ROLLING BEARINGS SELECTION BASED ON FUZZY LOGIC THEORY

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Abstract. *During a new mechanical product developing process, the design engineer faces with the problem of selecting different types of materials and mechanical components among those usually used on similar equipment available in industry. Usually, these elements are standardized and selected based on information available in manufacturers catalogs, presented in practical rules and strategies format. The rolling element bearings are extensively used in machinery and equipment; and are manufactured in a wide range of shapes, configurations, sizes, rolling and seals elements, that makes each bearing type more or less suitable for a given application or operating condition, considering specific design features, which characterizes the selection of bearings process as a multiple criteria decision making problem. Usually, the bearing characteristics and the importance of certain design requirements are described in a qualitatively mode by linguistic variables, making the usual selection process dependent on the engineer's theoretical knowledge and experience with similar projects. A modern approach supported on knowledge-based and expert systems, has been developed in order to provide the machines the capability of emulating human behavior in solving problems. Expert systems, based on the theory of fuzzy logic, are suitable for rolling bearing selection process, since this theory was developed based on the hypothesis that human thoughts are not numbers, but rather, linguistic terms. The fuzzy theory provides an effective method of translating inaccurate and qualitative verbal expressions in numerical values that can be arithmetically operated to provide comparative parameters that may be used in decision making process in a given problem. In present paper, the main purpose is to present an expert computer aided design system for a wide range rolling bearings selection and classification process for specific design requirements, based on fuzzy logic theory and using a multiple criteria decision making method, known as TOPSIS (Techniques for Order Preference by Similarity to an Ideal Solution).*

Keywords*: Rolling Bearing, Fuzzy Logic, Decision Making, TOPSIS*

1. INTRODUCTION

In a new mechanical product developing process, the design engineer faces with the problem of selecting different kinds of materials and mechanical components among those usually used on similar equipment available in industry. Usually, these elements, like screws, fasteners, couplings, seals and rolling bearings are standardized and selected based on information available in manufacturers catalogs, presented in practical rules and strategies format often used to reach a final solution.

The rolling element bearings are widely used in machinery and equipment and are manufactured in a wide range of shapes, configurations, sizes, rolling and seals elements, that makes each type of rolling bearing more or less suitable for a given application or operating condition, considering specific characteristics like the type of applied load, compensation of misalignment, running accuracy, speed limits, quiet operation, rigidity, axial displacement, mounting and dismounting. As a case study, deep groove ball bearings can support moderate radial loads and axial loads; they have low friction and can be produced with high precision as well to work in conditions of quiet operation, so they are preferred for small and medium electric motors. Spherical roller bearings with barrel-shaped rollers between the inner and outer rings are self-aligning, and these characteristics make them very useful, for example, in applications for heavy machines, where there are high loads, deflections of the shaft or housing, or misalignment of their axes.

Ordinarily, several factors must be considered and weighed together while choosing a type of bearing, so that no general rule can be formulated. Typically, this is a *multiple criteria decision making* (MCDM) problem which the main objective is to find the best solution among all feasible alternatives.

Rolling bearings manufactures provide in their catalogs a comprehensive overview of the standard types of rolling bearings, their design features, capabilities and suitability to demands made in a given application. Usually, these data are summarized in table format and, despite of their inherent limitations, those tables provide basic information to select the most suitable type of bearing for a given application. Once defined the more appropriate bearing type for a given project, some important quantitative criteria should be observed, including load capacity, expected life, friction, maximum angular velocity, internal clearance or preload and lubrication.

In bearing type selection process, rolling bearings characteristics and the importance of certain design requirements are described qualitatively by linguistic variables, like “poor”, “fair”, “good” and “excellent”, making the usual selection process dependent on the engineer's theoretical knowledge and experience with similar projects. This lack of precision in bearings selection, associated with other quantitative requirements of the project, indicates that the application of expert systems is suitable for this kind of problem.

Knowledge-based or expert computational systems emulate human behavior in solving problems, seeking to provide answers to specific situations, even if they are new or inaccurate. The use of these kinds of computer aided design systems in solving engineering problems has been widespread and popularized recently; however, most of the commercially available expert systems packages and associated knowledge bases are applicable only within a relatively narrow field for specific applications.

Many researchers have used a heuristic approach to the selection criteria and design of bearing components. Among the articles published on this topic, Fagan (1987) developed an expert system that helps designers to select the correct combination of ball and roller bearings for supporting a shaft subjected to a given set of operating conditions. The system outputs just a single bearing type, while designers may prefer to have more options to choose the best alternative which satisfy various additional criteria such as cost, maintainability, noise, life, availability and stiffness. Another expert system developed by Pathak and Ahluwalia (1990), considered all relevant aspects of the design, manufacture and assembly of roller bearings, although the design aspects were considered in greater detail than other factors. Sim and Chan (1991) and Rao (1992) reported knowledge-based expert systems integrated to database manipulation system in order to select an appropriate type and size of rolling bearing that meet a set of design specifications, considering the issues of cost and availability of the bearings. Ahluwalia *et al.* (1993) presented a methodology and an interactive program for the roller bearings evaluation in terms of convenience to a set of conditions and attributes of methodology, and the decision making process is based on an approach with multiple attributes.

Erden *et. al.* (2001) developed an intelligent software package to select bearings suitable for a given application in purpose of improving design efficiency and elimination of human dependent expertise for conceptual design. The work includes creation and development of a knowledge base and a classification of bearings based on operating and environmental factors within the knowledge base structure. Seo and Han (2001) compared three decision making approaches to find out a model that is appropriate to bearing selection problem. An artificial neural network, which is trained with real design cases, is used to select a bearing mechanism at the first step; then, the subtype of the bearing is selected by the weighting factor method. Pan *et. al.* (2003) proposed a new approach for implementing a Web-based bearing design support system to achieve agility in mechanical design and Suppapitnarm (2006) conducted a research for the development and implementation of a computational framework for catalogue-based bearing selection and design, based in knowledge and experience capture in two mechanical design companies and with a bearing supplier company cooperation.

Expert systems based on the theory of fuzzy logic are suitable for rolling bearing selection process, since this theory, originally formalized by Zadeh (1965), was developed based on the hypothesis that human thoughts are not numbers, but rather, linguistic terms. The fuzzy theory provides an effective method of translating inaccurate and qualitative verbal expressions, as well human knowledge and experience, into numerical values and strategies that can be arithmetically operated to produce comparative parameters for use in decision making process of complex problems by an expert computational system.

Chen (1996) presented fuzzy ratings in mechanical engineering design applications where qualitative linguistic variables are represented by fuzzy numbers. In the proposed system, the bearing design requirements were used as weights in the fuzzy average algorithm to produce ratings among bearings alternatives, but only a few numbers of rolling bearings types and user requirements were considered.

As explained before, the bearing type selection process is a typical MCDM problem and many classical solution methods were proposed, based on the procedure of finding the best option among all feasible alternatives, through the application of a multiplicity of judging criteria. The TOPSIS (Techniques for Order Preference by Similarity to an Ideal Solution), proposed by Hwang and Yoon (1981), is one of those methods and its basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution.

In present paper, the main purpose is to present an expert computer aided design system for a wide range rolling bearings selection and classification process for specific design requirements, based on fuzzy logic theory and using the TOPSIS method extended for MCDM problems with fuzzy data .

2. PRELIMINARIES

The fuzzy sets theory allows the mathematical modeling of imprecise and linguistic variables. A fuzzy number is a quantity or function which value is imprecise, rather than exact as the case of single real number, known like a crisp number. In the current section, it is presented a brief review of some essential definitions, notations and operations of fuzzy sets used in this paper (Zadeh, 1965; Dubois and Prade, 1981; Chen, 2000). Additionally, the fundamentals of TOPSIS method extended for decision making problems with fuzzy data are presented (Chen, 2000; Jahanshahloo, 2006; Hosseinzadeh, 2007).

2.1. Fuzzy Sets Fundamentals

Let X be a classical set of objects, called *universe*, which generic elements are denoted by x. The membership in a *crisp subset* of X is often viewed as characteristic function μ_A from X to $\{0,1\}$such that

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A, \end{cases} \tag{1}$$

where, {0,1} is called *valuation set*.

If the valuation set is allowed to be the real interval [0,1], A is called a *fuzzy set* and denoted by $\tilde{A}$ and $\mu_{\tilde{A}}$ is the *grade of membership* of x in $\tilde{A}$.

Definition 1. If $\tilde{A}$ is fuzzy set, then $\tilde{A}$ is completely characterized by a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\} \tag{2}$$

Definition 2. A fuzzy set $\tilde{A}$ is a *normal* if, and only if, $\exists x_i \in X \mid \mu_{\tilde{A}}(x_i) = 1$

Definition 3. A fuzzy set $\tilde{A}$ is *convex* if and only if for every pair of points x_1, x_2 in X, the membership function $\mu_{\tilde{A}}$ satisfies the inequality

$$\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad \text{where } \delta \in [0,1] \tag{3}$$

Definition 4. A *fuzzy number* is a convex normalized fuzzy set $\tilde{A}$ of the real line R with continuous membership function.

Definition 5. A fuzzy number $\tilde{A}$ is called positive fuzzy number and denoted by $\tilde{A} > 0$ if $\mu_{\tilde{A}}(x) = 0$ for all $x < 0$. A fuzzy number $\tilde{A}$ is called negative fuzzy number and denoted by $\tilde{A} < 0$ if $\mu_{\tilde{A}}(x) = 0$ for all $x > 0$.

Definition 6. A *triangular fuzzy number* $\tilde{A}$ (Fig. 1) can be denoted as $\tilde{A} = (a_1,a_2,a_3)$, where a_2 is the central value , $(\mu_{\tilde{A}}(a_2) = 1)$, a_2 is the left spread and a_3 is the right spread. The membership function $\mu_{\tilde{A}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ (x-a_1)/(a_2-a_1), & a_1 \leq x \leq a_2, \\ (x-a_3)/(a_2-a_3), & a_2 \leq x \leq a_3, \\ 0, & x > a_3 \end{cases} \tag{4}$$

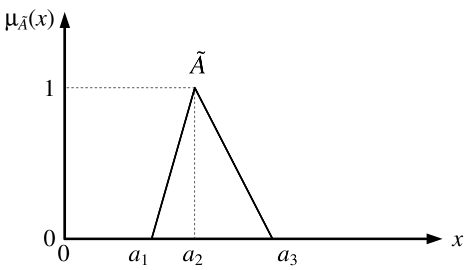

Figure 1. Triangular fuzzy number $\tilde{A}$

Definition 7. Consider two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$. The multiplication $\tilde{A}(x)\tilde{B}$ of the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as follow:

$$\tilde{A}(x)\tilde{B} = \begin{cases} (a_1b_1, a_2b_2, a_3b_3) & \text{if } \tilde{A} > 0, \tilde{B} > 0 \\ (a_3b_3, a_2b_2, a_1b_1) & \text{if } \tilde{A} < 0, \tilde{B} > 0 \\ (a_1b_3, a_2b_2, a_3b_1) & \text{if } \tilde{A} < 0, \tilde{B} < 0 \end{cases} \tag{5}$$

Definition 8. Let $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ be two triangular fuzzy numbers, then the distance between them using vertex method is defined as:

$$d(\tilde{A},\tilde{B})=\sqrt{\tfrac{1}{3}[(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2]} \tag{6}$$

2.2. Multiple criteria decision making method with fuzzy data

The solution of a multiple criteria decision making problem (MCDM) is the process of finding the best option among all feasible alternatives, through the application of a multiplicity of judging criteria. A MCDM problem can be concisely expressed in matrix form as follow

$$\boldsymbol{D}=\begin{matrix} & C_1 & C_2 & \cdots & C_n \\ A_1 & x_{11} & x_{12} & \dots & x_{1n} \\ A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_4 & x_{m1} & x_{m2} & \cdots & x_{mn} \end{matrix}, \quad \boldsymbol{W}=\begin{bmatrix} w_1, & w_2, & \dots & w_n \end{bmatrix} \tag{7}$$

where, $\boldsymbol{D}$ is the decision matrix, A_1, A_2, …, A_n are possible alternatives among which decision makers have to choose, C_1, C_2, …, C_n are criteria with alternative are measured, x_{ij} is the rating of alternative performance A_i with respect to criterion C_j, $\boldsymbol{W}$ is the weight vector and w_j is the weight of criterion C_j.

Classical MCDM methods consider that the ratings x_{ij} and weights w_j of the criteria are known precisely and one of those classical methods, known as the TOPSIS (Techniques for Order Preference by Similarity to an Ideal Solution), was proposed by Hwang and Yoon (1981). The basic principle of this method is that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). In TOPSIS process, the performance ratings and the weights of the criteria are given as crisp values.

The concept of TOPSIS method was extended for MCDM problems with fuzzy data and have been successfully used in the process of material selection (Jee and Kang, 2000; Libardi *et al*., 2008). The *decision matrix* $\boldsymbol{D}$ and the *weight vector* $\boldsymbol{w}$ are converted into a fuzzy format $\tilde{\boldsymbol{D}}$ and $\tilde{\boldsymbol{w}}$, respectively, resulting

$$\tilde{\boldsymbol{D}}=\begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \quad \tilde{\boldsymbol{w}}=\begin{bmatrix} \tilde{w}_1, & \tilde{w}_2, & \dots & \tilde{w}_n \end{bmatrix} \tag{8}$$

where $\tilde{x}_{ij}$ and $\tilde{w}_j$ are fuzzy numbers representing the linguistic variables and can be expressed by triangular fuzzy numbers, $\tilde{x}_{ij}=(x_{ij}^1,x_{ij}^2,x_{ij}^3)$ and $\tilde{w}_{ij}=(w_j^1,w_j^2,w_i^3)$.

The fuzzy ratings $\tilde{x}_{ij}$ and weights $\tilde{w}_j$ can be expressed in different quantities and scales, so in these cases, a normalization method must be applied to transform the various criteria scales and weight into a comparable scale and also to maintain the property that all normalized triangular fuzzy numbers belong to the interval [0,1]. The normalized fuzzy ratings and weights are represented by $\overline{\tilde{x}}_{ij}$ and $\overline{\tilde{w}}_j$, respectively. After the normalization process, the weight or importance of each criterion may be computed through the construction of the *weighted normalized fuzzy decision matrix* $\tilde{\boldsymbol{V}}$, defined as

$$\tilde{\boldsymbol{V}}=\left[\tilde{v}_{ij}\right]_{mxn}, \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \tag{9}$$

where $\tilde{v}_{ij}=\overline{\tilde{x}}_{ij\,(x)}\overline{\tilde{w}}_j$ are normalized positive triangular fuzzy numbers and their ranges belongs to the interval [0,1].

According to TOPSIS concept, a fuzzy positive ideal solution (FPIS), $\tilde{A}^+$, and a fuzzy negative ideal solution (FNIS), $\tilde{A}^-$, are defined as follow

$$\tilde{A}^+=(\tilde{v}_1^+,\tilde{v}_2^+,\dots,\tilde{v}_n^+) \quad \text{and} \quad \tilde{A}^-=(\tilde{v}_1^-,\tilde{v}_2^-,\dots,\tilde{v}_n^-) \tag{10}$$

where $\tilde{v}_j^+=(1,1,1)$ and $\tilde{v}_j^-=(0,0,0)$, $j=1,2,\dots,n$.

The distances d_i^+ and d_i^- of each weighted and normalized alternative A_i (i =1,2,...,m) from $\tilde{A}^+$ and $\tilde{A}^-$ are calculated applying the vertex method (Eq. 6)

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+) \quad \text{and} \quad d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1,2,\ldots,m \tag{11}$$

A *closeness coefficient* CC_i of each alternative A_i using the distances d_i^+ and d_i^- is defined and it approaches to 1 when the alternative A_i is closer to the FPIS ($\tilde{A}^+$) and farther from FNIS ($\tilde{A}^-$), so it can be used to determine the ranking order of all alternatives

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, 2, \ldots, m \tag{12}$$

3. THE PROPOSED ROLLING BEARING SELECTION PROCESS

The proposed application of TOPSIS method with fuzzy data for selecting the rolling bearing type is based on information provided by a manufacturer in its electronic catalog for more usual bearings (NSK, 2008), which are summarized in Tab. 1.

Table 1. Types and characteristics of rolling bearings (NSK, 2008)

Characteristics ➢ and Types ➣		C_1 Load Capacity			C_2	C_3	C_4	C_5	C_6
		Radial Loads	Axial Loads	Combined Loads	High Speeds	High Accuracy	Low Noise and Torque	Rigidity	Angular Misalignment
A_1	Single-Row Deep Groove Ball Bearing	○	↔ ○	○	◎	◎	◎		⊙
A_2	Magneto Bearing	●	← ●	●	⊙				●
A_3	Single-Row Angular Contact Ball Bearing	⊙	← ⊙	⊙	◎	◎			●
A_4	Double-Row Angular Contact Ball Bearing	⊙	↔ ⊙	⊙	○				●
A_5	Duplex Angular Contact Ball Bearing	⊙	↔ ⊙	⊙	⊙	◎		⊙	●
A_6	Four-Point Contact Ball Bearing	●	↔ ⊙	○	⊙	⊙			●
A_7	Self-Aligning Ball Bearing	○	↔ ●	●	⊙				◎
A_8	Single-Row Cylindrical Roller Bearing	⊙	×	×	◎	◎	⊙	⊙	○
A_9	Double-Row Cylindrical Roller Bearing	◎	×	×	⊙	◎		◎	●
A_{10}	Cylindrical Roller Bearing with Single Rib	⊙	← ○	○	⊙			⊙	○
A_{11}	Cylindrical Roller Bearing with Thrust Collar	⊙	↔ ○	○	⊙			⊙	○
A_{12}	Needle Roller Bearing	⊙	×	×	⊙			⊙	●
A_{13}	Single-Row Tapered Roller Bearing	⊙	← ⊙	⊙	○	⊙		⊙	○
A_{14}	Double-and Multiple-Row Tapered Roller Bearing	◎	↔ ⊙	◎	○			◎	●
A_{15}	Spherical Roller Bearing	◎	↔ ○	⊙	○				◎
A_{16}	Single-Direction Thrust Ball Bearing	×	← ⊙	×	×	⊙			×
A_{17}	Thrust Ball Bearing with an Aligning Seat	×	← ⊙	×	×				◎
A_{18}	Double-Direction Angular Contact Thrust Ball Bearing	×	↔ ⊙	×	○	◎		⊙	×
A_{19}	Cylindrical Roller Thrust Bearing	×	← ◎	×	●			◎	×
A_{20}	Tapered Roller Thrust Bearing	×	← ◎	×	●			◎	×
A_{21}	Spherical Thrust Roller bearing	●	← ◎	●	●				◎

◎ Excellent ⊙ Good ○ Fair ● Poor × Impossible ← One direction only ↔ Two directions

The characteristics C_1 to C_6 of different types of rolling bearings A_1 to A_{21} (Tab. 2), are expressed in linguistic variables: "Excellent", "Good", "Fair", "Poor", "Impossible", "One direction only" and "Two directions", pointing that the application of fuzzy theory is feasible for bearing selection. However, before the processing of these linguistic variables into fuzzy numbers and application of TOPSIS method, a preliminary analysis of table data must be performed, based on additional information available in the manufacture catalog, resulting in the following rules:

Rule 1. The "Load Capacity" (C_1) characteristic is differentiated in purely "Radial", purely "Axial" and radial-axial "Combined", therefore some bearings must be eliminated from analysis when the load is purely radial (A_{16} to A_{20}), purely axial (A_8, A_9 and A_{12}) or combined (A_8, A_9, A_{12} and A_{16} to A_{20}).

Rule 2. Additionally, if the load is purely axial or combined and the axial load occur in the two directions, bearings A_2, A_3, A_{10}, A_{13}, A_{16}, A_{17}, A_{19} to A_{21} also must be eliminated from the analysis.

Rule 3. For the "High Speeds" (C_2) characteristic, bearings A_{16} and A_{17} are indicated as "Impossible" for high speed applications, so they are eliminated from the analysis when this characteristic is required; otherwise, they are considered "Very Poor", once they can be used in very low speed condition.

Rule 4. In "High Accuracy" (C_3), "Low Noise and Torque" (C_4) and "Rigidity" (C_5) characteristics, the blank spaces for some bearings are interpreted as "Fair" for these design requirements.

Rule 5. The "Angular Misalignment" (C_6) characteristic indicates that the bearings A_{16} and A_{18} to A_{20} are not able to compensate angular misalignment, so they must be eliminated when a misalignment is expected; otherwise, the capability for misalignment compensation of cited bearings is assumed as "Very Poor".

The linguistic variables "Very Poor", "Poor", "Fair", "Good" and "Excellent" are expressed as positive triangular fuzzy numbers $\tilde{VP}$, $\tilde{P}$, $\tilde{F}$, $\tilde{G}$ and $\tilde{E}$, respectively (Fig. 2a), defined as follow

$$\tilde{VP} = (0,0,0.25)\,,\ \tilde{P} = (0,0.25,0.5)\,,\ \tilde{F} = (0.25,0.5,0.75)\,,\ \tilde{G} = (0.5,0.75,1)\,,\ \text{ and }\ \tilde{E} = (0.75,1,1) \qquad (13)$$

Similarly, the design requirements for bearings characteristics C_1 to C_6 are defined as the linguistic variables "Very Low", "Low", "Medium", "High" and "Very High", also expressed as positive triangular fuzzy numbers, denoted by $\tilde{VL}$, $\tilde{L}$, $\tilde{M}$, $\tilde{H}$ and $\tilde{VH}$, respectively (Fig 2b).

$$\tilde{VL} = (0,0,0.25)\,,\ \tilde{L} = (0,0.25,0.5)\,,\ \tilde{M} = (0.25,0.5,0.75)\,,\ \tilde{H} = (0.5,0.75,1)\ \text{ and }\ \tilde{VH} = (0.75,1,1) \qquad (14)$$

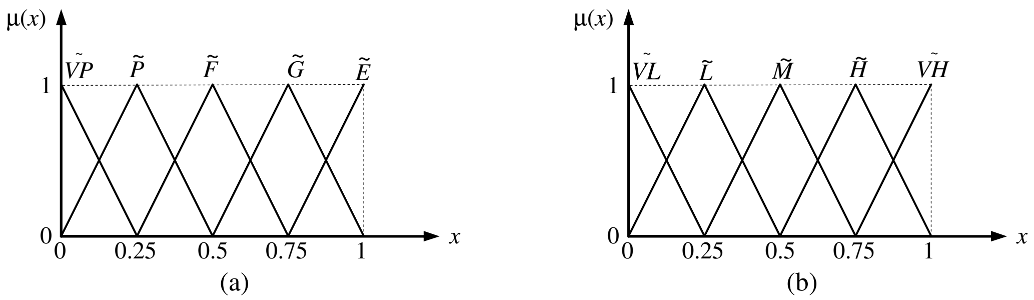

Figure 2. Membership functions of the fuzzy (a) rolling bearings characteristics and (b) design requirements

Based on previously defined rules and the fuzzy numbers for bearings characteristics and design requirements, one can conclude that the fuzzy decision matrix $\tilde{\boldsymbol{D}}$, that describe this problem, is not unique, but it is obtained from Tab. 2, taking into account the design requirements. After obtaining matrix $\tilde{\boldsymbol{D}}$ and weight vector $\tilde{\boldsymbol{w}}$, the TOPSIS method with fuzzy data may be applied. In this study, all fuzzy numbers were defined in the interval [0,1], so the normalization process mentioned earlier is unnecessary. The fuzzy positive ideal solution (FPIS), $\tilde{A}^+$, and a fuzzy negative ideal solution (FNIS), $\tilde{A}^-$, are defined as

$$\begin{cases} \tilde{A}^+ = [(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1)] \\ \tilde{A}^- = [(0,0,0),(0,0,0),(0,0,0),(0,0,0),(0,0,0),(0,0,0)] \end{cases} \qquad (15)$$

A computational algorithm for the proposed rolling bearing type selection process was implemented in MATLAB® and may be used as a computer aided system for preliminary mechanical engineering design.

4. NUMERICAL EXAMPLES

In this section, two numerical examples are presented to illustrate the application of the proposed system for rolling bearing type selection, using TOPSIS method with fuzzy data for decision making and considering different operation conditions and design requirements.

4.1. Case study #1

Assuming that the problem is to choose a suitable rolling bearings type for application in small electrical motor, with the operation conditions and the design requirements described as follow:

- Combined low radial and one direction axial load;
- Very high priority for high speed, rotation accuracy and low torque and noise emission;
- Very low priority for rigidity and misalignment compensation.

Applying rules 1 to 5 and the fuzzy triangular number previously defined, results in the fuzzy decision matrix $\tilde{\boldsymbol{D}}$ and *the* weight vector $\tilde{\boldsymbol{w}}$ presented in Tab. 2.

Table 2. The fuzzy decision matrix and fuzzy weights for rolling bearings selection for case study #1

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.25,0.5,0.75)	(0.75,1,1)	(0.75,1,1)	(0.75,1,1)	(0.25,0.5,0.75)	(0.5,0.75,1)
A_2	(0,0.25,0.5)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0,0.25,0.5)
A_3	(0.5,0.75,1)	(0.75,1,1)	(0.75,1,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0,0.25,0.5)
A_4	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0,0.25,0.5)
A_5	(0.5,0.75,1)	(0.5,0.75,1)	(0.75,1,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0,0.25,0.5)
A_6	(0.5,0.75,1)	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0,0.25,0.5)
A_7	(0,0.25,0.5)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)
A_{10}	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{11}	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{13}	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{14}	(0.75,1,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)	(0,0.25,0.5)
A_{15}	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)
A_{21}	(0,0.25,0.5)	(0,0.25,0.5)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0.75,1,1)
$\tilde{\boldsymbol{w}}$	(0,0.25,0.5)	(0.75,1,1)	(0.75,1,1)	(0.75,1,1)	(0,0,0.25)	(0,0,0.25)

As explained before, it is not necessary to perform a normalization process, once all fuzzy numbers were defined in the interval [0,1], therefore the weighted decision matrix $\tilde{\boldsymbol{V}}$ results from the direct fuzzy product of bearings characteristics (C_1, C_2, …, C_6) by the corresponding design requirements ($\tilde{w}_1$, $\tilde{w}_2$, …, $\tilde{w}_6$). After that, distances d_i^+ and d_i^- , and the closeness coefficients CC_i are computed by calculating the distance of all bearings alternatives to both FPIS and the FNIS simultaneously, in order to obtain the ranking order of these bearings, as given in Tab. 3.

Table 3. Distances, closeness coefficients and ranking for case study #1

	d_i^+	d_i^-	CC_i	Rank
A_1	21.2157	10.1995	0.3247	**1º**
A_2	23.4136	6.9379	0.2286	8º
A_3	22.1417	9.2946	0.2957	2º
A_4	23.5198	6.6219	0.2197	11º
A_5	22.3818	8.8241	0.2828	3º
A_6	22.6522	8.3236	0.2687	4º
A_7	23.2924	7.0842	0.2332	7º
A_{10}	23.1743	7.3103	0.2398	6º
A_{11}	23.1743	7.3103	0.2398	6º
A_{13}	22.9953	7.5285	0.2466	5º
A_{14}	23.4436	6.7230	0.2229	10º
A_{15}	23.3986	6.7682	0.2244	9º
A_{21}	24.2483	5.4096	0.1824	12º

In this case study, 13 different types of rolling bearings may be employed and according to the closeness coefficient CC_i, the first five options are bearings A_1, A_3, A_5, A_6 and A_{13}. The sixth position is shared by bearings A_{10} and A_{11}. The first bearing type option A_1, Single-Row Deep Groove Ball Bearing, has obviously the highest closeness coefficients CC_i=0.3247, and it results from the shortest distance $d_i^+ = 21.2157$, indicating its proximity to the fuzzy positive ideal solution (FPIS), $\tilde{A}^+$, and the longest distance $d_i^- = 10.1995$, indicating how far it is from the fuzzy negative ideal solution (FNIS), $\tilde{A}^-$.

4.2. Case study #2

Assuming now that it is necessary to choose an appropriate type of rolling bearing for a heavy machine under the following operation conditions and the design requirements described as follow:

- Very high radial load;
- Very low priority for high speed, rotation accuracy and low torque and noise emission;
- Very high priority for rigidity and misalignment compensation.

Once more, applying rules 1 to 5 and the fuzzy triangular number previously defined, results in the fuzzy decision matrix $\tilde{\boldsymbol{D}}$ and the weight vector $\tilde{\boldsymbol{w}}$ presented in Tab. 4. Distances d_i^+ and d_i^-, and the resultant closeness coefficients CC_i are presented in Tab. 5.

Table 4. The fuzzy decision matrix and fuzzy weights for rolling bearings selection for case study #2

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.25,0.5,0.75)	(0.75,1,1)	(0.75,1,1)	(0.75,1,1)	(0,0,0.25)	(0.5,0.75,1)
A_2	(0,0.25,0.5)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0,0.25,0.5)
A_3	(0.5,0.75,1)	(0.75,1,1)	(0.75,1,1)	(0,0,0.25)	(0,0,0.25)	(0,0.25,0.5)
A_4	(0.5,0.75,1)	(0.25,0.5,0.75)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0,0.25,0.5)
A_5	(0.5,0.75,1)	(0.5,0.75,1)	(0.75,1,1)	(0,0,0.25)	(0.5,0.75,1)	(0,0.25,0.5)
A_6	(0,0.25,0.5)	(0.5,0.75,1)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0,0.25,0.5)
A_7	(0.25,0.5,0.75)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0.75,1,1)
A_8	(0.5,0.75,1)	(0.75,1,1)	(0.75,1,1)	(0.5,0.75,1)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_9	(0.75,1,1)	(0.5,0.75,1)	(0.75,1,1)	(0,0,0.25)	(0.75,1,1)	(0,0.25,0.5)
A_{10}	(0.5,0.75,1)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{11}	(0.5,0.75,1)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{12}	(0.5,0.75,1)	(0.5,0.75,1)	(0,0,0.25)	(0,0,0.25)	(0.5,0.75,1)	(0,0.25,0.5)
A_{13}	(0.5,0.75,1)	(0.25,0.5,0.75)	(0.5,0.75,1)	(0,0,0.25)	(0.5,0.75,1)	(0.25,0.5,0.75)
A_{14}	(0.75,1,1)	(0.25,0.5,0.75)	(0,0,0.25)	(0,0,0.25)	(0.75,1,1)	(0,0.25,0.5)
A_{15}	(0.75,1,1)	(0.25,0.5,0.75)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0.75,1,1)
A_{21}	(0,0.25,0.5)	(0,0.25,0.5)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0.75,1,1)
$\tilde{\boldsymbol{w}}$	(0.75,1,1)	(0,0,0.25)	(0,0,0.25)	(0,0,0.25)	(0.75,1,1)	(0.75,1,1)

Table 5. Distances, closeness coefficients and ranking for case study #2

	d_i^+	d_i^-	CC_i	Rank
A_1	24.0008	6.0770	0.2020	7º
A_2	25.4702	3.8056	0.1300	14º
A_3	24.7975	5.0321	0.1687	11º
A_4	24.9039	4.8280	0.1624	12º
A_5	24.4498	5.5123	0.1840	9º
A_6	25.0701	4.3940	0.1491	13º
A_7	23.8355	6.3797	0.2111	2º
A_8	23.8200	6.2989	0.2091	3º
A_9	24.0423	6.1330	0.2032	6º
A_{10}	23.9371	6.1273	0.2038	5º
A_{11}	23.9371	6.1273	0.2038	5º
A_{12}	24.5021	5.4041	0.1807	10º
A_{13}	23.9438	6.1585	0.2046	4º
A_{14}	24.1487	5.9289	0.1971	8º
A_{15}	23.4919	7.1113	0.2324	**1º**
A_{21}	24.3386	5.7223	0.1904	15º

For these new design requirements, 16 different types of rolling bearings may be employed and according to the closeness coefficient CC_i, the first four options are bearings A_{15}, A_7, A_8 and A_{13}. The fifth position is shared by bearings A_{10} and A_{11}. The first bearing type option A_{15}, Spherical Roller Bearing, has the highest closeness coefficients CC_i=0.2324, resultant from the shortest distance d_i^+ =23.4919 and the longest distance d_i^- =7.1113.

In both classical study cases previously presented, the results produced by the computational algorithm for decision making process of the rolling bearing type chosen are in agreement with the design expectations, indicating that the proposed system is promising for the application in preliminary stage of machine design.

5. CONCLUSION

Fuzzy logic theory has been used in many different fields and in this paper was reported in detail the implementation of a computer aided design system for preliminary selection and ranking process of a rolling bearing type based on this theory. The bearing characteristics and design requirements are usually described by linguistic variables and translation of this qualitative information into fuzzy numbers allowed the processing of those data, through the application of TOPSIS method for an optimized selection and ordination process.

The proposed system has been tested on two typical applications and has proved to be efficient for the proposed task, but additional tests considering other different design requirements, as well as practical verifications with experienced design engineers are recommended for necessary adjusts of fuzzy numbers attributes and preliminary rules definitions.

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7. REFERENCES

Ahuwalia, J., Gupta, S.K. and Agrawal, V.P., 1993, "Computer-Aided Optimum Selection of Roller Bearing". Computer-Aided Design, Vol. 256, No. 8, pp.493-499.

Chen, C.-T., 2000, "Extensions of the TOPSIS for group decision-making under fuzzy environment". Fuzzy Sets and Systems, Vol. 114, pp.1-9.

Chen, Y.H., 1996, "Fuzzy ratings in mechanical engineering design – application to bearing selection". Proceedings of the Institution of Mechanical Engineers – Part B: Journal of Engineering Manufacture, Vol. 210, pp.49-53.

Erden, L.A, Akkök, M. and Taner, I.E., 2001, "Development of a knowledge-based system and a comparative study on the selection of bearings". Lubrication Engineering, Vol. 57, No. 6, pp. 7-15.

Fagan, M.J., 1987, "Expert Systems Applied to Mechanical Engineering Design – Experience with Bearing Selection and Application Program". Computer-Aided Design, Vol. 19, No. 7, pp. 361-367.

Jahanshahlloo, G.R., Lotfi, F.H. and Izadikhah, M., 2006, "Extension of the TOPSIS method for decision-making problems with fuzzy data". Applied Mathematics and Computation, Vol. 181, pp. 1544-1551.

Jee, D.-H. and Kang, K.-J., 2000, "A method for optimal material selection aided with decision making theory". Materials and Design, Vol. 21, pp. 199-206.

Libardi, W., Ferrante, M., Watanabe, F.Y., Rizziolli, E.C. and Vieira, J.P., 2008, "Processo de decisão pra a seleção de materiais baseado na teoria de conjuntos fuzzy". Annals of the 18th Congresso Brasileiro de Engenharia e Ciência dos Materiais – CBECiMat, Porto de Galinhas, Brazil.

Li, R.J., 1999, "Fuzzy method in group decision making". Computers and Mathematics with Applications, Vol. 38, pp. 91-101.

Lotfi, F.H., Allahviranloo, T., Jondabeh, M.A. and Kiani, N.A., 2007, "A New Method for Complex Decision Making Based on TOPSIS for Comples Decision Making Problems with Fuzzy Data". Applied Mathematics and Sciences, Vol. 1, No. 60, pp. 2981-2987.

NSK, 2008, Rolling Bearings Catalog - CAT. No. E1102f.

Pan, P.Y., Cheng, K., Harrison, D.K. 2003, "A Web-based agile system for rolling bearing design". Integrated Manufacturing Systems, Vol. 14, No. 6, pp.518-529.

Pathak, M.A. and Ahluwalia, R.S., 1990, "An Expert System for the Design and Selection of Ball Bearing Parameters". Materials & Design, Vol. 11, No. 6, pp.317-322.

Rao, P., 1992, "BEAS: Expert System for the Preliminary Design of Bearings". Advances in Engineering Software, Vol. 14, pp.163-166.

Seo, T.-S. and Han, S. 2001, "Artificial Intelligence-Based Stepwise Selectionof Bearings". Proceedings of the Pacific Asian Conference on Intelligent System (PAIS 2001), Seoul, Korea, pp. 219-223.

Simões, M.G. and Shaw, I.S., 2007, "Controle e Modelagem Fuzzy". São Paulo: Blucher: FAPESP.

Suppapitnarm, A., 2006, "Catalogue-Based Bearing Design by Capturing Mechanical Design Knowledge and Experience". Proceedings of the 7th WSEAS International Conference on Automation & Information, Cavtat, Croatia, pp. 32-37.

Zadeh, L.A., 1965, "Fuzzy sets". Information and Control, Vol. 8, pp. 338-353.

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