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CHECKERBOARD SUPPRESION FOR EVOLUTIONARY STRUCTURAL OPTIMIZATION WITH HEXAGONAL FINITE ELEMENT MESH

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Abstract. *Structural Topology Optimization is a powerful design tool in many fields of engineering such as mechanical, civil, aerospace, biomechanics and others. Several topology optimization techniques have been developed with the Finite Element Method consolidation and the evolution of computational capabilities. One of these techniques is called Evolutionary Structural Optimization (ESO), which has achieved promising results through the successive application of heuristic strategies in the structural analysis. The ESO method is labeled as "hard kill", because it is based on a rule of successive elimination of elements in a finite element mesh that covers the feasible solution space defined initially. One of the common problems in topology optimization methods is the "checkerboard", which consists in finding intermediaries solutions with solid-void alternated patterns that under or super estimate the stiffness of the structure and diverge from the optimal solution. To overcome this problem, high order finite elements and filters have been used. However, the computational cost of these alternatives could be high. It is proposed in this article the utilization of a hexagonal finite element mesh for the suppression of this problem in ESO method. Hexagonal elements share more edges and vertices with neighboring elements than the traditional quadrilateral ones. It reduces the probability that elements can be connected just by one node in the final topology (characteristic of "checkerboard" pattern), furthermore it minimizes stress concentrations. Additionally, the algorithm computational cost can be reduced with this hexagonal mesh because filters or high order elements will be no more in use.*

Keywords: *evolutionary structural optimization, hexagonal finite element, checkerboard suppression*

1. INTRODUCTION

Since the 80's many studies have been developed about the theory, methods and applications of structural topology optimization, which is a powerful technique applied to design new structures and materials. Among various numerical methods, the Evolutionary Structural Optimization (ESO) was proposed as a simple and heuristic algorithm of optimization. The scientific literature about ESO method started in 1993 with the first article written by Xie and Steven's and it is still an active area, Xie and Huang (2010). Great advances were obtained with this method and many discussions about its validity were presented, Zhou and Rozvany (2001). According to Tanskanen (2002), the ESO method has a very distinct theoretical basis and it is equivalent to the sequential linear programming (SLP)-based approximate optimization method followed by the Simplex algorithm if the strain energy rejection criterion in ESO is utilized. Despite the discussions, the method is nowadays well established and it reached more complex versions as its most known bi-directional evolutionary structural optimization (BESO), Huang and Xie (2007), the multiobjective and fixed elements based modification of ESO (MESO), Tanskanen (2006), genetic ESO (GESO), Liu *et al.* (2008), among others. Huang and Xie (2010) present a further review of ESO/BESO methods.

The evolutionary procedures are based in a simple and gradual removal of inefficient material from the predefined fixed region that is discretized using the type of approximation adopted in the finite element mesh, Cook *et al.* (2002), which plays a fundamental role in the algorithm. According to Xie and Steven (1993), the basic idea of ESO is to run a finite element analysis all over the structure's fixed design domain and, after a sensitivity number evaluation, to eliminate gradually the elements with low efficiency in the problem. For each type of analysis there will be one or more design criteria for elements elimination, which can be stiffness criterion, stress level, natural frequency, heat conduction, buckling and others. The problem presented here concerns about stiffness criteria with volume constraints. Figure 1 presents a general ESO method flowchart.

The most common problems in the application of the ESO method are the numerical instabilities like checkerboard patterns, Xie and Huang (2010), Li and Steven (2001), mesh dependency, Xie and Huang (2010), Chu *et al.* (1997) and local optimum, Huang and Xie (2010).

Checkerboard patterns are anomalies found in the resulting topology characterized by solid-void alternated patterns through the finite element mesh. In a continuum structure discretized by low order bilinear, in a 2D case, the sensitivity numbers can become C^0 discontinuous across the element boundaries. This leads to checkerboard patterns in the final topology, Jog and Haber (1996). The presence of the checkerboard causes difficulty in interpreting and manufacturing the 'optimal' solution, Xie and Huang (2010). Considering that this problem is caused by numerical instabilities of low order elements, high order elements can be applied as a solution for the problem in topology optimization, Sigmund and Peterson (1998). Filters have also been proposed by some authors as a technique to eliminate the checkerboard patterns.

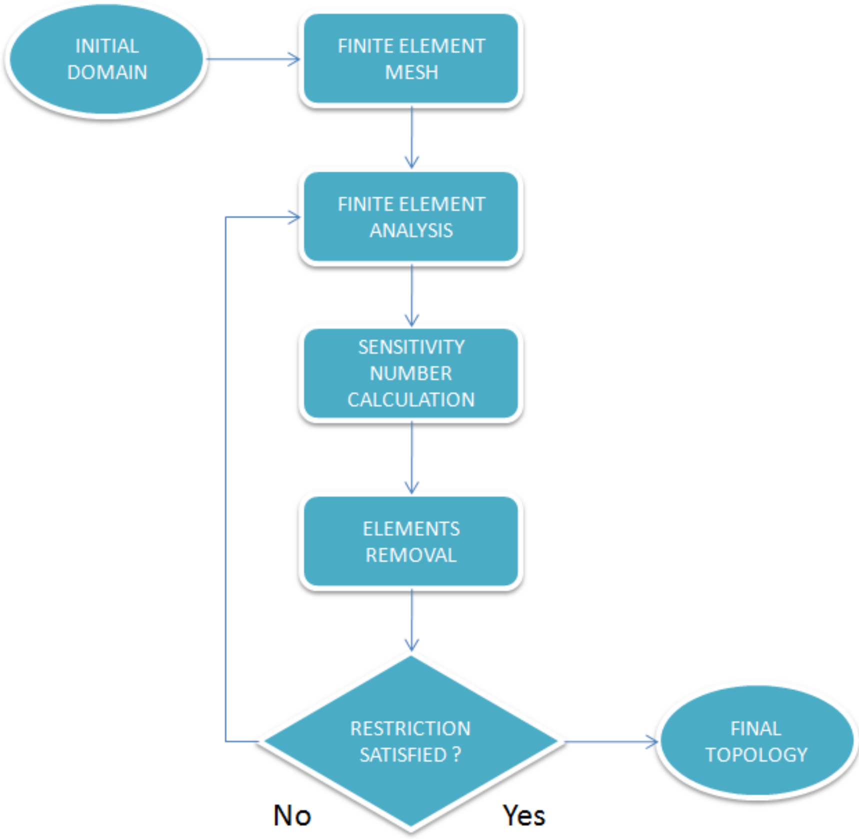

Figure 1. Flow chart of a general ESO algorithm.

Li and Steven (2001) proposed the first sensitivity number smoothing filter for ESO. However, the use of filters or high order elements increases the computational cost and it may be too high, which is a challenge in programming. Even though these techniques reach checkerboard-free designs, they do not obtain mesh independent solutions.

Mesh dependency refers to the problem of obtaining different topologies for different mesh-sizes or discretizations, Sigmund and Peterson (1998). When a finer mesh is used, the topology optimization algorithm will obtain a different topology with more members or smaller sizes in the final design, Xie and Huang (2010). Ideally, mesh-refinement should result in a better finite element modeling of the same optimal topology and a better description of boundaries, but not in a different structure. To overcome these problems, filters and techniques as perimeter control, restriction methods and others were applied, Sigmund and Peterson (1998). For ESO-based methods some of these techniques were applied. In the paper by Yang *et al.* (2003), a perimeter control in BESO algorithm has been presented. Mesh independent solutions of a hard-kill BESO is presented by Huang and Xie (2007). Similar to the checkerboard elimination process, the computational cost is increased with the extra algorithms to obtain mesh independent solutions in topology optimization and to solve both problems the required computational capability is high.

Through the studies and the cases mentioned above, others alternatives were proposed to solve some of these topology optimization problems. To obtain a checkerboard-free topology, Talischi and Paulino (2009) present Wachspress-type hexagonal elements applied to a SIMP-algorithm. They propose a honeycomb-like mesh and due to the element geometry they eliminate the chance of existence of one-node connections between elements that lead to the checkerboard patterns. Hexagonal elements share more edges and nodes with their neighboring ones than the traditional quad elements and have three edge-based symmetry lines. Other authors also use the idea of hexagonal mesh as Zhou (2010) in a hybrid discretization method for compliant mechanisms. Saxena and Saxena (2007) used a hexagonal mesh performed by blocks mounted with two quadrilateral elements. In ESO method, hexagonal approach has not been applied yet, which makes it reasonable to be explored.

This paper aims to present an alternative to obtain checkerboard-free topologies in ESO method with hexagonal meshes. The resulting topologies do not present checkerboard patterns nor one-node hinge connections when honeycomb-like meshes are applied. Furthermore, the topologies contours are smoothed due to hexagonal mesh geometry. The mesh dependency is also described. Even though evolutionary optimization methods are reaching more complex versions as BESO, basic ESO method is still under recent application, Silva and Pavanello (2010), Jia *et al.* (2011), Ghaffarianjam and Absolbashari (2010), Ansola *et al.* (2006) and so on. In future works hexagonal mesh with BESO method will be applied.

2. Sensitivity number for stiffness optimization under volume constraint

Stiffness is a key factor that must be taken into account in structural design. Commonly, compliance C is considered in the calculation, which means the inverse of general structure's stiffness. Some authors also define it as flexibility. The mean compliance C can be defined by the total strain energy of the structure or external work done by applied loads as:

$$C = \frac{1}{2} f^T u \tag{1}$$

in which f is the load vector and u is the displacement vector.

In finite element analysis, the static equilibrium equation of a structure, considering linear elastic material behavior, is expressed as:

$$Ku = f \tag{2}$$

in which K is the global stiffness matrix.

When the *i*th element is removed from the structure, the stiffness matrix will change by:

$$\Delta K = K^* - K = -K_i \tag{3}$$

in which K^* is the stiffness matrix of the resulting structure after the *i*th element removal and K_i is its stiffness matrix. It is assumed that the element removal does not affect the load vector f. By varying both sides of equation 2 and ignoring a higher order term, the change of displacement vector is obtained as:

$$\Delta u = -K^{-1} \Delta K u \tag{4}$$

From equations (1) and (4), the variation of the compliance ΔC can be evaluated by:

$$\Delta C = \frac{1}{2} f^T \Delta u = -\frac{1}{2} f^T K^{-1} \Delta K u = \frac{1}{2} u_i^T K_i u_i \tag{5}$$

in which u_i is the displacement vector for the *i*th element.

Thus, the sensitivity number for the mean compliance can be defined as:

$$\alpha_i^e = \frac{1}{2} u_i^T K_i u_i \tag{6}$$

The above equation indicates that the increase in the mean compliance as a result of the *i*th element removal is equal to its elemental strain energy. To minimize the mean compliance (which is equivalent to maximizing the stiffness) trough the removal of elements, it is intuitive to eliminate the elements with the lowest values of α_i, then the increase in C will be minimum as well.

The number of elements to be removed is determined by the element removal ratio (ERR) which is defined as the ratio of the number of elements removed at each iteration to the total number of elements in the initial FEA model. Elements will be removed until reaching a percentage predetermined for the final volume of the structure. The steps for evolutionary optimization with stiffness criterion under a volume constraint are:

- *Step 1*. Discretize the structure's design domain with a fine finite element mesh.
- *Step 2*. Carry out finite element analysis for the structure.
- *Step 3*. Calculate the sensitivity number for each element using equation 6.
- *Step 4*. Remove a number of elements with the lowest sensitivity numbers, according to the predefined removal ratio (ERR).
- *Step 5*. Repeat steps 2 to 4 until the volume of the final structure reaches the imposed limits.

In this work, this procedure has been implemented using MATLAB script for analysis, pre and post processing.

2.1 The checkerboard problem and the hexagonal mesh role

The "checkerboard" problem is presented in many structural topologic optimization methods which are based in a fixed finite element mesh. The problem is characterized by solid-void patterns ordered as a checkerboard in the final structure and it has its origin during the algorithm evolution. In the evolutionary structural optimization the problem has been detected under various analysis criteria and the alternated solid-void patterns strengthens the idea that the adopted criterion is over and under estimated through the elements.The presence of checkerboard pattern restrains the interpretation of the optimum solution and also the structure manufacturing.

The checkerboard has its origin in numerical approximation problems, Diaz and Sigmund (1995) and Jog and Haber (1996). When the traditional four-node quadrilateral elements are used, the kinematic restrictions of the discretized structure result in a artificially high stiffness numerically induced. These elements can present connections just by one node with your neighboring ones, which enables the *checkerboard* and/or *stiffness singularities* formation, Saxena and Saxena (2007). Figure 2 presents these patterns formation with the quadrilateral elements.

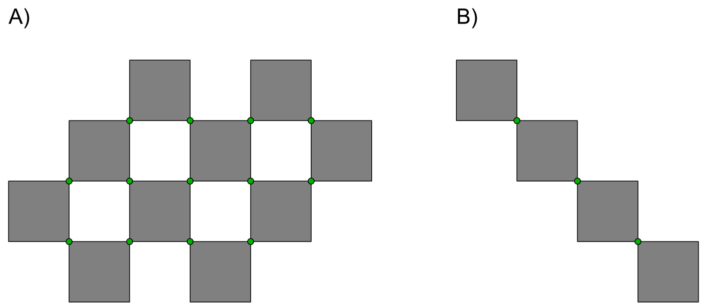

Figure 2. A) Checkerboard pattern; B) A continuum segment with one-node hinge sites.

To overcome these numerical difficulties in the context of the SIMP algorithm, researchers have proposed the utilization of high order elements which Diaz and Sigmund (1995) showed to be more stable, filters, Poulsen (2002) and Hu *et al.* (2009), or even elements with special functions, Talischi and Paulino (2009). However, these alternatives can not be the best ones because the increasing of the computational cost can be too high. In ESO method, checkerboard problems are also detected for stiffness criterion analysis based in four-node quadrilateral elements. Li and Steven (2001) propose a filter based in smoothing the sensitivity number over the structure to eliminate the problem in ESO method. An alternative for these complementary techniques (filters and high order elements) is based on the use of hexagonal elements. Talischi and Paulino (2009) propose the utilization of a special hexagonal element with Wachspress functions to eliminate the checkerboard patterns with no more filters or extra algorithms in SIMP method. Recently, other authors have also studied the combination of the traditional triangular and quadrilateral elements to create hexagonal elements (blocks) and/or hybrid meshes to eliminate these problems in traditional optimization methods, as Zhou (2010), Nguyen *et al.* (2010), Rahmatalla and Swan (2004) and so on.

This paper presents an alternative to suppress the checkerboard problem in ESO method using hexagonal blocks without needing extra filters or special elements. In the hexagonal tessellation, elements share more edges and nodes between them. The hexagonal blocks (honeycomb like) share at least 2 nodes and one edge with their neighboring ones, by the virtue of their geometry. This suppresses the possibility of a single one-node hinge formation and naturally excludes the formation of checkerboard. Furthermore, the final topology of the structure is obtained with smoother contours all over the mesh because hexagonal blocks have more lines of symmetry than the quadrilateral elements. Figure 3 presents the mesh generated in this work. It must be pointed out that the boundaries of the whole initial domain are supplemented with quadrilateral elements adapted to the mesh.

Here, the hexagonal block proposed is created by a combination of two four-node isoparametric quadrilateral elements. In practice, the finite element analysis is carried out with quadrilateral elements in the hexagonal mesh, but they are considered as a hexagonal block in the evolutionary algorithm. It is also important to point it out that there is one sensitivity number for each quadrilateral element in a hexagonal block. The choice of which element must be removed is based by an analysis of each individual quad one, considering this element is always attached to its parent one in the hexagonal block and they must be removed together. The mean of the sensitivity number was also tested, but it does not guarantee the topology's symmetry without an extra checking algorithm. This will be implemented in future works. Figure 4 presents the construction of the hexagonal element (block) used in this paper.

In this work, it was created automatic mesh generators for hexagonal meshes in MATLAB. Three kind of meshes were generated, a hexagonal one mounted by quad elements and supplemented with triangular and quad ones, a complete hexagonal element mesh supplemented with triangular and quad ones and the chosen hexagonal mesh mounted with quad

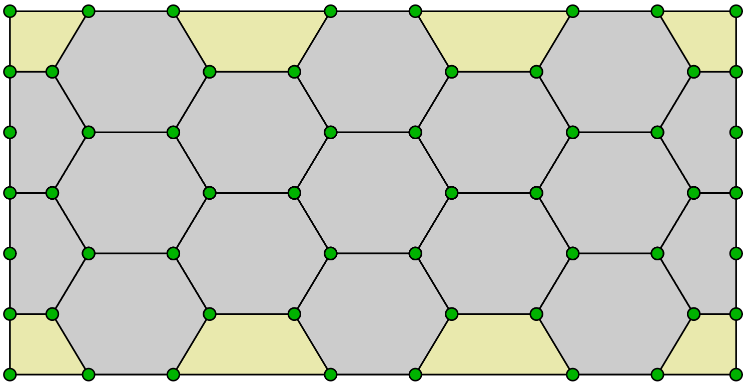

Figure 3. Domain with hexagonal mesh and boundaries supplemented with standard quadrilateral elements.

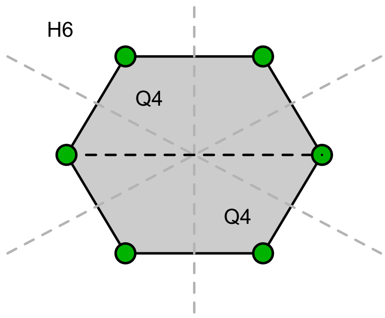

Figure 4. Combination with two quads into a hexagonal block and its three edge-based symmetry lines.

elements and supplemented only with other quad ones, showed in Fig. 3.

3. Numerical Results and Discussions

3.1 Checkerboard tests

A cantilever beam with aspect ratio $2H$xH is optimized by ESO method under a stiffness criterion and volume constraint. The Poisson's ratio of the material is taken to be $\nu = 0.3$ and its Young's modulus $E = 100GPa$. Figure 5 presents the $10m$x$5m$ domain with unit thickness.

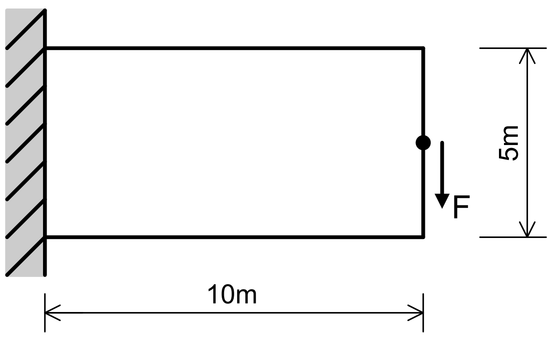

Figure 5. Design for a cantilever beam.

The domain was discretized by 53x26 hexagonal blocks and also by 53x52 regular quads, which are equivalent meshes considering the hexagon has two quadrilateral elements in your height in this approach. The objective here is to design the stiffest structure by minimizing compliance with 50% of material in the design domain. Figure 6 presents the evolution of the short beam optimization taken by ESO method with regular quad elements and Fig. 7 presents the same optimization using hexagonal blocks. Both graphics show volume fraction and mean compliance evaluated in each iteration. It is possible to note that the volume fraction decreases linearly during the method's evolution while the mean compliance increases with the elements removal, which it is compatible with ESO idea.

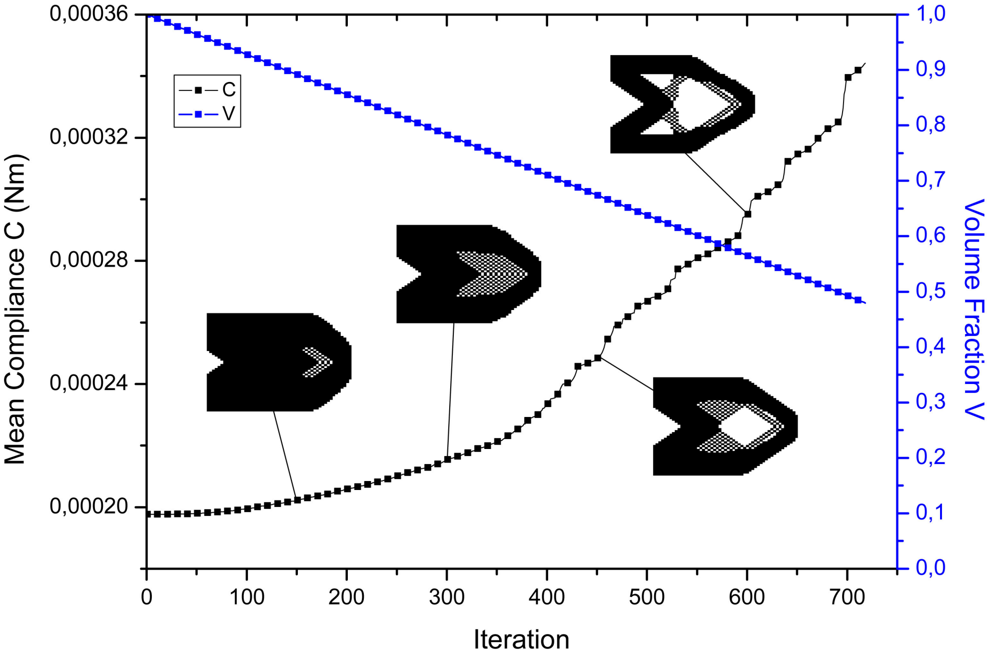

Figure 6. Evolutionary histories of the mean compliance, volume fraction and topology with regular quads mesh.

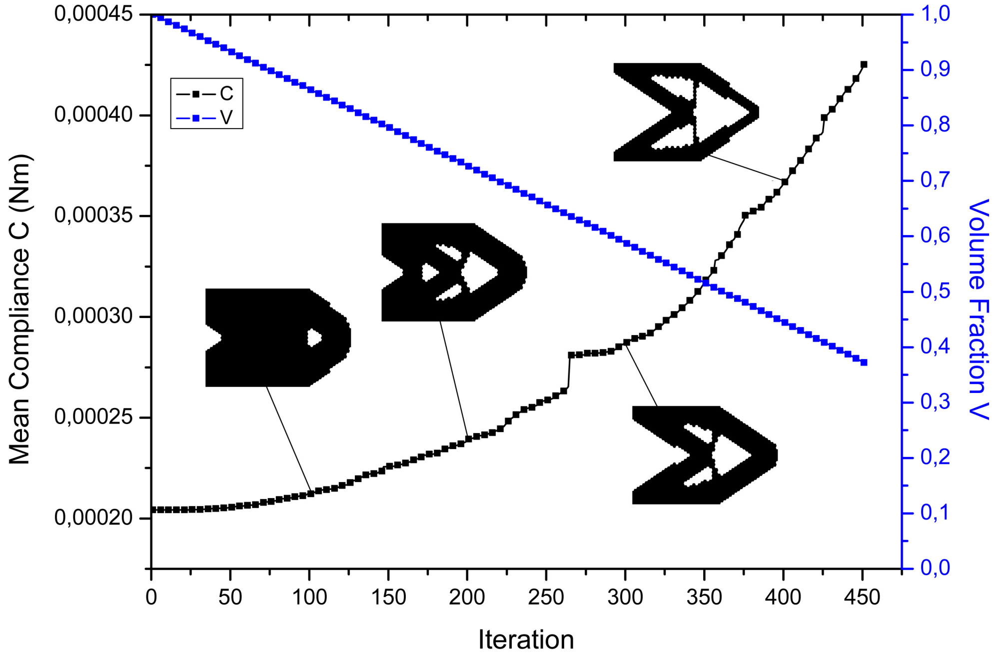

Figure 7. Evolutionary histories of the mean compliance, volume fraction and topology with hexagonal blocks mesh.

Both graphics also show the evolutionary histories for the structure's topology. It is possible to observe the checkerboard formation in the regular quad mesh while hexagonal block mesh does not give a chance to its formation. The explanation for why the evolution took so many iterations is because only two elements were removed per iteration, in order to guarantee some initial symmetry. Considering this, it must be pointed out that in the hexagonal mesh two blocks were removed, which is equivalent to remove four quadrilateral elements and then the hexagonal approach took much less (about a half less) iterations and time to run. Additionally, an important enlightening result is that for 50% of the initial structure's volume, the compliance C is about the same value for both cases, but the iterations in the hexagonal mesh were about the half of the iterations required by the quad mesh. Figure 8 presents the topologies obtained with 50% of the initial domain to compare both meshes optimization with a better visualization.

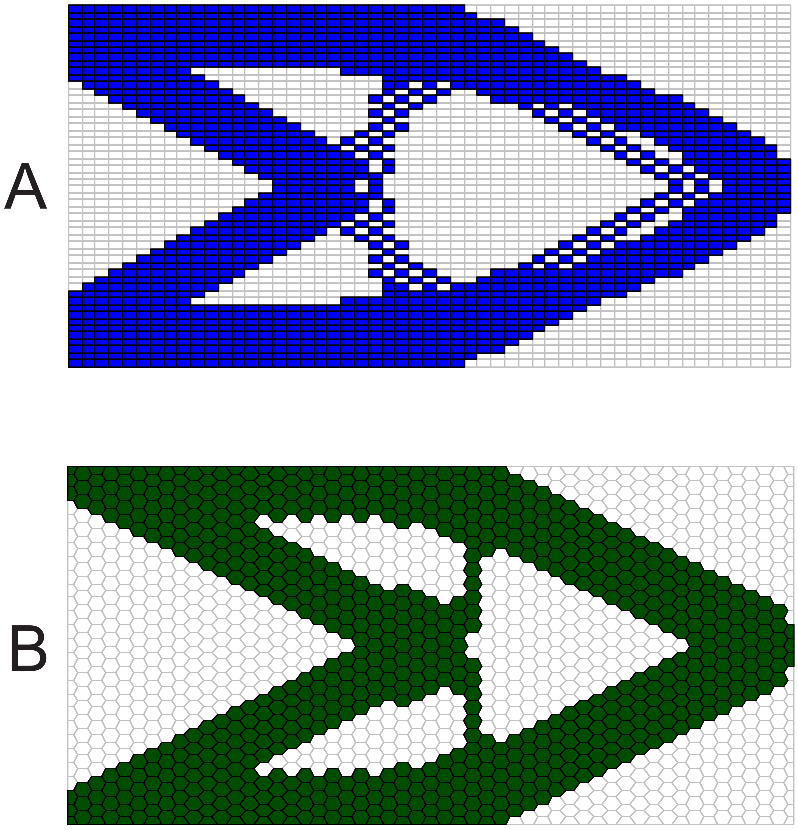

Figure 8. Cantilever beams optimized by ESO under stiffness criterion with 50% of the initial domain design. A) Mesh with regular quads. B) Hexagonal mesh.

In Fig. 8, the result with quadrilateral mesh contain patches of checkerboard while with hexagonal mesh no such patterns are observed. The edges of the hexagonal blocks have smoothed the contours of the final topology and no one-node hinges were obtained. No filtering techniques nor special elements were used and thus the checkerboard-free property of the hexagonal blocks is attributed to its geometry, which confirms its utilization.

3.2 Mesh dependency

Once the hexagonal mesh achieved checkerboard-free topologies with ESO method in the short beam example showed above, more analysis were carried out with different meshes to check its results and mesh dependency. It is shown that the ESO with hexagonal blocks is mesh dependent as well as the ESO with regular quad mesh, considering the method is based in a finite element analysis and no more extra techniques (filters, perimeter control and etc) were used. The method with hexagonal blocks and its mesh dependency is presented in Fig. 9 in line A.

The optimizations were carried out with the same properties as the short beam example, except their meshes. Figure 9 presents a comparison between 35x20 and 45x22 hexagonal blocks meshes and their equivalent for regular quad meshes 35x40 and 45x44, respectively. These analysis were carried out as: classical ESO for hexagonal mesh (A), classical ESO for quad mesh (B) and ESO with checkerboard filter for quad mesh (C). It is observed in the hexagonal blocks optimization that the mesh dependency problem is on as well as for regular quads analysis. Line B shows the mesh dependency for regular quads meshes with no filtering techniques while Line C presents the same analysis with a Li and Steven (2001) checkerboard suppression filter. It must be pointed out that with no filtering the checkerboard is in the structure and with filtering the problem is out. However, even though checkerboard is suppressed with filters, one-node hinges are obtained in the final topology, which is another advantage for the hexagonal mesh utilization. Thus, it is confirmed that evolutionary structural optimization with hexagonal mesh is a feasible alternative for the traditional method, but the mesh dependency problem persists.

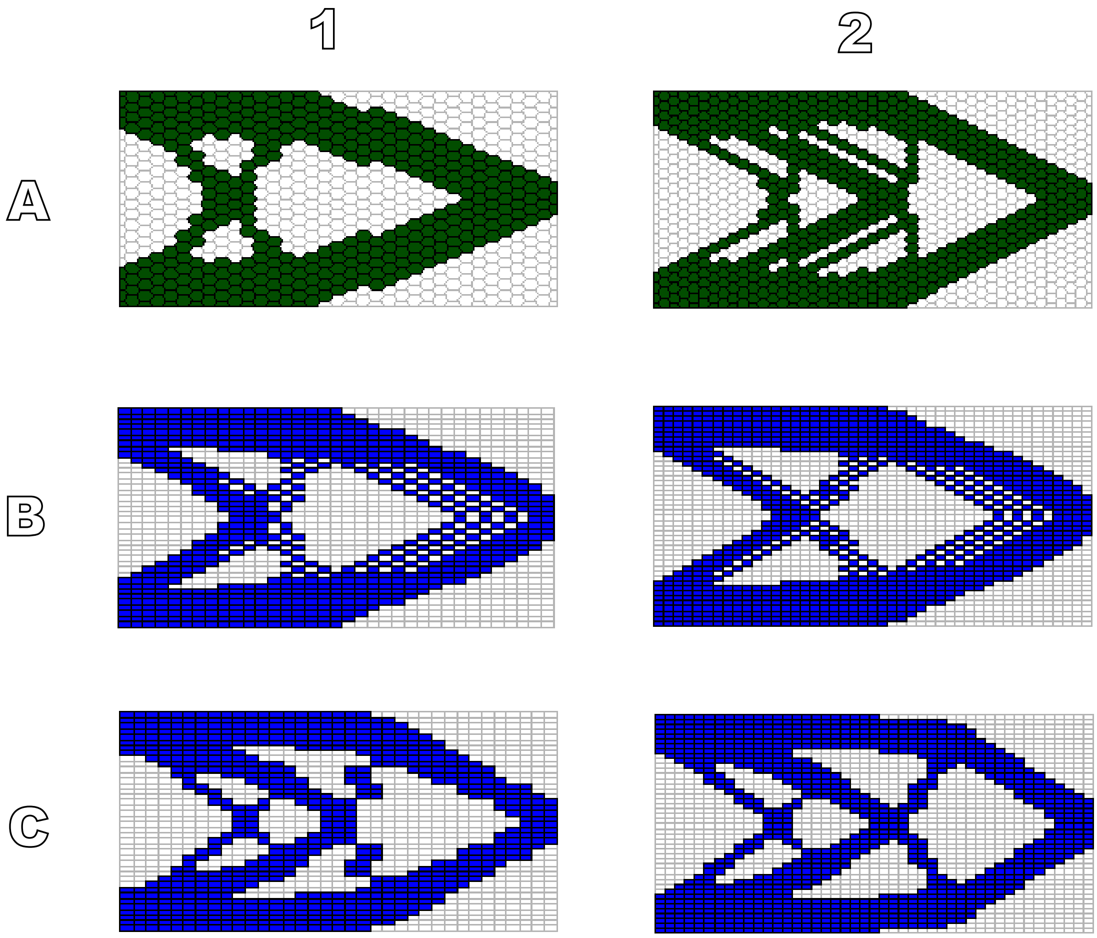

Figure 9. Cantilever beams optimized by ESO under stiffness criterion with 50% of the initial domain design and meshes comparison. A) Hexagonal mesh. B) Regular quads mesh. C) Regular quads mesh with Li and Steven (2001) checkerboard suppression filter. 1) 35x20 hexagonal blocks and 35x40 regular quads. 2) 45x22 hexagonal blocks and 45x44 regular quads.

4. Concluding remarks

This paper has presented an evolutionary structural optimization with hexagonal finite element mesh approach as a feasible alternative to the method. For a short cantilever beam problem the histories of the mean compliance were compared as well as the topologies for both hexagonal and standard quadrilateral meshes. Hexagonal mesh has achieved the checkerboard-free property attributed to the hexagonal blocks geometry and no one-node hinges were obtained. The computational cost with the hexagonal mesh was lower than with the regular quad one, considering it took about half less iterations to achieve the final topology with a very similar mean compliance for both approaches. It has confirmed the possibility of hexagonal ESO approach utilization.

For future works different criteria will be applied and also more complex ESO-based methods will be implemented with hexagonal mesh approach together with mesh dependency filters as well as a computational cost comparison. Comparisons with SIMP results will be done.

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