# A Fault Tolerant Flight Control System Using Dynamic Inversion with Control Allocation

#### Yasser Mahmud Abdallah, yasser.abdallah@embraer.com.br EMBRAER S.A.

Av. Faria Lima, 2170 – 12270-901 São José dos Campos/SP - Brazil

## Elder M. Hemerly, hemerly@ita.br

Technological Institute of Aeronautics - Electronics Division, Systems and Control Department Campo Montenegro - Vila das Acácias 12228-900 São José dos Campos/SP - Brazil

**Abstract.** This paper presents the design of a control algorithm for aircraft fault tolerant control. The approach is based on the blending of Dynamic Inversion and Control Allocation. The proposed control allocation is used to compute control surface deflections in order to produce certain desired aerodynamic moments in roll, pitch and yaw. The greatest benefit is to allow actuator failures to be handled in over-actuated systems. A fully coupled 6DOF model is employed to demonstrate an application on damaged aircraft and the controller was used to compensate variations due to control surface degradation.

Keywords: Fault tolerant, flight control, dynamic inversion, control allocation

## **1. INTRODUCTION**

Flight control under failure or damaged aircraft in off-nominal flight conditions poses significant technical challenges in many areas due to the dramatically change that could happen in control derivatives. According to Bodson (1993) a fault tolerant flight control must deal with challenges such as: 1) the aircraft is intrinsically a nonlinear system with changes in parameters with operating conditions; 2) the aircraft may become highly unstable after occurrence of a failure and 3) a strong cross-couplings, especially after an asymmetric failure, making it a multi-variable control problem.

In order deal with these challenges, we this paper propose a solution by combining dynamic inversion and control allocation techniques to automatically reconfigure part of the faulty control surface. The fault tolerant control system is composed by several key components. The core control is performed by a nonlinear dynamic inversion controller that generates a virtual control command in term of moment demands for roll, pitch and yaw torques. The control allocation then makes the combinations of control surface deflections that meet these moment demands.

Several control allocation methods have been described in the literature (Härkegård 2003): direct control allocation, daisy chaining and the linear programming method. As this work do not involve fault detection and identification, the daisy chaining method was choose, because the other methods do not taken into account the dynamics and limitations of the actuators after a failure, being necessary the switching of the control algorithm after the fault detection. One benefit of using daisy chain is that the controller remains the same and the control is distributed to all available actuators without reconfiguration. This is vital in terms of simplicity of design.

The main contributions with respect to this work are: a) A much more realistic aircraft model is used; b) the strategy do no requires changes of the control algorithm after the failure, and c) detailed analysis of failed operation under left aileron hard over is carried out. This paper is organized as follows: in section 2 the DI control strategy is summarized. The control allocation strategy is described in section 3. Simulations results are presented and discussed in section 4, followed by the conclusions in section 5.

## 2. DYNAMIC INVERSION CONTROLLER

Dynamic inversion is a form of feedback linearization, see Slotine and Li (1991) for details, which assumes that the exact form of a dynamic equations system is known and all states can be measured. A controller can be formulated to make the input-output behavior of the system as a set of integrators.

The DI flight control strategy was designed by considering two sets of equations. The first one composes the inner loop and deals directly with the aircraft angular rates. The second one is an outer loop linking the sideslip angle with the yaw angular rate. The basic diagram is shown in Fig. 1.



Figure 1. Baseline aircraft control system based on dynamic inversion.

All dynamic inversions were performed by supposing a standard x-y-z flat earth, rigid body and symmetrical airplane six-degree-of-freedom dynamic model. The six-degree-of-freedom aircraft dynamics can be expressed as, (Steven and Lewis, 2003)

$$\begin{split} \dot{p} &= \frac{-(-I_y I_z + I_z^2 + I_{xz}^2)qr + I_z(I_{xz} pq + \bar{q}SbC_l) + I_{xz}((I_x - I_y) pq + \bar{q}Sl_sC_n}{I_x I_z - I_{xz}^2} \\ \dot{q} &= \frac{\bar{q}Sl_\mu C_m - I_{xz}(p^2 - r^2) + (I_z - I_x) pr}{I_y} \\ \dot{r} &= \frac{I_{xz}(I_{xz} pq - (-I_y + I_x + I_z +)qr + \bar{q}Sl_sC_l) + I_x((I_x - I_y) pq + \bar{q}Sl_sC_n}{I_x I_z - I_{xz}^2} \\ \dot{\alpha} &= -\frac{\bar{q}S}{mV \cos\beta} C_L + q - tan \beta(p\cos\alpha + r\sin\alpha) + \frac{g}{V\cos\beta}(\cos\phi\cos\alpha + \sin\theta\sin\alpha) \\ \dot{\beta} &= -\frac{\bar{q}S}{mV} C_{Y_{wind}} + p\sin\alpha - r\cos\alpha + \frac{g}{V}\cos\beta\cos\alpha + \frac{g\sin\beta}{V}(\sin\theta\cos\alpha - \sin\alpha\cos\theta\cos\phi) \\ \dot{\phi} &= p + tan \theta(q\sin\phi + r\cos\phi) \\ \dot{\theta} &= q\cos\phi - r\sin\phi \end{split}$$
(1)

where  $C_{Y_{WND}} = C_Y \cos \beta + C_D \sin \beta$ . In addition  $C_L, C_D$  and  $C_Y$  are the lift, drag and side aerodynamic forces coefficients, and  $C_L, C_m$  and  $C_n$  are the rolling, pitching and yawing aerodynamic moment coefficients, respectively.

In equation (1),  $\alpha$  and  $\beta$  are the angle-of-attack and the angle-of-sideslip of the aircraft, respectively. The angular rates along the body axes are p,q and r and  $\phi$  and  $\theta$  are respectively the roll and pitch angles. These variables compose the state vector

$$x = [p, q, r, V_{tas}, \alpha, \beta, \phi, \theta, \psi]$$
<sup>(2)</sup>

The aerodynamics coefficients, and consequently the aerodynamic forces and moments, are function of the states of the vehicle and also function of the aerodynamic control surface deflections. For the flight controls problem in this work, three aerodynamic control surfaces are assumed: the aileron,  $\delta_a$ , for lateral control; the rudder,  $\delta_r$ , for directional control; and the elevator,  $\delta_e$ , for longitudinal control, composing the input vector  $u = [\delta_a \ \delta_r \ \delta_e]^T$ . These control surfaces are directly used in the inner linearization loop to control the output variables, assumed to be  $y = [p \ q \ r]^T$ 

Therefore, from equation (1) the inner linearization loop becomes linear in aerodynamic derivatives  $\Theta_1$  and  $\Theta_2$  as

$$\dot{\mathbf{y}} = F_1(\mathbf{x}) + F_2(\mathbf{x})\Theta_1 + M(\mathbf{u})\Theta_2 \tag{3}$$

were

$$F_{1} = \begin{pmatrix} \frac{q(-I_{x}I_{xz}p + I_{x}I_{y}p - I_{xz}I_{z}p + I_{x}^{2}r - I_{y}I_{z}r + I_{z}^{2}r)}{I_{xz}^{2} - I_{x}I_{z}} \\ \frac{(-I_{x} + I_{z})pr + I_{xz}(-p^{2} + r^{2})}{I_{y}} \\ \frac{q(-I_{x}^{2}p - I_{xz}^{2}p + I_{x}I_{y}p + I_{x}I_{x}r - I_{xz}I_{y}r + I_{xz}I_{z}r)}{I_{xz}^{2} - I_{x}I_{z}} \end{pmatrix}$$

$$(4)$$

The last 2 terms in (3) are the contributions of the aerodynamics characteristics of the aircraft, including the aerodynamic control surfaces. These portions depend on the modeling of  $C_L$ ,  $C_D$  and  $C_Y$  and  $C_l$ ,  $C_m$  and  $C_n$  as a function of the aircraft model states and inputs. They are omitted here, since they are quite large expressions.

### **3. CONTROL ALLOCATION**

In flight control system usually surface deflections are done in a manner to decoupling the axis. However when the aircraft experiences an actuator failure or airframe damage, the aircraft becomes asymmetric. Therefore, the actuator system must control the actuators fully independent, by which means that ailerons (or elevators) can individually move up, down, or together in the same direction. The propose of control allocation is to take advantage of aircraft over-actuated systems computing control surface deflections fully independently in order to produce certain desired aerodynamic moments in roll, pitch, and yaw axis. This configuration permits some pitch torque to be produced with ailerons or some roll torque to be produced with elevators.

By using control allocation with the daisy chain approach, the design of the flight control system can be separated into the derivation of the control laws and the design of a control allocator. There is no need to redesign the controller when such faults occur, since the control allocation compensates for the fault by reallocating the demand of the desired aerodynamic moment in other surface rather than the faulted one.

An advantage of performing control allocation separately, rather than letting the control distribution be decided by the feedback law, is that actuator position and rate limits can be considered. If one actuator saturates, the remaining actuators can be used to provide the difference (Durham 1993 and Wise et al. 1999).

In daisy chain control allocation (Buffington and Enns 1996, Bordignon 1996, Durham and Bordignon 1996), the allocator suite is divided into groups which are successively employed to generate the total control effort. The control allocation problem consists in solving

$$B_1 u_1 + B_2 u_2 + \ldots + B_M u_M = v \tag{5}$$

Where matrix  $B_i$ ,  $i = 1,..., M_i$  is the control effectiveness matrix of the control effectors in relation to the control variable  $v_i$ . The daisy chain control allocation procedure can be summarized as follows (Härkegård 2003):

$$u_{1} = sat_{u_{1}}(P_{1}v)$$

$$u_{2} = sat_{u_{2}}(P_{2}(v - B_{1}u_{1}))$$

$$\vdots$$

$$u_{M} = sat_{u_{M}}(P_{M}(v - \sum_{i=1}^{M-1} B_{i}u_{i}))$$
(6)

The daisy chain idea is to first try to satisfy this virtual control demand using only the first group of actuators by solving

$$B_1 u_1 = v \tag{7}$$

for  $u_1$ . If  $B_1$  have full rank, it is solved by

$$u_1 = P_1 v \tag{8}$$

where  $P_1$  is any right inverse of  $B_1$ . If  $u_1$  satisfies Eq. (7) as well as the actuator position and rate constraints, the allocation was successful and the procedure halts. Otherwise,  $u_1$  is saturated according to its position and rate constraints,

$$u_1 = sat_{u_1}(P_1 v) \tag{9}$$

and the second group of actuators is employed by solving

$$B_{2}u_{2} = v - B_{1}u_{1} \tag{10}$$

for  $u_2$ , yielding the solution  $u_2 = P_2(v - B_1u_1)$ . Again, if  $u_2$  fails to satisfy Eq. (10) or violates some constraint, the solution is saturated and  $u^3$  is employed to make up the difference. This procedure is repeated until either the virtual control demand is met, or all actuator groups have been employed.

## 4. RESULTS

The fault tolerant controller was simulated with a high fidelity regional aircraft model. It was simulated a hardover in left aileron to study the behavior of the controller under failure conditions (see figure 2). The effects of the aileron hardover induce a change in the roll moment coefficients as well as change in the lift coefficient. The consequence of this is that the aircraft would exhibit a pitch-roll coupling when the ailerons are deflected asymmetrically.



Figure 2. Surfaces position for aileron failure.



Figure 3. Attitude angles for aileron failure.

The stability of the aircraft can be regained if sufficient control powers are still available to overcome the rolling and yawing moments as well as to retrim the aircraft in the pitch axis (see figure 3, 4 and 5). To maintain a trim state, the flight control must compensate for the unwanted roll motion. It is possible to see that using only the dynamic inversion controller provide some compensation for the unwanted roll and yaw motion commanding the remain aileron and rudder, however the performance become degraded during pilot command (see figure 6). This happens because the DI only controls the aileron and rudder surface to control the roll-yaw axis (see figure 1). The hardover in one aileron implies that there is not enough control efficiency in the remaining aileron and rudder to achieve the same performance that existed prior the failure.

By using the control allocation together with dynamic inversion, it is possible to obtain the desired performance by commanding the elevator asymmetrically (see figure 1). Thus, the deflection of the elevator would result in a pitch-yaw coupling that must be compensated within the flight control system by adjusting the rudder control accordingly. Due to the asymmetry, the general motion of the aircraft is coupled in all the three axes.



Figure 4. Angular rates for aileron failure.



Figure 5. Drift angle for aileron failure.



Figure 6. Tracking for aileron failure.

From figure 6 is possible to see that the joint application of dynamic inversion and control allocation produces, under failure, performance similar to that obtained prior to the failure. Simulations with sensor noise were carried out and similar results were obtained.

#### **5. CONCLUSION**

This paper has investigated the joint application of dynamic inversion and control allocation strategies for dealing with aircraft flight control systems under failure condition. From the simulations, the following conclusion can be drawn: if there is performance degradation due to a failure, like a hardover in one of command surface, the dynamic inversion controller has enough robustness to retrim the aircraft. However, when the control allocation is introduced, the tracking and prediction errors are considerably reduced, as desired in such failure conditions.

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