EFFECT OF CIRCULAR HOLES DIMENSION AND LOCATION ON THE ELASTIC BUCKLING LOAD OF RECTANGULAR PLATES

Mauro de Vasconcelos Real, mauroreal@furg.br
Liércio André Isoldi, liercioisoldi@furg.br
Escola de Engenharia (EE), Universidade Federal do Rio Grande (FURG) – Av. Itália, km 8, Rio Grande-RS, Brazil

Abstract. *Thin steel plates are used in a great variety of engineering systems, such as deck and bottom of ship structures, and platforms of offshore structures. When these plates are subjected to compressive forces, they are prone to instability or buckling. Furthermore, it is very often necessary to have holes in the plate elements for inspection, maintenance, and service purposes. The presence of these holes may significantly alter the plate stability. The objective of the present study is to investigate the effect of the hole dimension and location on the elastic buckling load of rectangular plates subjected to uniform uniaxial end load. The ANSYS® software, based on Finite Element Method (FEM), is used for assessing the plate buckling load, and the Lanczos method is applied to the solution of the corresponding eingenvalue problem. The finite element studies demonstrate that holes can either decrease or increase a plate's critical elastic buckling load depending on the hole size and position, and that additional researches into this subject are justified.*

Keywords: *plate, elastic buckling, finite element method, ANSYS®*

1. INTRODUCTION

Many thin-walled structures contain holes. In bridges, inspection access holes are detailed in the flange and/or web of steel box girders. In airplanes, window cutouts extend along the full length of the fuselage and holes can also be found in the ribs attached to the main spar of an airplane wing (Moen and Schafer, 2009). In marine and offshore structures, the perforated panels are used in deck and bottom of ship hulls and in oil and gas platforms. Basically, cutouts are often located in plates to make a way of access or to reduce the total weight of the structure.

Therefore, there are several structural situations that use perforated plates. When those plates are subject to compression loads, the structure could buckle if the load exceeds the critical load. Thus, to know how this phenomenon occurs and to analyze the buckling behavior of those perforated panels has great importance for an efficient structural design.

The elastic buckling is an instability phenomenon that can occur if a slender and thin-walled plate (plane or curved) is subjected to axial pressure. At a certain given critical load the plate will suddenly bend in the out-of-plane transverse direction. The compressive force could besides coming from pure axial compression, also be generated by bending moment, shear or local concentrated loads, or through a combination among these (Real and Isoldi, 2010).

For plates without holes, the critical load can be evaluated analytically. Figure 1 considers a perfectly flat plate of length *a*, width *b* and thickness *t*, simply supported on all four sides and subjected to uniform compressive force, *Nx*, per unit length in the *x*-direction (uniaxial compression).

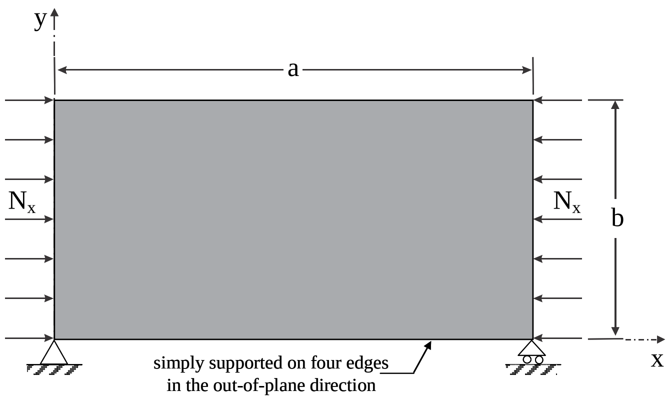

Figure 1. Plate geometry and loading

The critical buckling load per unit length, N_{cr}, can be written as (Wang et al., 2005):

$$N_{cr} = k\frac{\pi^2 D}{b^2} \tag{1}$$

where k is a dimensionless buckling coefficient of the plate and depends on the type of loading, edge support conditions, and the plate aspect ratio a/b, while D is the plate bending stiffness given by:

$$D = \frac{Et^3}{12\left(1-\nu^2\right)} \tag{2}$$

where E and ν are the Young's modulus and Poisson's ratio of the plate material, respectively.

Furthermore, if the critical laod, N_{cr}, is divided by the plate thickness, t, the critical buckling stress, σ_{cr}, is obtained.

For perforated plates, the critical buckling load can be obtained numerically, being the N_{cr} of Eq. (2) used to validate the computational modeling and also as a reference value for comparisons between the behavior of plates with and without holes.

As earlier mentioned, the presence of holes in structural plate elements is almost inevitable for inspection, maintenance and service purpose. The presence of these holes redistributes the membrane stresses in the plates, significantly altering their stability. For this reason, the buckling of such perforated panels has received the attention of many researchers over the past years.

El-Sawy and Nasmy (2001) have analyzed the influence of the hole dimension, shape and location in the buckling stress of thin plates with different aspect ratios. Moen and Schafer (2009) have developed closed-form expressions for approximating the influence of single or multiple holes on the critical elastic buckling stress of plates in bending or compression that were validated by the Finite Element Method (FEM). Cheng and Zhao (2010) have focused their studies on the cutout-strengthening of perforated steel plates subjected to uniaxial compressive loads. Shimizu (2007) has showed that, when a plate has a hole, compression stresses appear locally near the hole under a tensile load, and the compression stress may cause local buckling — the so-called tension buckling — of the plate.

The aim of this work is to study numerically the effect of circular hole size and location on the critical elastic buckling load of simply supported rectangular plates subjected to uniform uniaxial load. The results are compared with the critical load of the plate without holes. To analyze the influence of circular hole size in the critical load, plates with five different aspect ratio: a/b = 1, 2, 3, 4 and 5 (being the width b maintained constant and equal to 1.00 m) were used; where six different hole diameter were considered: 0.10, 0.20, 0.30, 0.40, 0.50 and 0.60 m. After that, to study the influence of hole location, the best result earlier obtained was adopted as reference, and the position of the hole was investigated. The elastic buckling behavior of simply-supported plates with and without holes was obtained with shell finite element eigenvalue buckling analyses using the ANSYS® package. The Lanczos Method was used to solve the corresponding eingenvalue problem.

The results obtained indicate that the presence of circular holes in the plates produce a new distribution of the membrane stress, comparing with the stress behavior of a plate without hole. Therefore, depending on the plate aspect ratio and hole dimension and position, the critical buckling load can be superior or inferior to the reference value (plate without hole).

2. METHOD OF ANALYSIS

Many problems in Structural Analysis are governed by differential equations. The solutions of these equations would provide an exact, closed-form solution to the particular problem being studied. However, such analytical solutions are only available for problems involving very simple geometry, loading and boundary conditions. Hence, for a more complex problem, the computational modeling can be employed to obtain an approximate solution (Real and Isoldi, 2010).

This study is interested in determining the load at which the perforated plate looses stability (buckles) using numerical simulation. Besides, the present work is only concerned with the elastic buckling load. Therefore the plate material is assumed to be linear elastic and the critical stress in the plate at buckling, generated by the critical load, is smaller than the material yield stress.

Then, with the ANSYS® software, based on the FEM, the approach adopted for buckling analysis was the eigenvalue buckling (linear). This numerical procedure is used for calculating the theoretical buckling load of a linear elastic structure. Since it assumes the structure exhibits linearly elastic behavior, the predicted buckling loads are overestimated (Madenci and Guven, 2006).

Therefore, if the component is expected to exhibit structural instability, the search for the load that causes structural bifurcation is referred to as a buckling load analysis. Because the buckling load is not known a priori, the finite element equilibrium equations for this type of analysis involve the solution of homogeneous algebraic equations whose lowest eigenvalue corresponds to the buckling load, and the associated eigenvector represents the primary buckling mode (Madenci and Guven, 2006).

The strain formulation used in the analysis includes both the linear and nonlinear terms. Thus, the total stiffness matrix, K, is obtained by summing the conventional stiffness matrix for small deformation, K_E, with another matrix, K_G, which is the so-called geometrical stiffness matrix (Przemieniecki, 1985). The matrix K_G depends not only on the geometry but also on the initial internal forces (stresses) existing at the start of the loading step, $\{P_0\}$. Therefore the total stiffness matrix of the plate with load level $\{P_0\}$ can be written as:

$$[K] = [K_E] + [K_G] \tag{3}$$

When the load reaches the level of $\{P\} = \lambda\{P_0\}$, where λ is a scalar, the stiffness matrix can be defined as:

$$[K] = [K_E] + \lambda[K_G] \tag{4}$$

Now, the governing equilibrium equations for the plate behavior can be written as:

$$[[K_E] + \lambda[K_G]]\{U\} = \lambda\{P_0\} \tag{5}$$

where $\{U\}$ is the total displacement vector, that may therefore be determined from:

$$\{U\} = [[K_E] + \lambda[K_G]]^{-1}\lambda\{P_0\} \tag{6}$$

At buckling, the plate exhibits a large increase in its displacements with no increase in the load. From the mathematical definition of the matrix inverse as the adjoint matrix divided by the determinant of the coefficients it is possible to note that the displacements $\{U\}$ tend to infinity when:

$$\det[[K_E] + \lambda[K_G]] = 0 \tag{7}$$

Equation (7) represents an eigenvalue problem, which when solved provides the lowest eigenvalue, λ_1, that corresponds to the critical load level $\{P_{cr}\} = \lambda_1\{P_0\}$ at which buckling occurs. In addition, the associated scaled displacement vector $\{U\}$ defines the mode shape at buckling. In the finite element program ANSYS®, the eigenvalue problem is solved by using the Lanczos numerical method (ANSYS, 2005).

3. RESULTS AND DISCUSSION

For this study, the plates are considered to be made of cold-formed steel. The material properties are assumed as E = 210 GPa and ν = 0.30, and the material yielding stress is σ_y= 250 MPa, for all plates.

In all numerical simulations the ANSYS® SHELL93 reduced integration eight-node thin shell element, shown in Fig. 2, was employed. This element has six degrees-of-freedom at each node: three translations (*u*, *v*, *w*) and three rotations ($\theta_x, \theta_y, \theta_z$).

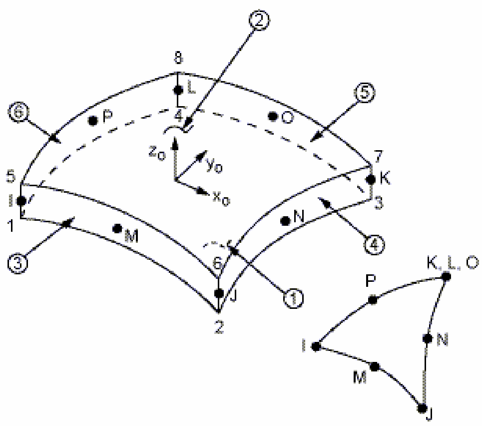

Figure 2. ANSYS SHELL93 8-node element geometry

To validate the computational modeling, the critical load of a non perforated plate was numerically evaluated, and the result was compared with the analytical solution given by Eq. (2). A steel plate with *a* = 2.00 m, *b* = 1.00 m and *t* =

0.01 m was discretized adopting a triangular element with side size of 0.05 m (*b*/20), generating a mesh with 1814 finite elements (Fig. 3). The analytical and numerical results are, respectively: 759,20 kN/m and 755,30 kN/m, showing a difference of 0,51 %. Figure 3 also presents the buckled shape of non perforated plate.

An important aspect is that the failure of plates subjected to uniaxial compression may be due to instability or material failure. For thin plates (i.e. large values of *b*/*t*) made from a typical strain hardening material with yield stress, σ_y, instability occurs at an average stress σ_{cr} that is much less than the yield stress, especially if the plate has no holes. For the present case, the critical stress σ_{cr} is equal to 75,92 MPa, while the material yield stress σ_y is equal to 250 MPa.

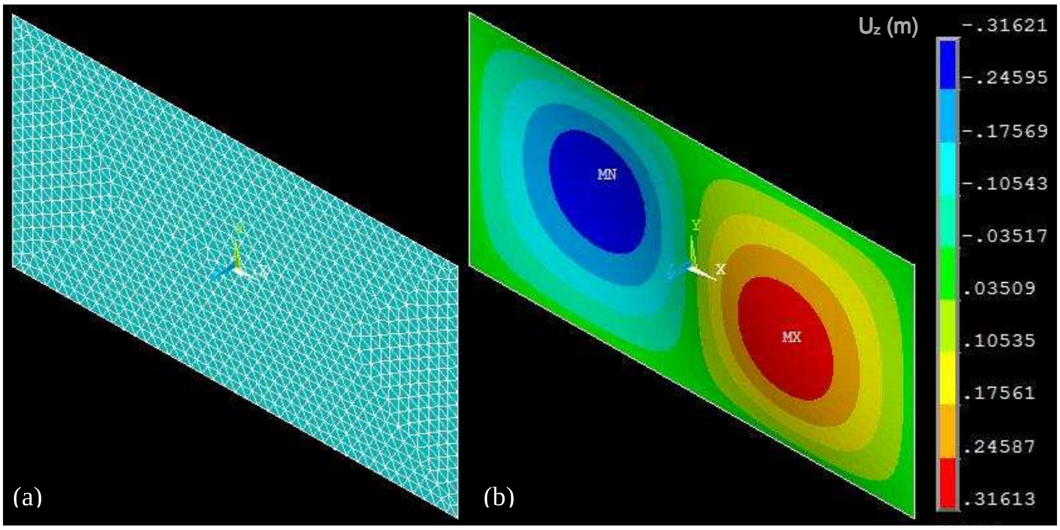

Figure 3. Plate without hole: (a) Finite element mesh; (b) Buckled shape

Another characteristic that has great influence on the buckling stress, σ_{cr}, are the boundary conditions of the plate. For a real plate, the actual boundary conditions may be somewhere between the all fixed and the all simply supported extremes. In general, for the same plate geometry, the boundary condition of simply supported edges is more critical with respect to buckling than the condition of fixed edges. Therefore, in the present study only the critical case of plate with simply supported edges is analyzed.

As previously said, the problem considered in this work is the elastic buckling of a simply supported rectangular perforated plate (circular hole) subjected to uniaxial end compression along its longitudinal direction. Firstly, the hole size influence in the critical load was analyzed. After that, an investigation about the hole position influence was made.

3.1. Hole size influence

All the plates analyzed in this study have the following constant dimensions: b = 1.00 m and t = 0.01 m. The aspect ratio is variable: a/b = 1, 2, 3, 4 and 5. The hole diameter also varies: d = 0.10, 0.20, 0.30, 0.40, 0.50 and 0.60 m. For this initial numerical simulations the hole position along de *x* axis is constant: $x_{hole} = 0.50a$, as shown in Fig. 4. In all cases the loads were applied along the plate's minor side, i.e. the vertical edge.

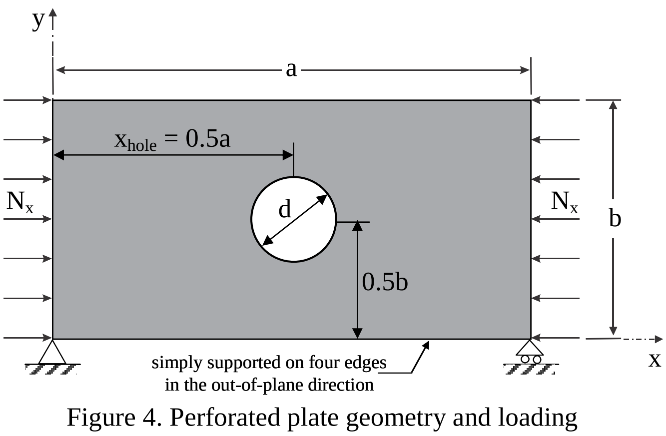

Figure 4. Perforated plate geometry and loading

The results obtained for the critical buckling load of perforated plates are presented in Fig. 5. The critical load of a plate with no hole, analytically evaluated by Eq. (2), was used to normalize critical load and the width *b* was adopted to normalize the hole diameter.

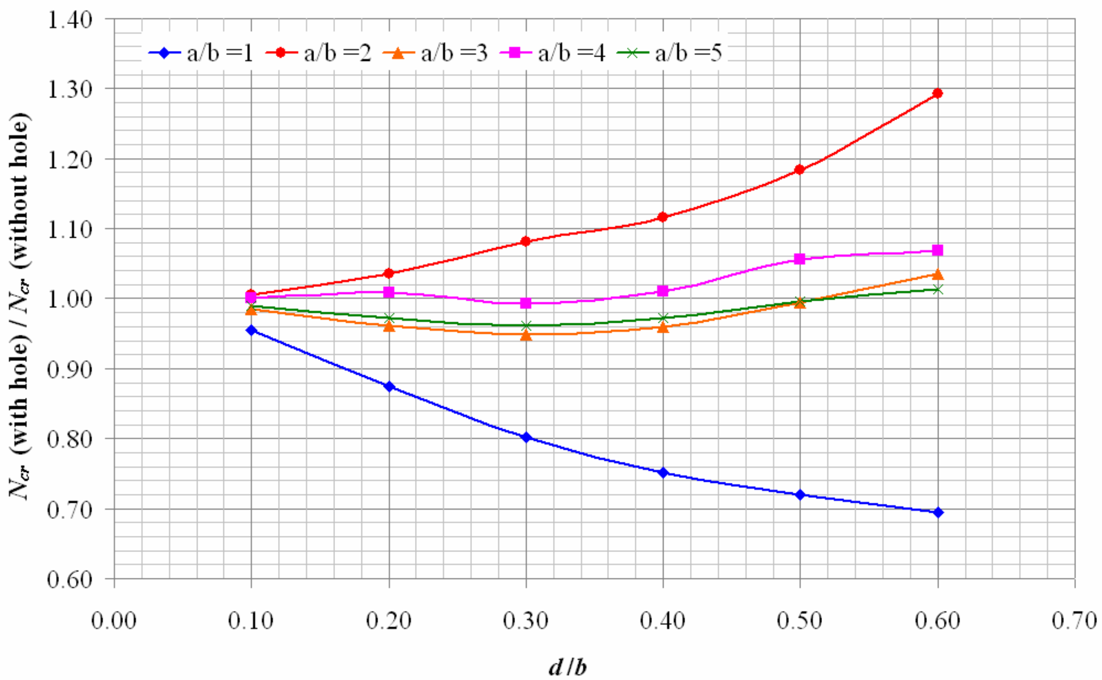

Figure 5. Normalized critical load x hole diameter

In Fig. 5 is possible to observe that the plate with aspect ratio *a*/*b* = 1 has the worst behavior in the elastic buckling analyses accomplished. In Fig. 6 the buckled shapes of this case are showed, indicating the formation of only one half-wave for all hole diameters used. Therefore, being the center of the plate the center of the half-wave, as the hole in this region increases, the stiffness of the plate decreases, resulting in a decreasing critical load.

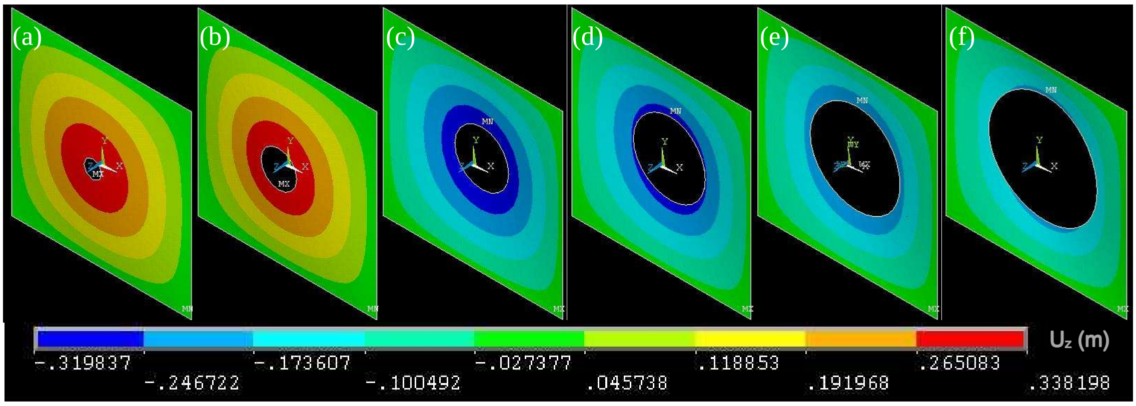

Figure 6. Buckling mode shape - plate with *a*/*b* = 1 and *d*/*b* of: (a) 0.10; (b) 0.20; (c) 0.30; (d) 0.4; (e) 0.50; (f) 0.60

The perforated plate with aspect ratio *a*/*b* = 2 has the best behavior among the studied cases. There is an increase in the critical buckling load as the hole size also increases. This trend could be explained if one considers the buckled mode shapes of the plate, which are presented in Fig. 7. In fact, the buckling resistance increases due to a redistribution of the membrane stresses towards the laterally supported side edges of the plate. When the ratio *d*/*b* increases the plate buckled shape changes from two half-waves to three half-waves. This explains the increasing of the buckling load in this case.

In an eigenvalue buckling analysis, only the critical load can be precisely determined. The buckling mode shape can also be evaluated through the eingenvectors associated with each eigenvalue. But only the shape of the buckling surface can be estimated. The real values of the displacements result undetermined in this kind analysis. As in the finite element method the stress at a point is a function of the displacement at this point, the stress distribution cannot be evaluated during an eigenvalue buckling analysis.

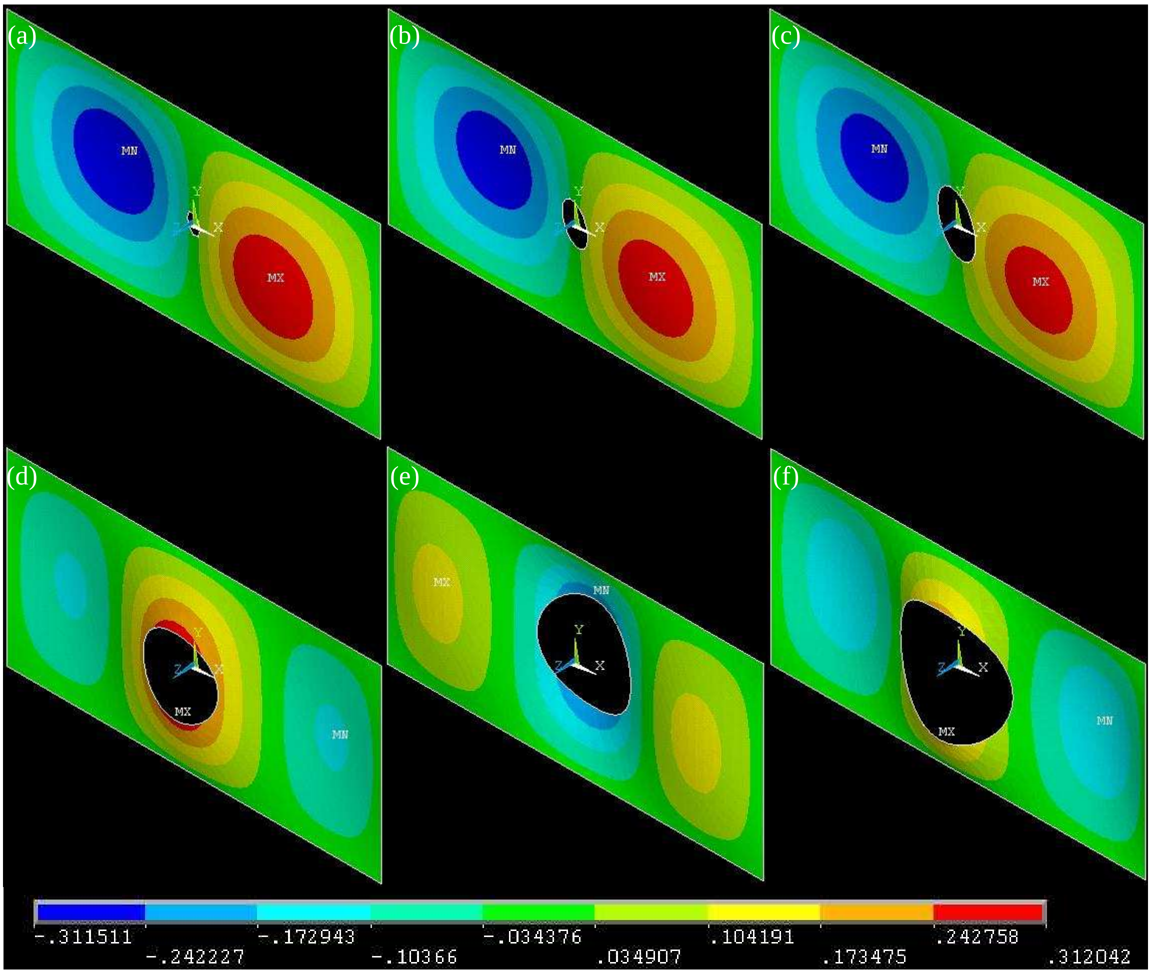

Figure 7. Buckling mode shape - plate with *a*/*b* = 2 and *d*/*b* of: (a) 0.10; (b) 0.20; (c) 0.30; (d) 0.4; (e) 0.50; (f) 0.60

However, for a certain buckling mode shape, it may be established the working hypothesis that the region of the surface that presents the larger displacements must be submitted to the higher stresses, although theses stresses cannot be determined. This kind of behavior can be observed for the square plate with a circular hole, as it is shown in Fig. 6. This working hypothesis will be tested in the future by imposing the buckling mode shape as an initial imperfection on the plate and making a nonlinear finite element analysis to determine the real stress field.

For the perforated plate with aspect ratio *a*/*b* = 3 (Fig. 8), it can be observed that for the cases when *d/b* = 0.1, 0.2 and 0.3, the hole coincides with the region of the higher displacements (internal half-wave). But for the cases when *d/b* = 0.4, 0.5 and 0.6, the displacements in the externals half-waves are equal or higher than those of the internal half-wave. This can indicate a stress transference from de central part of the plate to its extremities which still have a large load carrying capacity. This kind of behavior would permit the increasing of the critical load of the plate, as it is observed in Fig. 5.

The plate with aspect ratio *a*/*b* = 4 (Fig. 9) with *d/b* = 0.1 has almost the same critical load that the plate with no hole. This hole is small and it is located on a region of small displacements, and, assuming the validity of the above working hypothesis, on a region of small stresses. When the relation *d/b* is increased to 0.2, the buckling mode shape changes from four equal half-waves to five different half-waves. The larger displacements are in the central portion of the plate, and they diminish in the direction of the plate extremities. The change from four to five half-waves bluckling mode shape can explain the increasing of the critical load. The reduction of the stiffness of the central portion of the plate can justify the reduction in the value of the buckling load for *d/b* = 0.3. For the cases when *d/b* = 0.4, 0.5 and 0.6, the displacements in the central half-waves diminish, while the displacements in the externals half-waves increase, so the critical load may increase again.

The buckling mode shape for the plate with aspect ratio *a*/*b* = 5 (Fig. 10) is formed by five half-waves. For the cases when *d/b* = 0.1, 0.2 and 0.3, the region of the higher displacements coincides with the central portion of the plate. When the relation *d/b* is increase from 0.4 to 0.6, the region of the higher displacements moves from the center to the extremities of the plate. This behavior explains at first the reduction of the critical load and its later increasing value.

It is important to emphasize that the results obtained in the present work were in agreement with those presented by El-Sawy and Nasmy (2001).

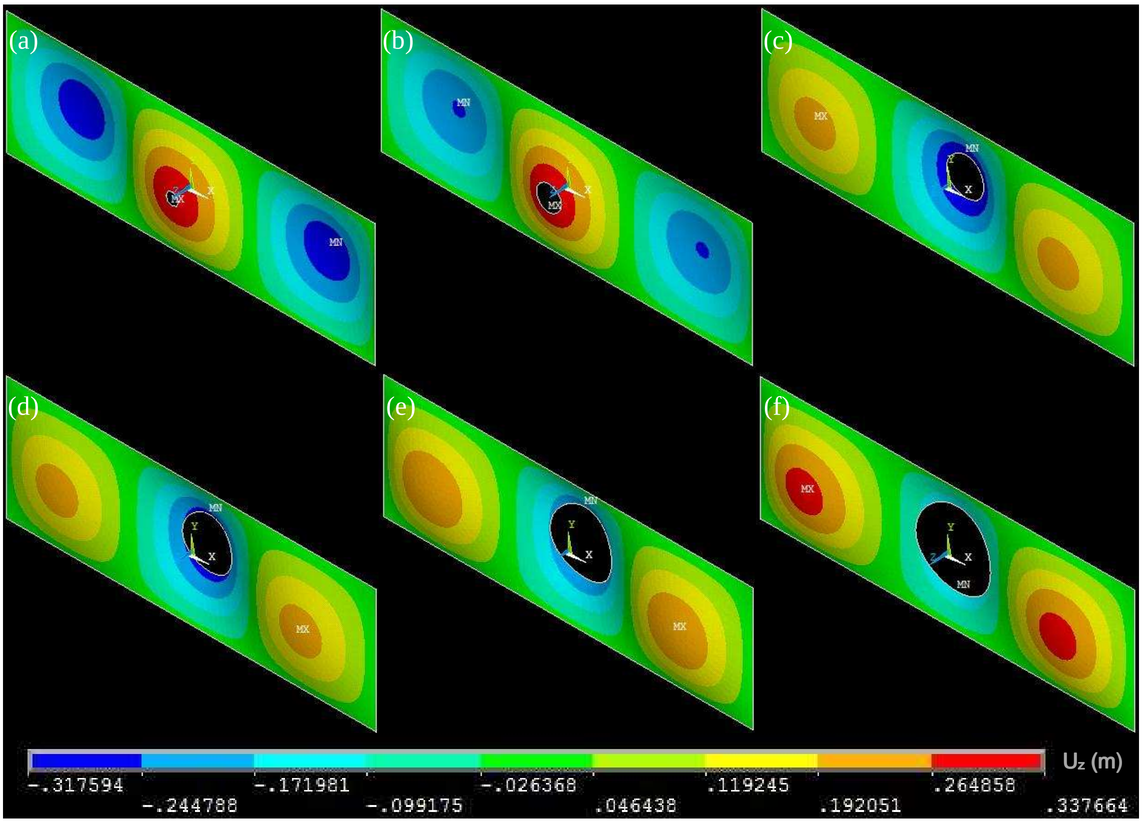

Figure 8. Buckling mode shape - plate with *a*/*b* = 3 and *d*/*b* of: (a) 0.10; (b) 0.20; (c) 0.30; (d) 0.4; (e) 0.50; (f) 0.60

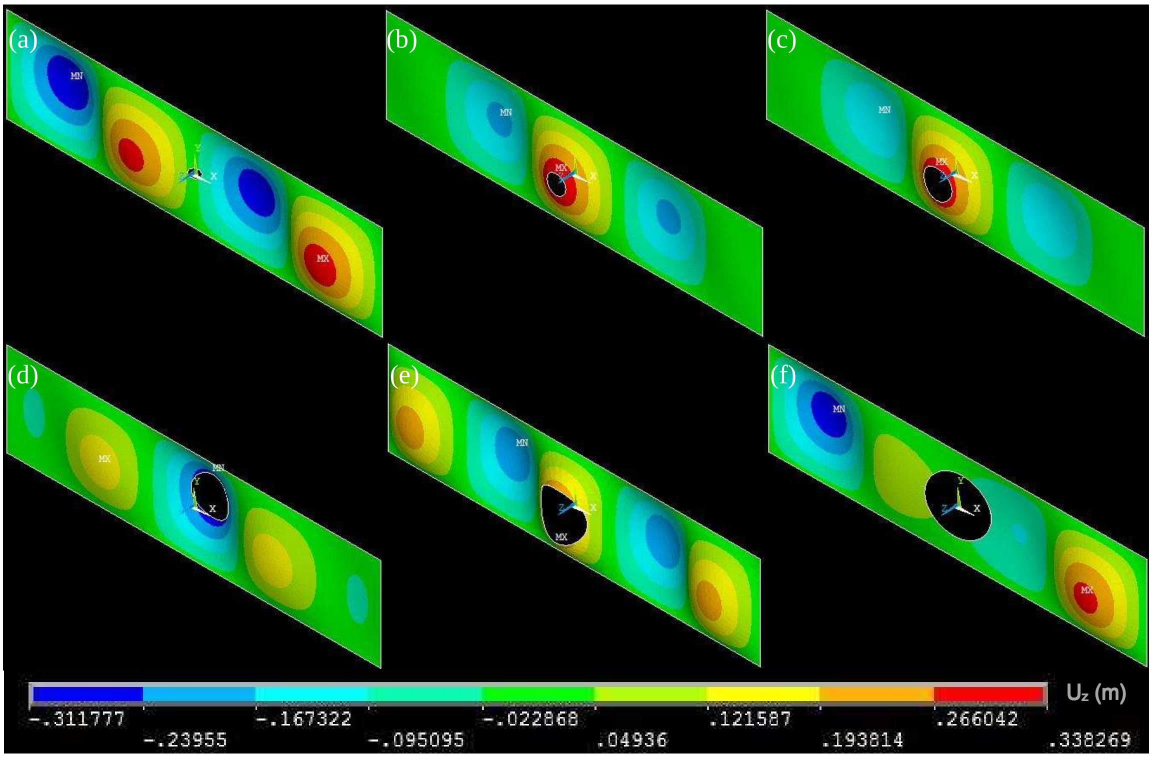

Figure 9. Buckling mode shape - plate with *a*/*b* = 4 and *d*/*b* of: (a) 0.10; (b) 0.20; (c) 0.30; (d) 0.4; (e) 0.50; (f) 0.60

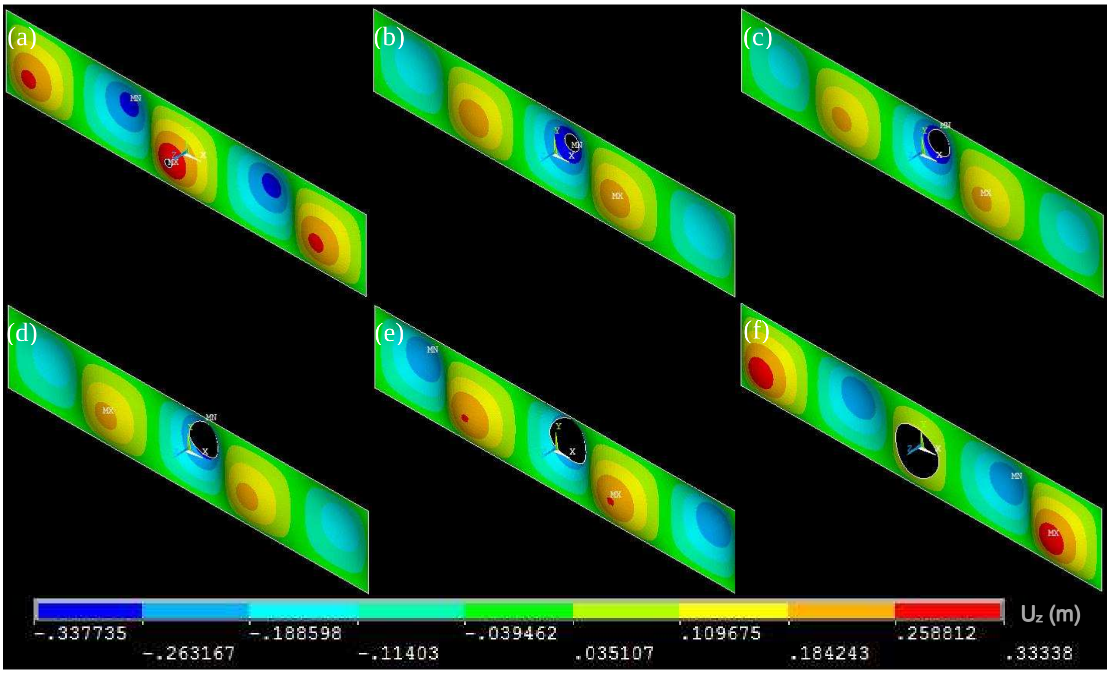

Figure 10. Buckling mode shape - plate with a/b = 5 and d/b of: (a) 0.10; (b) 0.20; (c) 0.30; (d) 0.4; (e) 0.50; (f) 0.60

3.2. Hole location influence

Finally, an investigation about the hole location influence was performed. This study was developed considering the best result previously obtained, i.e., the case with the largest critical load (plate with aspect ratio a/b = 2) was used for analyzing the hole position effects in elastic buckling. For this purpose, based on the Fig. 4, the longitudinal hole position, x_{hole}, was varied: x_{hole} = 0.125a, 0.250a, 0.375a and 0.500a. The numerical results are plotted in Fig. 11, where the plate width b was employed to normalize the hole location.

One can observe in Fig. 11 that for small diameters the influence of hole location in longitudinal direction is not significant for the critical buckling load. However, as the hole size increases its position becomes important in the elastic buckling load. Figure 11 indicates that the case with d/b = 0.60 has the largest variation for the critical load. Therefore, the buckling modes of this case are presented in Fig. 12.

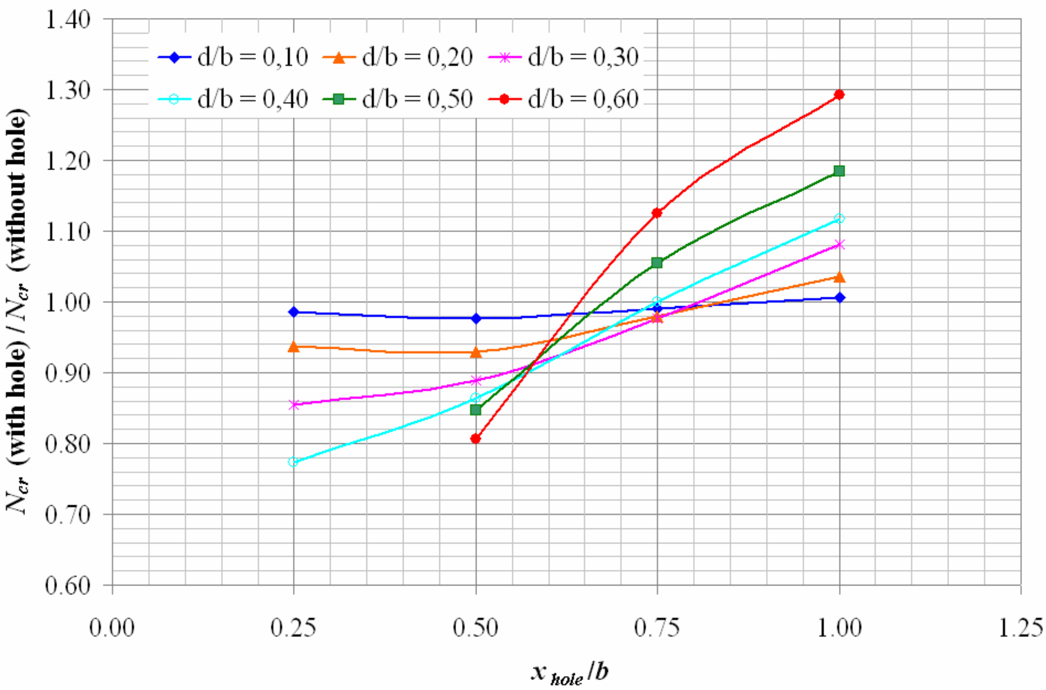

Figure 11. Normalized critical load x hole location

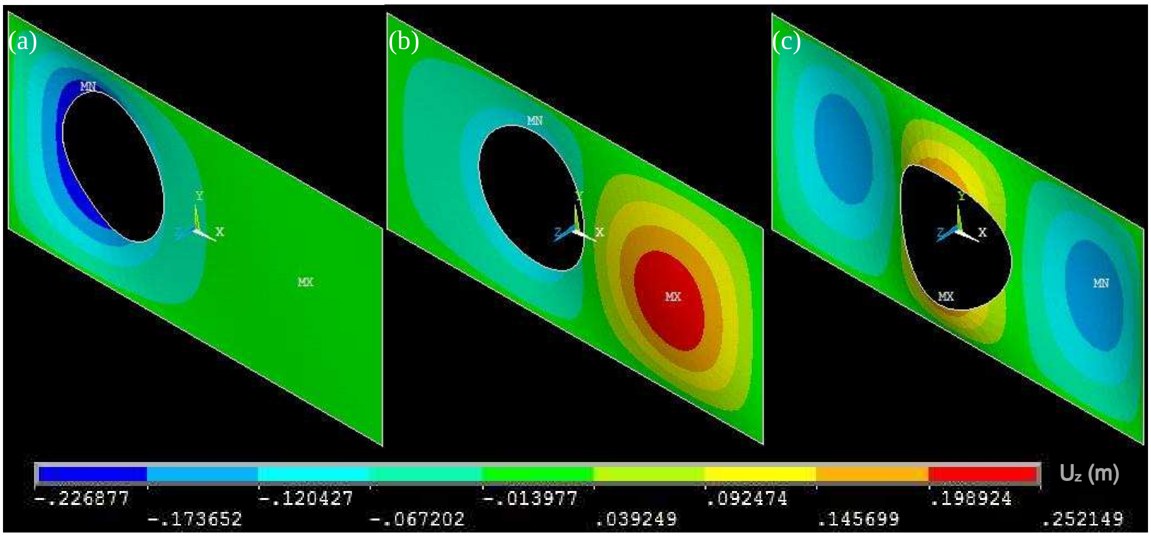

Figure 12. Buckling mode shape - hole size d = 0.60 m and x_{hole} of: (a) 0.250a; (b) 0.375a; (c) 0.500a

Again, the results generated in this work were in agreement with those obtained by El-Sawy and Nasmy (2001).

4. CONCLUSION

The influence of plate aspect ratio, hole diameter and hole location on the critical buckling load of rectangular perforated plates subjected to uniform uniaxial end compression were numerically studied. For the elastic buckling analysis the general-purpose finite element program ANSYS® was used and the eigenvalue buckling (linear) approach was adopted.

A plate with no hole that has a critical load evaluated analytically was considered to validate the computational model and as a reference value.

Several cases were studied and the results demonstrated that plates with centered circular holes can present a better or worse behavior than a plate without hole, if comparing the elastic buckling load. One of the factors that interfere in this behavior is the plate aspect ratio. The perforated plate with aspect ratio a/b = 1 had the worst behavior of all cases, reaching a minimum value of critical load about 30% smaller than the reference value. Moreover, the plate with aspect ratio a/b = 2 presented the best behavior for all holes sizes analyzed, reaching a critical load 30% larger than the reference value. The others aspect ratio investigated (a/b = 3, 4 and 5) had similar behaviors amongst themselves and the values of the critical load were around the reference value.

Besides, the effect of the hole position in the elastic buckling was investigated. The longitudinal location was changed and the results indicated that for small circular holes the position in not an important factor for the critical load. However, for larger holes the location has a great influence in the critical load. It was observed that for big holes, the position where the maximum critical load is obtained is the center of the plate.

All the analyzed cases show that the presence of circular holes (centered or eccentric) generates a redistribution of the membrane stresses in the plates. This phenomenon causes a significant change in the structural stability of the plates. For this reason, the buckling of perforated panels has great importance in structural design and additional researches into this subject are justified.

5. ACKNOWLEDGEMENTS

The authors thank UFRGS for the support.

6. REFERENCES

ANSYS, 2005, “User’s manual (version 10.0)”, Swanson Analysis System Inc, Houston, USA.

Cheng B. and Zhao J., 2010, “Strengthening of Perforated Plates under Uniaxial Compression: Buckling Analysis”, Thin-Walled Structures, Vol. 48, pp. 905–914.

El-Sawy, K.M. and Nazmy, A.S., 2001, “Effect of Aspect Ratio on the Elastic Buckling of Uniaxially Loaded Plates with Eccentric Holes”, 39, pp. 993-998.

Madenci, E. and Guven, I., 2006, “The Finite Element Method and Applications in Engineering Using ANSYS®”, Ed. Springer, New York, USA, 686 p.

Moen, C.D. and Schafer, B.W., 2009, “Elastic Buckling of Thin Plates with Holes in Compression or Bending”, Thin-Walled Structures, Vol. 47, pp. 1597-1607.

Przemieniecki, J.S., 1985, “Theory of Matrix Structural Analysis”, Ed. Dover Publications, New York, USA.
Real, M.V. and Isoldi, L.A., 2010, “Finite Element Buckling Analysis of Uniaxially Loaded Plates with Holes”, Proceedings of the 4th Southern Conference on Computational Modeling, Rio Grande, Brazil, pp. 69-73
Shimizu, S., 2007, “Tension Buckling of Plate Having a Hole”, Thin-Walled Structures, Vol. 45, pp. 827–833.
Wang, C.M., Wang, C.Y. and Reddy, J.N., 2005, “Exact Solutions for Buckling of Structural Members”, Ed. CRC Press, Boca Raton, USA, 209 p.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.