ACTIVE VIBRATION CONTROL OF STRUCTURES WITH PARAMETRIC UNCERTAINTIES

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Abstract. *The aim of this paper is to apply the techniques of active control in flexible structures for a cantilever beam considering parametric uncertainties in the model. The uncertainties was considered in damping matrix and modulus of elasticity of the material. Uncertainties are handled by using the polytopic form and the $\mathcal{H}_\infty$ control technique, formulated by linear matrix inequalities - LMI, was utilized.*

Keywords: *Parametric uncertainties, $\mathcal{H}_\infty$ control, LMI.*

1. INTRODUCTION

Active vibration control of structures has its main interest in several applications such as aerospace structures, antennas, noise attenuation of panels and others. The structural model used for the active vibration control presents some uncertainties and the active control system needs to be effective considering them. Two kinds of uncertainties are usually considered: dynamic and parametric. The parametric uncertainties will be the issue of this work and it is related to unknown values of parameters such as mass, stiffness and damping of the structure. The dynamic uncertainties may be originate from imperfections in the model structure, from the lack of representation of some physical effect or from simplifying assumptions (Mazoni, 2008).

The uncertainties in parameters ought to be estimated in order to be considered in the active control design. The control design employs a structural model, which can be experimentally identified if the structure is available or determined using modeling techniques such as a finite element model. When dealing with flexible structures, parameters such as damping coefficient and elastic modulus of the material are usually known within a finite precision, so these parameters need to be modeled as presenting parametric uncertainties.

The parametric uncertainties can be considered in the model of the structure in polytopic form or by Δ methodology (Mazoni, 2008). For this study, the parametric uncertainties will be considered in polytopic form.

Modern control techniques, such as the $\mathcal{H}_\infty$ control, are applied to some mechanical problems. The objective of the $\mathcal{H}_\infty$ technique is to minimize the worst case of vibration (maximum peak) in terms of frequency response and can be formulated using linear matrix inequalities that transform the design in a convex optimization problem. This optimization problem can be solved using software packages as the Robust Control Toolbox of MATLAB (Gahinet *et al.*, 1995)

2. STRUCTURAL MODEL

For the present case, the beam finite element was considered with 2 nodes and 3 degrees of freedom in each node, as shown in Fig. 1. From the considered beam element and using the Hermitian formulation, the stiffness and the mass matrices of a 2-D frame element can be found as shown repectively in Eq. (1) and Eq. (2) (Kwon and Bang, 1997). The stiffness and mass matrices of the considered problem can be found through the *assembly*, which is the sum, in correct allocation, of each element matrix. The next step of the finite element method is to apply the boundary conditions.

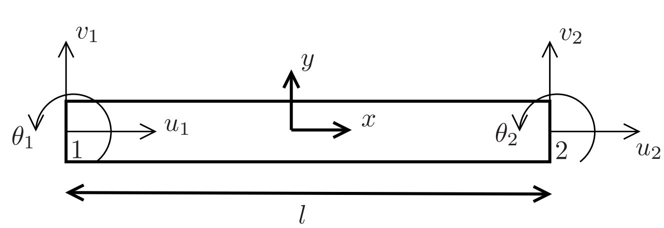

Figure 1. Beam element.

$$K^e = \frac{E}{l^3}\begin{bmatrix} Al^2 & 0 & 0 & -Al^2 & 0 & 0 \\ 0 & 12I & 6Il & 0 & -12I & 6Il \\ 0 & 6Il & 4Il^2 & 0 & -6Il & 2Il^2 \\ -Al^2 & 0 & 0 & Al^2 & 0 & 0 \\ 0 & -12I & -6Il & 0 & 12I & -6Il \\ 0 & 6Il & 2Il^2 & 0 & -6Il & 4Il^2 \end{bmatrix} \quad (1)$$

$$M^e = \rho \text{Al}\begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 13/35 & (11/210)l & 0 & 9/70 & -(13/420)l \\ 0 & (11/210)l & (1/105)l^2 & 0 & (13/420)l & -(1/140)l^2 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 9/70 & (13/420)l & 0 & 13/35 & -(11/210)l \\ 0 & -(13/420)l^2 & -(1/140)l^2 & 0 & -(11/210)l & (1/105)l^2 \end{bmatrix} \quad (2)$$

The studied model is performed from an aluminium cantilever beam (L x h x b = 1.10 x 0.032 x 0.003 m) divided into 24 nodes, 74 degrees of freedom. The fixed end constrained the first two nodes (0.10 m from left edge). After the application of the constrained conditions the studied case will now have 44 degrees of freedom and with a properly enumeration the problem will be as Fig. 2 shows.

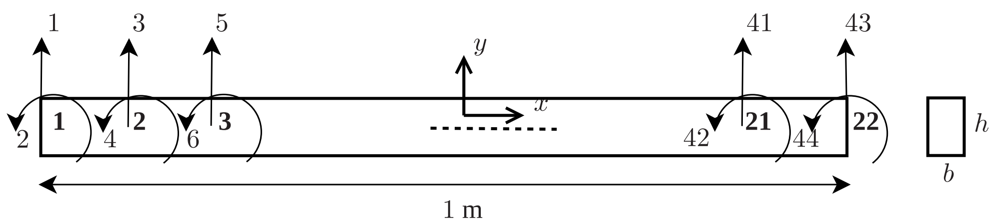

Figure 2. Considered beam.

The inputs and the outputs for the active control problem are:

- The disturbance on the input, *d*, will be a momentum, degree of freedom 6,
- The control effort, *u*, will be a momentum, degree of freedom 24,
- The performance, *z*, and the measurement, *y*, signal output will be a vertical displacement, degree of freedom 43.

Figure 3 shows the final configuration for vibration control.

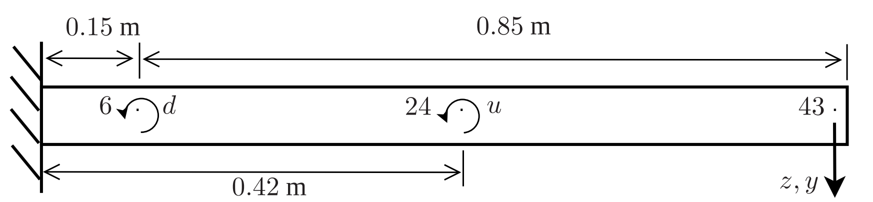

Figure 3. Considered beam for control.

The dynamic equation that describes the beam vibration is:

$$M\ddot{x} + \overline{C}\dot{x} + Kx = u(t) + d(t) \quad (3)$$

where *M*, $\overline{C}$ e *K*, are respectively the mass, damping and stiffness system matrices. The mass and the stiffness matrices were found with a program performed by the authors using finite element model of the beam element as shown in Eq. (1) and (2). The damping matrix will be found through the model of proportional damping that is a function of mass and stiffness matrices, Eq (4).

$$\overline{C} = \alpha(M) + \beta(K) \quad (4)$$

The uncertainties was considered for all the beam in the damping matrix and in the elastic modulus of the material. The uncertainty in damping was considered because this is a difficult parameter to be known precisely for flexible structures, and the elastic modulus was considered uncertain, because it may assume different values with the variation on the alloy compounds.

2.1 PARAMETRIC UNCERTAINTIES MODELED IN POLYTOPIC FORM

In this kind of representation it is assumed that there is a set of systems that can be found by knowing the variation of uncertain parameters. These set of systems is written as a convex combination of state matrices and form a polytope. In this polytope, because the variation of parameters are known, the vertices are known. It can be written in standard state space notation that the Ψ domain is formed by:

$$\Psi = \{(A_i, B_i, C_i, D_i) \mid i = 1, \ldots, j\} \tag{5}$$

with j the number of uncertain systems and A_i, B_i, C_i e D_i the state space matrices. Figure 4 shows a polytope with the considered uncertainty parameters. The vertices of the polytope are the minimum and the maximum variation around the nominal value of the elastic modulus (E) and the damping matrix ($\overline{C}$) (Santos, 2010). In this study it will be assumed that the parameters of the system do not variate on the time.

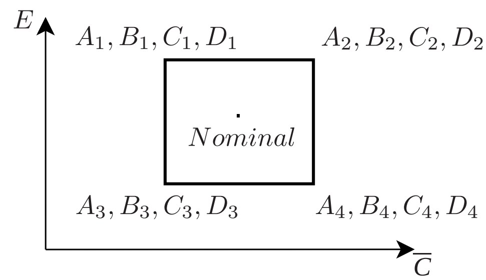

Figure 4. Polytope formed by the uncertainties in the system.

The number of vertices of the polytope can be found using 2^c, with c, the number of uncertain parameters. In this case, $c = 2$, and 4 possible plants (vertices) are possible.

2.2 FINDING THE $\mathcal{H}_\infty$ CONTROLLER

The $\mathcal{H}_\infty$ controller problem consists of finding the controller that minimizes the H-infinity norm, $\|H(s)\|_\infty$, of the closed loop system. This problem can be written as a optimization problem with constrains and can be solved using LMI formulation (Scherer *et al.*, 1997).

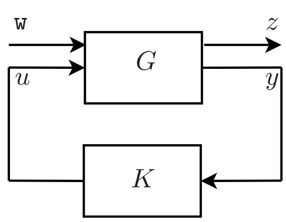

Figure 5. Representation for control.

The closed loop of Fig. 5 is shown in Eq. (6).

$$\begin{aligned} \dot{x} &= Ax + B\mathtt{w} \\ z &= Cx + D\mathtt{w} \end{aligned} \tag{6}$$

and *A*, *B*, *C*, *D* can be found with the open loop of the system and the controller matrices.

From Lyapunov stability concepts, a linear systems will be stable with $P > 0$ when Eq. (7) are satisfied.

$$A'P + PA < 0 \tag{7}$$

Considering $\gamma = \|H(s)\|_\infty$ and $\mu = \gamma^2$, the optimization problem that need to be solved to find the $\mathcal{H}_\infty$ controller are shown in Eq. (8).

$$
\begin{array}{ll}
\textit{minimize} & \mu \\
\textit{with} & \left\{ \begin{array}{c} \left[\begin{array}{ccc} A'P + PA & PB & C' \\ B'P & -I & D' \\ C & D & -\mu I \end{array} \right] < 0 \\ P > 0 \end{array} \right.
\end{array}
\tag{8}
$$

According to (Santos, 2010), a usual way of solving the problem with polytopic uncertainties is to take the extreme systems of the polytopic representation and determine those who compose their convex hull. This consideration increases the area on which the stability are considered and consequently increases the conservatism of the controller to be designed.

For the case with parametric uncertainties, the Eq. (7) can be used as a constraint for each vertex of the polytopic region considered. In this way, the Eq. (9) satisfies the condition of stability for systems with uncertainties.

$$
PA_i + A_i'P < 0, i = 1, ..., j. \tag{9}
$$

Some terms on problem Eq. (8) are nonlinnear and to apply the LMI concept, these restrictions need to be linearized. Applying some changes of variables , the Eq. (8) can be written as Eq. (10) (Scherer *et al.*, 1997).

$$
\begin{array}{ll}
\textit{minimize} & \mu \\
\textit{with} & \left\{ \begin{array}{c} LMI_i \\ \left[\begin{array}{cc} Y & I \\ I & X \end{array} \right] > 0 \end{array} \right.
\end{array}
\tag{10}
$$

In Eq. (10), the LMI_i term is the Eq. (11) for each vertex of the considered polytope. The optimization variables are μ, X, Y, F, L e M. The solution of Eq. (10) should find a $\mathcal{H}_\infty$ stable controller for all the polytopic region.

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix} AY + B_2F & A + B_2D_cC_2 \\ M & XA + LC_2 \end{bmatrix} + \begin{bmatrix} YA' + F'B_2' & M' \\ A' + C_2'D_c'B_2' & A'X + C_2'L' \end{bmatrix} & \begin{bmatrix} B_1 + B_2D_cD_{21} \\ XB_1 + LD_{21} \end{bmatrix} & \begin{bmatrix} YC_1' + F'D_{12}' \\ C_1' + C_2'D_c'D_{12}' \end{bmatrix} \\
\hline
sym. & -I & D_{11}' + D_{21}'D_c'D_{12}' \\
\hline
sym. & sym. & -\mu I
\end{array}\right] < 0 \qquad (11)
$$

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2.3 WEIGHTING FUNCTIONS DESIGN

Spillover phenomenon, which is a result of uncontrolled or unmodelled modes in the plant, represents a significant factor of instability in control system. To avoid this effect, weighting functions are designed in order to truncate the frequency range of vibration that it is not important for the system. For that, it is necessary to set a frequency limit for the signal to be controlled. A low pass filter, Wz, will be used in the low frequency range in order to give a high weigth to these region. In the other hand, the high frequency region will be attenuated with a high pass filter, Wu, in order to avoid the control effort outside the region of interest (Balas, 1978).

With Wu and Wz, the augmented plant can be found and a controller that stabilizes the plant could be designed. Since the filters designed are able to guarantee the stability of uncontrolled modes, the spillover phenomenon could be avoided and a robustness of the structure could be performed. It is important to say that the closed loop does not include the weighting filters.These filters are only weightings to shape the desired design requirements during the design phase. Fig. 6 shows the control scheme with the filters inclusion.

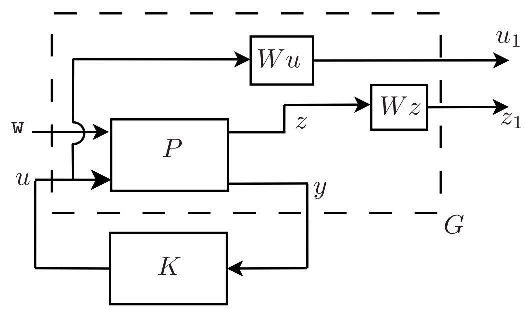

Figure 6. Control scheme with weighting functions.

With all those considerations, the choice of weighting functions for best performance of the controller is shown in Eq. (12).

$$Wz = \frac{200}{s+2} \; and \; Wu = \frac{s}{s+200} \tag{12}$$

3. RESULTS

For vibration control, it was considered as nominal value, from Eq. (4), that α and β are, respectively, $2e^{-5}$ and $1e^{-1}$. The nominal value of elastic modulus was considered as 70 GPa. The variation was considered hypothetically as 30% for the damping matrix and 10% for the elastic modulus. With these considerations, the politope formed by the variation in the system parameters will have 4 vertices, as shown in Tab. 1.

Table 1. Vertices of the politope.

Vertex	E (GPa)	$\overline{C}$	
		α	β
1	77	$1.4e^{-5}$	$7e^{-2}$
2	77	$2.6e^{-5}$	$1.3e^{-1}$
3	63	$1.4e^{-5}$	$7e^{-2}$
4	63	$2.6e^{-5}$	$1.3e^{-1}$

The model found by finite element method was truncated in 12 modes and considered as the real plant. Figure 7 shows the considered real plant without uncertainties (with nominal values) and the real plants with uncertainties (each vertex of the politope). For simulation purposes, to find the $\mathcal{H}_\infty$ controller, the plant considered in project is the truncated plant, with 2 modes. The considered weighting functions for designed can be found in Eq. (12).

To solve the LMI problem and find the $\mathcal{H}_\infty$ controller, the function *mincx* from Robust Control MATLAB Toolbox (Gahinet *et al.*, 1995) was used. The input options parameters set for *mincx* was respectively: $1e^{-9}$, 200, $1e^{6}$, 12 and 1.

The controlled system is the closed loop of each real plant with the found controller. Figure 8 shows the frequency response of the controlled and uncontrolled real plants with uncertainties. The closed loop of the real plant without uncertainties are shown in Fig. 9. Table 2 shows the attenuation in the frequency response for the first five peaks of vibration comparing the uncontrolled and controlled real plant without uncertainties. It could be seen that a good attenuation was achieved, specially on the first mode of vibration, which is the peak of vibration.

The closed loop response to a chirp signal, which is a linear swept frequency signal on time, in this case from 0 to $\frac{1000}{2\pi}$ Hz until 100 s, for each controlled and uncontrolled plants are shown in Fig. 10.

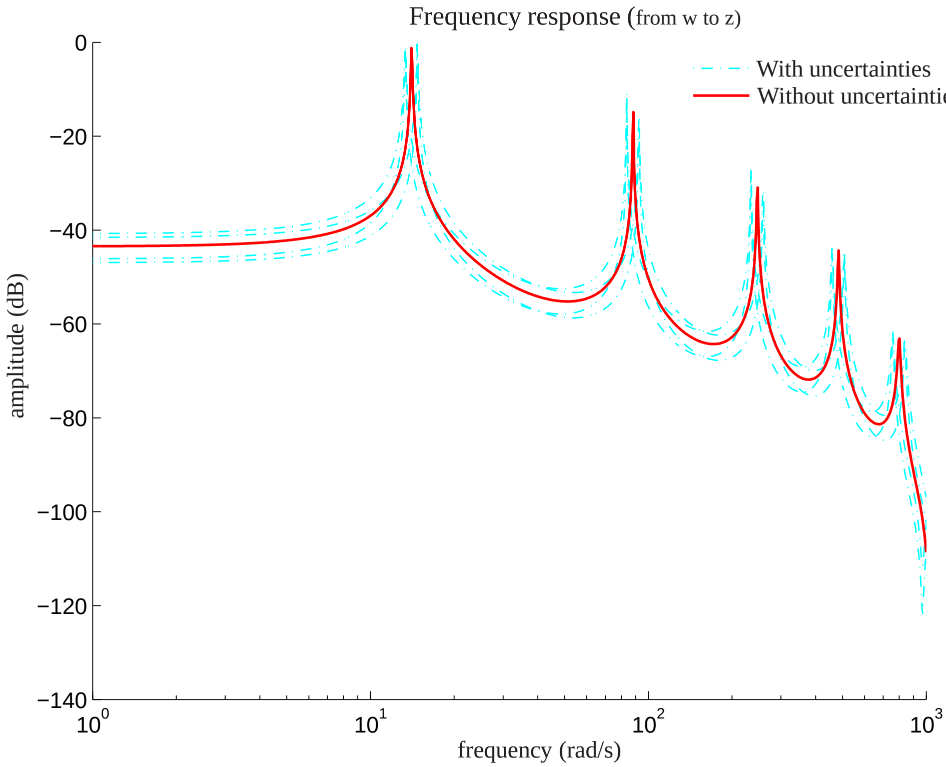

Figure 7. Frequency response of the real plants (with and without uncertainties).

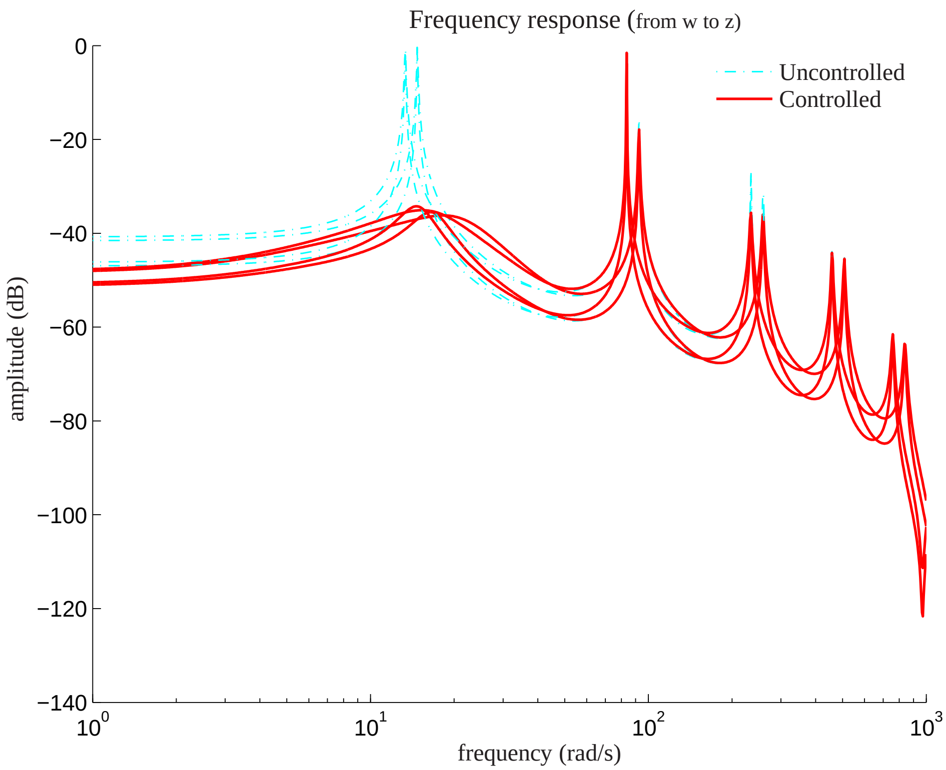

Figure 8. Frequency response of controlled and uncontrolled real plants with uncertainties.

Table 2. Controlled and uncontrolled real plant without uncertainties - vibration attenuation and damping factor - according to Fig. 9

Peak number	peak at w (rad/s)		ξ		Attenuation (db)
	Uncontrolled	Controlled	Uncontrolled	Controlled	
1	14.1	16.8	0.0037	0.3310	-33.6
2	88.1	88.1	0.0014	0.0029	-1.8
3	247	247	0.0026	0.0053	-7.2
4	484	484	0.0049	0.0050	-0.2
5	800	800	0.0080	0.0079	+0.1

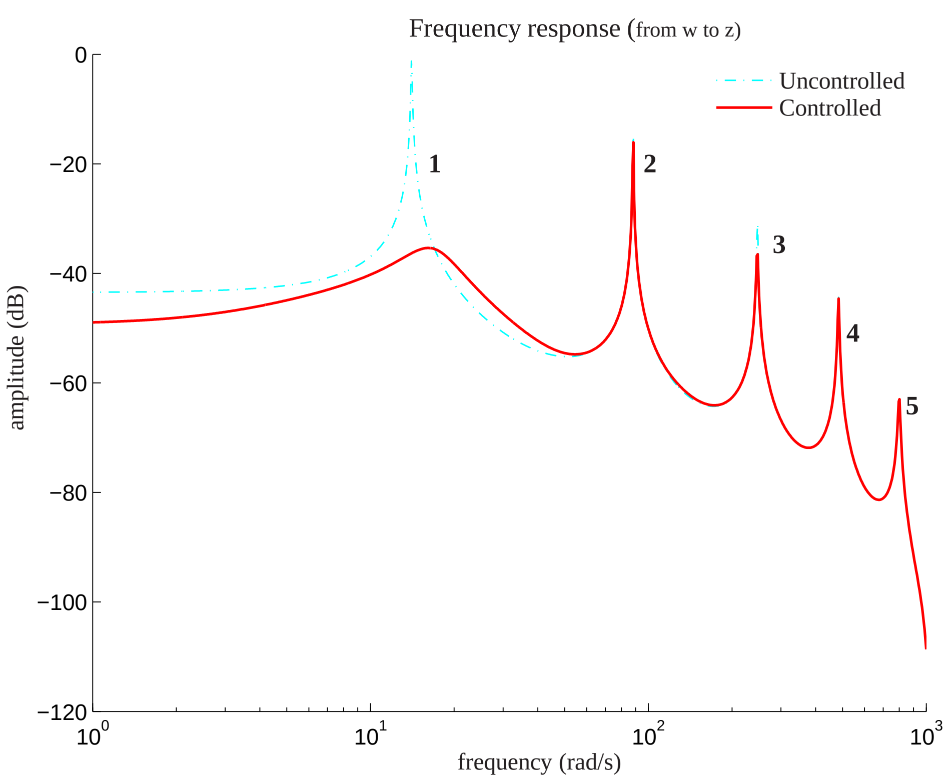

Figure 9. First five peak of vibration in frequency response of controlled and uncontrolled real plant without uncertainties.

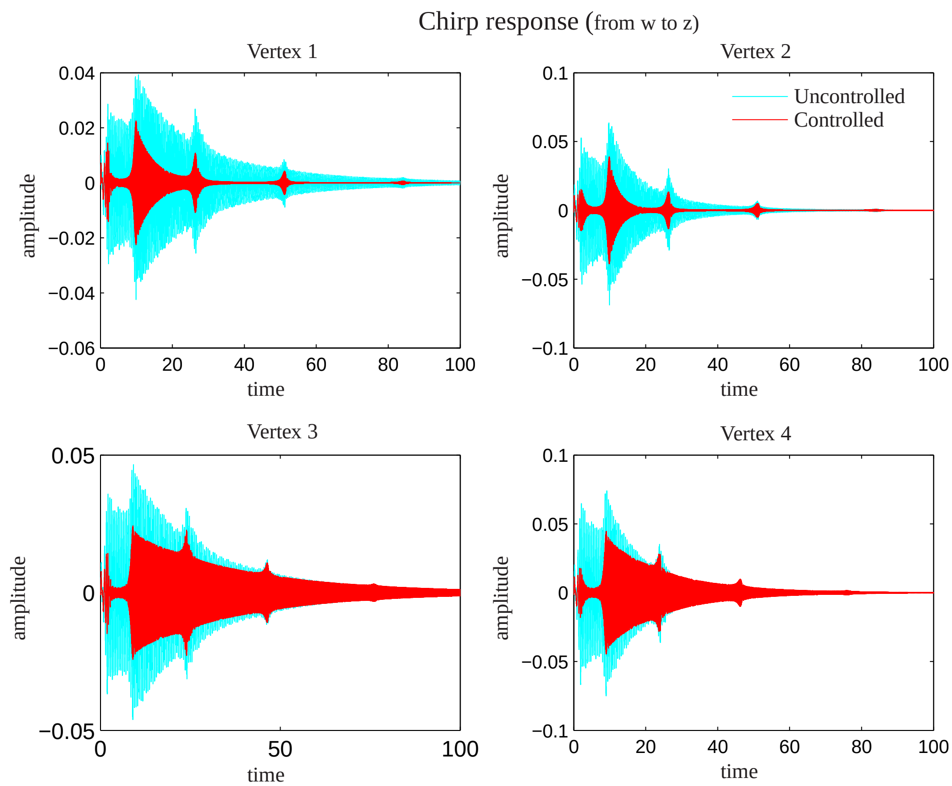

Figure 10. Chirp response of controlled and uncontrolled real plants with uncertainties.

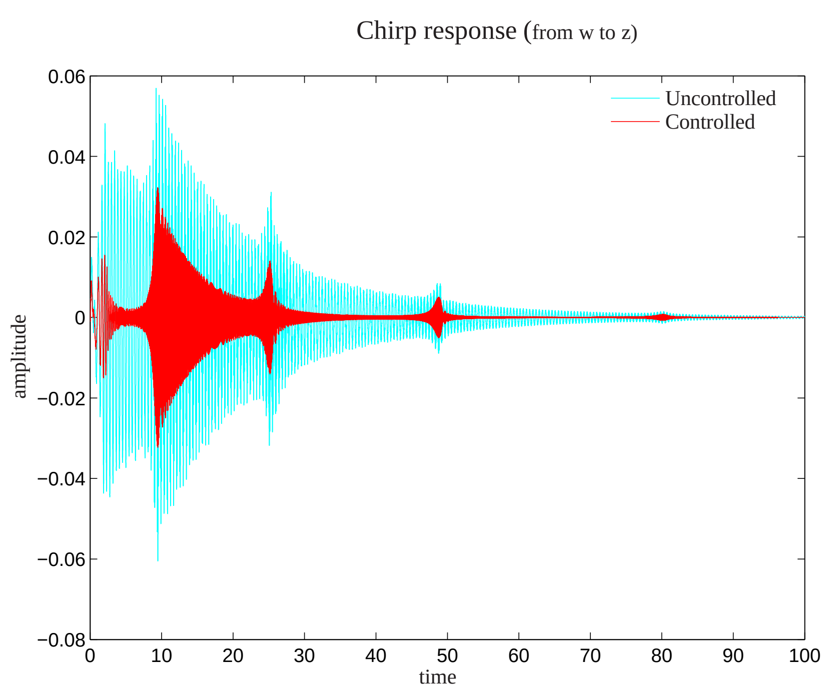

Figure 11. Chirp response of controlled and uncontrolled real plants without uncertainties.

4. CONCLUSIONS

When dealing with flexible structures, there are some problems that may appear from the modeling of the system. One of them is the uncertainty of the system's parameters such as damping matrix and elastic modulus of the material. The possible variation of these parameters can be considered in the controller design. One way of considering these variations within finite precision is to consider them as parametric uncertainties using the polytopic form.

The aim of this paper was to show how to deal with the parametric uncertainties written in polytopic form in the $\mathcal{H}_\infty$ controller design, which have the objective of minimizing the higher peak of vibration in frequency response. Figure 8 showed that all the vertices, or all the possible plants considered in the design, had a considerable attenuation in the vertical vibration. Table 2 showed that a attenuation of -33.6 db was achieved in the first mode of vibration - which is the peak of vibration - when comparing the uncontrolled and controlled real plant without uncertainties.

Possible future investigation could consider the parametric uncertainties in the Δ methodology (Gu *et al.*, 2005), which is a different form of dealing with uncertainties, and its results could be compared.

5. ACKNOWLEDGEMENTS

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