PROPELLER COMPUTACIONAL ANALYSIS UTILIZING BLADE ELEMENT THEORY

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Abstract *The proposal of the present work is show the development and application of a simple tool able to analysis of the propulsion system constitute by propeller. The idea is utilize this program to class education during propulsion theory studies. There are several theories whose intent to previous propeller behavior in real conditions. The classical is the momentum theory which gave estimates for power and induced velocity in terms of the thrust. After, it is the blade element theory, which allowed to incorporate features such as number of blades, airfoil section drag and lift characteristics, taper and twist, etc.. According to results presented by the Simple Blade-Element Theory, in order to determine the efficiency of a propeller it is necessary to previously obtain some aerodynamic values - due to the presence of lift and drag coefficients - and also the knowledge about the spatial distribution of that propeller. To simplify this calculation, it is necessary to implement an algorithm of numerical integration. The numeric integration using Simpson's rule becomes applicable to this situation, as it is adopted a finite number of elements to approximate the desired integrals. Moreover, when developing an algorithm that estimates of efficiency, is necessary consider the available database of the aerodynamic properties regarding airfoils in use. Therefore, it is adopted the program Xfoil in which it is possible to determine the lift and drag coefficients with the method of panels. Thus, the present work integrates Excel and Xfoil in order to obtain analysis of the propulsion system.*

Keywords: *Propeller, Blade Element Theory Simulation*

1. INTRODUCTION

The purpose of motor propulsive system in a particular vehicle is primarily the conversion of a specific type of energy – such as the chemical energy of fuel – to the kinetic energy of translation. So the propeller represents an element of that system, whose goal is to provide traction by the cost of engine power. This procedure will only be accomplished through a fluid, as it provides, for a certain amount of mass, movement in the opposite direction to that of the vehicle being propelled. However, throughout this process there are certain energy losses such as friction with the fluid. So there appears the need to determine the efficiency of propellers regarding specific operation conditions.

From an educational viewpoint, the introduction of basic models associated with propellers not only represents the basis for more complex approaches but also a variety of development tools that can be later used. Thus, the comprehension of the Simple Blade-Element theory presents itself constructively in the learning process. Furthermore, the implementation of a computational approach, as subsequently developed, enables both the execution of mathematical operations with greater speed and the combination of available programs to estimate aerodynamic parameters. Also it should be taken into account that the chosen programming environment enables more interactivity with the user in order to favor understanding of the data content.

The results given by the Simple Blade-Element theory consists in estimating the distribution of traction and torque along the propeller radius and calculating the efficiency in certain conditions. Having knowledge about the theoretical behavior of these data regarding the experimental data, one can also examine the viability of applying the same practice. From an educational standpoint, this project has aimed to show how important is the development of physical modeling together with computer implementation. Hence it is possible to estimate values without the need for a high operating cost.

2. SIMPLE BLADE-ELEMENT THEORY

Table 1. Definition of variables

ρ	Air density
μ	Air viscosity
Di	Diameter of the propeller
R	Radius of the propeller
b	Cord of the element
n	Rotation frequency
α	Angle of attack of the element

β	Angle of geometric pitch
ϕ	Angle of effective pitch
C_D	Drag coefficient
C_L C_L	Lift coefficient
T_C, C_T	Thrust coefficient
Q_C, C_Q	Torque coefficient
η	Propeller efficiency
B	Number of blades
L	Lift
D	Drag

The theoretical model presented by the Simple Blade-Element Theory, although first published by W. Froude (1810-1879) in 1878 and later improved by W. Lanchester (1868-1946) in 1885, is primarily awarded to the Polish scientist S. Drzewiecki (1844-1938), due to the fact that he was responsible for the dissemination and practical application of that theory. The Simple Blade-Element Theory considers the propeller formed, along the radius of each blade, by infinitesimal elements in shape of airfoils. Therefore this description is illustrated in Fig. 1.

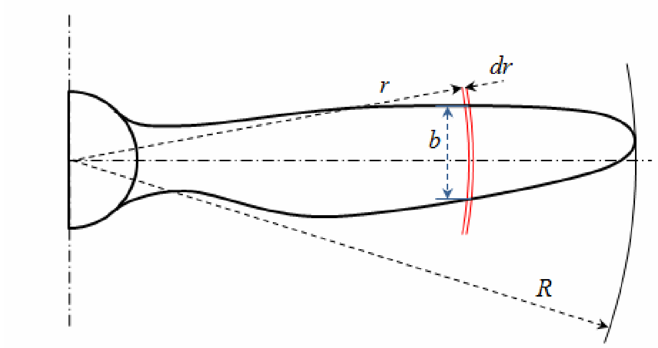

Figure 1. Infinitesimal airfoils along the blade radius

It is supposed that the air flow can be considered two-dimensional and thus unaffected by neighboring elements of the blade. The evidence supporting this statement is shown by experiments made by Lock *et al.* (1921). It should also be noted that for each element the resultant velocity (V_r) can be decomposed in rotation (V_{rot}) and translation (V_{trans}) components. Each element has an aerodynamic resultant reaction (*R*) can be decomposed in the aerodynamic coordinate system, resulting the drag (*D*) and lift (*L*) forces. However, when decomposed in the plane of rotation, it gives rise to the thrust (T) and the force (F) responsible for producing torque. Nevertheless these aerodynamic reactions refers to an blade element and must be treated as infinitesimal, as sketched in Fig.2

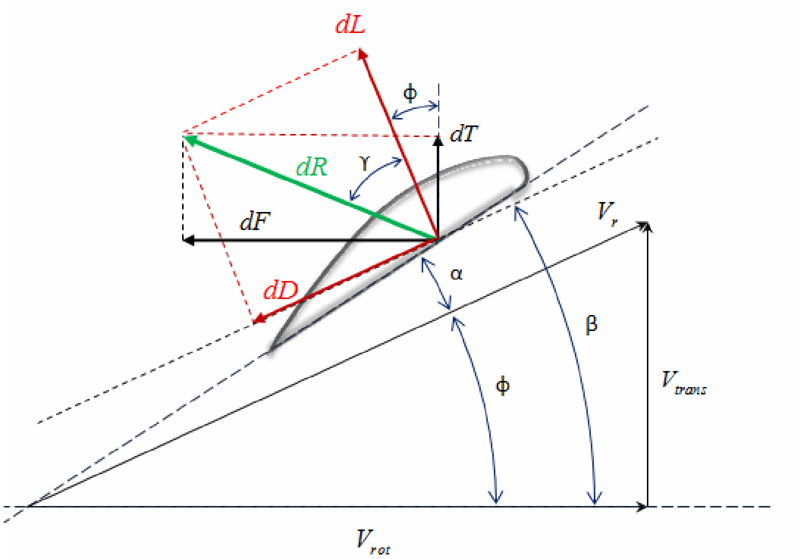

Figure 2. Different settings concerning each element

The path developed in space by a generic element is displayed in Fig. 3. Also in Fig. 3, bellow the path sketch, there is corresponding planned path over a period of time in order to illustrate the angle of geometric pitch and the angle of effective pitch for that given element.

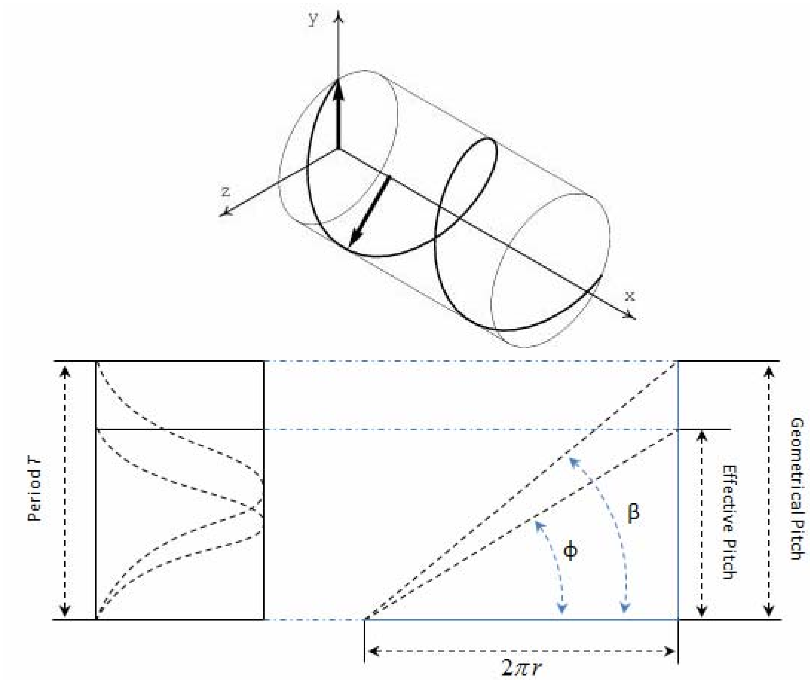

Figure 3. Path developed by na element together with corresponding geometric and effective pitch

It can be applied, the expression for the lift in the elementary profile represented by Eq. (1).

$$dL = \frac{1}{2}\rho V_r^2 C_L b dr \tag{1}$$

By the geometry in Fig. 2, it is given that:

$$dL = dR\cos\gamma \tag{2}$$

$$dR = \frac{1}{2}\frac{\rho V_{trans}^{2} C_L b dr}{sen^{2}\phi \cos\gamma} \tag{3}$$

$$\gamma = \arctan\left(\frac{dD}{dL}\right) \tag{4}$$

Thus, the infinitesimal traction produced by the considered element is given by:

$$dT = dR\cos(\phi+\gamma) \tag{5}$$

$$dT = \frac{1}{2}\frac{\rho V_{trans}^{2} C_L b \cos(\phi+\gamma) dr}{sen^{2}\phi \cos\gamma} \tag{6}$$

In order to simplify expressions, the following terms are defined from Eq. (6):

$$K = \frac{C_L b}{sen^{2}\phi \cos\gamma} \tag{7}$$

$$T_C = K\cos(\phi+\gamma) \tag{8}$$

The Eq. (9) expression for total thrust results from the integration of Eq. (6) over the radius, then a corrective factor that takes account the number of blades is introduced:

$$T = \frac{1}{2}\rho V_{trans}^{2} B\int_{0}^{R} T_C dr \tag{9}$$

According to the infinitesimal treatment adopted, the horizontal component *dF* of total air forces is responsible for the torque. Thus, it is possible to write Eq. (10) and Eq.(11).

$$dQ = rdF \tag{10}$$

$$dF = dRsen(\phi+\gamma) \tag{11}$$

Using Eq. (3) and Eq. (6), is similarly possible to determine the expression for the total torque, resulting in Eq. (12).

$$Q = \frac{1}{2}\rho V_{trans}^{2} B\int_{0}^{R} Q_C dr \tag{12}$$

Eq. (13) results from the definition of efficiency for a propeller:

$$\eta = \frac{\text{Rate of useful work of the propeller}}{\text{Rate of energy supplied by the motor}} = \frac{\dot{W}_{use}}{\dot{E}_{\sup}} \tag{13}$$

Given the properties of the system, it is know the rate of work:

$$\dot{W}_{util} = TV_{trans} \tag{14}$$

The rate of energy supplied by the motor matches the power of it:

$$\dot{E}_{\sup} = 2\pi n Q \tag{15}$$

$$\eta = \frac{TV_{trans}}{2\pi n Q} \tag{16}$$

It can be noted that expressions Eq. (9), Eq. (12) and Eq. (16) estimate properties of the propeller in a specific state depending exclusively on geometry, fluid dynamics and kinematics of that propeller.

Despite the Simple Blade-Element Theory describes the behavior of a propeller moving through a certain type of fluid, there are considerations and features that are not taken into account, some are listed below:

a) It does not take into account the interference between the blades, which is directly proportional to the rotation frequency, specially in regions close to the root, as stated in Weick (1930);
b) The effects of wing tip are disregarded, and the values of torque and traction computed in regions close to the tip of the wing are larger than those derived experimentally according to Weick (1930);
c) The fluid dynamic calculations should performed based on the average direction that the fluid flows, however it is only applied to the direction of incoming flow. It is noted however that the profiles change the direction of incoming flow, experimental results from Fage and Howard (1921) explain the difference between the experimental aerodynamic coefficients and those calculated assuming that the direction of flow is not modified;
d) Analysis of experimental results in Durand and Lesley (1924) demonstrate the low accuracy of this model when applied, so that the use of this theory is restricted to comparison and estimate;
e) This Theory is only valid for incompressible flow.

3. DEVELOPMENT

According to results presented by the Simple Blade-Element Theory, in order to determine the efficiency of a propeller it is necessary to previously obtain some aerodynamic values - due to the presence of lift and drag coefficients - and also knowledge about the spatial distribution of that propeller. In a way, the expressions given by Eq. (9), Eq. (12) and therefore Eq. (16) present laborious mathematical based on integration over the radius of a propeller.

To simplify this calculation, it is necessary to implement an algorithm of numerical integration. The limitation lies precisely in the determination of an analytical expression for the traction and torque coefficient relative to the variable *r*, however one can calculate the value of these functions in certain sections of the blade, since the aerodynamic coefficients are previously known. Therefore, the numeric integration using Simpson's rule becomes applicable to this situation, as it is adopted a finite number of elements to approximate the desired integrals. The general expression of the known Simpson's rule regarding *n* ordinates whose abscissas are spaced Δr is given below:

$$\int_0^R F(r)dr = \frac{\Delta r}{3}\left[y_1 + 4y_2 + 2y_3 + 4y_4 + ... + 2y_{n-2} + 4y_{n-1} + y_n\right] \tag{17}$$

Thus, considering the restriction of that rule, it should be taken sections of the blade that are distributed approximately equidistant.

Moreover, when developing an algorithm that estimates of efficiency, one must consider the available database of the aerodynamic properties regarding airfoils in use. After all, the use of aerodynamic factors is observed in equations Eq. (1) and Eq. (4), which are the basic mathematical concepts regarding the Simple Blade-Element Theory. Therefore, it is adopted the program *Xfoil* in which it is possible to determine the lift and drag coefficients with the method of panels, and also determine the angle related to Eq. (4). However, it should be noted that the link established between the developed algorithm and the program in question restricts the algorithm itself to the convergence conditions and the available airfoils in *Xfoil*. If the profile is not available in *Xfoil*, it is possible to load from database files that contain the curve coordinates of a certain airfoil. Despite the operating conditions, the practical approach regarding the Simple Blade-Element Theory is valid from an educational standpoint, because it develops a different presentation of theoretical introduction involving propellers.

The *Xfoil* input data are type of airfoil, angle of attack and Reynolds number, all of these referring to the element under study. It should be noted that the profile size is standardized, so it is necessary to calculate the Reynolds number. So the Eq. (18) and Eq. (19) are used:

$$\alpha = \beta - \phi \tag{18}$$

$$\mathrm{Re} = \frac{\rho}{\mu} b V_r \tag{19}$$

Thus, with the lift and drag coefficients determined from *Xfoil*, Eq. (4) can be modified taking into account Eq. (1) and the Eq. (20):

$$dD = \frac{1}{2}\rho V_r^2 C_D b dr \tag{20}$$

The Eq. (21) result is obtained:

$$\gamma = \arctan\left(\frac{C_D}{C_L}\right) \tag{21}$$

The programming environment selected is the Excel platform so that data can be organized in the form of tables and it is also possible the development of macros in VBA environment (Microsoft Visual Basic for Applications). The input data are classified according to Fig. 4 and they are inserted directly into the Excel spreadsheet such as Table I. If not previously known, the geometric data may be obtained through detailed measurements.

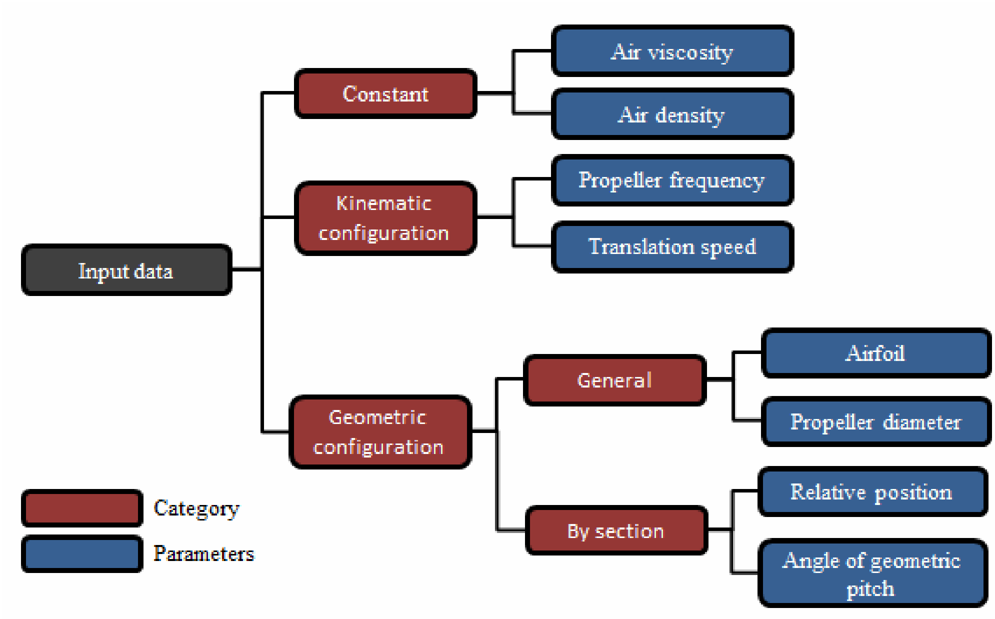

Figure 4. Input data diagram

Through VBA were developed two macros named: *ComandoXfoil* and *IntegralSimpson*. It is necessary that the user execute these macros in order to expected results.

The macro *ComandoXfoil* is responsible for calculating lift and drag coefficients of the sections executing an external link with the program *Xfoil*. The data required for this process are α and *Re* of the corresponding elements, which are calculated from input data contained in Table I with Excel spreadsheet formulas. To this end, two files are created respectively containing the appropriate commands to be executed for Xfoil (. vbs) and the boot in MS-DOS (. bat) to run *Xfoil*. The results generated by *Xfoil* are exported to a text file, and the algorithm performs the reading of this file to assign the corresponding values of the coefficients in the Excel spreadsheet as shown in Table I. This procedure, described in Figure 5, is then performed iteratively to perform calculations on all elements defined.

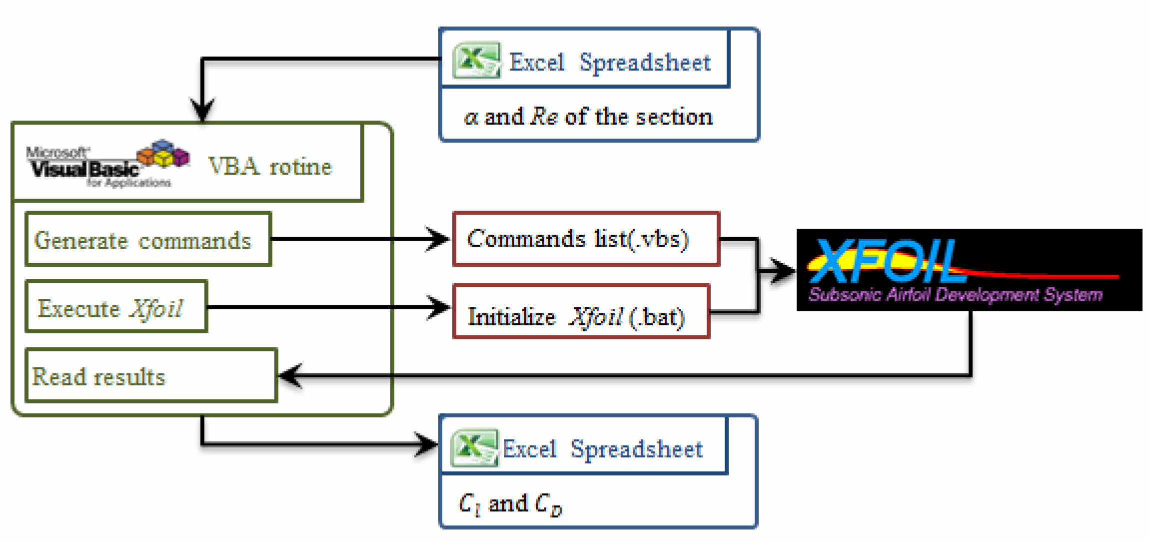

Figure 5. Method used to calculate aerodynamic coefficients

Also it is necessary to configure some *Xfoil* and *ComandoXfoil* options before starting to execute the macros. This can be done by filling a reserved space in the Excel spreadsheet represented by Fig. 6. It is important to notice that when the user fills the airfoil name or directory it automatically completes Table I.

VBA Program Configuration			
Directories		Examples	
Directory used to create *Xfoil* input TXT archive	D:\naca2412\xfoilinput.txt	D:\xfoilinput.txt	
Directory used to create *Xfoil* results TXT archive	D:\naca2412\results.txt	D:\results.txt	
Disk location *Xfoil* EXE	D:\xfoilP4.exe	D:\xfoilP4.exe	
Xfoil Configuration		Examples	
Viscous-solution iteration limit	10000		
Airfoil exists in Xfoil library? (Y/N)	Y	Y	N
(Y) Airfoil name (N) Airfoil directory	naca 2412	naca 2412	d:\raf6.dat

Figure 6. Configurations

On the other hand the macro *IntegralSimpson* execute the expression given by Eq. (17) using the traction and torque coefficients already calculated by spreadsheet formulas with the output of *ComandoXfoil*. Finally, the results are exposed in Table II. In fact this routine was developed so that the variable n is given by the user, thus the number of sections that represent the propeller blade can be determined accord to the amount of information the user has. When increasing n, it is only necessary to extend Table I so that it can support more sections.

After connecting these routines, the final algorithm can be represented by the Fig.7.

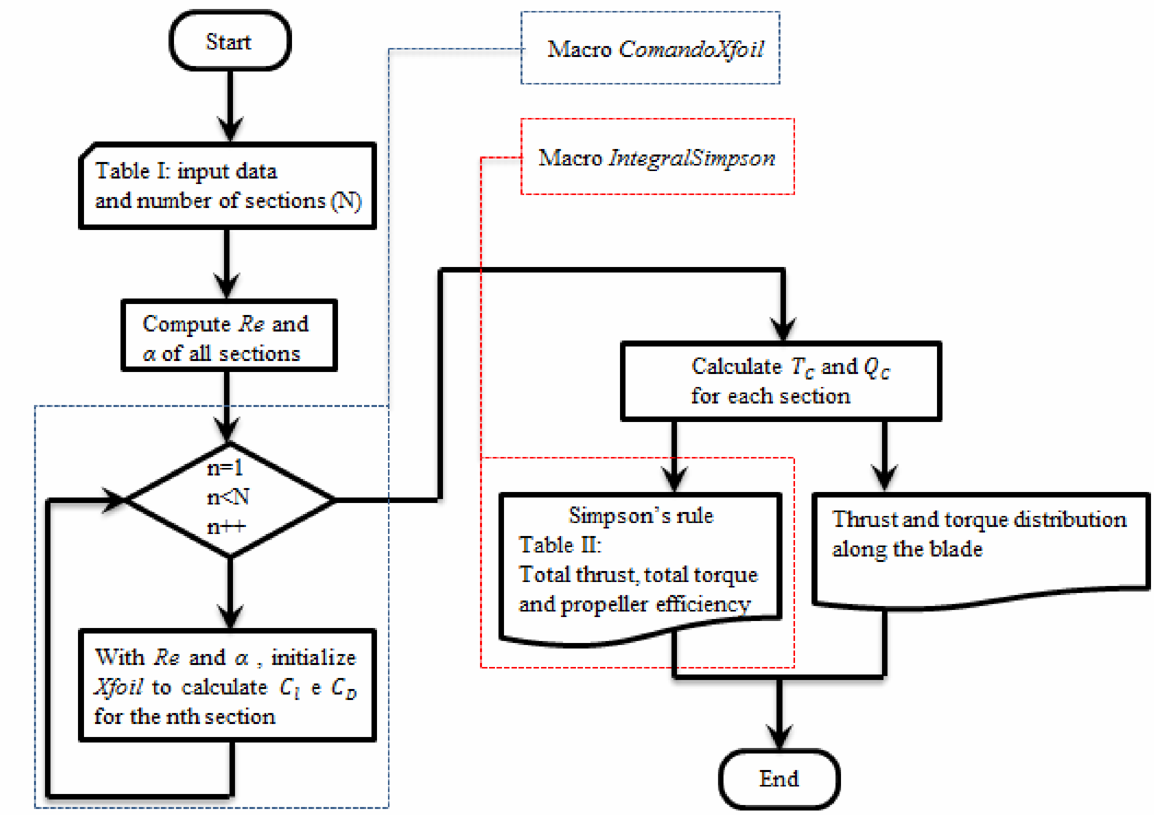

Figure 7. Complete algorithm flowchart

Figures denoted in Fig. 8 and Fig. 9 represents a part of the worksheet in Excel that correspond to the tables listed as Table I and Table II. In order to assist the interpretation of these tables, Fig. 10 contains subtitles that are also present in the Excel file. In addition, Fig. 11 represents another part of the spreadsheet that contains input data regarding general constants.

Table I. Excel and Xfoil Computations for Propeller Analysis regarding the Simple Blade Element Theory							
Number of sections	7						
Section Data	1	2	3	4	5	6	7
r/R	0	0,15	0,3	0,45	0,6	0,75	0,9
$Airfoil$	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412
$D(m)$	0,9144	0,9144	0,9144	0,9144	0,9144	0,9144	0,9144
$b(m)$	0	0,06858	0,071933	0,0762	0,071933	0,06035	0,041148
$\beta(rad)$	0	0,97912971	0,638790506	0,460766923	0,356047167	0,289724656	0,242600766
$V_{trans}(m/s)$	17,87652	17,87652	17,87652	17,87652	17,87652	17,87652	17,87652
$n(Hz)$	30	30	30	30	30	30	30
$r(m)$	0	0,06858	0,13716	0,20574	0,27432	0,3429	0,41148
$J = \frac{V_{trans}}{nD}$	0,651666667	0,651666667	0,651666667	0,651666667	0,651666667	0,651666667	0,651666667
$V_{rot} = 2\pi rn(m/s)$	0	12,92702545	25,8540509	38,78107635	51,7081018	64,63512725	77,56215271
$V_{total} = \sqrt{V_{trans}^2 + V_{rot}^2}\ (ft./s)$	17,87652	22,06077864	31,43249776	42,70294897	54,71103874	67,06168535	79,59558719
$\phi = arctag\left(\frac{V_{trans}}{V_{rot}}\right)(rad)$	0	0,944715776	0,604957736	0,431930687	0,33285674	0,269830743	0,226524163
$\alpha = \beta - \phi(rad)$	0	0,034413934	0,033832771	0,028836235	0,023190428	0,019893913	0,016076603
$Re = \frac{\rho}{\mu} b\, V_{total}$	0	96298,69329	143916,0209	207116,6033	250498,5462	257604,719	208468,1913
C_L	0	0,5063	0,5108	0,483	0,4517	0,4338	0,4145
C_D	0	0,01604	0,01198	0,00979	0,00891	0,00877	0,00961
$\gamma = arctag\left(\frac{C_D}{C_L}\right)(rad)$	0	0,031670229	0,023449108	0,020266376	0,019722924	0,020213936	0,023180407
$K = \frac{C_L b}{cos\gamma\, sin^2\phi}$	0	0,052905273	0,113629292	0,210058893	0,304401439	0,368500876	0,338222099
$T_c = K\cos(\phi+\gamma)$	0	0,029628028	0,09192213	0,188945723	0,285676192	0,353108999	0,327732299
$Q_c = K\, r \sin(\phi+\gamma)$	0	0,003005923	0,009161978	0,018883584	0,028835389	0,036138033	0,034391773
$\frac{dT}{dr} = \frac{1}{2}\rho V_{trans}^2 T_c$	0	5,60471747	17,38885801	35,74275706	54,04120594	66,79743235	61,99693624
$\frac{dQ}{dr} = \frac{1}{2}\rho V_{trans}^2 Q_c$	0	0,568628813	1,73316628	3,572196994	5,454774465	6,836211514	6,505872572
	Boss						

Figure 8. Table I content

Table II. Thrust, Torque and efficiency computations using Simpson's rule	
B (number of blades)	2
$\int_0^R T_c dr$	0,077030426
$\int_0^R Q_c dr$	0,007829474
$T = \frac{1}{2}\rho V_{trans}^2 B \int_0^R T_c dr\ (N)$	29,14360554
$Q = \frac{1}{2}\rho V_{trans}^2 B \int_0^R Q_c dr\ (N.m)$	2,962194381
$P = 2\pi n Q\ (W)$	558,3604864
$C_P = \frac{P}{\rho n^3 D^5}$	0,027324662
$\eta = \frac{TV_{trans}}{P} = \frac{C_T}{C_P}\left(\frac{V_{trans}}{nD}\right)$	0,93306432

Figure 9. Table II content

Table I information		Tabela II Information	
	Geometric input data		Geometric input data
	Kinematics input data		Excel computation results (IntegralSimpson Macro)
	Excel computation results (ComandoXfoil Macro)		Results
	Xfoil computation results		

Figure 10. Table I and II subtitles

Constants							
ρ (air density Kg/m^3)	1,1839	1,1839	1,1839	1,1839	1,1839	1,1839	1,1839
μ (air viscosity $Pa.s$)	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05

Figure 11. Input constants

4. RESULTS

In order to illustrate the results provided by the algorithm, an example is developed using most of input data present in Weick (1930), only modifying the airfoil type of the propeller. Thus, the input data are shown in Figure 12.

Constants							
ρ (air density Kg/m^3)	1,1839	1,1839	1,1839	1,1839	1,1839	1,1839	1,1839
μ (air viscosity $Pa.s$)	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05	1,86E-05
Number of sections	7						
Section Data	**1**	**2**	**3**	**4**	**5**	**6**	**7**
r/R	0	0,15	0,3	0,45	0,6	0,75	0,9
$Airfoil$	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412	naca 2412
$D(m)$	0,9144	0,9144	0,9144	0,9144	0,9144	0,9144	0,9144
$b(m)$	0	0,06858	0,071933	0,0762	0,071933	0,06035	0,041148
$\beta(rad)$	0	0,97912971	0,638790506	0,460766923	0,356047167	0,289724656	0,242600766
$V_{trans}(m/s)$	17,87652	17,87652	17,87652	17,87652	17,87652	17,87652	17,87652
$n(Hz)$	30	30	30	30	30	30	30

Figure 12. Input data of the example execution

The settings used are the same as described in Fig. 6 for both *Xfoil* and *ComandoXfoil*. Thus, it is possible to proceed with the execution of *ComandoXfoil*. With the results automatically exported to Fig. 8, two graphs shows the behavior of the distribution of traction and torque along the blade. In the case of input data presented in Fig. 12, the graphs in Fig.13 and Fig. 12 are obtained by means of polynomial approximation of degree 6 provided by Excel. It is also possible to verify computations since incorrect points usually do not form smooth curves.

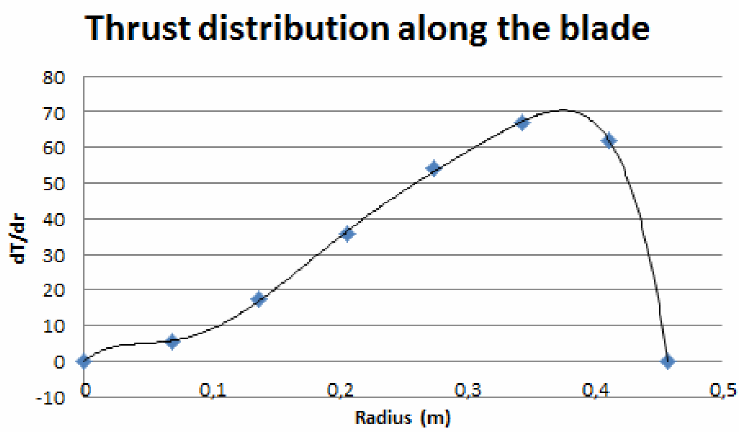

Figure 13. Thrust distribution along the blade

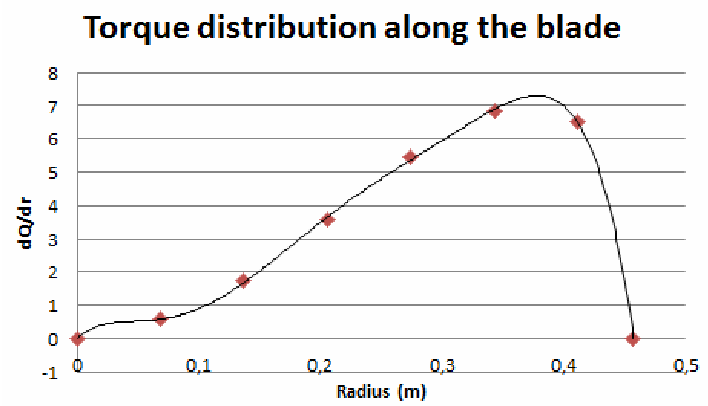

Figure 13. Torque distribution along the blade

Finally, after running the macro *IntegralSimpson* the data contained in Table I are used to calculate the resulting traction and torque, and efficiency of the propeller in question as well. It is necessary to provide the number of blades in of the system, which in this case are 2. The results from this last run are shown in Fig. 14.

Table II. Thrust, Torque and efficiency computations using Simpson's rule	
B (number of blades)	2
$\int_0^R T_c dr$	0,077030426
$\int_0^R Q_c dr$	0,007829474
$T = \frac{1}{2}\rho V_{trans}{}^2 B \int_0^R T_c dr\ (N)$	29,14360554
$Q = \frac{1}{2}\rho V_{trans}{}^2 B \int_0^R Q_c dr\ (N.m)$	2,962194381
$P = 2\pi n Q\ (W)$	558,3604864
$C_P = \frac{P}{\rho n^3 D^5}$	0,027324662
$\eta = \frac{T V_{trans}}{P} = \frac{C_T}{C_P}\left(\frac{V_{trans}}{nD}\right)$	0,93306432

Figure 14. Results after executing IntegralSimpson

5. CONCLUSION

It is observed that the calculated efficiency is high, a fact which in reality is questionable. The resulting value shows explicitly the limits of the model designed with the Simple Blade-Element theory. The goal, however, consists in presenting the theory through a computational approach providing the user a more detailed understanding of the theoretical foundations of the propulsion system in question.

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