A KINEMATIC AND DYNAMIC STUDY OF THE LOWER LIMB

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Abstract. *The biomechanical studies of fundamental concepts in human movements can be faced as kinematics and dynamics of mechanisms and design of machine elements. The objective of this paper is to present a study complaining the kinematics and dynamics of human lower limb through the formulation of the mathematical equations describing the human gait. This study can be used to other researches in some different sub-fields of biomechanics (prosthetic technologies and gait analysis), considering experimental or computational approaches. This study is essential for understanding experimental researches using force platforms and gait laboratory.*

Keywords*: kinematic, dynamic, biomechanic, gait human*

1. INTRODUCTION

The human locomotion is represented through the walk or the run including variations of velocity and acceleration. The study of human walking is called the gait analysis.

The gait cycle describes the motions from initial placement of the supporting heel on the ground to when the same heel contacts the ground for a second time. The gait cycle is divided into two phases: stance (defined as interval in which the foot is on the ground (60% of the gait cycle)) and. swing (defined as the interval in which the foot is not contact with the ground (40% of the gait cycle)). The stance is divided into four phases: heel strike to foot flat, foot flat through midstance, midstance through heel off and heel off toe off. The swing is divided into two phases: acceleration to midswing and midswing to deceleration (Fig. 1).

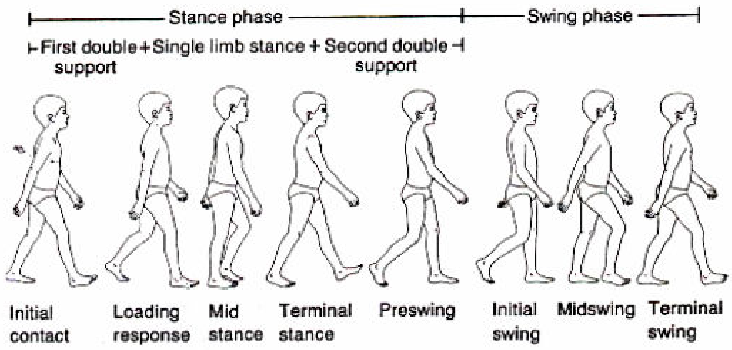

Figure 1. Gait Cycle (Vaughan *et al.*, 1992).

The anatomical planes of motion are three perpendicular planes. Each plane divides the human body into two halves of equal mass and intersect ate the center of mass of the body. This planes are called the sagital plane, that divided vertically the body, the coronal plane, that divided the body in anterior and posterior, and the transverse plane, that divided horizontally the body. The study of anatomical directional terms and body planes will help to visualize positional and spatial locations and describe the body movements (McGinnis, 2002).

In the Figure 2 is shown anatomical planes.

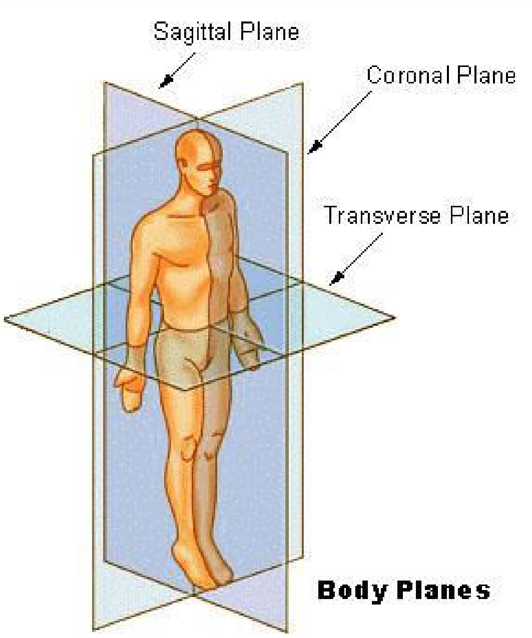

Figure 2. Body Planes (McGinnis, 2002).

2. KINEMATICS

In the Figure 3 is shown the kinematics of the lower limbs scheme.

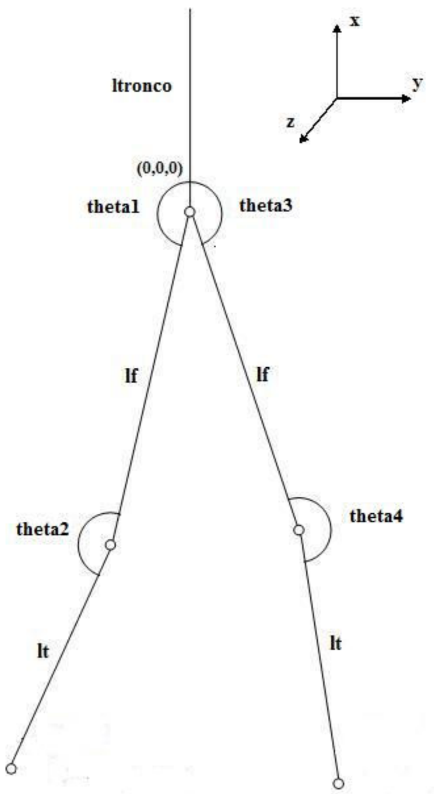

Figure 3. The modeling of the kinematics scheme.

The human locomotion involves a very large number of degrees of freedom (Forsyth *et al.*, 2006). So for this were made the following considerations: modelling 2D (sagital plane), four degrees of freedom were used, *lt* is a tibia length, *lf* is a femur length, *Θ1* and *Θ3* are the angular displacement of the hip joint, and *Θ2* e *Θ4* are the angular displacement of the knee joint.

The following are presented the transformation matrices of the system, where *MPE* is the transformation matrix of left leg and *MPD* is the transformation matrix of right leg.

$$MPE = R1.T1.R2.T2 \quad (1)$$

$$MPD = R5.T5.R6.T6.R7.T7.R8.T8 \quad (2)$$

Where *Rn* is a rotation matrix and *Tn* is a translation matrix.

$$Rn = \begin{vmatrix} \cos(\theta_n) & -sen(\theta_n) & 0 & 0 \\ sen(\theta_n) & \cos(\theta_n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (3)$$

$$Tn = \begin{vmatrix} 1 & 0 & 0 & l_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (4)$$

Ankle position – left leg

$$X_{TE} = l_f.sen(\theta_1) + l_t.sen(\theta_1 + \theta_2) \quad (5)$$

$$Y_{TE} = -l_f.\cos(\theta_1) - l_t.\cos(\theta_1 + \theta_2) \quad (6)$$

$$Z_{TE} = 0 \quad (7)$$

Knee position – left leg

$$X_{JE} = l_f.sen(\theta_1) \quad (8)$$

$$Y_{JE} = -l_f.\cos(\theta_1) \quad (9)$$

$$Z_{JE} = 0 \quad (10)$$

Ankle position – right leg

$$X_{TD} = l_f.sen(\theta_3) + l_t.sen(\theta_3 + \theta_4) \quad (11)$$

$$Y_{TD} = -l_f.\cos(\theta_3) - l_t.\cos(\theta_3 + \theta_4) \quad (12)$$

$$Z_{TD} = 0 \quad (13)$$

Knee position – right leg

$$X_{JD} = l_f.sen(\theta_3) \tag{14}$$

$$Y_{JD} = -l_f.\cos(\theta_3) \tag{15}$$

$$Z_{JD} = 0 \tag{16}$$

The velocity the each point is obtained through the first derivative of position and the acceleration through the second derivative of position.

In the Figure 4 is showed the angular displacements of joints during the human gait.

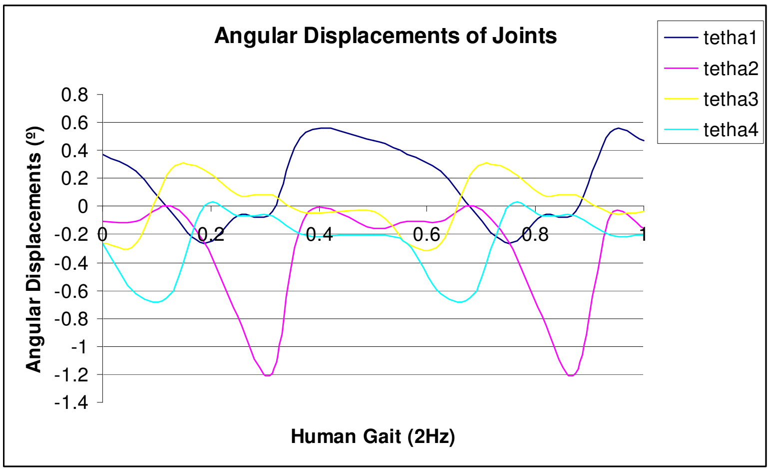

Figure 4. The angular displacements during the human gait.

In the Figure 5 is showed the comparison between the gait cycle (Fig. 1) and the kinematics.

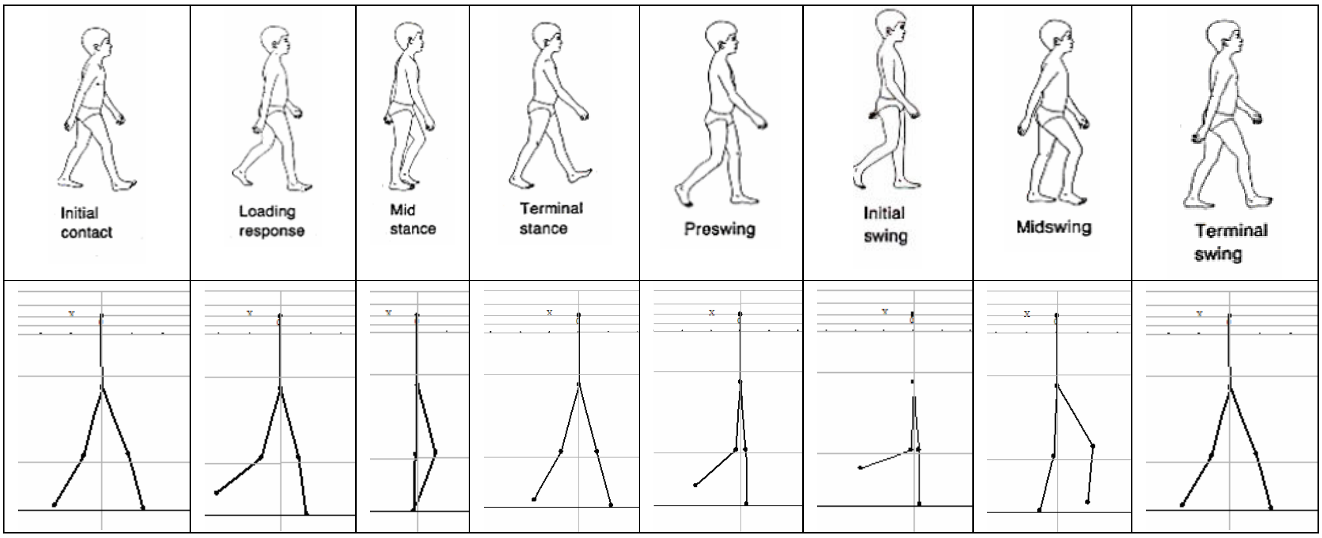

Figure 5. The kinematics scheme.

3. DYNAMIC

The scheme to calculate the dynamic modelling from human locomotor is shown in the Fig. 6.

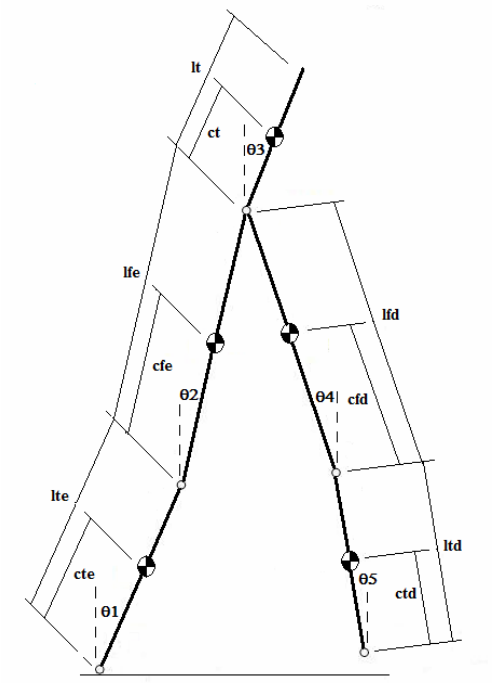

Figure 6. The lower limbs scheme.

The dynamic modelling is based on the Lagrange formulation. The energy balance is shown in Eq. (17).

$$L = \sum_{i=1}^{5}(K_i) - \sum_{i=1}^{5}(U_i) \tag{17}$$

The kinetic and potential energy are presented in Eq. (18) and (19) respectively.

$$K_i = \frac{1}{2}.m_i.v_{ci}^2 + \frac{1}{2}.I_i.\dot{\theta}_i^2 \tag{18}$$

$$U_i = m_i.g.y_{ci} \tag{19}$$

In the Equations (20), (21), (22), (23), (24), (25), (26), (27), (28) e (29) are shown the kinetic and potential energy to each link.

$$K_1 = \frac{1}{2}.m_{te}.v_{cte}^2 + \frac{1}{2}.I_{te}.\dot{\theta}_1^2 \tag{20}$$

$$U_1 = m_{te}.g.y_{cte} \tag{21}$$

$$K_2 = \frac{1}{2}.m_{fe}.v_{cfe}^2 + \frac{1}{2}.I_{fe}.\dot{\theta}_2^2 \tag{22}$$

$$U_2 = m_{te}.g.y_{cfe} \tag{23}$$

$$K_3 = \frac{1}{2}.m_t.v_{ct}^2 + \frac{1}{2}.I_t.\dot{\theta}_3^2 \qquad (24)$$

$$U_3 = m_t.g.y_{ct} \qquad (25)$$

$$K_4 = \frac{1}{2}.m_{fd}.v_{cfd}^2 + \frac{1}{2}.I_{fd}.\dot{\theta}_4^2 \qquad (26)$$

$$U_4 = m_{fd}.g.y_{cfd} \qquad (27)$$

$$K_5 = \frac{1}{2}.m_{td}.v_{ctd}^2 + \frac{1}{2}.I_{td}.\dot{\theta}_5^2 \qquad (28)$$

$$U_5 = m_{td}.g.y_{ctd} \qquad (29)$$

In the Equations (30) and (31) are presented the Lagrange formulation from movement equation.

$$\frac{d}{dt}\{\frac{\partial L}{\partial \dot{\theta}_i}\} - \frac{\partial L}{\partial \theta_i} = T_i \qquad (30)$$

$$D(\theta)_{5x5}\,\ddot{\theta} + h(\theta,\dot{\theta})_{5x1} + G(\theta)_{5x1} = T(\theta)_{5x1} \qquad (31)$$

$D(\theta)_{5x5}$ is the mass matrix, $h(\theta,\dot{\theta})_{5x1}$ is the vector of centripetal and Coriolis forces, $G(\theta)_{5x1}$ is the vector of gravitational forces and $T(\theta)_{5x1}$ is the vector of torques.

In a Table 1 are presented mass matrix elements.

Table 1. The mass matrix elements.

$D_{11} = I_{te} + m_{te}c_{te}^2 + (m_{fe} + m_t + m_{fd} + m_{td})l_{te}^2$	$D_{12} = [m_{fe}l_{te}c_{fe} + (m_t + m_{fd} + m_{td})l_{te}l_{fe}]\cos(\theta_1 - \theta_2)$
$D_{13} = [m_t l_{te} c_t]\cos(\theta_1 - \theta_3)$	$D_{14} = [m_{fd}l_{te}(l_{fd} - c_{fd}) + m_{td}l_{te}l_{fd}]\cos(\theta_1 + \theta_4)$
$D_{15} = [m_{td}l_{te}(l_{td} - c_{td})]\cos(\theta_1 + \theta_5)$	$D_{21} = [m_{fe}l_{te}c_{fe} + (m_t + m_{fd} + m_{td})l_{te}l_{fe}]\cos(\theta_1 - \theta_2)$
$D_{22} = I_t + m_{fe}c_{fe}^2 + (m_t + m_{fd} + m_{td})l_{fe}^2$	$D_{23} = [m_t l_{fe} c_t]\cos(\theta_2 - \theta_3)$
$D_{24} = [m_{fd}l_{fe}(l_{fd} - c_t) + m_{td}l_{fe}l_{fd}]\cos(\theta_2 + \theta_4)$	$D_{25} = [m_{td}l_{fe}(l_{td} - c_{td})]\cos(\theta_2 + \theta_5)$
$D_{31} = [m_t l_{te} c_t]\cos(\theta_1 - \theta_3)$	$D_{32} = [m_t l_{fe} c_t]\cos(\theta_2 - \theta_3)$
$D_{33} = I_t + m_t c_t^2$	$D_{34} = 0$
$D_{35} = 0$	$D_{41} = [m_{fd}l_{te}(l_{fd} - c_{fd}) + m_{td}l_{te}l_{fd}]\cos(\theta_1 + \theta_4)$
$D_{42} = [m_{fd}l_{fe}(l_{fd} - c_{fd}) + m_{td}l_{fe}l_{fd}]\cos(\theta_2 + \theta_4)$	$D_{43} = 0$
$D_{44} = I_{fd} + m_{fd}(l_{fd} - c_{fd})^2 + m_{td}l_{fd}^2$	$D_{45} = [m_{td}l_{fd}(l_{td} - c_{td})]\cos(\theta_4 - \theta_5)$
$D_{51} = [m_{td}l_{te}(l_{td} - c_{td})]\cos(\theta_1 + \theta_5)$	$D_{52} = [m_{td}l_{fe}(l_{td} - c_{td})]\cos(\theta_2 + \theta_5)$
$D_{53} = 0$	$D_{54} = [m_{td}l_{fd}(l_{td} - c_{td})]\cos(\theta_4 - \theta_5)$
$D_{55} = I_{td} + m_{td}(l_{td} - c_{td})^2$	

In a Table 2 are presented the vector of centripetal and Coriolis forces elements.

Table 2. The vector of centripetal and Coriolis forces elements.

$h_{11} = [m_{fe}l_{te}c_{fe} + (m_t + m_{fd} + m_{td})l_{te}l_{fe}]sen(\theta_1 - \theta_2) + [m_{te}l_{te}c_t]sen(\theta_1 - \theta_3) - [m_{fd}l_{te}(l_{fd} - c_{fd}) + m_{td}l_{te}l_{fd}]sen(\theta_1 + \theta_4) - [m_{td}l_{te}(l_{td} - c_{td})]sen(\theta_1 + \theta_5)$
$h_{21} = [m_{fe}l_{te}c_{fe} + (m_t + m_{fd} + m_{td})l_{te}l_{fe}]sen(\theta_1 - \theta_2) + [m_t l_{fe}c_t]sen(\theta_2 - \theta_3) - [m_{fd}l_{fe}(l_{fd} - c_{fd}) + m_{td}l_{fe}l_{fd}]sen(\theta_2 + \theta_4) - [m_{td}l_{fe}(l_{td} - c_{td})]sen(\theta_2 + \theta_5)$
$h_{31} = -[m_{te}l_{te}c_t]sen(\theta_1 - \theta_3) + [m_t l_{fe}c_t]sen(\theta_2 - \theta_3)$
$h_{41} = -[m_{fd}l_{te}(l_{fd} - c_{fd}) + m_{td}l_{te}l_{fd}]sen(\theta_1 + \theta_4) - [m_{fd}l_{fe}(l_{fd} - c_{fd}) + m_{td}l_{fe}l_{fd}]sen(\theta_2 + \theta_4) + [m_{td}l_{fd}(l_{td} - c_{td})]sen(\theta_4 - \theta_5)$
$h_{51} = -[m_{td}l_{te}(l_{td} - c_{td})]sen(\theta_1 + \theta_5) - [m_{td}l_{fe}(l_{td} - c_{td})]sen(\theta_2 + \theta_5) - [m_{td}l_{fd}(l_{td} - c_{td})]sen(\theta_4 - \theta_5)$

In a Table 3 are presented the vector of gravitational forces elements.

Table 3. The vector of gravitational forces elements.

$G_{11} = -[m_{te}c_{te} + m_{fe}l_{te} + m_t l_{te} + m_{fd}l_{te} + m_{td}l_{te}]g.sen(\theta_1)$
$G_{21} = -[m_{fe}c_{fe} + m_t l_{fe} + m_{fd}l_{fe} + m_{td}l_{fe}]g.sen(\theta_2)$
$G_{31} = -[m_t c_t]g.sen(\theta_3)$
$G_{41} = [m_{fd}(l_{fd} - c_{fd}) + m_{td}l_{fd}]g.sen(\theta_4)$
$G_{51} = [m_{td}(l_{td} - c_{td})]g.sen(\theta_5)$

Substituing in the Eq. (31) the parameters of Tab. 1, 2 and 3 are obtained equations related to torques at each joint of the leg.

4. CONCLUSION

In this study are presented the mathematical modeling of kinematics and dynamics of lower limbs. Through this modeling can be determined the positions and the value of the torques of the legs joints. These parameters are very important for the analysis of human gait. This study can be employed in various areas of biomechanical such as transfemoral prosthesis, rehabilitation and biped robots. In this case the study will be applied in research on development of intelligent transfemoral prosthesis.

5. REFERENCES

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