ON A NONLINEAR DYNAMICS OF A NON-IDEAL OSCILLATOR, WITH A SNAP-THROUGH TRUSS ABSORBER (STTA).

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Abstract. *This work considers the vibrating system that consists of a snap-through truss absorber (STTA) coupled to an oscillator under excitation of an electric motor with an eccentricity and limited power, characterizing a non-ideal oscillator (NIO). It is aimed to use the absorber STTA to establish the conditions that we have the maxim atenuation the jump phenomenon.*
In this study, we have interest in determining the conditions of the vibrating system in which there is reduced motion amplitudes of the oscillator, when it passes through the region of resonance.

Keywords*: vibration absorber, Sommerfeld Effect, Non-ideal oscillator.*

1. INTRODUCTION

This work considers a non-ideal vibration system, which is characterized by the interaction between system's response and excitation. The response influences system's excitation, as opposite ideal systems. For this reason, this model is much closer to real situations.

In this non-ideal system, besides the interaction of the response and excitation, the fact that the excitation source has limited power makes the study more complex and challenging. Studies about this type of model have recently increased, once the ideal system behavior is already known and studied.

When the system is considered together with a limited power source, it requires another equation that describes how the power supply feeds the system. So an extra 1-DOF is added to the work.

The non-ideal systems theories can see details in: (Balthazar et al., 2003) and (Nayfeh and Mookl, 1979), and a number of others authors.

For this study, we used a system formed by an oscillating block and a DC motor with limited power source, which works as an excitation source. This situation characterizes the behavior of a non-ideal system. The passage through resonance reveals interesting behaviors, when many real machines can suffer damage, high-energy wastes and/or further consequences, once the engine is able to transfer a big part of its energy to carry out system oscillations, generating large movement's amplitude. Thereby, we used a motor as a model of linear torque. This torque was used as a control parameter, in order to obtain the passage through resonance and control motor's frequency.

The main phenomenon for non-ideal systems is the Sommerfeld effect, in which the oscillator presents unstable movements at resonance regions. A jump can be seen in the frequency-response curve, revealing the conditions where there is no permanent state

Thus, the application of vibration absorbers presents an interesting alternative in reducing large amplitudes and in saving energy at the regions of resonance we have studied. Besides, it is a passive controller that does not wastes energy to work.

In this work, we use the snap-through truss in the absorption of longitudinal vibrations of the non-ideal oscillating system. In this situation, the oscillation energy of the main system is transferred to STTA, which fluctuates around an equilibrium point. The analysis of this system free oscillation was recently studied by (Avramov and Mikhlin, 2004). Later, the forced system coupled STTA was studied by (Avramov and Mikhlin, 2006). Also, in recent work (Avramov and Gendelman, 2009) analyzed the interaction among STTA and an elastic system.

This objective of this article is to understand the interaction between the non-ideal oscillator and the STTA, so that the vibration amplitudes are reduced at the passage through resonance and the jump phenomenon is attenuated.

The work is organized as the following:

In Section 2, the mathematical model represents the non-ideal system connected to STTA. In Section 3, results and numerical simulations will be shown and in Section 4, conclusions will be presented.

2. MATHEMATICAL MODEL

The system considered here is based on previous works from (Avramov and Mikhlin, 2004) and (Felix and Balthazar), and can be described by Fig. 1. In this study, we aim to show how the coupling of a small snap-through truss mass can absorb part of the non-ideal system vibration, considering especially the passage through resonance and Sommerfeld.

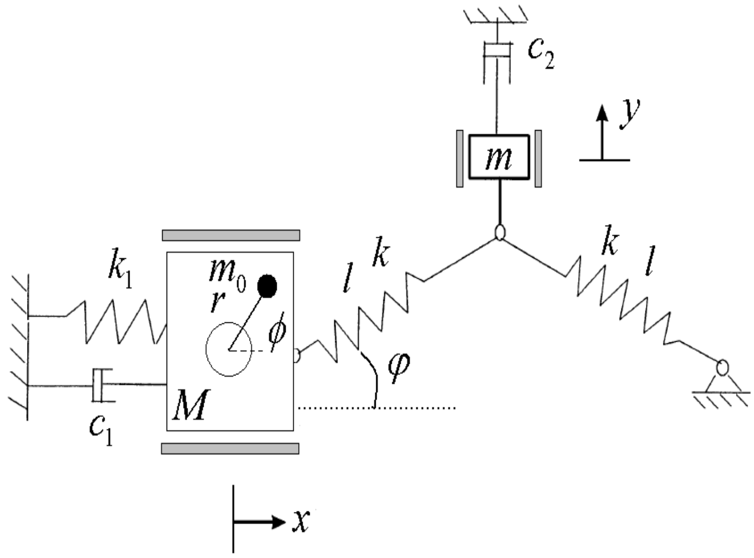


Figure 1. A non-ideal structure attachment coupled to snap-through truss absorber.

The motion equations, which represent the mathematical model of a non-ideal oscillator, are shown below:

$$m_t\ddot{x} + f(x,\dot{x}) + \frac{\partial U(x)}{\partial x} = m_0 r(\dot{\phi}^2 \sin\phi - \ddot{\phi}\cos\phi)$$

$$I\ddot{\phi} = L(\dot{\phi}) - H(\dot{\phi}) - m_0 r\ddot{x}\cos\phi \tag{1}$$

On the mathematical model, x is the horizontal displacement of the NIO (non-ideal oscillator), $m_t = M + m_0$ is NIO total mass, φ is the rotation angle of the DC engine shaft, r and m_0 are the eccentricity and unbalanced mass of the electric engine considered. I is the moment of inertia of the rotor. The function $H(\dot{\phi})$ is the resistive torque applied to engine, and the function $L(\dot{\phi})$ is the driving torque of the engine.

By using a linear torque model, we consider that $L(\dot{\phi}) - H(\dot{\phi}) = u_1 - u_2\dot{\phi}$, in which u_1 is related to the voltage applied to the engine and acts as a control parameter for our problem, and u_2 is constant for each type of engine.

The non-linear and non-conservative restoring force is given by $f(x,\dot{x})$, while $\frac{\partial U(x)}{\partial x}$ represents the conservative force and $U(x)$ is the potential.

Now considering the NIO (non-ideal oscillator) coupled to STTA, we obtain the motion equations as follows:

$$m_1\ddot{x}+c_1\dot{x}+k_1x+2k\left[x-l\cos\varphi+\frac{l(l\cos\varphi-x)l^2}{\sqrt{l^2+2l(y\sin\varphi-x\cos\varphi)+x^2+y^2}}\right]=m_0r(\dot{\phi}^2\sin\phi-\ddot{\phi}\cos\phi)$$

$$I\ddot{\phi}=\Gamma(\dot{\phi})-m_0r\ddot{x}\cos\phi \tag{2}$$

$$m\ddot{y}+c_2\dot{y}+2k\left[y+l\sin\varphi+\frac{l(l\sin\varphi+y)}{\sqrt{l^2+2l(y\sin\varphi-x\cos\varphi)+x^2+y^2}}\right]=0$$

in which (x,y,ϕ) are the generalized coordinates of NIO, STTA and rotor, respectively. $(\dot{\phi})$ is the angular velocity of the rotor. (m_0, M, m) are the unbalanced mass, NIO mass and STTA mass, respectively. (k, k_1) are the linear stiffness of the springs. (c_1, c_2) are NIO and STTA linear dampings. Finally, l is the spring length and φ is the angle that defines STTA equilibrium position.

2.1 DIMENSIONLESS SYSTEM

Using the following dimensionless parameters:

$$\tau=\omega_n t \qquad \omega_n=\sqrt{\frac{k_1}{m_1}} \qquad \Gamma(\phi^{"})=a-b\phi^{'} \qquad m_1=m_0+M$$

$$\alpha_1=\frac{c_1}{m_1\omega_n} \qquad \alpha_2=\frac{c_2}{m\omega_n} \qquad a=\frac{u_1}{I\omega_n^2} \qquad \mu=\frac{m_1}{m} \tag{3}$$

$$\eta_1=\frac{m_0r}{m_1l} \qquad \eta_2=\frac{m_0rl}{I} \qquad b=\frac{u_2}{I\omega^2{}_n} \qquad c=\cos\varphi$$

$$\gamma=\frac{k}{k_1} \qquad u=\frac{x}{l} \qquad v=\frac{y}{l} \qquad s=\sin\varphi$$

We can rewrite the motion equations as:

$$\begin{aligned}&u''+u+2\gamma(u-c)K(u,v)=\eta_1(\phi'^2\sin\phi-\phi''\cos\phi)-\alpha_1u'\\&\phi''=a-b\phi'-\eta_2u''\cos\phi\\&v''+2\mu\gamma(s+v)K(u,v)=-\alpha_2v'\end{aligned} \tag{4}$$

In which

$$K(u,v)=1-\frac{1}{\sqrt{1+2(vs-uc)+u^2+v^2}}$$

3. NUMERICAL SIMULATIONS RESULTS

In this work numerical simulations were carried out by using Matlab® using as the numerical integrator ode113, Adams-Bashforth-Moulton PECE solver algorithm with variable step-length. In the Tab. 1 we find the parameters adopted for numerical simulations of this work. All initial conditions are zero. Values s=0.39 and c=0.92 were obtained from φ. Here, two situations are presented, especially for different parameters absorber mass and spring stiffness in order to demonstrate the influence that these parameters can have on the main system.

Table 1. Dimensionless parameters of the systems (Felix and Balthazar, 2009).

Variables	Symbol	Values system(1)	Values system(2)
Interaction coefficients	η_1 , η_2	0.05, 0.35	0.05, 0.35
Damping coefficients	α_1, α_2	0, 0.03	0.01, 0.03
Stiffness coefficient	γ	0.01	0.1
Mass coefficient	μ	100	10
Angle	φ	0.405	0.405

3.1. First case (system 1 values):

Considering that the purpose of this work is to study the system behavior when the STTA is coupled to it and also verify the reduction of motion amplitudes compared to the system without STTA, we are going to analyze how the system works before, during and after passing through the resonance region.

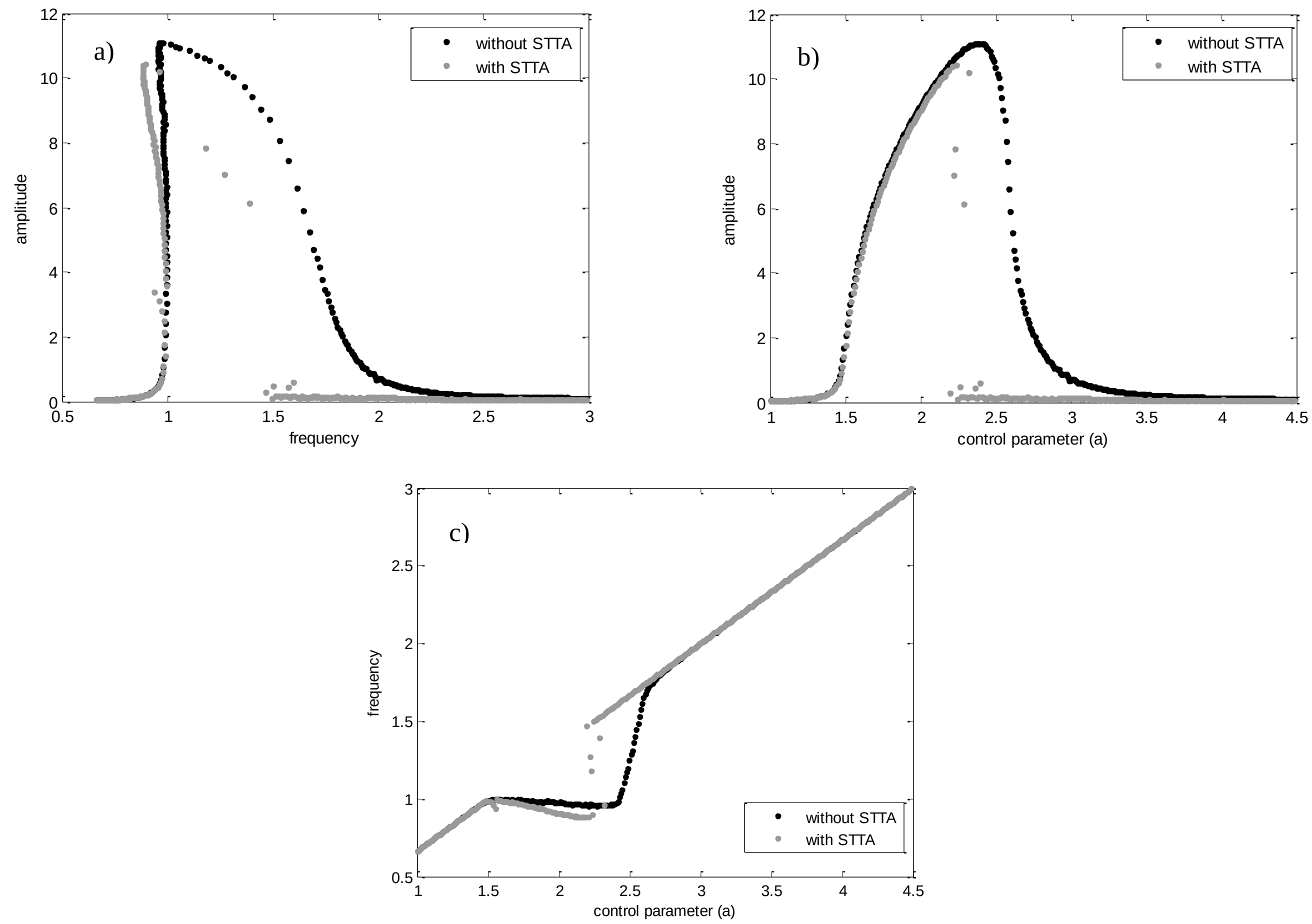


Figure 2. Sommerfeld effect (STTA coupled to NIO: grey point) and (NIO uncoupled: black point): with $1 \leq a \leq 4.5$ and Δa = 0.01.

Figure 2a shows the maximal amplitude for each average value of angular velocity. Thus, we see that the maximal amplitudes are lower when the non-ideal oscillator (NIO) is coupled to the snap-through absorber (gray point) compared to amplitudes of NIO without coupling (black point). Figure 2b shows the maximal amplitudes versus the control parameter a. It is also evident that NIO leaves the resonant state much earlier when STTA is coupled. Finally, Fig. 2c, allows to visualize a sudden increase of the resonance frequency through the variation of a. In Fig. 2, $\Delta a = 0.01$ refers to the step change in the parameter control.

The displacement of the NIO, the rotor angular velocity and STTA displacement are shown in Figures 3, 4 and 5 in three cases: a before the resonance, a inside the resonant region and a outside the region. Note that the value of a may be different for the two systems in many situations, with the STTA active and not active. This is done because the resonance curves can vary a lot from positions, with we consider the control parameter and it does not allows us to use the same value for a to demonstrate the three cases described above. Therefore, $a1$ and $a2$ refer to the control parameters of coupled STTA and uncoupled STTA, respectively.

Therefore:

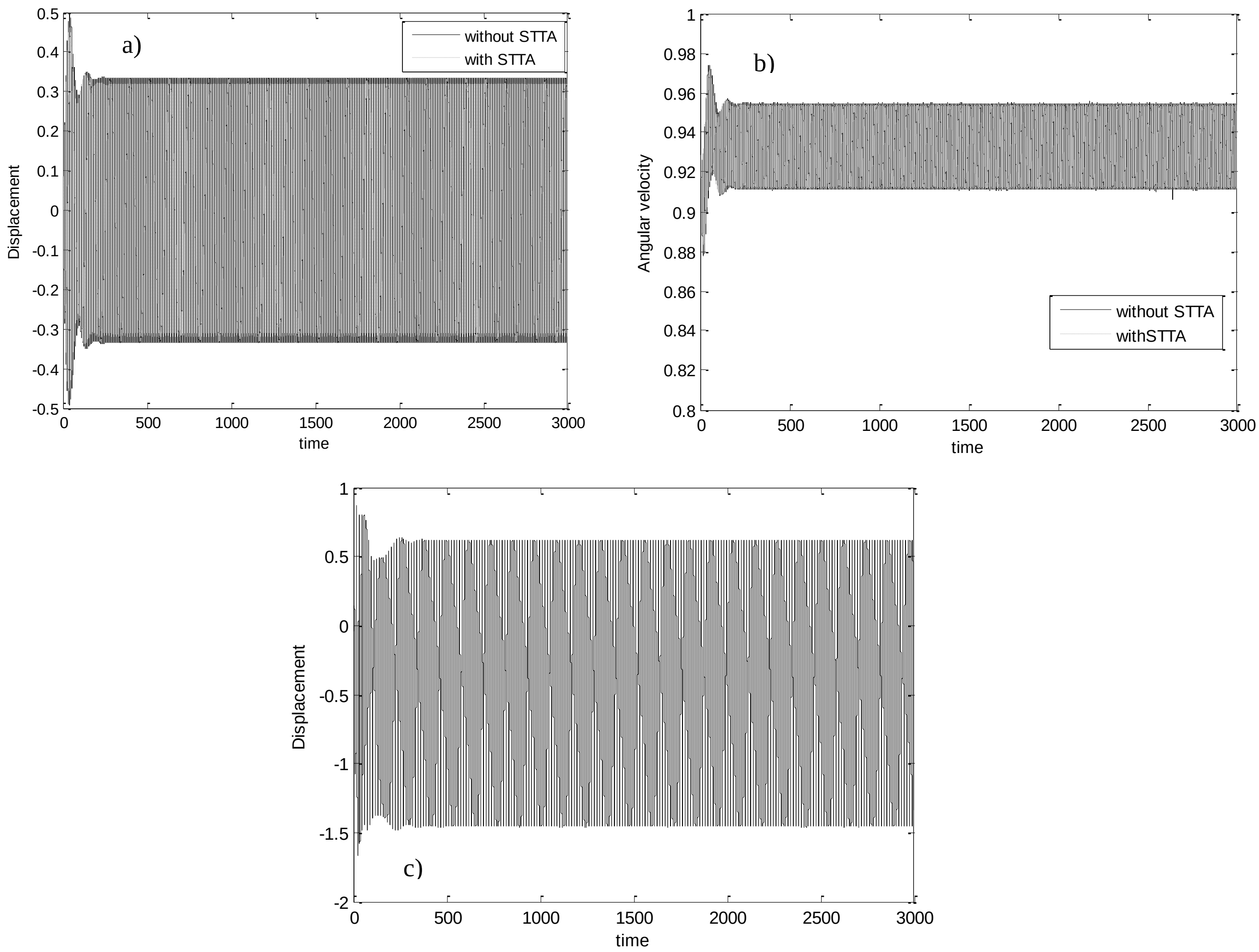


Figure 3. Time histories ***a*1 = 1.4 (grey line) and *a*2 = 1.4 (black line)**: a) Displacement of NIO; b) Angular Velocity; c) Displacement STTA.

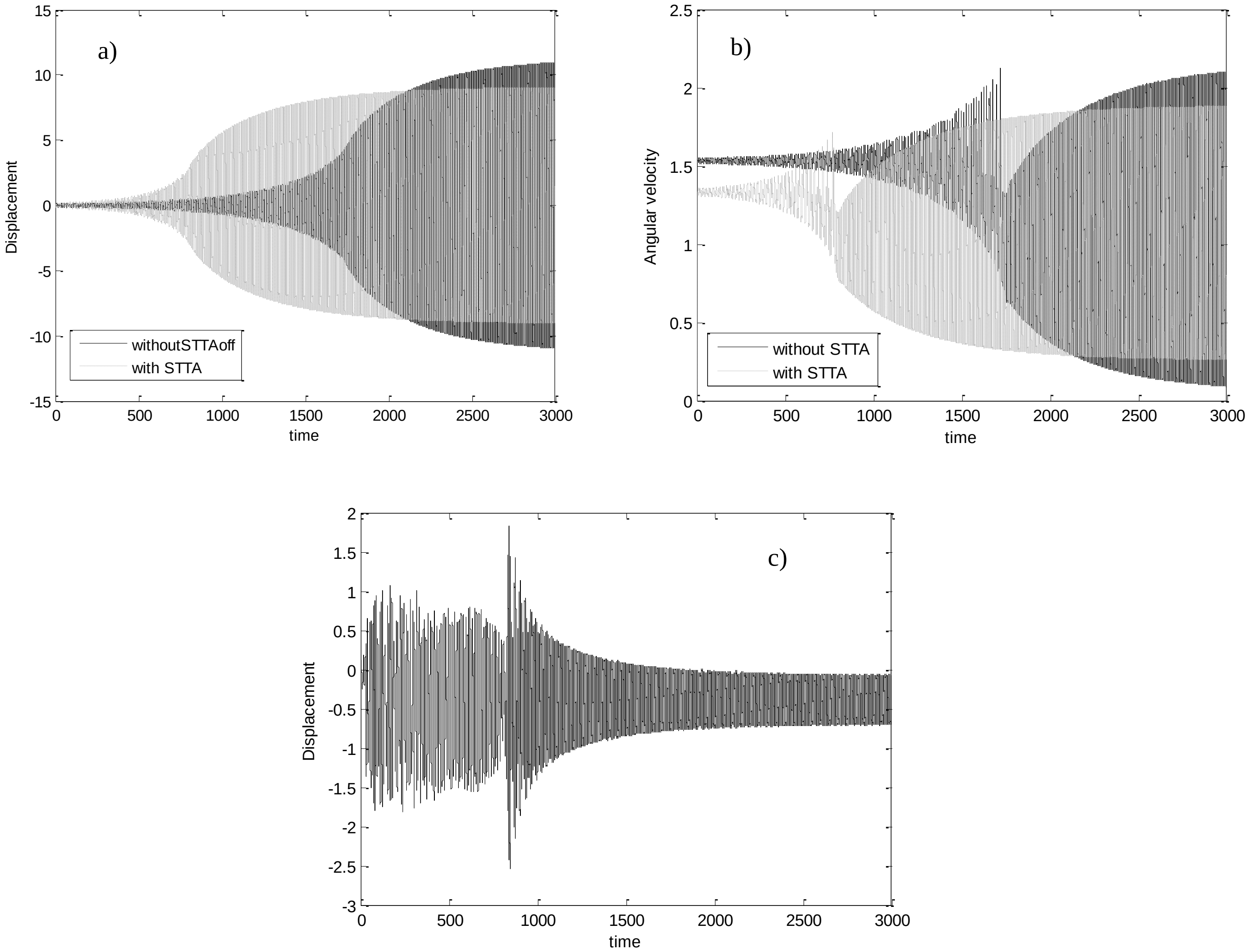
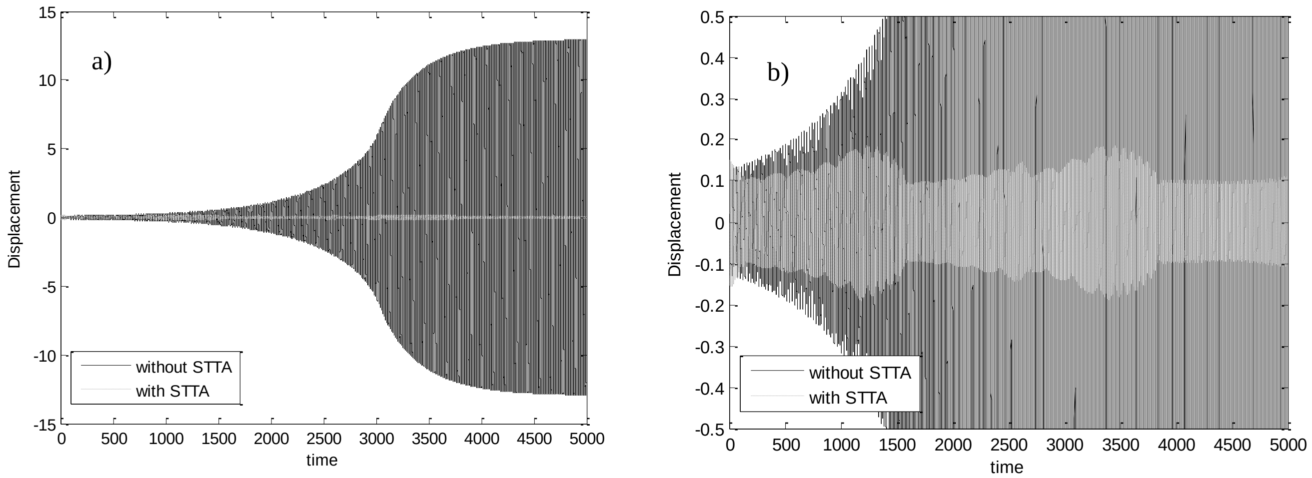


Figure 4. Time histories ***a*1 = 2.0 (grey line) and *a*2 = 2.3 (black line)**: a) Displacement of NIO; b) Angular Velocity; c) Displacement STTA.

The comparisons between the two systems using time histories from NIO, show that the absorber has been able to reduce considerably the amplitudes to the values of *a* in the resonance region. Comparing Fig. 5b with Fig. 5d we can also see that the STTA undergoes large displacements when NIO amplitudes start to increase, making the oscillator remains with small fluctuations.



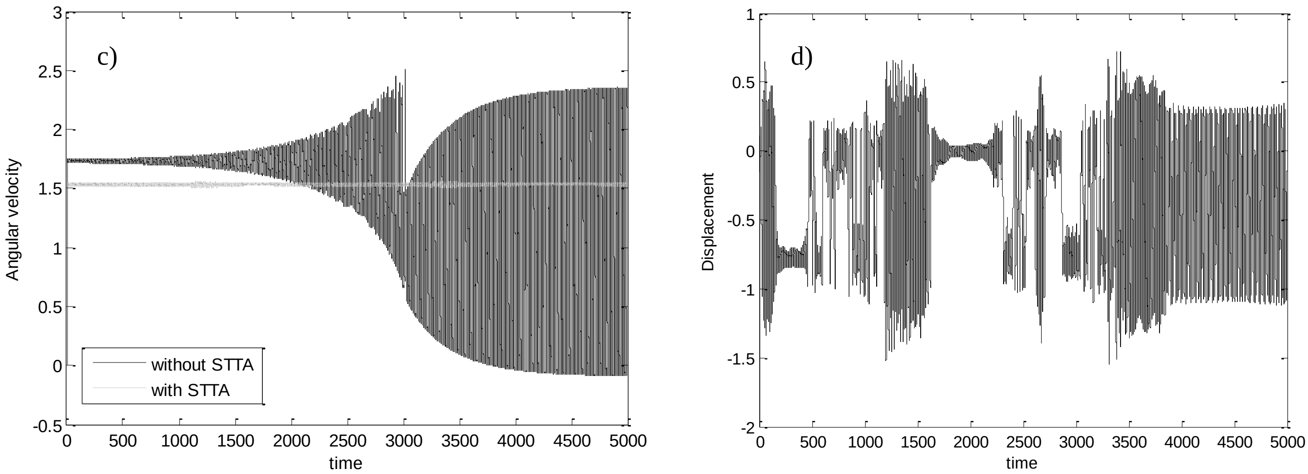


Figure 5. Time histories $a1$ **= 2.3 (grey line) and** $a2$ **= 2.6 (black line)**: a) Displacement of NIO; b) expanded displacement of NIO related to the coupled system; c) Angular Velocity; d) Displacement STTA.

3.2. Second case (system 2 values):

In our second case, following the same principles mentioned above, we perform the system simulation for the values of the Tab.1. Again, in Fig. 6, $\Delta a = 0.01$ refers to the step change in the parameter control.

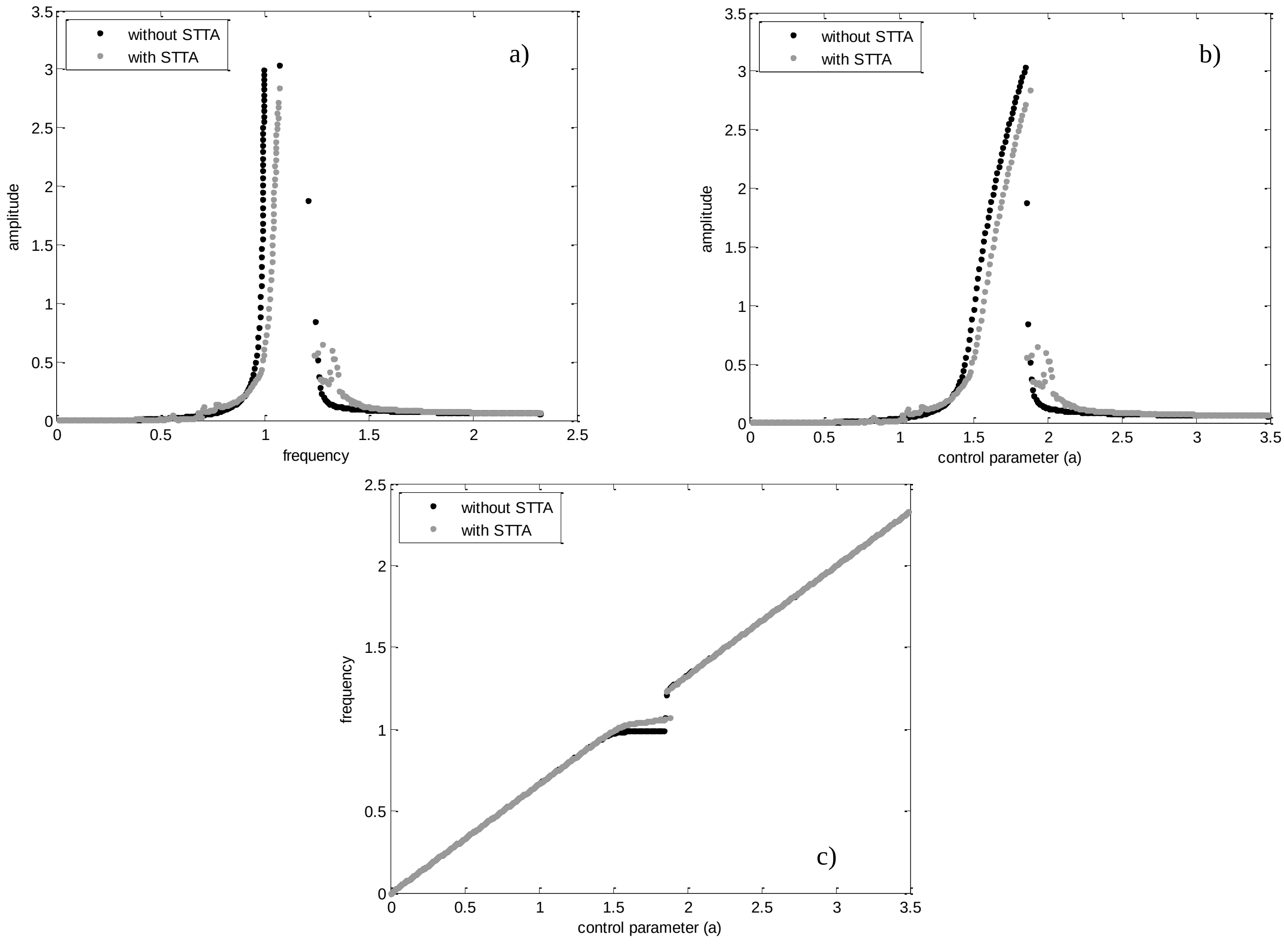


Figure 6. Sommerfeld effect (STTA coupled to NIO: grey point) and (NIO uncoupled: black point): with $0 \leq a \leq 3.5$ and $\Delta a = 0.01$.

For control parameter values within the resonance region by Fig 6 we can see that the amplitudes of the NIO coupled to STTA compared to NIO isolated are smaller in the resonance region. On top of that, when the NIO leaves the resonance region, we can observe that the amplitudes of coupled system are slightly higher compared to the NIO isolated, indicating that the isolated system can leave the region of large amplitudes earlier than the coupled system.

Now, as done before, we are going to analyze the historical data for the three regions mentioned.

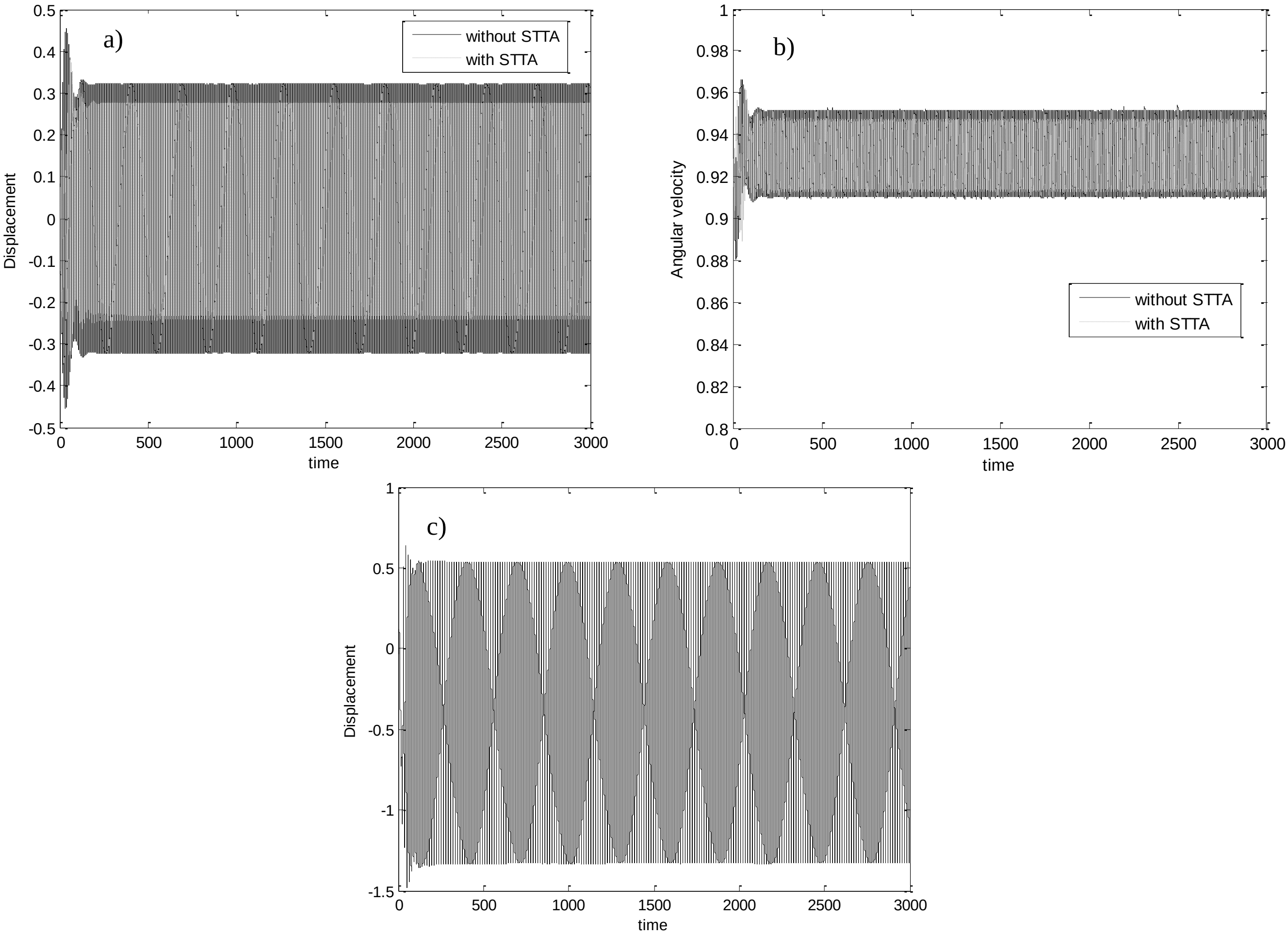
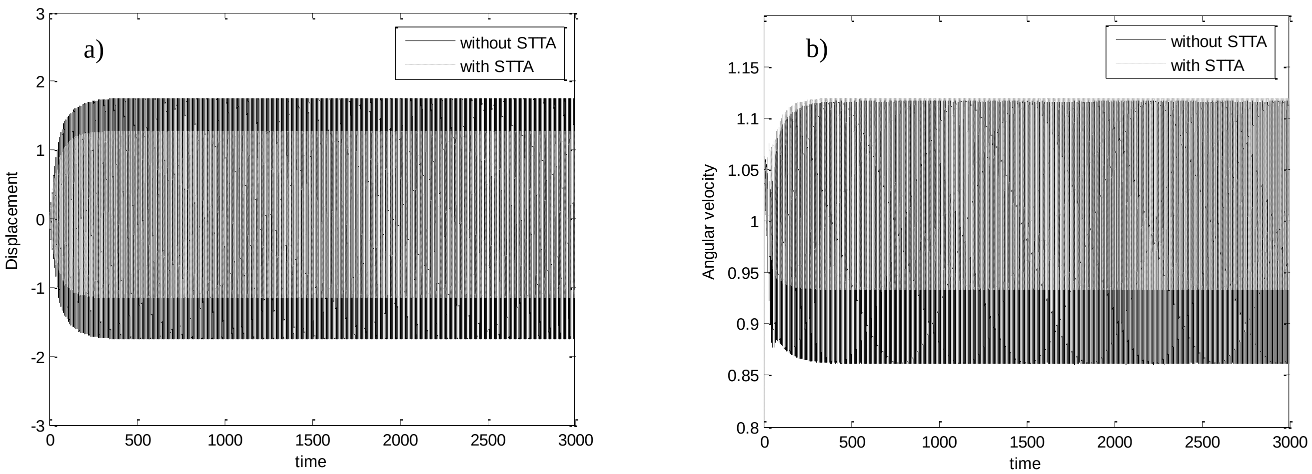


Figure 7. Time histories ***a*1 = 1.4 (grey line) and *a*2 = 1.4 (black line)**: a) Displacement of NIO; b) Angular Velocity; c) Displacement STTA.



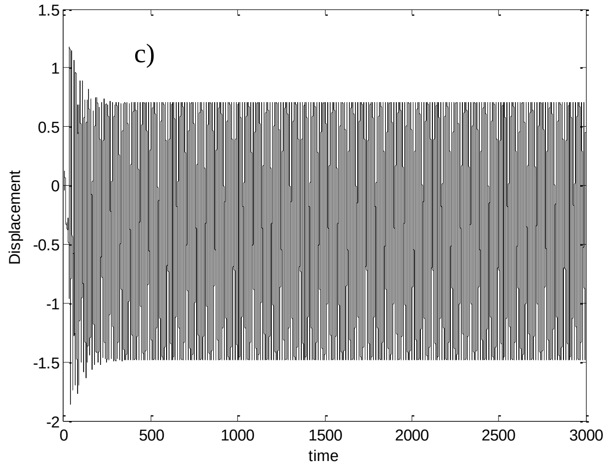


Figure 8. Time histories ***a*1 = 1.6 (grey line) and *a*2 = 1.6 (black line)**: a) Displacement of NIO; b) Angular Velocity; c) Displacement STTA.

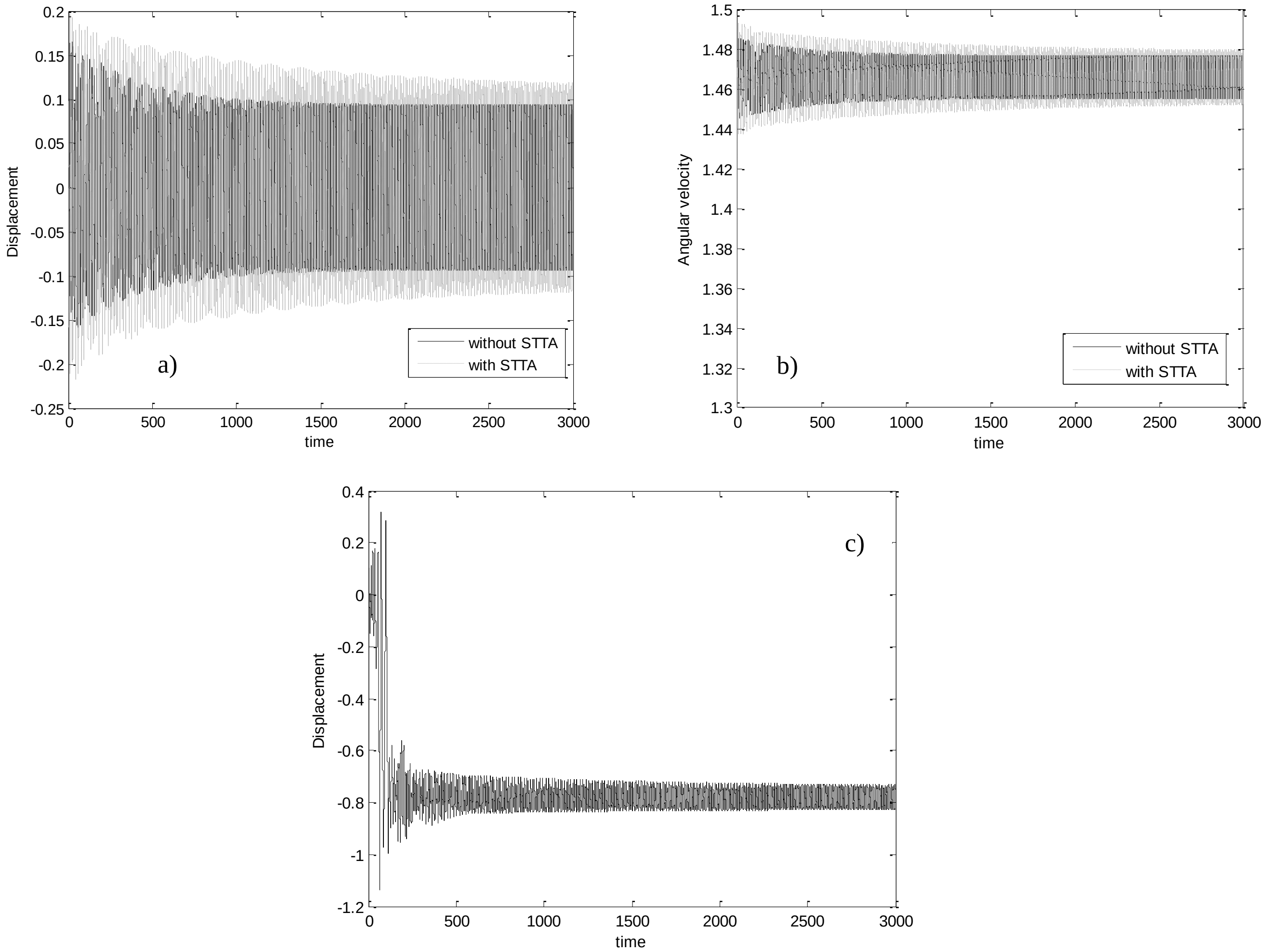


Figure 9. Time histories ***a*1 = 1.6 (grey line) and *a*2 = 1.6 (black line)**: a) Displacement of NIO; b) Angular Velocity; c) Displacement STTA.

Observing Fig. 7, Fig. 8 and Fig. 9 we see how the amplitudes are smaller in cases where the control parameter is approaching to the jump region or when is in resonance. However, when it leaves the resonance, the NIO coupled to the STTA reaches larger amplitudes than the system without STTA.

4. CONCLUSIONS

The system of non-ideal oscillator coupled to the STTA proposed in this work has shown that this type of passive controller is very effective when applied to vibrant systems. The main interest of this application in non-ideal systems is the reduction of vibration amplitudes, especially when the excitation frequency approaches to the natural frequency of the system, which creates the jump phenomenon. Therefore, the absorber generated the desired results by reducing effectively the amplitudes of this region.

In future works, starting from the parameters of mass and stiffness, we will try to set the conditions in which occurs the highest energy transfer from the oscillator to the absorber “energy pumping”.

5. ACKNOWLEDGEMENTS

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