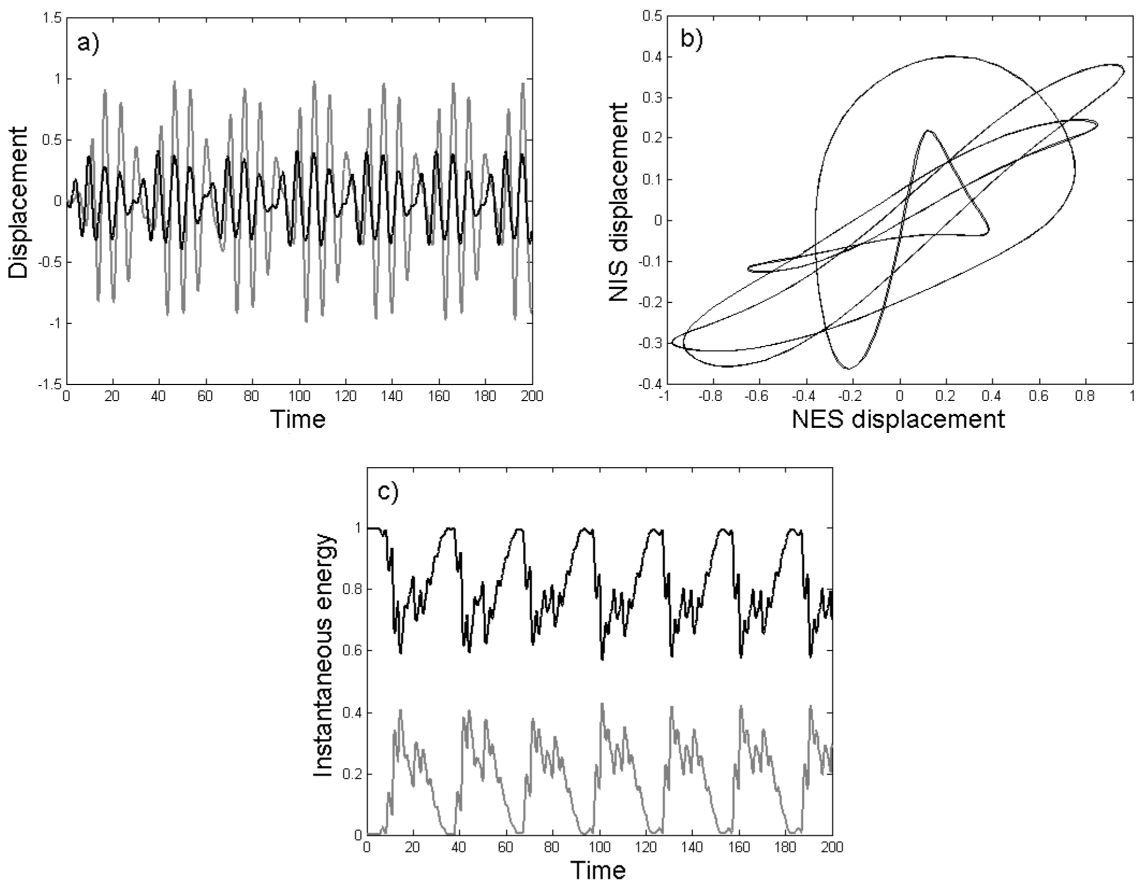


Figure 8. Beat phenomenon and energy transfer to a =1.6: a) displacements of NIS u (black line), NES v (gray line); b) motion in phase plane (u , v); c) instantaneous energy in each NES (gray line) and NIS (black line).

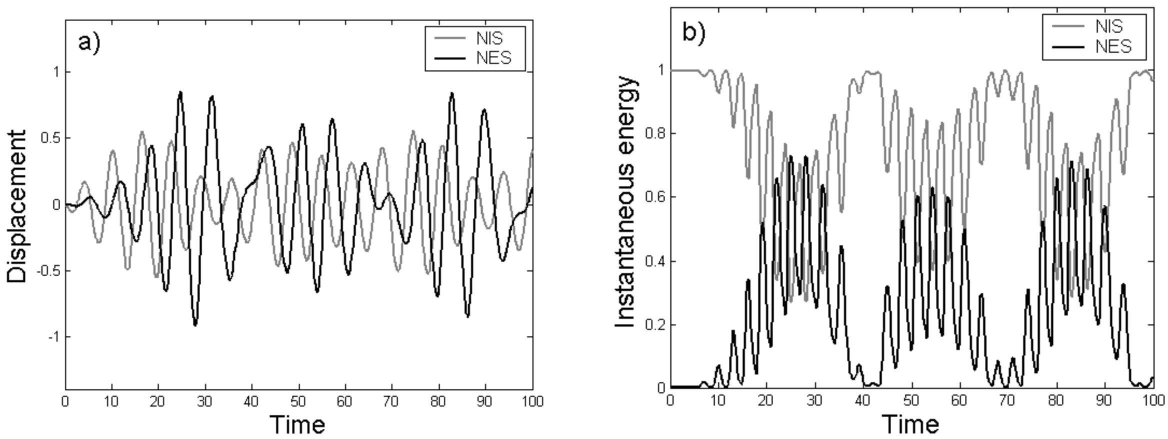


Figure 9. Energy transfer of system (6) to a =1.5, α_2 =0.05 and α_3 =0.2: a) displacements of NIS u (gray line), NES v (black line); b) instantaneous energy in each NES (black line) and NIS (gray line).

5. CONCLUSIONS

This paper has dealt with the numerical simulation study of a non-ideal structure (consists of a simples portal frame under excitation of an unbalanced DC motor with limited power supply) coupled to a nonlinear essentially oscillator (with properties of nonlinear energy sink).

We observed that the phenomenon of energy pumping, beat and synchronization is present in the passage through resonance with the influence of the damping of NES.

The effects of the damping and initial conditions of the system were investigated. Considering as control the NES we can to reduce the amplitudes of oscillations of the NIS and to eliminate and to reduce the Sommerfeld effect inside and outside resonance region, respectively.

We announced the future works (Felix and Balthazar, 2011):

Using perturbation methods, announced that during transient motion the ***phenomenon of fluttering***, may be present, as it is shown in Fig. 10(a) while the angular velocity tends to a constant value V_0 and the steady state motion sshowed that the response of the angular velocity when it is choosing the control parameter a =1.5, small parameter $\hat{\varepsilon}$ =0.0025 and the initial conditions: [0.08, 0, 0.02, 0, 0.8], ***it is captured in resonance***, in the time interval $\tau \in$ [800, 2000] and the relative displacement has dynamical behaviour of ***quasi-periodic motion***. The same result was verified by using numerical simulations of Eq. (9).

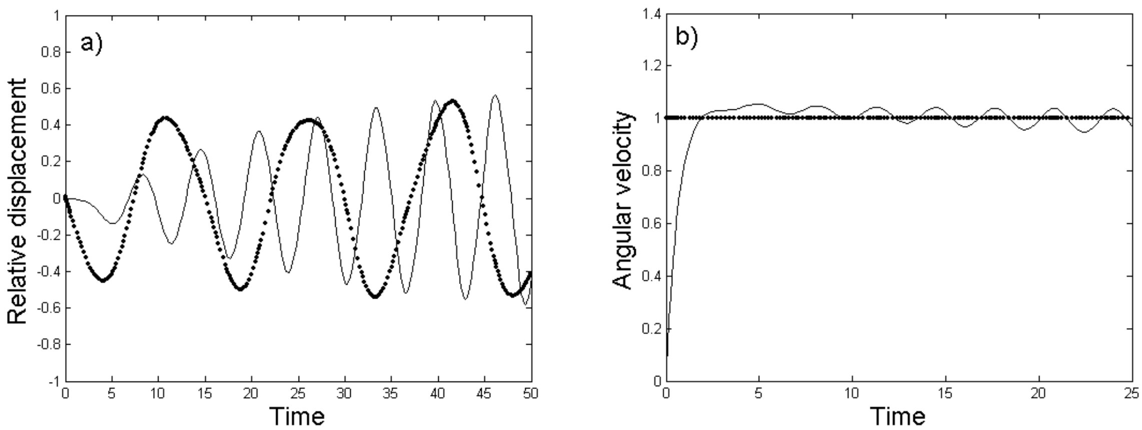


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Figure 10. Response of Perturbation analysis (point line) and Eq. (8) (solid line) for a =1.5:

a) relative displacement, b) angular velocity.

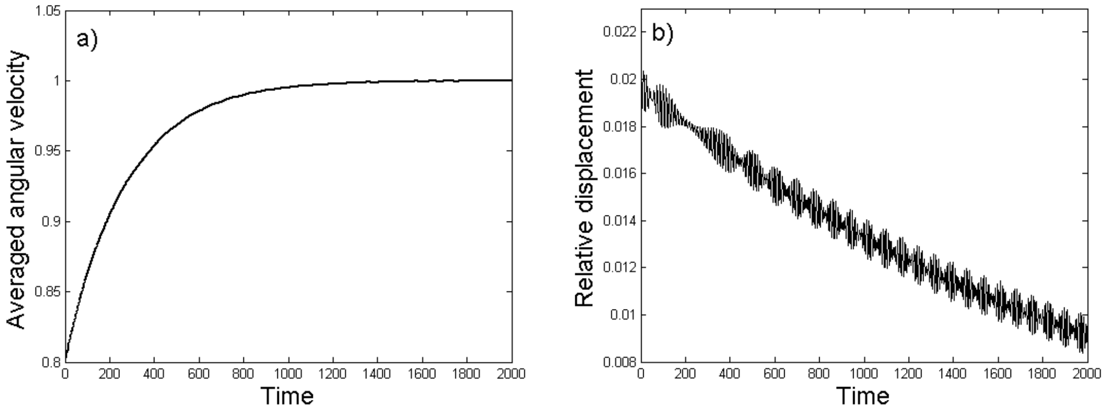

Figure 11. Perurbation Analysis Response for $\hat{\varepsilon}$ =0.0025, IC: [0.08, 0, 0.02, 0, 0.8], a =1.5: a) Angular velocity, b) relative displacement

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Balthazar, J.M., Mook, D.T., Weber, H.I., Brasil, R.M.L.R.F., Fenili, A., Belato, D., Felix, J.L.P, 2003," An overview on non-ideal vibrations". Meccanica, Vol. 38, No. 6, pp.613-621.

Felix, J.L., Balthazar, J.M., Dantas, M.J.H., 2009," On energy pumping, synchronization and beat phenomenon In a non-ideal structure coupled to an essentially nonlinear oscillator", Nonlinear Dynamics. 56, 1-2, pp. 1-11

Felix, J.L., Balthazar, J.M, 2011, On an Analytical solution to a NES Absorber in a Non-ideal Building Structure, Preprint

Vakakis A.F., Gendelman, O.V., Bergman, L.A., MacFarland D.M., Kerscher G., Lee Y.S, 2008," Nonlinear Target Energy Transfer in Mechanical and Structural Systems. Ed. Springer Verlag Holland, 800p

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