FEEDBACK LINEARIZATION CONTROL WITH FRICTION COMPENSATION APPLIED TO A PNEUMATIC POSITIONING SYSTEM

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Abstract. *This work presents a nonlinear control algorithm to be applied to the trajectory tracking of a pneumatic positioning system. The objective is to compensate the highly nonlinear effects that are inherent to such systems due to the dynamic behavior of pressurized air inside the chambers and to friction forces. In order to accomplish this task, the proposed algorithm employs the feedback linearization control technique, equiped with a friction-compensating scheme based on a continuous version of the LuGre friction model. A mathematical model of the controlled system that includes nonlinear friction effects is presented. The proposed controller is described and simulation results are presented in order to illustrate its main features.*

Keywords: *pneumatic servo-system, feedback linearization, friction compensation, LuGre friction model*

1. INTRODUCTION

Pneumatic positioning systems are very attractive for many applications because they are cheap, clean, lightweight, and easy to assemble. Also, they present good ratio between transported load and supplied power, and its operating fluid (compressed air) is largely available in industrial environments. In spite of these advantages, such systems present highly nonlinear behavior, so that controlling their operation in high precision applications is a very difficult task. If linear controllers are employed, for instance, trajectory-tracking performance is highly influenced by the position at which its approximate models are defined (Virvalo, 1995; Nouri *et al.*, 2000). Thus, the efficiency of a linear controller degrades rapidly as the range of operation of the system is increased, so that high-precision positioning can only be achieved for a very small part of the cylinder stroke. Also, these systems suffer from the highly nonlinear behavior associated to compressiblity effects (Bobrow and McDonell, 2002) and dry-friction forces at near-zero velocities (Guenther *et al.*, 2006; Khayati *et al.*, 2009). Such undesirable features limit the use of these systems in wide-range positioning applications that require a fast and precise response, especially when fixed-gain linear controllers are employed.

In order to cope with the difficulties posed by pneumatic systems, many novel control techniques have been proposed. Most of them are based on nonlinear approaches such as adaptive control (Kaitwanidvilai and Parnichkun, 2005; Zhang *et al.*, 2007), feedback linearization (Brun *et al.*, 2002; Smaoui *et al.*, 2006), neural networks (Song *et al.*, 2006; Hong and Yao, 2007) and variable structure control (Pandian *et al.*, 2002; Bone and Ning, 2007). As for the compensation of friction effects in servomechanisms, the use of model-based control schemes is found to be an effective solution (Guenther *et al.*, 2006; Jamaludin *et al.*, 2008; Khayati *et al.*, 2009). In this context, the LuGre model (Canudas de Wit *et al.*, 1995) is employed in several control algorithms (Xie, 2007; Zhang *et al.*, 2008; Khayati *et al.*, 2009). In order to calculate the net friction force between two contacting surfaces, this model uses a nonlinear first-order state observer that emulates the average elastic displacement of the bristles that exist on these surfaces in microscopic scale. By means of such observer, this model is capable of representing many friction phenomena with satisfactory accuracy. Additionally, the mathematical properties of the LuGre model include invariance and passivity, which are very useful in determining stability characteristics of controlled systems. Nevertheless, as discussed with more detail in Section 3, application of friction-compensating schemes bansed on the LuGre model to the specific case of a pneumatic servo system is restricted by the existence of a discontinuous term in the mathematical structure of the state observer upon which such model is based. Therefore, a continuous approximation of this model must be used. In Guenther *et al.* (2006), it is employed a continuous approximation of the LuGre model, but the important properties of invariance and passivity are not proven to hold for such approximate model. To overcome this difficulty, a new continuous version of the LuGre friction model was proposed by Sobczyk (2009), in which such properties are guaranteed to be preserved.

In this work, it is proposed a nonlinear control algorithm based on the feedback linearization technique, aiming at minimizing the undesirable effects inherent to a pneumatic positioning system due to the nonlinear dynamics of its air mass flow rates and of friction forces at near-zero velocities. The objective is to cancel such nonlinear effects by means of a proper linearizing-control action, so that its closed-loop performance characteristics can be determined by employing linear control design techniques without significant degradation in wide-range positioning tasks. A nonlinear model of the pneumatic positioning system is presented, including a detailed descripition of the employed approximation of the LuGre friction model. The proposed controller is described and the convergence properties of the closed-loop system are outlined. Finally, simulation results are used to illustrate the main characteristics of the closed-loop system when controlled by means of the proposed algorithm.

This paper is organized as follows: in Section 2, the theoretical model of the pneumatic positioning system is described without expliciting friction dynamics, which is discussed in Section 3. Section 4 is dedicated to presenting the proposed control strategy, whereas simulation results are given in Section 5. Finally, the main conclusions drawn from this work are outlined in Section 6.

2. DYNAMIC MODEL

A typical pneumatic servo system is depicted in Fig. 1. Except for the representation of friction effects, which will be discussed in Section 3, its mathematical model follows the same approach employed in many different works (see, for instance, Bobrow and McDonell, 1998, Kazerooni, 2005, or Ning and Bone, 2005). The development of the specific model that is used in this work is discussed in further detail in Sobczyk and Perondi (2005). Its formulation is based on the following physical phenomena:

(i) the relationship between the air mass flow rate and the pressure changes in the cylinder chambers, obtained by energy conservation;

(ii) the equilibrium of the forces acting on the piston, given by Newton's second law.

(iii) the description of friction forces as the net result of elastic displacements of the asperities that are present on the contacting surfaces of the cylinder and its internal piston, estimated by means of a continuous approximation of the LuGre friction model proposed in Sobczyk (2009).

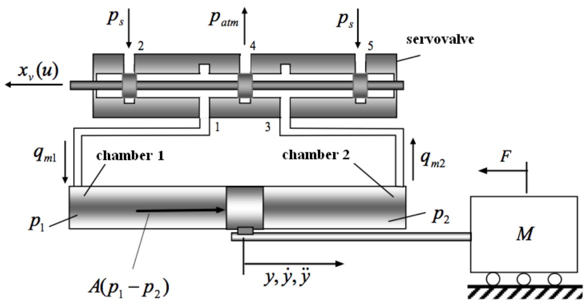

Figure 1. Pneumatic positioning system

Based on energy conservation arguments (Sobczyk and Perondi, 2005), the dynamics of the pressures in the two chambers of the pneumatic cylinder are given by:

$$\dot{p_1} = -\frac{Ar\dot{y}}{V_{10} + Ay}p_1 + \frac{rRT}{V_{10} + Ay}q_{m1} \tag{1}$$

$$\dot{p_2} = \frac{Ar\dot{y}}{V_{20} + A(L-y)}p_2 + \frac{rRT}{V_{20} + A(L-y)}q_{m2} \tag{2}$$

where r is the ratio between the specific heat values of the air, $\dot{y}$ is the piston velocity, R is the universal gas constant, T is the air supply temperature, y is the position of the piston, V_{10} and V_{20} are the dead volumes of air in the lines and at the extremities of both chambers, L is the cylinder stroke, A is the cylinder cross-section area, p_1 and p_2 are the pressures in the two chambers, and $q_{m1} = q_{m1}(u, p_1)$ and $q_{m2} = q_{m2}(u, p_2)$ are the mass flow rates of air into or out of each chamber. Such flow rates are nonlinear functions that depend on the pressures in each chamber, on the supply pressure and on the control voltage u applied to the servovalve. The behavior of such flow rates is modeled by means of $3rd$ order polynomial functions whose coefficients were determined experimentally. The general format of the mathematical expressions that model these mass flow rates is given by

$$q_m(p_i, u) = [q_m]_{max} f_p(p_i) f_u(u) \tag{3}$$

where $[q_m]_{max}$ is the maximum absolute value of air mass flow through the valve, $f_p(p_i)$ is a $3rd$ order polynomial in terms of the pressure inside the considered chamber (1 or 2), and $f_u(u)$ is another $3rd$ order polynomial that depends on the control signal that is applied to the servovalve. These polynomial functions were determined experimentally by evaluating the dependence on each variable (p_i and u) while each of the chambers was first filled and then exhausted.

Therefore, eight polynomial functions are employed (four that depend on the control signal plus four that model the effect of the pressures in the chambers), with two of them being combined for each chamber in a given instant according to the process that is occurring. The complete set of polynomial functions and the experimental procedure that was employed for determining them are described in Perondi (2002).

The dynamics of the strictly mechanical part of the system is modeled by applying Newton's second law directly on the piston-load assembly, resulting

$$M\ddot{y} + \sigma_2\dot{y} + F = A(p_1 - p_2) \tag{4}$$

where M is the mass of the piston-load assembly σ_2 is the viscous friction coefficient, and F is the dry-friction force.

Accompanied by the eight polynomial fucntions that are employed in Eq.(3), equations (1), (2) and (4) represent the fourth-order nonlinear model of a pneumatic positioning system without expliciting the dynamics of dry-friction effects. As the piston is moved due to the difference of pressure between the two chambers, it is convenient to rewrite these equations in terms of a differential pressure $p_\Delta = p_1 - p_2$. Thus, subtracting Equations (1) and (2), one obtains:

$$\dot{p}_\Delta = \hat{h}(p_1, p_2, y, \dot{y}) + \hat{u}(p_1, p_2, u, y) \tag{5}$$

where $\hat{h}(p_1, p_2, y, \dot{y})$ and $\hat{u}(p_1, p_2, u, y)$ represent the grouped terms that are functions only of the states of the system and those that are affected by the input voltage u, respectively. Such grouped terms are defined as:

$$\hat{u}(p_1, p_2, u, y) = RrT\left(\frac{q_{m1}(u, p_1)}{V_{10} + Ay} - \frac{q_{m2}(u, p_2)}{V_{20} + A(L - y)}\right) \tag{6}$$

$$\hat{h}(p_1, p_2, y, \dot{y}) = -rA\dot{y}\left(\frac{p_1}{V_{10} + Ay} - \frac{p_2}{V_{20} + A(L - y)}\right) \tag{7}$$

As it can be observed from equations (4) and (5), the control action that is applied to the system affects only the time derivative $\dot{p}_\Delta$ of the differential pressure that is responsible for positioning the piston-load assembly. Therefore, in order to employ the proposed control scheme that will be discussed in Section 4, it is necessary to rewrite the equations that model the dynamics of the system so that the dependence of its translational states (y, $\dot{y}$ and $\ddot{y}$) on the control action can be written in an explicit way. This can be carried out by deriving Eq.(4) and substituting Eq.(5) into the resulting function, obtaining:

$$\dddot{y} = -\frac{\sigma_2}{M}\ddot{y} + \frac{A}{M}(\hat{u} + \hat{h}) \tag{8}$$

3. FRICTION MODEL

In this work, it is employed a continuous version of the The LuGre model to represent the effects of dry friction forces in the pneumatic servo actuator. This model is well known in control literature and its most important features are discussed in detail in Canudas de Wit *et al.* (1995) and Barabanov and Ortega (2000), among other works. Its representation of friction effects is based on the interaction between two contacting surfaces in microscopic scale. In this context, the surfaces are rough, and the complex nature of friction is determined by the relationship between the asperities of these surfaces. Mathematically, this relationship is modeled by means of the elastic deformations of microscopic elements (*bristles*). Such deformations are represented by an average deflection z, which is treated as an internal state of the system. As this state is not directly measurable, it must be observed by means of a suitable algorithm. Using this state, the dry-friction force F acting on two bodies presenting relative motion can be written as:

$$F = \sigma_0 z + \sigma_1 \dot{z} \tag{9}$$

where σ_0 is a stiffness coefficient and σ_1 is a damping coefficient related to the time derivative of z. In its original form, the state observer that describes the dynamics of z is given by:

$$\dot{z} = \dot{y} - \frac{|\dot{y}|}{f(\dot{y})}z \tag{10}$$

where $f(\dot{y})$ is defined as (Canudas de Wit *et al.*, 1995)

$$f(\dot{y}) = \frac{1}{\sigma_0}\left[F_C + (F_S - F_C)\, e^{-(\dot{y}/\dot{y}_S)^2}\right] \tag{11}$$

where F_C is the Coulomb friction force, F_S is the static friction force, and $\dot{y}_S$ is the Stribeck velocity.

One of the most important features of the LuGre model lies in its capability of representing with satisfactory accuracy many nonlinear effects that are inherent to dry friction, such as varying break-away forces, stick-slip motion and hysteresis (Xie, 2007; Astrom and Canudas de Wit, 2008). Also, it allows the prediction of undesirable effects that could arise in closed-loop systems due to friction, such as limit cycles. Furthermore, this model presents three analytical properties that are of special importance in the context of control systems: **(i)** the internal state $z(t)$ is limited (Canudas de Wit *et al.*, 1995); **(ii)** the model defines a passive mapping when the system input is given in terms of velocity and the output is the internal state $z(t)$ (Canudas de Wit *et al.*, 1995); **(iii)** with a proper choice of the values of its parameters, the model is passive with respect to a velocity input and an output in the form of friction force (Barabanov and Ortega, 2000). These properties are relevant to the synthesis of control algorithms that include friction compensation schemes because they are useful in ensuring the stability of the closed-loop plant (Janiec, 2004; Guenther *et al.*, 2006; Khayati *et al.*, 2009).

Despite its usefulness as a tool for developing friction-compensating control algorithms, there are important difficulties that arise from the use of the LuGre model in the case of pneumatic positioning system that is studied in this work. Specifically, these difficulties occur because there is a dynamic relationship between the control signal u applied to the servovalve and the corresponding control action p_Δ that is responsible for the actual movement of the piston-load assembly. In this case, as indicated in Eq. (8), the time derivative $\dot{F}$ of the friction force must be explicitly known. According to expressions (9) and (10), the original LuGre model represents friction forces in a way that depends directly on $\dot{z}(t)$, which is a function of $|\dot{y}|$, whose time derivative is discontinuous in $\dot{y} = 0$. Thus, $\ddot{z}(t)$ does not exist at this point, and the time derivative of the estimated friction force cannot be represented by means of a continuous function for all operating conditions of the controlled system. Therefore, as already pointed out in Makkar *et al.* (2005) and also in Guenther *et al.* (2006), the use of this model in its original form is not possible in the case of this class of systems, and a continuous approximation must be employed. One such approximation was used in Guenther *et al.* (2006) with satisfactory practical results. However, it has not been proven that such approximation is guaranteed to preserve the aforementioned analytical properties possessed by the original model. As these properties have expressive physical meanings and play important roles in ensuring the stability of the controlled system, this problem can be a severe hindrance to the utilization of such a modified model as part of a control algorithm. If the model is actually not passive, for instance, it may cause the control signal applied to the plant to be excessively large, leading to equally large tracking errors or even operation accidents.

In order to develop a continuous approximation of the LuGre model and still keep the most important analytical properties given by its original form, it was proposed in Sobczyk (2009) that Eq. (10) should be modified as follows:

$$\dot{z} = \dot{y} S_1(\dot{y}) - \frac{S_2(\dot{y})}{f(\dot{y})} z \tag{12}$$

where $S_1(\dot{y})$ and $S_2(\dot{y})$ are defined with the aid of an auxiliary function $S_0(\dot{y})$. These new terms are given by:

$$\begin{cases} S_0(\dot{y}) = \frac{2}{\pi}\arctan(k_v \dot{y}) \\ S_1(\dot{y}) = [S_0(\dot{y})]^2 \\ S_2(\dot{y}) = \dot{y} S_0(\dot{y}) \end{cases} \tag{13}$$

By analyzing Eq. (12), and its auxiliary functions, it is observed that the structure of the proposed approximation tends to the original observer given in Eq. (10) as the value of k_v is increased. Thus, provided that k_v is chosen to be sufficiently large, the numerical predictions made by means of this newly proposed approximation can be made very similar to those obtained by using the original LuGre model. In Sobczyk *et al.* (2009), simulation studies are presented regarding the value of k_v that is necessary for achieving such good approximations. In the same work, it is concluded that such value is of the order of $10^7 [s/m]$ or greater.

As this approximation can be proven to retain all three analytical properties of interest (see Sobczyk, 2009, for details), its use with an appropriate value of k_v allows the application of the LuGre model to the case in study. However, in order to accomplish this task, it is still necessary to obtain the explicit formulation of the time derivative $\dot{F}$ of the dry-friction force that is present in the system (see Eq. (8)). This is carried out by deriving Eq. (9) with respect to time, obtaining:

$$\dot{F} = \sigma_0 \dot{z} + \sigma_1 \ddot{z} \tag{14}$$

The term $\ddot{z}$ employed in this expression can be calculated by taking the time derivative of Eq. (12):

$$\ddot{z} = \dot{S}_1 \dot{y} + S_1 \ddot{y} - (\dot{g} z + g \dot{z}) \tag{15}$$

where $g(\dot{y}) = S_2(\dot{y}) / f(\dot{y})$. The derivatives of the intermediate terms involved in this equation are given by the following expressions:

$$\dot{S}_1 = 2 S_0 \dot{S}_0 \tag{16}$$

$$\dot{S}_0 = \frac{2k_v}{\pi\left[1 + (k_v\dot{y})^2\right]}\ddot{y} \quad (17)$$

$$\dot{g} = \frac{\dot{S}_2 f - S_2\dot{f}}{f^2(\dot{y})} \quad (18)$$

$$\dot{S}_2 = \dot{S}_0\dot{y} + S_0\ddot{y} \quad (19)$$

$$\dot{f} = \frac{-2\left(F_s - F_c\right)}{\sigma_0 v_s^2}\dot{y}\ddot{y}e^{-(\dot{y}/v_s)^2} \quad (20)$$

4. CONTROL ALGORITHM

The proposed controller employs a feedback linearization technique in association with a linear control law that employs the position, velocity and acceleration of the piston as state variables. This approach aims at cancelling the nonlinear effects that derive from the dynamics of dry-friction forces and of the pressures in the two chambers as they are filled or exhausted, so that the desired dynamic behavior of the system can be entirely defined by means of standard pole-placement considerations. In order to establish the proposed control law, it is necessary to define the trajectory tracking errors of the controlled system as:

$$\begin{cases} \tilde{y} = y - y_d \\ \dot{\tilde{y}} = \dot{y} - \dot{y}_d \\ \ddot{\tilde{y}} = \ddot{y} - \ddot{y}_d \\ \dddot{\tilde{y}} = \dddot{y} - \dddot{y}_d \end{cases} \quad (21)$$

where y, $\dot{y}$, $\ddot{y}$ and $\dddot{y}$ are the position, velocity, acceleration and jerk of the piston, respectively, y_d, $\dot{y}_d$, $\ddot{y}_d$, and $\dddot{y}_d$ are the correspondingly desired values for the same variables. Using this definition, the proposed control law is given by

$$\hat{u} = \alpha - \beta \quad (22)$$

where α is the linear part of the proposed controller and β is the feedback linearization term. These two functions are defined as follows:

$$\alpha = \frac{M}{A}\left(- k_1\tilde{y} - k_2\dot{\tilde{y}} - k_3\ddot{\tilde{y}} + \dddot{y}_d + \frac{\sigma_2}{M}\ddot{y}\right) \quad (23)$$

where α is the linear part of the proposed controller and β is the feedback linearization term. These two functions are defined as follows:

$$\beta = \hat{h} + \dot{\hat{F}} \quad (24)$$

In Eq. (23), k_1, k_2 and k_3 are the feedback gains, whose values will be determined in Section 5. In the case of Eq. (24), $\hat{h}$ is the same term given in Eq. (6). Its purpose is to cancel the nonlinear effects of the controlled system due to the dynamics of compressed air inside of the actuator cylinder, whereas $\dot{\hat{F}}$ is an estimate of the time derivative of the dry-friction forces acting on the piston-load assembly. This estimate is obtained by means of a state observer that has the same structure of the friction model described in Section 3, that is:

$$\dot{\hat{F}} = \sigma_0\dot{\hat{z}} + \sigma_1\ddot{\hat{z}} \quad (25)$$

$$\dot{\hat{z}} = \dot{y}S_1\left(\dot{y}\right) - \frac{S_2\left(\dot{y}\right)}{f\left(\dot{y}\right)}\hat{z} \quad (26)$$

$$\ddot{\hat{z}} = \dot{S}_1\dot{y} + S_1\ddot{y} - (\dot{g}\hat{z} + g\dot{\hat{z}}) \quad (27)$$

where $\hat{z}$, $\dot{\hat{z}}$ and $\ddot{\hat{z}}$ are the observed counterparts of z, $\dot{z}$ and $\ddot{z}$, respectively.

Assuming that all parameters of the system are exactly known, the nonlinear effects due to the term $\hat{h}$ (Eq. (6)) are entirely cancelled. Thus, substitution of Eq. (22) into Eq. (8) yields

$$\dddot{\tilde{y}} = -k_1\tilde{y} - k_2\dot{\tilde{y}} - k_3\ddot{\tilde{y}} + \dot{\tilde{F}} \tag{28}$$

In Eq. (28), $\dot{\tilde{F}}$ stands for the residue of the attempt of cancelling the term $\dot{F}$ by means of its observed conterpart $\dot{\hat{F}}$. As it can be observed from the same expression, if $\dot{\tilde{F}}$ is suficiently small so that its influence can be neglected, the dynamics of the controlled system in state-space form can be approximated by

$$\dot{\mathbf{x}} = \mathbf{A_m}\mathbf{x} \tag{29}$$

with $\mathbf{Am}$ and $\mathbf{x}$ defined as

$$\mathbf{A_m} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \tilde{y} \\ \dot{\tilde{y}} \\ \ddot{\tilde{y}} \end{bmatrix} \tag{30}$$

Thus, the dynamic behavior of the trajectory tracking errors of the system is entirely specified by the roots of the characteristic equation associated to the matrix $\mathbf{A_m}$, and such behavior can be adjusted as desired by suitably choosing the values of the feedback gains k_1, k_2 and k_3.

Remark #1: To this moment, a complete stability analysis of the closed-loop pneumatic servo actuator when the proposed controller is employed is not available. However, due to the characteristics presented by the friction model in study, it is possible to infer some preliminary conclusions regarding the convergence properties of the trajectory-tracking errors for this system. In particular, because the internal state z that characterizes the friction model is limited, it can be shown that the dry-friction force F and its derivative $\dot{F}$ are both limited. Thus, by employing a pair of definite positive, symmetrical matrices $\mathbf{P}$ and $\mathbf{Q}$ that satisfy Lyapunov's relation $\mathbf{A}^T\mathbf{P} + \mathbf{PA} = -\mathbf{Q}$, combined with the non-negative function $V(t) = \mathbf{xPx}^T$, it can be proven that the vector of closed-loop trajectory-tracking errors $\mathbf{x} = [\tilde{y} \quad \dot{\tilde{y}} \quad \ddot{\tilde{y}}]^T$ for this system is restricted to a region around the origin of its state space that is given by

$$\| \mathbf{x} \| \leq \frac{2\lambda_{max}(\mathbf{P})}{\lambda_{min}(\mathbf{Q})}\Gamma \tag{31}$$

where $\lambda_{min}(\mathbf{Q})$ is the minimum eigenvalue of $\mathbf{Q}$, $\lambda_{max}(\mathbf{P})$ is the maximum eigenvalue of $\mathbf{P}$, and Γ is an upper bound for the residue $\dot{\tilde{F}}$ from an imperfect cancellation of friction effects. The process for obtaining such limit is completely developed in Perondi *et al.* (2010), for the closely related case of an imperfect cancellation of the term $\hat{h}$ that models the nonlinear dynamics of the pressures in the chambers of the actuating cylinder.

Remark #2: It is important to observe that the control law $\hat{u}$ defined in Eq. (22) does *not* represent the input signal u that is actually applied to the servovalve. As seen in Eq. (7), these two variables are related one to each other by means of a nonlinear equation that depends on the values of the air mass flow rates across the orifices of the valve. Thus, once a desired value $\hat{u}$ is calculated, it is necessary to obtain the corresponding input voltage u to be applied to the servovalve by means of a proper inversion process. In order to execute such operation, it is necessary the knowledge of the auxiliary $3rd$ order polynomial functions, as mentioned in Section 2. This process is summarized in Fig. 2, and the complete set of instructions in order to perform it can be found in Perondi (2002).

5. SIMULATION RESULTS

In the simulations, the system was required to track a sinusoidal trajectory with frequency 2 [rad/s] and amplitude 450 [mm], originated at the center of the pneumatic cylinder. The values of all necessary parameters of the controlled system were taken from a real pneumatic workbench, as described in Perondi (2002): $A = 4.19 \cdot 10^{-4}$ $[m^2]$, $r = 1.4$, $R = 286.9$ $[Jkg/K]$, $T = 293.15$ $[K]$, $L = 1$ $[m]$, $V_{10} = 1.96 \cdot 10^{-6}$ $[m^3]$, $V_{20} = 4.91 \cdot 10^{-6}$ $[m^3]$ and $M = 3.66$ $[kg]$. Simulations were carried out by means of a Matlab/Simulink package, using the *Runge-Kutta* integration method with a time step of $1 \cdot 10^{-4}$ [s]. In order to keep the simulation conditions as close as possible to those of an experimental evaluation of the proposed controller, the simulated model includes a set of low-pass filters that are similar to those to be encountered in the instrumentation apparatus of the real system. These filters are all of the Butterworth type, $2nd$ order, with the following cutoff frequencies: 350 [rad/s] for the position signal, 60 [rad/s] for the velocity signal and 40 [rad/s] for the acceleration signal.

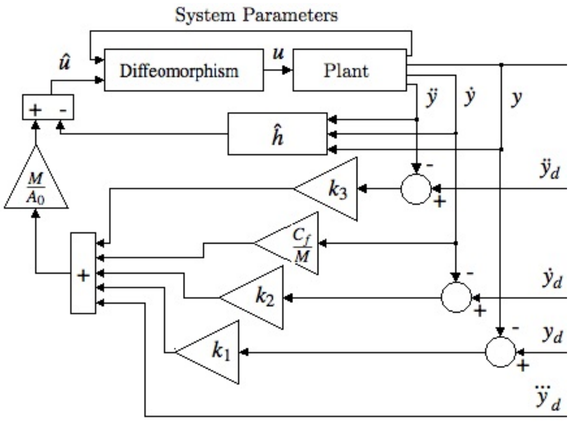

Figure 2. Proposed control scheme

The performance of the pneumatic positioning system was evaluated for three different control algorithms, defined as follows:

(i) the proposed feedback linearization controller, denominated FL-DF (Feedback Linearization with Dry-Friction compensation);

(ii) a simpler version of the proposed algorithm, in which only the nonlinear dynamics of the pressures in the chambers are compensated, named as FL-PD (Feedback Linearization with Pressure Dynamics compensation);

(iii) a classical linear state-feedback control law, based on the signals of Position, Velocity and Acceleration, commonly known as PVA controller.

The first control algorithm is described by means of Eq. (22) with all the auxiliary definitions given in Section 4. The second controller is obtained by setting $\dot{\hat{F}} \equiv 0$ in the linearizing part β of the proposed control law (see Eq. (24)). A detailed discussion about this algorithm can be found in Perondi *et al.* (2010). The third control law is a standard linear full-state-feedback controller, defined as:

$$u = -k_p\tilde{y} - k_v\dot{\tilde{y}} - k_a\ddot{\tilde{y}} \tag{32}$$

For the nonlinear controllers, the values of the gains employed in the linear parcel α (Eq. (23)) were determined by means of pole-placement considerations, in which the desired dynamic behavior was designed to resemble that of a $2nd$-order system with settling time of 0,6 [s] and maximum allowed overshoot of 2%. Since the error dynamics of the system is of $3rd$ order (see Eq. (29)), in order to approximate such behavior, the third pole was placed 8 times further from the origin than the real part of the dominant complex-conjugate poles that correspond to the desired dynamics. Also, as the error dynamics is expressed in the canonical form, the values of the gains are equal to the corresponding coefficients of the characteristic equation that satisfies the desired dynamic behavior. Thus, the gains of the linear part of the FL-PD and FL-DF controllers are: $k_1 = 3,9 \cdot 10^3$, $k_2 = 784,22$ and $k_3 = 66,67$. The gains of the PVA controller were defined so as to make the system present the same dynamic behavior that was specified for the case of the nonlinear algorithms. In this case, however, the values of the gains must be obtained by employing a linearized model of the system. Such approximate model was defined by linearizing the state equations presented in Section 2 in the central position of the pneumatic cylinder. The resulting linear model can be conveniently represented by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s(s + a_2 s + a_1)} \tag{33}$$

where s is the Laplace variable, $b_0 = 617.69$, $a_1 = 325.54$ and $a_2 = 19.13$. The values of the corresponding gains of the PVA controller are $k_p = 6,31$, $k_v = 1,76$ and $k_a = 0,14$.

The results obtained in the simulations can be observed in figures 3 and 4. In both cases, part (a) is devoted to presenting the entire trajectories, whereas part (b) illustrates the corresponding tracking errors. It can be observed that, when compared to the classical PVA controller that was developed to fulfill the same performance requirements, the proposed controller yields significant reduction of the tracking errors both with and without the dedicated friction-cancelling term.

Even though it allows a more significant reduction of the tracking errors, the presence of the friction-cancelling term in the proposed control law causes system response to become less smooth. This effect is noticed especially at the extremities of the cylinder, when the velocity response presents sharp variations. As it can be verified in Fig. 5, this

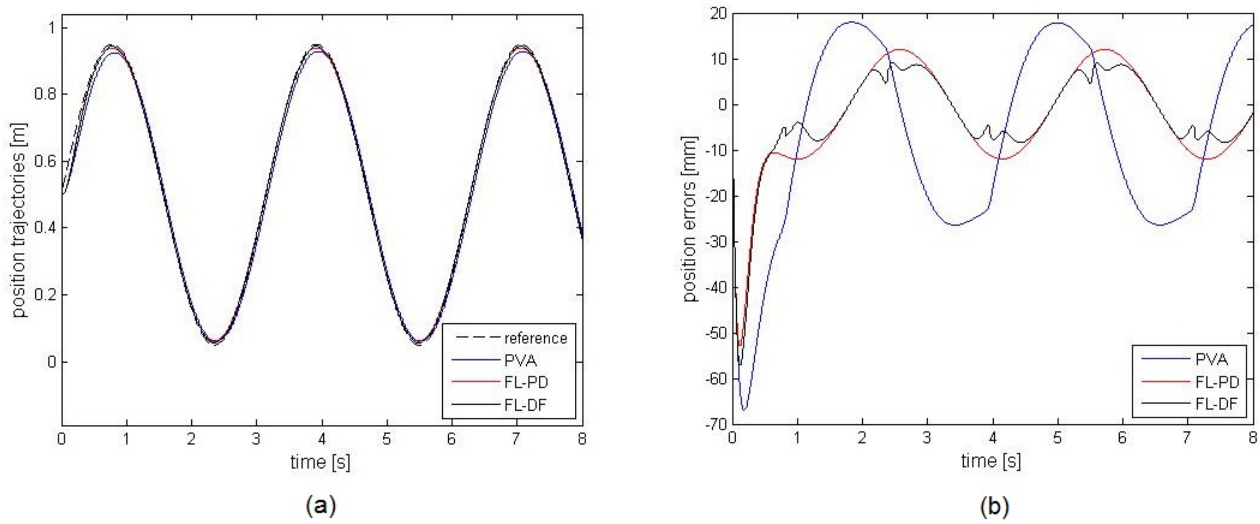

Figure 3. Simulation results - position tracking

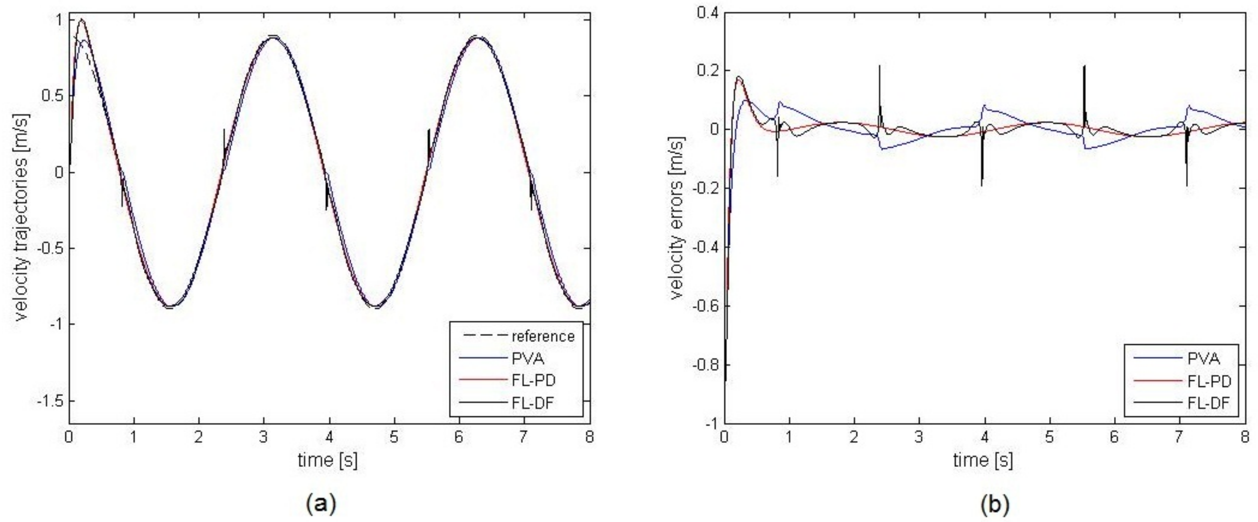

Figure 4. Simulation results - velocity tracking

effect occurs because the friction force calculated by means of the proposed observer suffers from the delays caused by the filters in the signal chain of the monitored variables. If the filters are removed, the errors in the compensation of friction effects become very small, and the resulting trajectory-tracking errors are significantly reduced. This fact is confirmed by the results presented in Tab. 1: when no filters are employed, the FL-DF controller yields a reduction of about 44% in the RMS value of postion tracking errors; when the filters are present, the value of such reduction falls to 29%. In the case of the PVA controller, the resulting errors are almost insensitive to the presence of the filters.This indicates that the proposed controller tends to be more difficult to implement in an experimental system, especially if its friction-compensating capabilities are to be employed. Thus, in order to be fully explored in terms of its potential benefits, the practical utilization of the proposed controller may require the development of alternative noise-cancelling techniques, such as state observers or more elaborate filtering algorithms.

Table 1. RMS values - position tracking errors

Filters	RMS error per controller [mm]		
	PVA	FL-PD	FL-DF
on	17,82	8,01	5,69
off	17,70	6,11	3,38

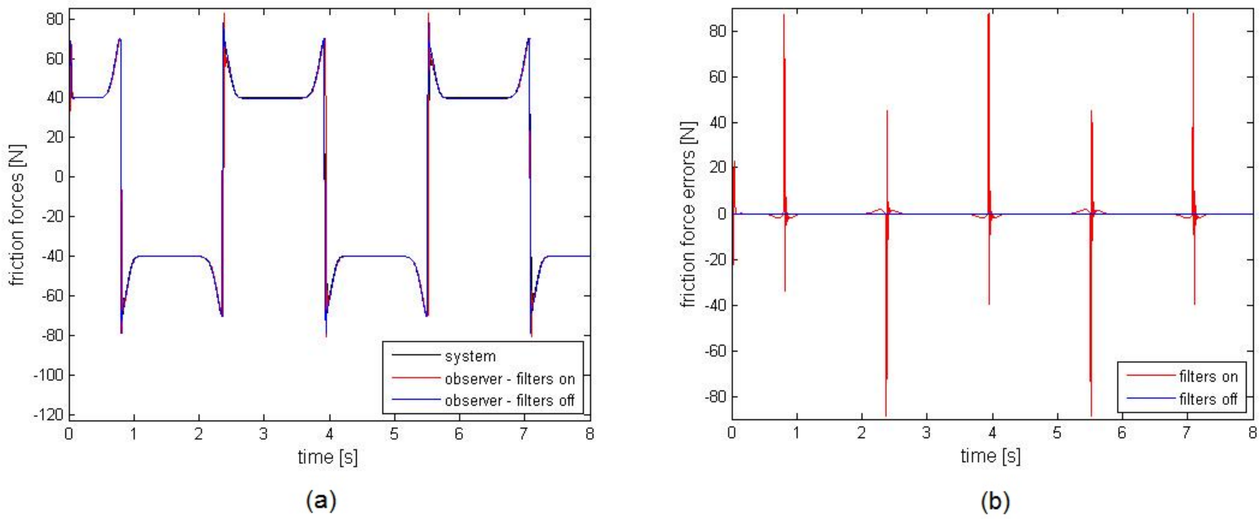

Figure 5. Friction compensation - effect of filters

6. CONCLUSIONS

In this work, a novel control algorithm to be applied to a pneumatic positioning system was proposed, aiming at compensating the undesired nonlinear effects due to the dynamics both of pressurized air and dry-friction forces. The most important features of the proposed controller were discussed and illustrated by means of simulation results. It was observed that the proposed algorithm allows significant reductions in the amplitude of the trajectory-tracking errors of the controlled system when compared to a classical control approach. However, the proposed control law is also more sensitive to the effect of delays due to the filtering of measured signals, a feature that might limit its utility in the case of practical applications.

Future work will focus on the development of a complete stability analysis of the controlled system and on the experimental evaluation of the proposed algorithm.

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