SHORT CRACK ARREST IN FRETTING FATIGUE: COMPARATIVE ANALYSIS BETWEEN MULTIAXIAL STRESS AND STRESS INTENSITY FACTOR BASED MODELS

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Abstract. *This paper presents two models to estimate short crack arrest in fretting fatigue. The stress-based model involves the computation of a multiaxial parameter at a critical distance from the contact. The critical distance is identified by means of standard fatigue tests, namely two plain fatigue tests and the threshold test for long crack propagation. The fracture mechanics model compares the mode I stress intensity factor range at a critical distance from the contact with the threshold stress intensity for crack propagation. In this case, the critical distance is defined by the crack transition length between the short and long crack regimes, which is obtained from the Kitagawa-Takahashi diagram. In order to carry out a comparative analysis between the models, fretting fatigue tests were conducted. A cylinder on plane contact configuration, made of 7050-T7451 aluminum alloy, was considered. Twelve tests were performed with the same experimental parameters except the mean bulk load, which varied from a tensile to a compressive value until run-out was achieved. For the amount of experimental data produced in this work, the stress-based model provided slightly better estimates of fatigue resistance than the stress intensity factor-based model. The potentialities and limitations of both models in practical applications are discussed.*

Keywords: *fretting fatigue, multiaxial fatigue, short crack arrest, 7050 aluminum alloy*

1. INTRODUCTION

Fretting fatigue is a failure mechanism which occurs in mechanical couplings subjected to a combination of (i) relative tangential displacement between the contacting surfaces and (ii) cyclic loading in one of the components. Failures by fretting fatigue are observed in engineering assemblies under vibration, such as riveted or bolted connections, dovetail joints in turbine engines, and overhead conductors, among others.

The fretting fatigue problem involves wear and a strong stress concentration which speed up the crack initiation stage. In addition, the rapid stress decay inwards the contact has been associated with a size effect observed in many fretting fatigue studies (Nowell, 1988; Amargier et al., 2010). As in plain fatigue, safe design against fretting fatigue can focus on the resistance to crack initiation. In practice, however, the aggressive contact conditions caused by the fretting process invariably leads to the formation of embryo cracks. Therefore, an alternative strategy is to guarantee that, even when embryo cracks are formed, they become non-propagating due to the rapid stress decay in the fretting region. Although such an approach can be based either on the arrest of short or long cracks, the former choice is more conservative.

This paper presents a new version of the crack arrest model proposed by Araújo and Nowell (1999). Such a model has relied on the stress intensity factor to quantify short crack arrest. However, as pointed out by some researchers (Smith, 1977; Miller, 1982, 1993; Suresh, 1998), the use of Linear Elastic Fracture Mechanics in the short crack regime is questionable. The new model is based only on the comparison of the mode I stress intensity factor range at a critical distance from the contact with the threshold stress intensity for crack propagation. The critical distance is chosen as the crack transition length between the short and long crack regimes obtained from the Kitagawa-Takahashi diagram. Hence, the stress intensity factor is computed only at the minimum crack length which guarantees the validity of Linear Elastic Fracture Mechanics.

As an alternative to fracture mechanics models, a stress-based model is also investigated in this work. In order to incorporate the stress gradients and multiaxial stresses beneath the contact, an average stress and a multiaxial critical plane criterion are considered, respectively. The average stress is represented by the stress at a critical distance from the contact. If the stress history at the critical distance is such that the multiaxial criterion is satisfied, it means that either there will be no crack initiation or short crack arrest. It should be noted that Araújo et al. (2007) and Castro et al. (2009) have already proposed a similar model in the context of fretting and notch fatigue, respectively. The modification in the present model concerns the definition of the equivalent shear stress amplitude in a material plane, which is now based on the Maximum Prismatic Hull concept (Araújo et al., 2011).

The proposed models can, in principle, be considered to estimate short crack arrest in mechanical problems involving the presence of stress concentration/stress gradient. It remains to respond whether they provide similar estimations and, if not, which one provides the most accurate estimates. To carry out the comparative analysis, fretting fatigue tests were performed on an Al 7050-T7451 aeronautical alloy.

2. EXPERIMENTAL PROGRAM

The material used in this work was a 7050-T7451 aluminum alloy provided by EMBRAER-LIEBHERR (ELEB). The mechanical properties of this metal are given in Table 1.

Fretting fatigue tests were carried out with a pair of cylindrical pads pressed against a dog-bone specimen. Figures 1a and 1b show the fretting fatigue test setup and a scheme of the applied loads, respectively. The normal load P is applied by hydraulic cylinders and is monitored by two load washers mounted on drawbars. The fretting device, mounted on a MTS 810 servo-hydraulic machine, works as a “spring” that reacts to the motion of the pads while the specimen is subjected to a cyclic bulk load $B(t)$. The reaction of the "spring" results in the cyclic tangential load $Q(t)$ whose value is half of the difference between the forces measured at the lower and upper load cells of the servo-hydraulic machine. Full details about the fretting fatigue test setup are described by Martins et al. (2008).

The loading history (Fig. 1c) was programmed as follows: First, a mean bulk load B_m was applied to the specimen. Next, the pads were clamped producing a constant normal load P. A sinusoidal bulk loading of amplitude B_a was then applied to the specimen which, due to the stiffness of the fretting device, also experienced a sinusoidal tangential loading of amplitude Q_a. All tests were conducted in the partial slip regime ($Q_a \leq fP$ where f is the friction coefficient) under the same experimental conditions, except the mean bulk stress which varied from a compressive to a tractive value. Fatigue failure was defined as the complete fracture of the specimen, and run-out was set at 10^7 cycles. Table 2 lists the parameters of the fretting fatigue tests and observed lives, where p_o is the peak contact pressure, a is the contact semi-width, ω is the bulk loading frequency, σ_a is the bulk stress amplitude, and σ_m is the bulk mean stress. All tests were replicated at least three times, except the ones with $\sigma_m = 30$ MPa which were replicated two times. Although no statistical procedure has been carried out to evaluate the experimental tests, the consistency of the observed lives is clear and indicates that fatigue endurance occurs for a mean stress within the range $(0, -30)$ MPa.

Table 1. Mechanical properties of Al 7050-T7451.

Yield strength σ_Y, MPa	453.8 ± 2.8
Tensile strength, MPa	513.3 ± 4.1
Elastic modulus, GPa	73.4 ± 2.0
Elongation, %	11.1 ± 0.6
Microhardness, (HV)	153.6 ± 2.6
Poisson’s ratio	0.3
Grain size, μm	4-7
Plain fatigue strength σ_{-1}, MPa ($R = -1, 10^7$ cycles)	161.3
Plain fatigue strength σ_0, MPa ($R = 0, 10^7$ cycles)	120.0
Threshold stress intensity range, MPa$\sqrt{\text{m}}$ ($R = 0.1$)	2.5

Table 2. Parameters of fretting fatigue tests and observed lives.

Test #	1	2	3	4	5	6	7	8	9	10	11	12
σ_m (MPa)	50	50	50	30	30	30	0	0	0	0	-30	-30
Life (10^6 cycles)	2.66	1.50	0.94	2.24	1.24	1.18	3.77	10	4.62	10	10	10

For all tests: $p_o = 175$ MPa, $a = 0.61$ mm, $Q_a/fP = 0.62$, $f = 0.6$, $\omega = 15$ Hz, $\sigma_a = 35$ MPa.

3. MODELS FOR SHORT CRACK ARREST IN FRETTING FATIGUE

3.1 Model based on the stress intensity factor

A number of works (Araújo and Nowell, 1999; Nowell et al., 2006; Fouvry et al., 2008) have shown that short crack arrest models can be successfully applied to describe fretting fatigue. The starting point of such models is the diagram introduced by Kitagawa and Takahashi (1976), which represents the threshold condition for crack propagation as a function of crack length. Accordingly, for long cracks the threshold condition for propagation is observed to be independent of the crack length, that is

$$\Delta K = \Delta K_{\text{th}} \quad (1)$$

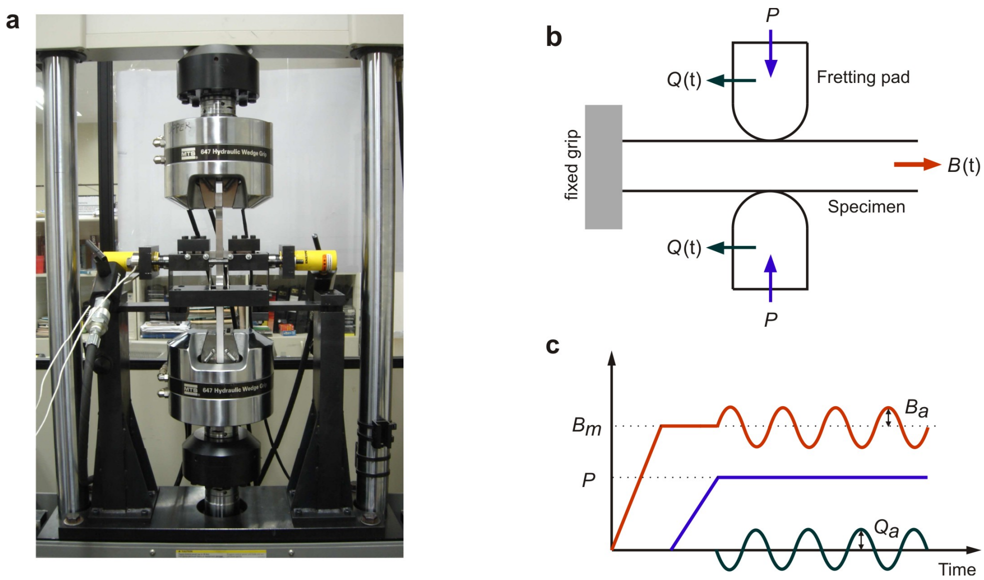

Figure 1. (a) Fretting fatigue test setup, (b) scheme of contact model and (c) loading history.

where ΔK and ΔK_{th} are the applied and threshold stress intensity ranges under mode I loading, respectively. On the other hand, the threshold condition for short crack propagation is given by

$$\Delta\sigma = \Delta\sigma_{-1} \tag{2}$$

where $\Delta\sigma$ is the applied stress range and $\Delta\sigma_{-1}$ is the plain specimen fatigue limit. Alternatively, expression (2) can be written in terms of the stress intensity range as

$$\Delta K = Y\Delta\sigma_{-1}\sqrt{\pi b} \tag{3}$$

where Y is a geometrical correction factor and b is the crack length. The crack transition length b_0, which divides the short and long crack regimes, can be obtained by equating expressions (1) and (3):

$$b_0 = \frac{1}{\pi}\left(\frac{\Delta K_{\text{th}}}{Y\Delta\sigma_{-1}}\right)^2 \tag{4}$$

The Kitagawa-Takahashi diagram is a graphical representation of the threshold conditions (1) and (3), as illustrated in Fig. 2. To estimate crack arrest in fretting fatigue, the stress intensity range of a crack growing inwards the contact is compared with the threshold condition for crack propagation. In fretting fatigue tests of some metal alloys, slant cracks have been observed usually at the trailing edge of the contact or within the slip zone. However, as a first approximation, we will consider a model where a straight crack of length b is located at the trailing edge of the contact ($x = -a$) and is driven by the mode I stress intensity range (Fig. 2a). Some cracking behaviors and their corresponding stress intensity variations are illustrated in Figure 2b: in curve A a short crack becomes non-propagating, curve B represents the threshold condition for crack propagation, whereas in curve C the crack achieves the long crack regime.

An alternative, simple and mechanically consistent model to estimate short crack arrest in fretting fatigue, hereafter called ΔK-based point model, is now proposed:

$$\Delta K \leq \Delta K_{\text{th}} \quad \text{at} \quad b = b_0 \qquad \Leftrightarrow \qquad \text{short crack arrest} \tag{5}$$

The interpretation of the model is as follows: short crack arrest occurs if the inequality is satisfied, and the threshold condition for short crack arrest is attained when the equality holds true. A short crack will evolve to a long crack whenever the inequality is not satisfied. It is claimed here that the ΔK-based point model improves similar models presented in the literature (Araújo and Nowell, 1999), since the questionable use of Linear Elastic Fracture Mechanics in the short crack regime is circumvented.

The ΔK-based point model is motivated by the examination of the cracking behaviors A, B, and C shown in Fig. 2b. Clearly, in such cases the proposed model is valid. On the other hand, it could cease to function in the presence of abrupt changes of ΔK in the short crack regime, as also shown in Fig. 2b: in curve D, a threshold condition is estimated instead of short crack arrest, whereas, in curve E, long crack growth is estimated instead of short crack arrest. However, we shall assume that abrupt ΔK variations, with a strong decay immediately followed by rapid recovery within such small crack length, are spurious situations. Indeed, the normal stress component along the line of the crack — produced by the normal and tangential contact loads and the bulk load — is not observed to vary in such a irregular manner.

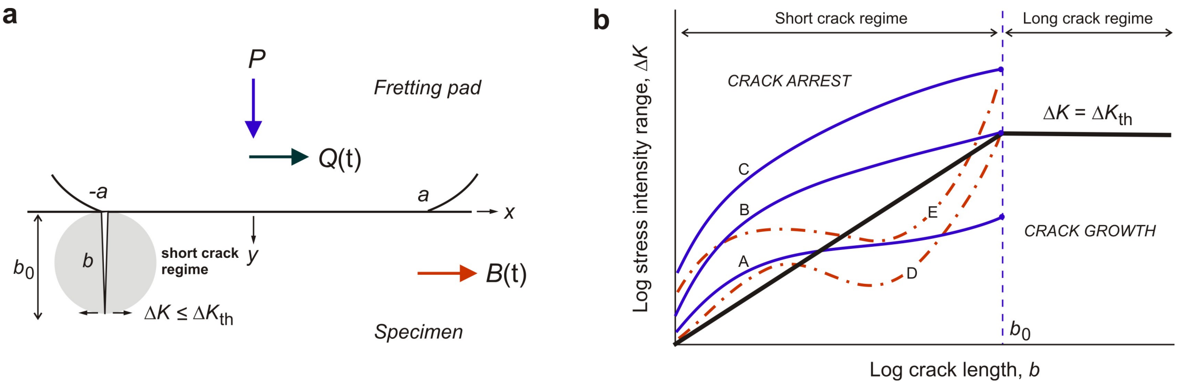

Figure 2. (a) Scheme of the ΔK-based point model. (b) Cracking behaviors in a Kitagawa-Takahashi diagram: short crack arrest (A), threshold condition (B), long crack initiation (C), and spurious situations (D and E).

3.2 Model based on non-local multiaxial parameter

Fatigue models (Neuber, 1958; Peterson, 1959; Taylor, 2007; Susmel, 2009) based on non-local stress-based parameters have been used to estimate the fatigue strength of notched components . In such models, the effective stress which produces fatigue damage is an average value of the stresses around a stress concentrator. Hence, the stress gradient effect on the fatigue resistance can be taken into account in a simple manner, but useful for engineering purposes. Although the effective stress can be defined on volume or line elements, the most common procedure is to consider the stress at a critical distance from the stress concentrator (Point Method). For simplicity, we will only deal with the Point Method in this paper.

The main issue in non-local models concerns the identification of the critical distance. Taylor (1999) devised a new strategy to identify the critical distance by assuming that non-local models can be applied to any kind of stress concentrator (including sharp notches and cracks). In this setting, and when a multiaxial model based on the maximum principal stress amplitude is considered, it can be shown that the critical distance which provides an exact fit between estimated and observed threshold loading for long crack propagation is given by

$$L = \frac{1}{2\pi}\left(\frac{\Delta K_{\text{th}}}{\Delta\sigma_{-1}}\right)^2 \tag{6}$$

where ΔK_{th} is the mode I threshold stress intensity range and $\Delta\sigma_{-1}$ is the plain fatigue limit range, both under fully reversed loading. In order to extend Taylor's method to notches under complex multiaxial loading, Susmel (2004) applied the Modified Wöhler Curve Method (Susmel and Lazzarin, 2002) in terms of the Point Method. In this case, it can be shown that the same critical distance given by Eq. (6) is obtained. In addition, Susmel (2004) introduced the structural volume concept as the process zone where micro/meso-cracks are arrested in fatigue threshold conditions, and estimated that its size is correlated to L.

Estimation of fretting fatigue thresholds with notch methodologies has been investigated in the literature (Araújo et al., 2007; Rossino et al., 2009). This is motivated by the similarities between fretting and notch fatigue, since both have stress distributions characterized by multiaxial stresses and stress gradients. However, fretting fatigue encourages premature crack initiation and formation of material debris, due to the surface damage caused by the relative displacement between the contacting surfaces. Based on the previous remarks, notch methodologies could, in principle, be applied to fretting fatigue if the surface damage could be regarded as negligible. Some researchers (Nowell and Dini, 2003) have sustained such a viewpoint when the fretting damage occurs in the partial slip regime . Nevertheless, more conclusive studies should be conducted in order to place the notch analogy on a scientific basis.

Hereafter, the notch analogy hypothesis is assumed to hold true. In this context, a fretting fatigue model has only to take account of the stress gradients and multiaxial stresses beneath the contact, by means of (i) an average stress and (ii) a multiaxial parameter, respectively. In present paper, a short crack arrest model based on the Point Method and a critical plane parameter is proposed as follows:

$$F(\tau_{ac}, \sigma_{n\,max}) := \tau_{ac} + \kappa\frac{\sigma_{n\,max}}{\tau_{ac}} - \lambda \leq 0 \quad \text{at} \quad (x, y) = (-a, L) \tag{7}$$

The critical plane is defined as the material plane where the *equivalent shear stress amplitude* τ_a attains its maximum value τ_{ac}. Formally, $\tau_{ac} := \max_{\theta,\phi} \tau_a$, where θ and ϕ are the spherical angles associated with the unit normal vector to an arbitrary material plane. In order to take into account the *mean stress effect*, a non-linear relationship between τ_{ac} and the *maximum normal stress* $\sigma_{n\,max}$ on the critical plane is considered as proposed by Susmel and Lazzarin (2002). The material parameters are represented by κ, λ and L. The model is illustrated in Fig. 3a and can interpreted as follows: if the stress history at the critical distance L is such that the multiaxial criterion (7) is satisfied, it means that either there will be no crack initiation or short crack arrest within a process zone of radius L. When the multiaxial criterion is exactly satisfied, the threshold condition for short crack arrest occurs, whereas long crack growth is estimated if it is not satisfied.

The crucial aspect in expression (7) is how to properly define the equivalent shear stress amplitude. Susmel (2009) presents a review of the most common equivalent shear stress amplitudes in fatigue problems. In the present paper, the equivalent shear stress amplitude is associated with the *Maximum Rectangular Hull* of the shear stress vector path Ψ in a material plane, as proposed by Araújo et al. (2011). In what follows, the main steps of the method are described.

As shown in Fig. 3b, for each *φ-oriented rectangular hull*, described with respect to a basis $\{\boldsymbol{\tau}_1(\varphi),\ \boldsymbol{\tau}_2(\varphi)\}$, its amplitude can be defined by

$$\tau_a(\varphi) := \sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \tag{8}$$

where

$$a_1(\varphi) = \frac{1}{2}\left[\max_t \tau_1(\varphi,t) - \min_t \tau_1(\varphi,t)\right] \qquad a_2(\varphi) = \frac{1}{2}\left[\max_t \tau_2(\varphi,t) - \min_t \tau_2(\varphi,t)\right] \tag{9}$$

are halves of the sides of the φ-oriented rectangular hull. Then, the equivalent shear stress amplitude is given by the maximum value of expression (8):

$$\tau_a := \max_{\varphi\in[0,\frac{\pi}{2})} \tau_a(\varphi) \tag{10}$$

As highlighted by Araújo et al. (2011), the Maximum Prismatic Hull method is very simple to implement, and more accurate than traditional approaches when compared with combined axial (or bending) and torsion experimental data.

The procedure to identify the model parameters is now described. The parameters κ and λ were obtained from an exact fit between the model and the fatigue strengths σ_{-1} and σ_0 of plain specimens subjected to load ratios $R = -1$ and $R = 0$. The results are:

$$\kappa = 0.5(\sigma_{-1} - \sigma_0) \qquad \lambda = \sigma_{-1} - 0.5\sigma_0 \tag{11}$$

The parameter L was identified so as to obtain an exact fit between estimated and observed threshold stress intensity ranges. Such a strategy is fully described by Castro et al. (2009) and yields

$$L = \frac{1}{32\pi}\left(\lambda - \frac{2\kappa}{1-R}\right)^{-2} \Delta K_{\mathrm{th},R}^2 \tag{12}$$

where $\Delta K_{\mathrm{th},R}$ is the threshold stress intensity range at a load ratio R.

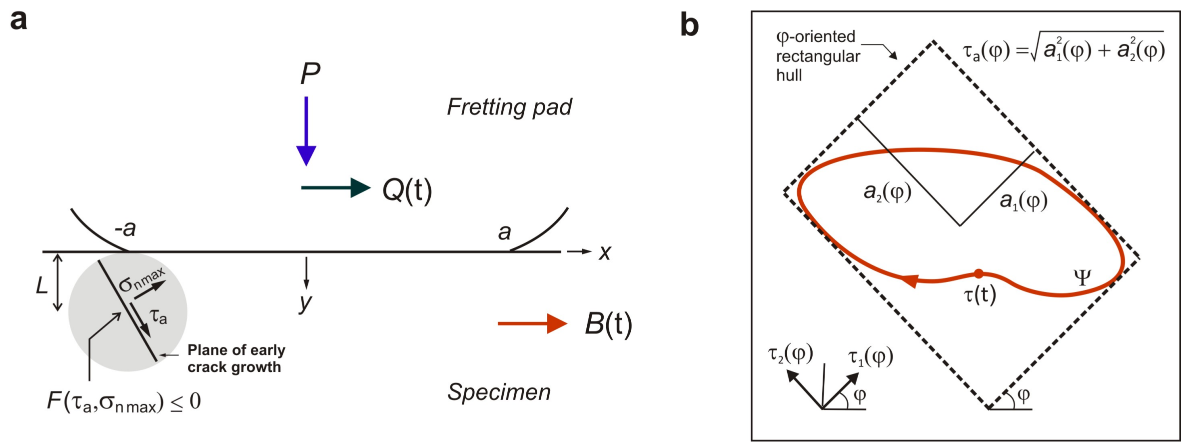

Figure 3. (a) Scheme of short crack arrest model based on multiaxial parameter. (b) Computation of the equivalent shear stress amplitude in terms of the Maximum Rectangular Hull.

4. MODELS ASSESSMENT

As a first step towards the assessment of the models, the elastic contact problem associated with the fretting fatigue test setup has to be solved. In this respect, the cylindrical pad/dog-bone specimen configuration is particularly attractive, because the stresses can be calculated analytically. A brief summary of the contact model is given next, while a full description of the analytical formulas are presented by Hills and Nowell (1994) and references therein. The contacting bodies are idealized by a plane contact model of a cylinder against a half-plane, both made of the same material and under plane strain conditions. The contact pressure (normal traction) is given by the elliptical Hertz distribution. The shear tractions are similar to the Mindlin–Cattaneo distribution, with the stick zone shifted due to the alternating bulk stress in the specimen. Once the surface tractions are found, the subsurface stresses can be obtained by means of a Muskhelishvili's potential. It should be remarked that, in the present loading history, the specimen is first strained before being clamped. As a result, a constant mean stress produced by the initial straining must be superposed to the stress field.

To implement the ΔK-based short crack arrest model, a straight crack of length b was assumed to have originated at the trailing edge of the contact surface ($x = -a$). Following Hills and Nowell (1994), the mode I stress intensity factor was computed with the Distributed Dislocation Method . The stress intensity range was defined as $\Delta K := K_{\text{max}} - K_{\text{min}}$, where K_{max} and K_{min} are the maximum and minimum stress intensities experienced by the crack tip during one loading cycle, respectively. It was assumed that negative stress intensities do not contribute to crack propagation; hence, the stress intensity range was given by $\Delta K = K_{\text{max}}$ whenever $K_{\text{min}} < 0$.

The crack transition length was estimated as $b_0 = 62\,\mu$m based on the assumptions that $Y = 1$ and $\Delta K_{\text{th}} = 4.5\,\text{MPa}\sqrt{\text{m}}$, at a load ratio $R = -1$. It should be noted that expression (4) requires the threshold stress intensity at $R = -1$, but the authors only measured such parameter at $R = 0.1$. In order to extrapolate the observed parameter to $R = -1$, the empirical relation between ΔK and R for aluminium alloys provided by Kujawski (2001) was considered.

Figure 4a shows the ΔK variation as a function of crack length for each experimental condition. A good agreement with the experimental results is observed. Indeed, for mean stresses equal to $\sigma_m = -30$ MPa and $\sigma_m = 50$ MPa, infinite life due to short crack arrest and fracture of the specimen are correctly estimated, respectively. The estimated threshold condition is close to $\sigma_m = 30$ MPa, whereas the tests suggest that it occurs between $\sigma_m = -30$ MPa and $\sigma_m = 0$. More important, the ΔK-based point model proposed in this paper is clearly confirmed: every time that $\Delta K < \Delta K_{\text{th}}$ at $b = b_0$, short crack arrest occurred.

The results for the multiaxial stress-based model are shown in Fig. 4b. The parameters given by expressions (11) and (12) to calibrate the threshold line are: $\kappa = 20.8$ MPa, $\lambda = 101.5$ MPa, and $L = 20\,\mu$m (based on the threshold stress intensity factor range for $R = 0.1$). Note that the experimental conditions were such that the critical plane always experienced the same equivalent shear stress amplitude, but different maximum normal stresses. A sound agreement between the estimates and the experimental data is observed. Long crack growth in the specimens subjected to positive mean stresses was correctly estimated, whereas a safe condition characterized by micro/short crack arrest was estimated for the run-out tests with $\sigma_m = -30$ MPa. The data dispersion for $\sigma_m = 0$ (two run-outs and two fractures) may be caused by the proximity of the experimental conditions to the fretting fatigue threshold.

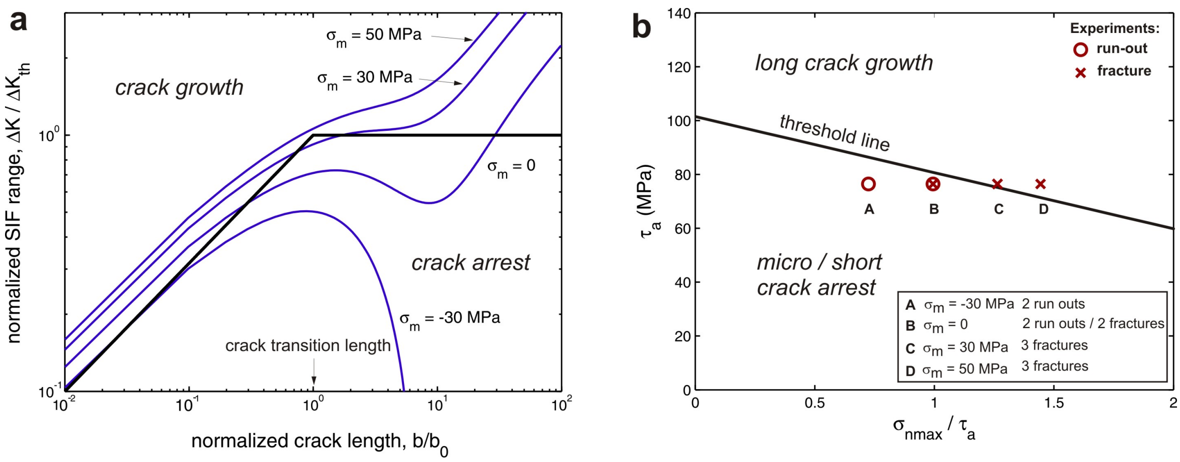

Figure 4. Comparison between experimental data and estimations of (a) ΔK-based and (b) stress-based models.

Figure 5 shows a fretting fatigue threshold diagram, where both models are compared with the experimental data. The procedure to draw the diagram was as follows: The normalized stress amplitude was fixed at $\sigma_a/fp_o = 0.33$. Then, for ten Q_a/fP values, the maximum bulk stress $\sigma_{B\,\text{max}}$ was varied so that equality of expressions (5) and (7) was achieved. Finally, linear interpolation of the points provided the curves. Although such a diagram only represents the experimental conditions described in Section 2, it allow us to appreciate that the safe loading domains estimated by the two methods are clearly distinct. In addition, it can be seen that the estimates of the multiaxial model were slightly more accurate: two fractured specimens fell within the safe domain of the multiaxial model, whereas five fell in the safe domain estimated by the ΔK-based point model.

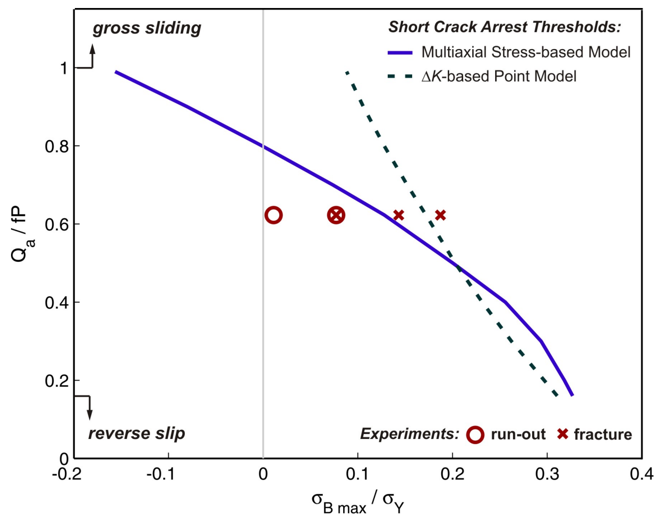

Figure 5. Short crack arrest thresholds estimated by the stress and ΔK-based models.

5. DISCUSSION AND CONCLUSIONS

Two models to estimate short crack arrest in fretting fatigue have been proposed. The models are based on the mode I stress intensity factor range and on a multiaxial stress based parameter, both computed at a critical distance. In order to assess the models, new fretting fatigue tests were carried out with a cylinder on plane contact configuration made of an Al 7050-T7451 alloy. Experiments were conducted so that all test parameters remained constant but the mean bulk load. The threshold level of mean bulk load dividing run-out and complete fracture of the specimens was observed to be within 0 and -30 MPa.

The proposed ΔK-based point model estimates short crack arrest by considering only the evaluation of ΔK at the crack transition length between the short and long crack regimes. Therefore, it does not require any type of correction of Linear Elastic Fracture Mechanics parameters as a mean to reproduce the so called anomalous short crack behavior. For design purposes, the ΔK-based point model could be regarded as a conservative approach. Indeed, at the estimated threshold condition, the real non-propagating crack length could be less than the crack transition length due to, for instance, microstructural barriers such as grain boundaries.

The models presented in this paper are mainly suited for estimating safe loadings (or design allowables) associated with the existence of non-propagating short cracks. However, no information is given about the number of cycles which is necessary for a crack to grow until a certain length or to become non-propagating. In designs where such information is important, explicit modeling of the short crack regime has to be considered. For instance, short crack growth laws based on a multiparameter characterization of the crack tip stress field (e.g. incorporating the T-stress) could be considered for this purpose; see the work by Hamam et al. (2005) and references therein.

Looking at the fretting fatigue threshold diagram shown in Fig. 5, it becomes clear that both models provide different safe domains for the same contact configuration. For the limited amount of experimental data here produced, the multiaxial stress-based model provided slightly better estimates of fatigue resistance than the ΔK-based point model. More experimental data considering different contact configurations and other alloys are necessary to further challenge the accuracy of these models.

For two-dimensional short crack arrest analysis, the computation of both models is relatively straightforward. However, for real three-dimensional geometries, the calculation of the stress intensity factor is not well-established and usually demands special techniques. On the other hand, the results of non-local stress-based models can, in principle, be visualized in a post-processing procedure of an elastic finite element analysis. Hence, it seems that stress-based models can potentially provide more rapid and simple design solutions than a fracture mechanics model. As concerns the validity of both models to estimate short crack arrest in three-dimensional configurations, further experiments comprising complex geometries and loading histories should be carried out.

6. ACKNOWLEDGEMENTS

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