DISCRIMINANT ANALYSIS TO CRACK INITIATION IN FATIGUE TESTS USING IMPEDANCE-BASED SHM

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Abstract. *Discriminant analysis is a statistical technique used to classify a set of observations based on a previously defined probability distribution of the influenced parameters. Basically, it consists of a maximum likelihood estimation of the predefined groups from a set of observations. Besides, fatigue testing takes an important role in many industrial developments to identify the life cycle of system components and products. In this context, the present contribution focuses on proposing a method of continuous impedance-based structural health monitoring of a structure under fatigue tests and the use of a discriminant function analysis as a warning technique for crack initiation. Two PZT patches were bonded on the surface of the coupon and fatigue tests were performed in parallel with impedance-based health monitoring. Then, damage metrics for both sensors were obtained and correlated to the number of cycles of the test and used as parameters of the discriminant function. The tests presented here were made in coupons but can be extended to more complex structures and industrial products.*

Keywords*: impedance-based health monitoring, fatigue tests, discriminant analysis.*

1. IMPEDANCE-BASED STRUCTURAL HEALTH MONITORING

The technique of impedance-based structural health monitoring utilizes the material's piezoelectric character and is a non-destructive evaluation method (RAJU, 1997). The basic concept of this technique is the monitoring of changes in the structure's mechanical impedance caused by the presence of damage. Because the direct measurement of the structure's impedance mechanical is a difficult task, the method uses piezoelectric materials (PZT) bonded on or incorporated into the structure, allowing the measurement of electrical impedance. It is related to the structure mechanical impedance, which is affected by the presence of damage. Evidently, it is considered that the piezoelectric material used as a sensor of electrical impedance remains intact during the test.

The development theorists who proposed the use of impedance as a technique for structural monitoring was (by) Liang *et al.* (1994) and subsequently the theory was extended by Chaudhry (1995, 1996), Sun *et al.* (1995), Park *et al.* (1999, 2000, 2001, 2003), Giurgiutiu and Zagrai (2000), Soh *et al.* (2000), Bhalla *et al.* (2002), Giurgiutiu *et al.* (2002, 2003), Moura and Steffen (2004), Peairs (2000), Tsuruta (2008) and Moura (2008).

The health monitoring method utilizes impedance sensors to monitor changes in structural stiffness, damping and mass. The impedance sensors consist of small piezoelectric patches, usually smaller than 25x25x0.1mm, which are used to measure directly the local dynamic response.

The piezoelectric material acts directly producing an electric change when a mechanical stress is applied on the material. Conversely, a mechanical stress is produced when an electric field is applied. The impedance-based method uses simultaneously both versions, direct and inverse, the piezoelectric effect for measurements of impedance (Park *et. al*, 1999).

Liang *et. al* (1994) demonstrated that the admittance Y (ω) of the actuator PZT can be written as a function of the combined actuator PZT's and structure's mechanical impedance, as shown in Eq. (1):

$$Y(\omega) = i\omega a\left(\bar{\varepsilon}_{33}{}^{T}(1-i\delta) - \frac{Z_s(\omega)}{Z_s(\omega)+Z_a(\omega)} d_{3x}{}^{2}\hat{Y}_{xx}^{E}\right) \quad (1),$$

where $Y(\omega)$ is the electrical admittance (inverse of impedance), $Z_a(\omega)$ and $Z_s(\omega)$ are the PZT material's and the structure's mechanical impedances, respectively. $\hat{Y}_{xx}^{E}$ is the complex Young's modulus of the PZT with zero electric field, d_{3x} is the piezoelectric coupling constant in the arbitrary x direction at zero electric field, $\bar{\varepsilon}_{33}^{T}$ is the dielectric constant at zero stress, δ is the dielectric loss tangent of the PZT, and a is a geometric constant of the PZT. Assuming that the mechanical properties of PZT do not vary over time used for monitoring, Eq. (1) shows that the electrical impedance of PZT is directly related to the structure's impedance. Damage causes changes in structure's mechanical impedance, changing local dynamics features. Hence, the PZT's electrical impedance is used to monitor the structure health represented by the structure's mechanical impedance.

The graph of response from impedance provides a qualitative assessment of the damage. For a quantitative assessment of the failure a damage metric is used (RAJU, 1997). The damage metric used in this work is given by the average difference square metric [20], and its mathematical formulation is presented by Eq. (2)

$$ASD = \sum_{i=1}^{n}\left[Re(Z_{1,i}) - \left(Re(Z_{2,i}) - \delta\right)\right]^2 \qquad (2),$$

where,

$$\delta = (\bar{Z}_1) - (\bar{Z}_2) \qquad (3),$$

Where, ASD is the average difference square metric, $Re(Z_{1,i})$ is the impedance of the PZT measure at healthy conditions, and $Re(Z_{2,i})$ is the impedance for the comparison with the baseline measurement at frequency interval i, $Re\bar{Z}_1$ is the average value of the initial curve and $Re\bar{Z}_2$is the average value of the subsequent curve.

2. DISCRIMINANT ANALYSIS METHOD

The discriminant analysis is a technique that can be used for element classification of a sample or population, but differs from the cluster analysis. For its implementation to classify each element of the groups it is necessary to be predefined the classification. This knowledge allows the development of a mathematical function called the rule of classification of existing groups. In the discriminant analysis, the comparison between the sample element and candidate groups is generally done through construction of a mathematical rule for classification or discrimination based on the theory of probability. For each new element sample, the rule of classification allows researchers to decide which is most likely to have generated their values in p-characteristics.

Initially, only two populations are considered and a set of independent observations from each population. If the probability distribution of measurements of the elements' characteristics from each sample population is known the principle of maximum likelihood can be used to construct a classification rule that minimizes the chance of incorrectly classifying a sample.

To calculate the ratio between the two probabilities distributions of two populations, the maximum likelihood is used, defined by Eq. (4).

$$\lambda(x) = \frac{density\ function\ x\ population 1}{density\ function\ x\ population 2} = \frac{f_1(x)}{f_2(x)} \qquad (4),$$

in the case of normal distribution, it becomes Eq.(5).

$$\lambda(x) = \frac{\frac{1}{\sqrt{2\pi\sigma}}\exp\left\{\frac{-1}{2}\left(\frac{x-\mu_1}{\sigma}\right)^2\right\}}{\frac{1}{\sqrt{2\pi\sigma}}\exp\left\{\frac{-1}{2}\left(\frac{x-\mu_2}{\sigma}\right)^2\right\}} = \exp\left\{\frac{-1}{2}\left[\left(\frac{x-\mu_1}{\sigma}\right)^2 - \frac{-1}{2}\left(\frac{x-\mu_2}{\sigma}\right)^2\right]\right\} \qquad (5),$$

where μ_1 and μ_2 are the mean values of the population 1 and 2, σ^2 is the variance of the two populations, and x is the sample of each population.

For a fixed sample x, when $\lambda(x) > 1$, the value of the density function calculated for the population 1 is greater than that obtained using the distribution of the population 2. When $\lambda(x) = 1$, the two density functions have numerically the same value and, therefore, the sample can be classified either as belonging to population 1 or (and) population 2. However, in cases like these, more additional information is obtained to take more appropriate decision.

Back in the Eq.(5), taking the neperian logarithm and multiplying $\lambda(x)$ by -2, the Eq.(6) is obtained.

$$-2\ln(\lambda(x)) = \left(\frac{x-\mu_1}{\sigma}\right)^2 - \left(\frac{x-\mu_2}{\sigma}\right)^2 = \frac{1}{\sigma^2}\left[(x-\mu_1)^2 - (x-\mu_2)^2\right] \qquad (6),$$

which is related to the difference of the weighted squared Euclidean distance of x and μ_1 and x and μ_2, and the weighing factor equal to the inverse of the varianceσ^2. Considering (the) Eq.(7):

$$\lambda(x)=\frac{\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{\frac{-1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right\}}{\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left\{\frac{-1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right\}}=\frac{\sigma_1}{\sigma_2}\exp\left\{\frac{-1}{2}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]\right\} \tag{7}$$

For $-2ln(\lambda(x))$, then Eq.(8) is obtained.

$$-2\ln(\lambda(x))=-2\ln\left(\frac{\sigma_1}{\sigma_2}\right)+\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right] \tag{8}$$

And again, it classifies the sample as part of the population 1, if $-2ln(\lambda(x))$is smaller than zero, $-2ln(\lambda(x))$as the population 2, if greater than zero, and any of the two populations if $-2ln(\lambda(x))$ is equal zero.

The rule presented so far for the two populations can be extended to the case where $p > 1$variables and have measurements in each element of each sample and population data are derived from p-normal distributions p-varied.

3. FATIGUE TESTS USING THE IMPEDANCE-BASED STRUCTURAL HEALTH MONITORING METHOD

Previous tests were done to determine the load applied and the number of cycles, two 10x10x0.1mm PZT patches were bonded to a test sample. The PZT patches were bonded outside the area of greatest stress concentration, as shown in Fig.1. This caution avoided high stresses that could jeopardize the integrity of the PZT patches.

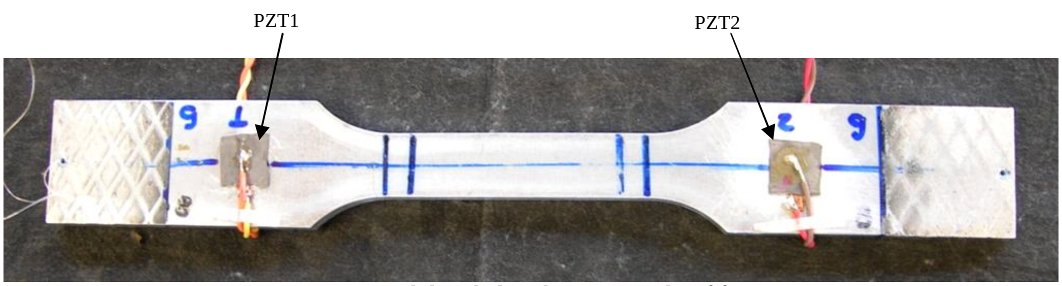


Figure 1. PZT patch bonded in the test sample of fatigue test.

The test sample presented a visible crack after 50.636 cycles. Six impedance measurements were taken at each of 8000 cycles until reaching 48000, plus six additional measurements that were made when the crack became visible (50,636 cycles). In summary, the procedure was as follows:

a) First, measurements were taken to evaluate/determine the soundness state of the structure (before the fatigue test);

b) Then, the test samples were placed in the fatigue test machine and 8000 force cycles were applied;

c) The test sample was withdrawn from the fatigue test machine for the impedance measurements;

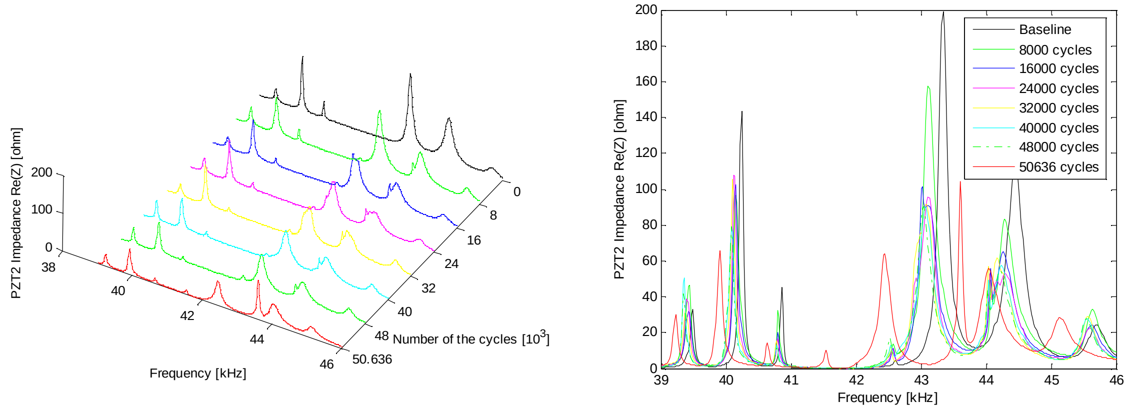
d) Again the test sample was placed in the fatigue test machine for the application of other 8000 cycles;

e) The experiment was repeated until the specimen had a visible crack.

4. RESULTS AND DISCRIMINANT ANALYSIS EXPERIMENT

The frequency range of 30 kHz to 50 kHz is the most commonly used by the impedance-based method(MOURA, 2008). In the present case, the range used was 39 kHz to 46 kHz. The real part of the impedance was measured for PZT2 as shown in Fig.2.

The signal is very different when the crack is visible. This can also be seen through the signals measured for PZT2 for the same frequency range (Fig. 2), moreover that PZT2 is more distant from the crack than PZT1.

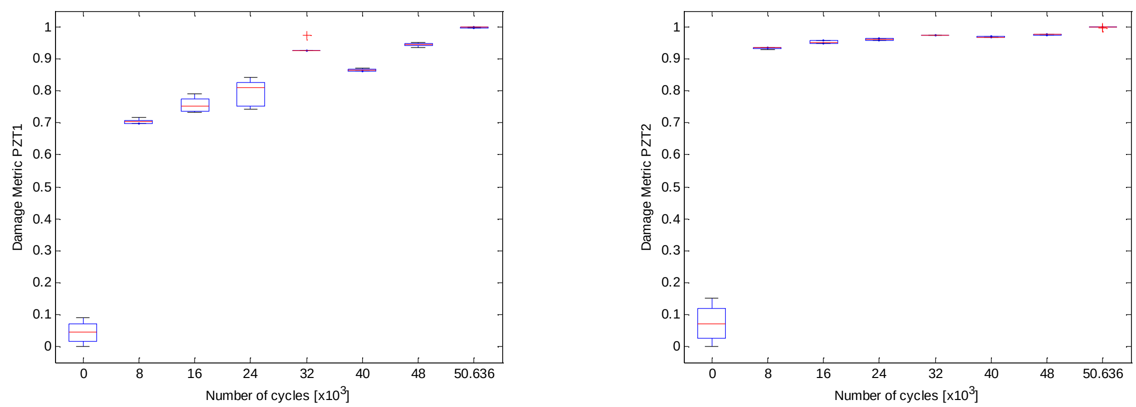


a - 3D Impedance Curves

b - Impedance curves

Figure 2. The electrical impedance measurements of PZT2 signs for the frequency range of 39 kHz to 46 kHz.

The ASD metric was applied to the impedance signals. They were normalized with the highest value. The results presented in Fig. (3) were obtained. An increase in the value of the ASD metric with the increase in the number of cycles for PZT1 and PZT2 is observed.



(a) Damage Metric PZT1

(b) Damage Metric PZT2

Figure 3. Damage Metric (ASD) for the frequency range of 39 kHz to 46 kHz.

Fig, (3a) shows that the damage metric (ASD) for the 32000 cycles does not follow same the behavior observed for smaller numbers of cycles. This may be related to some problem in data acquisition at the 32000 cycles measurement.

Based on the experimental data, a discriminant analysis procedure was evaluated to identify if by the use of the ASD metrics one is capable of identifying a crack. For this purpose, it was considered both PZT patches and 32, 40 and 48 x 10^3 cycles as "before" crack group and 50.636 x 10^3 for "after" crack group.

First, a procedure was made to identify the normality of the data. Fig. (4) shows the normality test for the group "before" crack due to the ASD impedance metrics.

After the test of normality of all groups, i.e. "before" and "after", considering PZT1 and PZT2, it is possible to assume a Gaussian curve (for them) for discriminant analysis purposes, as shown in Fig.(5).

Figure (5) illustrates both PZT patches for the two conditions or groups: before and after the initial crack. As it can be seen, the probability distributions for ASD metrics for the "before" group are wider than for the "after" one This happens probably because the size of sampling groups and a greater number of cycles is considered for the "before" group (32-48 x 10^3 cycles).

Based on those distributions a discriminant analysis was done using the software Minitab. For this purpose, the raw data was divided in two sets: classification (84%) and test (16%). The first one was used to build a quadratic discriminant function, the classifier, while the second one was used to test the classifier. The final result shows a hit of 100% of the correct answer, both classification and test groups. For comparison, the classification of the groups "before" and "after" was checked by another multivariate method: K-Means Clustering Analysis. The ASD impedance metrics for both PZT patches (two variables) were inserted into a K-Means algorithm, also in Minitab, which evaluated the classification of the total metrics in two groups, supposedly, groups "before" and "after" initial crack.

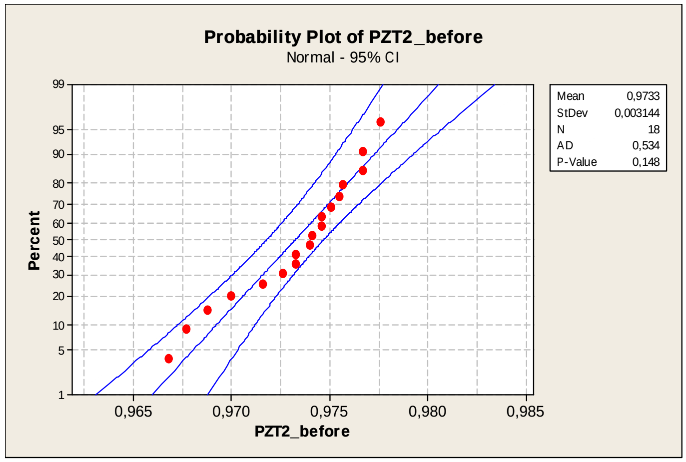
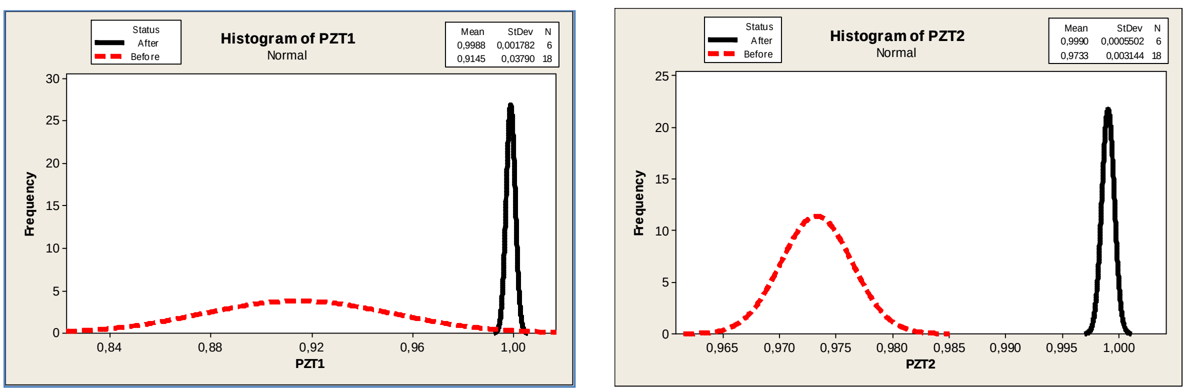


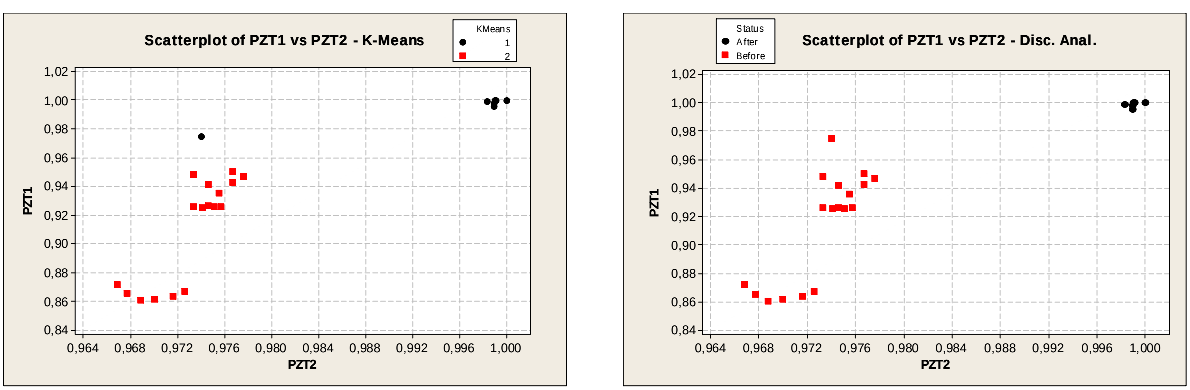
Figure 4. An example of the normality test for ASD metrics from PZT2.



(a) ASD metric distributions – PZT1 (b) ASD metric distributions – PZT2

Figure 5. ASD impedance metric distributions for both PZT patches.

The two results are presented in Fig.(6). Fig.(6(a)) presents the classification by the K-Means method and a misclassification of one element (black circle). In Fig.(6b()) this mistake can be identified (now this element is a red square) while the Discriminant Analysis correctly classified it.



(a) K-Means Classification (b) Discriminant Analysis Classification

Figure 6. Classification of the ASD impedance metrics.

5. CONCLUSIONS

With this contribution the applicability of the discriminant analysis method for SHM systems was shown. The difference between both multivariate methods was presented, because while the Clustering K-Means method uses only the distances to calculate if an element is part of one group, the discriminant analysis by the use of maximum likelihood takes into account the probabilistic/stochastic behavior of the variables.

The results show a good application of discriminant analysis for monitoring systems applications. The ASD impedance metric was used for a single coupon but the proof of the concept can be extended to a complex structure for fatigue crack identification.

Another positive aspect of the method is that the procedure can be implemented by the use of different damage metrics and others techniques of monitoring.

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