GENERALIZED NEWTONIAN FLUID FLOWS IN DUCTS WITH PERMEABLE WALLS

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Abstract. *The flow of a generalized Newtonian incompressible fluid in a permeable wall channel limited by two impermeable flat plates is studied by considering two distinct flow regions: the former representing the fluid and the latter the fluid flowing through a porous medium. A mixture theory model is employed in a local description of the two adjacent flow regions in which both the fluid and the fluid constituent of a binary mixture (representing the flow throgh the saturated porous matrix) are considered as power-law fluids. Compatibility conditions at the interface (pure fluid-mixture) for momentum transfer are considered. Numerical simulations employing a Runge-Kutta methodology, coupled with a shooting strategy, have been performed and compared with some exact solutions. The shear-thickening influence was also investigated.*

Keywords: *Mixture theory, generalized Newtonian flow, dilatant behavior, two-regions flow*

1. INTRODUCTION

Ducts with permeable walls are present in many relevant engineering situations such as flow of perforation mud in oil wells and porous bearing lubrication. Transport phenomena in a permeable duct are studied by considering two distinct flow regions: one inside the duct where an incompressible generalized Newtonian fluid flows (the so-called pure fluid region) and another representing the duct's permeable wall – a saturated porous medium through which this generalized Newtonian fluid flows. In this later region – named as mixture region, a mixture theory approach is used, instead of the volume-averaging technique, employed in most of the works dealing with transport in porous media (see Alazmi and Vafai, 2000 and references therein). The Continuum Theory of Mixtures has been specially developed to describe multiphase phenomena. It models fluid saturated porous media by considering the fluid and the porous matrix as superimposed continuous constituents of a chemically non-reacting binary mixture – each of them occupying its whole volume. The mixture theory leads to an apparent independence among the constituents, requiring additional terms, playing the role of momentum and energy sources, to account for the thermomechanical coupling among the constituents in the balance equations. Thermodynamically consistent constitutive relations for these sources are used. Compatibility conditions at the interface pure fluid-mixture must be imposed in order to allow the solution of the problem. For instance, supposing no flow across the interface, the velocity must be zero on the solid parts of the boundary and should match the fluid diffusing velocity on the fluid parts of the boundary and the shear stress at the pure fluid region is balanced by a multiple of the partial shear stress at the mixture region. Numerical simulations, employing a fourth-order Runge-Kutta methodology coupled with a shooting strategy, have been performed and compared with the exact solution for a fluid flowing though a plane channel and also with the exact solution for a Newtonian fluid flowing through the permeable wall channel limited by two impermeable flat plates. These simulations show the influence of he shear thickening of the fluid on the behavior of the power-law fluid in the two adjacent flow regions.

2. MECHANICAL MODEL

The open sets Ω_1 and Ω_2, with boundaries $\partial\Omega_1$ and $\partial\Omega_2$ represent the regions occupied by the pure fluid and the mixture, respectively. Assuming the porous matrix rigid and at rest, it suffices to solve mass and momentum equations for the fluid constituent in Ω_2. The mass balance equations, assuming absence of mass generation in the mixture region are given by (Gurtin, 1981; Atkin, and Craine, 1976; Rajagopal and Tao, 1995)

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{in } \Omega_1 \\ &\frac{\partial \rho_F}{\partial t} + \nabla \cdot (\rho_F \mathbf{v}_F) = 0 \qquad \text{in } \Omega_2 \end{aligned} \tag{1}$$

in which the actual fluid parameters – defined in Ω_1 – are ρ standing for its mass density and $\mathbf{v}$ for its velocity. In the mixture region Ω_2, the fluid constituent parameters are ρ_F its mass density – representing locally the ratio between its mass and the respective volume of mixture and $\mathbf{v}_F$ standing for its velocity. The momentum balance is given by (Gurtin, 1981; Atkin, and Craine, 1976; Rajagopal and Tao, 1995)

$$\begin{aligned} &\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\nabla \mathbf{v})\mathbf{v}\right]=\nabla\cdot\mathbf{T}+\rho\mathbf{g}, && \text{in } \Omega_1 \\ &\rho_F\left[\frac{\partial \mathbf{v}_F}{\partial t}+(\nabla \mathbf{v}_F)\mathbf{v}_F\right]=\nabla\cdot\mathbf{T}_F+\mathbf{m}_F+\rho_F\mathbf{g}, && \text{in } \Omega_2 \end{aligned} \tag{2}$$

where $\mathbf{T}$ represents Cauchy stress tensor (in Ω_1) and $\mathbf{T}_F$ is the partial stress tensor associated with the fluid constituent (in the mixture region Ω_2). The body force per unit mass is represented by $\mathbf{g}$ everywhere while $\mathbf{m}_F$, defined in the mixture region Ω_2, is the momentum supply due to the interaction between the fluid and solid constituents. It arises from the possible existence of n distinct velocity fields in an n-constituents mixture at each spatial point, so that the net momentum supply to the mixture is zero, in other words, $\sum_{i=1}^{n}\mathbf{m}_i=0$ (in Ω_2), where i represents the i-th constituent of the mixture. Since the porous medium is assumed saturated by the fluid constituent in Ω_2 the field ρ_F is given by $\rho_F=\varphi\rho$, in which φ represents the porosity and ρ stands for the actual fluid density.

Constitutive relations for the stress tensor in Ω_1 and the partial stress tensor and momentum source in Ω_2 are required to solve the problem considering a generalized Newtonian fluid flowing in a domain $\Omega_1\cup\Omega_2$ (see Martins-Costa et al., 2000 and references therein).

In this work a power-law constitutive model is employed. The stress tensor for the power-law incompressible fluid flowing in Ω_1 (Bird et al., 1977) and the partial stress tensor for the fluid constituent, flowing in the region Ω_2, obtained by considering an analogy with the equation proposed for Newtonian fluids by Williams (1978), are given by

$$\begin{aligned} &\mathbf{T}=-p\mathbf{I}+2\eta(\mathbf{D}\cdot\mathbf{D})^n\mathbf{D} && \text{in } \Omega_1 \\ &\mathbf{T}_F=-p\varphi\mathbf{I}+2\lambda\varphi^2\eta(\mathbf{D}_F\cdot\mathbf{D}_F)^n\mathbf{D}_F && \text{in } \Omega_2 \end{aligned} \tag{3}$$

in which p is the hydrostatic pressure acting on the fluid, η and n are the classical power-law material parameters that characterize the fluid behavior which will be considered positive-valued in this work. If n=0 the fluid has a Newtonian behavior. Also, φ is the porous matrix porosity and λ is a scalar parameter depending on the porous matrix microstructure. Also, $\mathbf{D}$ and $\mathbf{D}_F$ are the strain rate tensors acting on the fluid and on the fluid constituent, respectively. It is important to note that the usual power-law equation, given by $\boldsymbol{\tau}=2K(\dot{\gamma})^m\mathbf{D}$ (Bird et al., 1977), would be recovered in Ω_1, for $\eta=2^{(m-1)/2}K$ and $n=(m-1)/2$.

The momentum source $\mathbf{m}_F$, representing the interaction force acting on the power-law fluid constituent due to its interaction with the solid constituent (representing the porous matrix) is given by (see Martins-Costa et al., 2000 and references therein)

$$\mathbf{m}_F=-\frac{\varphi^2\eta}{K}\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\varphi}{6K}\right)^n\left\|\mathbf{v}_F\right\|^{2n}\right]\mathbf{v}_F \qquad \text{in } \Omega_2 \tag{4}$$

in which K is the porous matrix permeability.

The problem requires not only boundary conditions at the impermeable boundaries but also compatibility conditions at the interface between pure fluid and mixture – namely between the regions Ω_1 and Ω_2 – defined by the set $\partial\Omega_I\equiv\bar{\Omega}_1\cap\bar{\Omega}_2$. Assuming no flow across the interface, the following relations are imposed on $\partial\Omega_I$

$$\begin{aligned} &\mathbf{v}\cdot\mathbf{n}=\mathbf{v}_F\cdot\mathbf{n}=0 && \text{on } \partial\Omega_I \\ &\mathbf{v}=\varphi\mathbf{v}_F && \text{on } \partial\Omega_I \\ &\varphi\mathbf{T}\mathbf{n}\cdot\mathbf{t}=\mathbf{T}_F\mathbf{n}\cdot\mathbf{t} && \text{on } \partial\Omega_I \end{aligned} \tag{5}$$

where $\mathbf{n}$ is a unit outward normal to $\partial\Omega_I$ and $\mathbf{t}$ is any tangent to $\partial\Omega_I$. These compatibility conditions, based on the work of Beavers and Joseph (1967), subsequently confirmed by other authors (Nield and Bejan, 1992), have been obtained from the solution of thermodynamically consistent equations in both regions and do not suffer from the difficulty of combining the Navier-Stokes equation with the equation for the flow through the porous medium (Nield and Bejan, 1992). Equations (5) are discussed by Martins-Costa and Saldanha da Gama (1994), but it is worth mentioning that at the interface the fluid velocity is distinct from the fluid constituent velocity (modeled by a mixture theory viewpoint).

Assuming a steady-state fully developed flow of a power-law fluid flowing though both the pure fluid and the mixture regions, the governing equations for the flow in a rigid porous medium within the context of Mixtures theory can be summarized as follows

$$
\begin{aligned}
&\nabla\cdot\mathbf{v}=0 && \text{in } \Omega_1\\
&\nabla\cdot\mathbf{v}_F=0 && \text{in } \Omega_2\\
&\nabla\cdot\left[-p\mathbf{I}+2\eta(\mathbf{D}\cdot\mathbf{D})^n\mathbf{D}\right]+\rho\mathbf{g}=0 && \text{in } \Omega_1\\
&\nabla\cdot\left[-p\varphi\mathbf{I}+2\lambda\varphi^2\eta(\mathbf{D}_F\cdot\mathbf{D}_F)^n\mathbf{D}_F\right]-\frac{\varphi^2\eta}{K}\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\varphi}{6K}\right)^n\left\|\mathbf{v}_F\right\|^{2n}\right]\mathbf{v}_F+\rho_F\mathbf{g}=0 && \text{in } \Omega_2\\
&\mathbf{v}\cdot\mathbf{n}=\mathbf{v}_F\cdot\mathbf{n}=0 && \text{on } \Omega_I\\
&\mathbf{v}=\varphi\mathbf{v}_F && \text{on } \Omega_I\\
&\varphi\mathbf{T}\mathbf{n}\cdot\mathbf{t}=\mathbf{T}_F\mathbf{n}\cdot\mathbf{t} && \text{on } \Omega_I\\
&\mathbf{v}=0 && \text{on } \Omega_{S1}\\
&\mathbf{v}_F=0 && \text{on } \Omega_{S2}
\end{aligned}
\tag{6}
$$

where Ω_{S1} and Ω_{S2} denote the impermeable surfaces for pure fluid and mixture, respectively and the last two equations of Eq. (6) are the classical no-slip boundary conditions imposed at these impermeable boundaries.

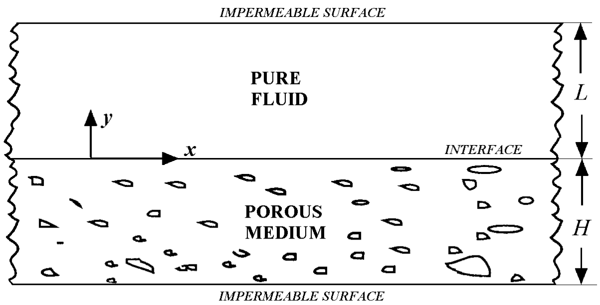

Figure 1. Problem statement

At this point some simplifying assumptions are made, so that the problem to be considered in this work, illustrated in Fig. 1, supposes fully developed steady-state flow between two impermeable flat plates in the horizontal direction, allowing gravitational effects to be neglected. Since no flow across the interface $\partial\Omega_I$ is assumed, it comes that $\mathbf{v}=v_x(y)=v$ and $\mathbf{v}_F=v_{Fx}(y)=v_F$ which, could be expressed as $\mathbf{v}=v\mathbf{i}$ and $\mathbf{v}_F=v_F\mathbf{i}=wi$. Also, making $\beta=\lambda\varphi\eta$ and $\alpha(n)=\frac{\varphi\eta}{K}\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\varphi}{6K}\right)^n\right]$ it can be shown that the system presented in Eq. (6) is reduced to

$$
\begin{aligned}
&-\frac{dp}{dx}+\frac{2n+1}{2^n}\eta\left|\frac{dv}{dy}\right|^{2n}\frac{d^2v}{dy^2}=0 && -H<y\leq 0\\
&-\frac{dp}{dx}+\frac{2n+1}{2^n}\beta\left|\frac{dw}{dy}\right|^{2n}\frac{d^2w}{dy^2}-\alpha|w|^{2n}w=0 && 0\leq y<L\\
&v=\varphi w && \text{at} \quad \mathrm{y}=0\\
&\frac{\partial v}{\partial y}=\lambda\varphi\frac{\partial w}{\partial y} && \text{at} \quad \mathrm{y}=0\\
&v=0 && \text{at} \quad \mathrm{y}=-H\\
&w=0 && \text{at} \quad \mathrm{y}=L
\end{aligned}
\tag{7}
$$

3. NUMERICAL PROCEDURE

Considering the second equation of system (7), for the fluid flowing through the porous matrix, given by

$$\frac{dp}{dx}=\frac{2n+1}{2^n}\beta\left|\frac{dw}{dy}\right|^{2n}\frac{d^2w}{dy^2}-\alpha|w|^{2n}w \tag{8}$$

the following convenient redefinition is proposed

$$z_1=w \quad ; \qquad z_2=\frac{dw}{dy} \tag{9}$$

Substituting these new variables in equation (8) it comes that

$$\frac{dp}{dx}=\frac{2n+1}{2^n}\beta|z_2|^{2n}\frac{dz_2}{dy}-\alpha|z_1|^{2n}z_1 \tag{10}$$

So, the following system of first order ordinary differential equations is obtained

$$
\begin{aligned}
&\frac{dz_2}{dy}=\frac{2^n}{(2n+1)\beta}|z_2|^{-2n}\left(\frac{dp}{dx}+\alpha|z_1|^{2n}z_1\right)\\
&\frac{dz_1}{dy}=z_2
\end{aligned}
\tag{11}
$$

Now, considering the first equation of system (7), given by

$$\frac{dp}{dx}=\frac{2n+1}{2^n}\eta\left|\frac{dv}{dy}\right|^{2n}\frac{d^2v}{dy^2} \tag{12}$$

a convenient redefinition, analogous to the one presented in Eq. (9), is proposed

$$\hat{z}_1=v \quad ; \qquad \hat{z}_2=\frac{dv}{dy} \tag{13}$$

Substituting these new variables in equation (12) it comes that

$$\frac{dp}{dx} = \frac{2n+1}{2^n}\eta\left|\hat{z}_2\right|^{2n}\frac{d\hat{z}_2}{dy} \tag{14}$$

So, the following system is obtained

$$\begin{aligned} \frac{d\hat{z}_2}{dy} &= \frac{2^n}{(2n+1)\eta}\left|\hat{z}_2\right|^{-2n}\left(\frac{dp}{dx}\right) \\ \frac{d\hat{z}_1}{dy} &= \hat{z}_2 \end{aligned} \tag{15}$$

In order to obtain the velocity profiles at the channels, it is necessary to solve the following two-point boundary value problem: Find z_1:[$-H$, 0]$\rightarrow \mathbb{R}$, z_2:[$-H$, 0]$\rightarrow \mathbb{R}$ and $\hat{z}_2$:[0,L]$\rightarrow \mathbb{R}$ and $\hat{z}_1$:[0,L]$\rightarrow \mathbb{R}$ that verify the following systems of ordinary differential equations

$$\begin{aligned} &\left.\begin{aligned} \frac{dz_2}{dy} &= \frac{2^n}{(2n+1)\beta}\left|z_2\right|^{-2n}\left(\frac{dp}{dx} + \alpha\left|z_1\right|^{2n} z_1\right) \\ \frac{dz_1}{dy} &= z_2 \end{aligned}\right\} \quad \text{for } -H \le y < 0 \\ &\text{with } \begin{cases} z_1 = \varphi\hat{z}_1 & \text{at} \quad y = 0 \\ z_2 = \lambda\varphi\hat{z}_2 & \text{at} \quad y = 0 \end{cases} \\ &\left.\begin{aligned} \frac{d\hat{z}_2}{dy} &= \frac{2^n}{(2n+1)\eta}\left|\hat{z}_2\right|^{-2n}\left(\frac{dp}{dx}\right) \\ \frac{d\hat{z}_1}{dy} &= \hat{z}_2 \end{aligned}\right\} \quad \text{for } 0 \le y < L \\ &\text{and such that } \begin{cases} z_1 = 0 & \text{at} \quad y = -H \\ \hat{z}_1 = 0 & \text{at} \quad y = L \end{cases} \end{aligned} \tag{16}$$

The problem stated in Eq. (16) is equivalent to finding the root of a scalar function represented as

$$\begin{aligned} &\Phi : \mathbb{R} \rightarrow \mathbb{R} \\ &t \rightarrow \Phi(t) = \hat{z}_1(y = L; t) \end{aligned} \tag{17}$$

where for a given $t \in \mathbb{R}$, representing an initial estimate, the value $\Phi(t)$ is the value of the variable $\hat{z}_1$ at point $y=L$ obtained by solving the following initial boundary value problem

$$\left.\begin{aligned} \frac{dz_2}{dy} &= \frac{2^n}{(2n+1)\beta}\left|z_2\right|^{-2n}\left(\frac{dp}{dx} + \alpha\left|z_1\right|^{2n} z_1\right) \\ \frac{dz_1}{dy} &= z_2 \end{aligned}\right\} \quad \text{for } -H \le y < 0 \tag{18}$$

$$\text{with } \begin{cases} z_1 = \varphi\hat{z}_1 & \text{at} \quad y = 0 \\ z_2 = \lambda\varphi\hat{z}_2 & \text{at} \quad y = 0 \end{cases} \tag{19}$$

$$\left.\begin{aligned}\frac{d\hat{z}_2}{dy} &= \frac{2^n}{(2n+1)\eta}\left|\hat{z}_2\right|^{-2n}\left(\frac{dp}{dx}\right)\\ \frac{d\hat{z}_1}{dy} &= \hat{z}_2\end{aligned}\right. \quad \text{for} \quad 0 \le y < L \tag{20}$$

$$\text{and such that} \quad \begin{cases} z_1 = 0 & \text{at} \quad y = -H \\ z_2 = t & \text{at} \quad y = -H \end{cases} \tag{21}$$

Essentially, this procedure is called a shooting technique with *t* being the initial estimate of the derivative $(\partial w / \partial y)$ at the point $y = -H$. The initial boundary value problem may be approximated by many different techniques for initial boundary value problems in ordinary differential equations, such as the Rosembrock methods and its extensions to Runge-Kutta methods (Dahlquist and Bjorc, 1969). Also, the root of the function Φ can be obtained by using distinct unconditionally convergent procedures, such as Regula-Falsi or Bisection methods (Dahlquist and Bjorc, 1969). It is important to remark that the above proposed change of variables is only adequate when $z_2 \neq 0$ and $\hat{z}_2 \neq 0$. It is important to remark that the above proposed change of variables is only adequate when $z_2 \neq 0$ and $\hat{z}_2 \neq 0$. It is possible to prove that $z_2 = \frac{dv}{dy} \neq 0, \ \forall \ y$ since

$$\frac{dp}{dx} = \frac{2n+1}{2^n}\eta\left|\frac{dv}{dy}\right|^{2n}\frac{d^2v}{dy^2} \Rightarrow \frac{dv}{dy} \neq 0; \ \frac{d^2v}{dy^2} \neq 0, \ \forall \ y \tag{22}$$

It is also possible to prove that if $\hat{z}_2 = \frac{dw}{dy} = 0$ or $\frac{d\hat{z}_2}{dy} = \frac{d^2w}{dy^2} = 0$ then $w = -\frac{1}{\alpha}\frac{dp}{dx}$, since

$$\frac{dp}{dx} + \alpha\left|w\right|^{2n} w = \frac{2n+1}{2^n}\beta\left|\frac{dw}{dy}\right|^{2n}\frac{d^2w}{dy^2} \Rightarrow w = -\frac{1}{\alpha\left|w\right|^{2n}}\frac{dp}{dx} \qquad \text{when} \quad \frac{d^2w}{dy^2} = 0 \quad \text{or when} \quad \frac{dw}{dy} = 0 \tag{23}$$

The limitation of the change of variables can be circumvented numerically when $\hat{z}_2 = 0$ using the previous relation.

4. NUMERICAL RESULTS

4.1. Validation

Initially, to check the accuracy of the proposed procedure to circumvent the singularity when $z_2 = 0$, the problem is studied for different values of the material parameter n , considering absence of porous matrix – namely a simple fluid would be flowing in the both regions of the channel (which is equivalent of making $\beta = \eta$ and $\alpha = 0$ in Eqs. (16) and (18)) and using a classic fourth-order Runge-Kutta method. In this case, the problem presents an analytical solution given by

$$v(y) = \frac{1}{2^{n+1}}\left(\frac{2n+1}{n+1}\right)\left[-\frac{1}{\eta}\frac{dp}{dx}\right]^{\frac{1}{2n+1}}\left[1-\left(y^2\right)^{\frac{n+1}{2n+1}}\right] \tag{24}$$

Figure 2 shows a comparison between the above analytical solution and a numerical approximation, for different values of the material parameter n , ranging from 0 to 0.5 and considering $dp/dx = 10^{-3}$ and $\eta = 10^{-3}$. As observed no difference can be detected for all considered values of the power-law index, showing the adequacy of the employed numerical procedure.

As expected, as n increases the maximum velocity increases too, as a consequence of the shear-thickening increase.

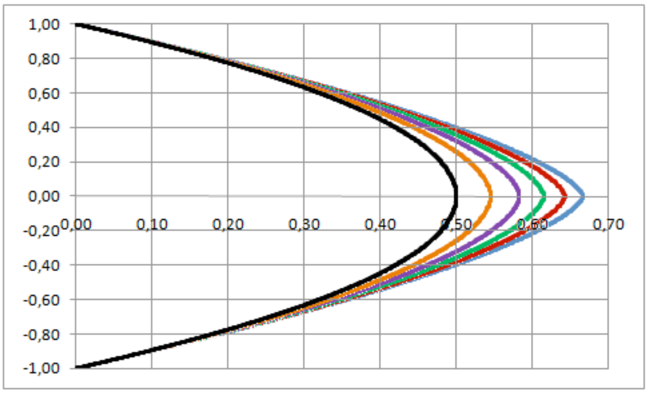

Figure 2. Flow of a power-law fluid in a plane channel

The second step is to consider a power law fluid flowing through a porous matrix limited by two impermeable flat plates – namely a porous plane channel. Figure 3 shows numerical results obtained considering the following parameters: $dp/dx = 10^{-2}$ Pa/m, $\eta = 10^{-3}$ Pa.s^n, $\varphi = 0.5$, $\lambda = 1$, $\beta = \varphi\eta = 0.5 \times 10^{-3}$ Pa.s^n and $K = 10^{-3}$ m^{-2}. It may be noted that the velocity profile becomes flatter as n decreases.

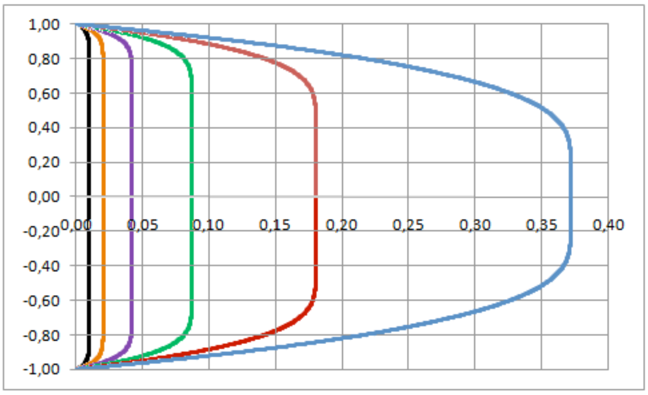

Figure 3. Flow of a power-law fluid through a porous plane channel

4.2. Results for fluid flowing in two distinct flow regions

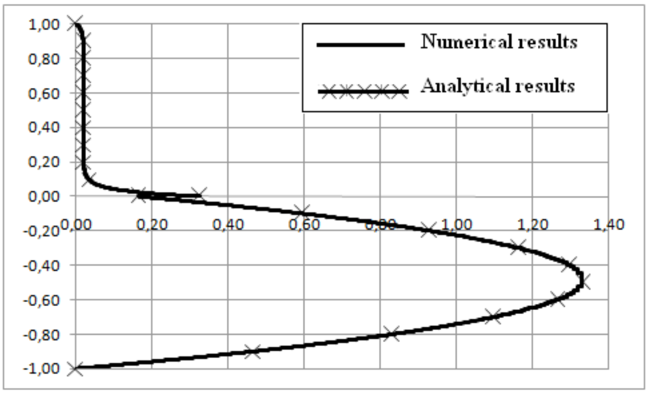

Figure 4. Newtonian fluid flowing through a permeable wall channel limited by impermeable flat plates

First, a Newtonian fluid (n=0) flowing through the two distinct flow regions depicted in Figure 1 is considered. This problem presents the following exact solution (see Martins-Costa and Saldanha da Gama, 1994)

$$v = -\frac{1}{2\eta}\frac{dp}{dx}\left[(L+H)^2-(y+H)^2\right]+\frac{1}{2\eta}\frac{dp}{dx}(y-L)\left[\frac{2\left(\dfrac{\cosh\kappa_2-1}{\cosh\kappa_2}\right)\dfrac{K}{L}-2H-L-\dfrac{2H}{L}\kappa_1\tanh\kappa_2}{1+\dfrac{1}{L}\kappa_1\tanh\kappa_2}\right]$$

for $0 \le y < L$

$$w = \frac{K}{\eta\varphi}\frac{dp}{dx}\left[\cosh\left(\frac{y+H}{\sqrt{K\lambda}}\right)-1\right]-\frac{K}{\eta\varphi}\frac{dp}{dx}\sinh\left(\frac{y+H}{\sqrt{K\lambda}}\right)\left[\frac{\kappa_1\left(\dfrac{\cosh\kappa_2-1}{L\cosh\kappa_2}\right)+\tanh\kappa_2+\dfrac{L}{2\sqrt{K\lambda}\cosh\kappa_2}}{1+\dfrac{1}{L}\kappa_1\tanh\kappa_2}\right] \quad (25)$$

for $-H < y \le 0$

with $\kappa_1 = \sqrt{\dfrac{K}{\lambda}}$ $\quad \kappa_2 = \dfrac{H}{\sqrt{K\lambda}}$

Figure 4 shows the velocity profiles in the two distinct flow regions, considering the approximation employing the numerical procedure described in this work (depicted by the continuous lines) and the analytical solution presented in Eq. (25). The results have been obtained for the following parameters: $dp/dx = 10^{-2}$ Pa/m, $\eta = 10^{-3}$ Pa.s^n, $\varphi = 0.5$, $\lambda = 1$, $\beta = \varphi\eta = 0.5\times10^{-3}$ Pa.s^n and $K = 10^{-3}$ m^{-2}. It is worth noting that both solutions are indiscernible within the precision of the graph.

Now distinct shear-thickening fluids are considered. Figure 5 shows velocity profiles for both the pure fluid and the fluid constituent for distinct of the power-law parameter n – namely for n=0 (Newtonian), n=0.1, n=0.2, n=0.3 and n=0.5, and the remaining parameters are the same employed in Fig. 4.

The fluid constituent maximum velocity increases as the shear-thickening increases (as the parameter n increases). The pure fluid maximum velocity is less influenced by the shear-thickening variation, than the results for the pure fluid presented in Fig. 2, due to the strong influence of the compatibility conditions. It is important to note that at the interface the fluid constituent velocity is twice the fluid velocity (since $\varphi = 0.5$ was considered in the simulations). This compatibility condition pushes away the fluid constituent velocity, preventing the formation of a velocity profile tending to a parabolic shape (as depicted in Fig. 3). For all considered values of n, an inflexion on the fluid constituent velocity curve is observed. For instance, for n=0, the inflexion is for y<0.2, while for n=0.3 the inflexion is for y>0.4.

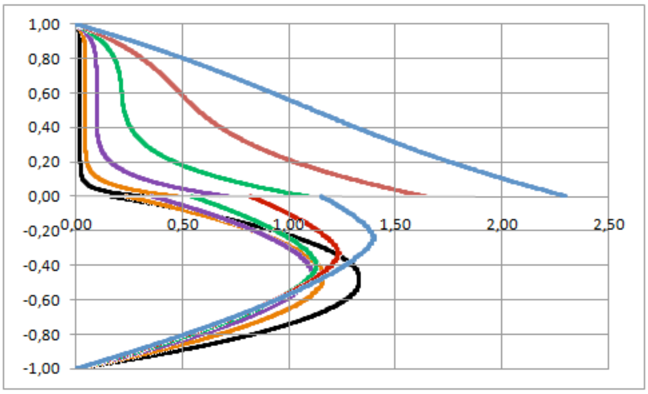

Figure 5. Flow of a power-law fluid through a permeable wall channel limited by impermeable flat plates

5. ACKNOWLEDGEMENTS

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