GNSS AMBIGUITY RESOLUTION WITH LEAST SQUARES METHODS

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Abstract. The attitude determination based on GNSS is just an extension of the method for differential positioning with the carrier phase, since it enables the obtention of the relative position, with sub-centimeter-level accuracy, between a pair of antennas. However, multiple receivers are needed and a special configuration of the antennas, which is known as multi-antenna GNSS system. Two baselines composed by three antennas fully defines the Euler angles associated with the vehicle attitude, but the baseline determination requires the solution of the integer ambiguity problem, from a basic relation envolving also the carrier phase measurements. Least Squares (LS) methods are the most used techniques to solve the ambiguity problem. Two LS algorithms for solve the integer ambiguity problem are implemented and tested with realistic simulated data.
Keywords*: GNSS, Ambiguity Resolution, Least Squares, On-The-Fly*

1. INTRODUCTION

Carrier phase measurements are inevitably required in order to estimate high accuracy attitude parameters using GNSS. Since GNSS receivers provide very accurate measurements of fractional carrier wave cycles plus the total number of integer cycle counts from the start of tracking, carrier phase measurements are ambiguous by an unknown number of integer cycles, the so-called phase ambiguity, before they give meaningful range information for positioning.

The carrier phase ambiguity resolution is a key problem which has to be solved in GNSS static and kinematic precise positioning. However, for the real time application the solution on-the-fly of the ambiguity is necessary, but this is not an easy task. The concept of ambiguity resolutions on-the-fly was first introduced by Seeber and Wübbena (1989) and Hatch (1989, 1990). Over the past few years, many different on-the-fly ambiguity resolution techniques have been developed. Among them are the ambiguity function technique (Remondi, 1984; Mader, 1990), the narrow-lane and extrawide-lane technique (Wübbena, 1989), the Hatch's least squares ambiguity search method (Hatch, 1989, 1990), the fast ambiguity search filter technique (Chen and Lachapelle, 1994; Landau and Vollath, 1994), Least Squares by Cholesky Decomposition (Landau and Euler, 1992; Lu, 1995), Least Squares Ambiguity Decorrelation Adjustment (LAMBDA) (Teunissen, 1994) and Mixed Integer Least Squares (MILES) (Chang and Zhou, 2006).

The ambiguity function technique requires extensive computation time. For this reason, the ambiguity function method is not suitable for use in GNSS multi-antenna systems which aim at real-time applications. The extrawide-lane technique is primarily designed for working only with dual frequency P-code receivers, which are rarely used for platform attitude determination tasks. The most appealing techniques capable of use in GNSS multi-antenna systems are therefore the Hatch's least squares ambiguity search method, the fast ambiguity search filter method, Least Squares by Cholesky Decomposition, LAMBDA method and MILES. All of them are based on the least squares adjustment and upon the assumption that within a properly defined ambiguity search space and under normal error distributions, the correct ambiguity set will always be included in the search space and give the smallest sum of squares of carrier phase residuals among all the potential ambiguity sets but, this assumption is not necessarily true, as will be shown in this paper.

Two methods were selected for implementation, namely, the Least Squares by Cholesky Decomposition and MILES. The Least Squares by Cholesky Decomposition has the advantage of using *prior* information of the construction of the platform to reduce the search space of phase ambiguity. The method for solving the ordinary integer least squares problem implemented by the MILES is a modification of the modified LAMBDA method present in Chang et al. (2005) and provides fast and numerically reliable routines to solve the ambiguity problem. These two methods were selected for performance comparison because of the intended application in real time GNSS/INS fusion without additional attitude aid, such as magnetometers, wich are difficult to calibrate and prone to external interference. Moreover, the attitude reading for composing the measurements of the navigator Kalman Filter has to come also from GNSS. In this case, accuracy and efficient computation are required in the real time ambiguity problem solution. The data used for testing the algorithms comes from Dai *et al* (2008) and are realistic simulated data. The paper is organized as follows in section 2 describes the methodology of implementation methods used, in section 3 the results are presented and section 4 presents the conclusions from the results obtained.

2. METHODOLOGY

The basic idea of attitude determination using GNSS carrier phase measurements is similar to the principle used in interferometry and differential positioning. The system consists of a reference or master receiver whose location is well known and one or more receivers embedded in the vehicle, known as slaves, from which the relative position is obtained. The Differential Global Positioning System (DGPS) is based on the principle that receivers in the same vicinity will simultaneously experience common errors on a particular satellite ranging signal. In general, the user employs measurements from the pseudorange and carrier phase to remove such errors. For this to be resolved, the user must employ the same data from satellites that the used reference receiver. DGPS positioning equations are formulated in order to cancel such errors. Therefore, it is assumed that for a short baseline (few meters) the unit vectors from both receivers to a given satellite are the same. This is based on the fact that the baseline length is negligibly small compared to the distance between GNSS satellites and the user, approximately 22,000 km. This is shown in Fig. 1.

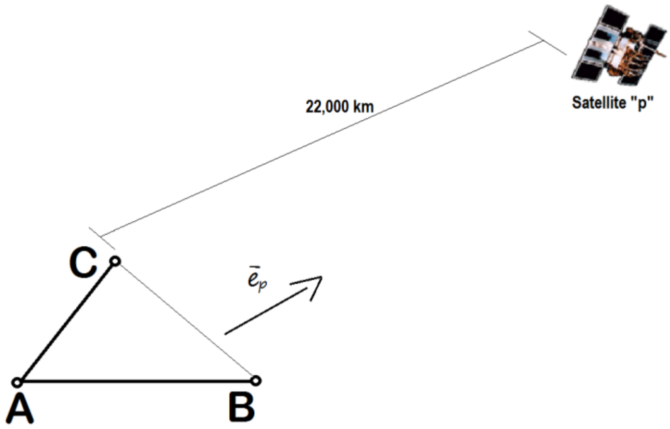

Figure 1 - Interferometry principle

The difference between the true ranges from satellite *"p"* (p-th satellite) to antennae *A* and *B*, or *A* and *C*, can be expressed as

$$|AB| = R_n^T \vec{e}_p = \rho_A^p - \rho_B^p \tag{1}$$

where R_n is the baseline vector determined by antenna *A* and *B*, $\vec{e}_p$ is a unit directional vector from antenna *A* and *B* to satellite "p", ρ_A^p and ρ_B^p are the distances between the antennae *A* and *B* and satellite *"p"* respectively. The double-differenced observation equation can be expressed as (Fan, 2005)

$$\begin{aligned} \lambda DD_{AB}^{pq} &= R_n^T \vec{e}_p - R_n^T \vec{e}_q + \lambda (N_0)_{AB}^{pq} + \lambda \Delta v_{AB}^{pq} \\ &= (\vec{e}_p - \vec{e}_q)^T R + \lambda (N_0)_{AB}^{pq} + \lambda \Delta v_{AB}^{pq} \end{aligned} \tag{2}$$

where DD_{AB}^{pq} is the phase double-differenced measurement between antenna *A* and *B* for the satellites *p* and *q*, λ is the wavelength of the GNSS signal, $(N_0)_{AB}^{pq}$ is the integer double-differenced carrier phase ambiguity, and Δv_{AB}^{pq} is the measurement noise. The use of double difference phase has significant advantages, such as removal of orbital and clock errors of satellites.

If there are *M* satellites in view, all the measurements can be written in the following matrix from,

$$DD_{AB} = HR_{AB} + \lambda N_{AB} + V \tag{3}$$

with

$$\begin{aligned} DD_{AB} &= \lambda \left[DD_{AB}^{21} \quad DD_{AB}^{31} \quad DD_{AB}^{41} \quad \ldots. \quad DD_{AB}^{M1} \right]^T \\ N_{AB} &= \left[(N_0)_{AB}^{21} \quad (N_0)_{AB}^{31} \quad (N_0)_{AB}^{41} \quad \ldots. \quad (N_0)_{AB}^{M1} \right]^T \end{aligned} \tag{4}$$

where R_n is a baseline vector in the local level system, *H* is the design matrix of line of sight from antennae to GNSS satellites in the local level system, *V* is the carrier phase difference-doubled measurement noise vector. In equation (3), if the ambiguities have been fixed to integers, there will be only 3 unknowns (three components of the baseline vector

$R_n(x_n, y_n, z_n)$). Therefore, if there are at least 4 satellites in view, there will be 3 or more independent double-differenced observations, and the baseline vector can be estimated. The next section presents summaries for the methods selected for in this paper.

2.1. LEAST SQUARES BY CHOLESKY DECOMPOSITION

The fixed positions between the antennas impose some geometrical constraints, which are to be met by the integer ambiguities and the most powerful and widely used seems to be the fixed baseline length (Lu, 1995). This method is a modification of the Hatch's least square ambiguity search method (Hatch, 1989, 1990) and the initial ambiguity search space for the slave GNSS antenna is usually defined within the uncertainty space corresponding pseudorange solution (Lachapelle *et al*, 1993). If the baseline length between the master and the slave antennas is known, as is the case, the potential solutions for the slave antenna are confined to the surface of a sphere whose radius is equal to the fixed baseline length. The Cholesky decomposition makes the search process for the phase ambiguity computationally fast and simple in derivation. A very recent improvement over Lu´s method is the Constrained LAMBDA (Teunissen, 2011), in which the baseline constraint is considered in the search method. However, Lu´s method is still relevant in applications with very short baselines, as is the case in this paper, since the potential set has few candidates and is easy to generate.

Suppose that four primary satellites have been chosen and the baseline vector from the antenna A (master) to the antenna B (slave) is represented for R_{AB}. Then, Eqn. (3) can be modified as follows,

$$0 = HR_{AB} + W \tag{5}$$

where $W = \lambda N - DD_{AB}$.

The goal is to get the values of baseline vector components from knowledge of the values of phase ambiguity. For this Eqn. (5) can be written as

$$-H^{-1}W = R_{AB} \tag{6}$$

from which, by squaring,

$$R_{AB}^2 = R_{AB}^T R_{AB} = W^T \left(AA^T\right)^{-1} W \tag{7}$$

where R_{AB}^2 is the square of the known baseline length and AA^T is a 3x3 positive definite matrix, which can be Cholesky decomposed into the product of a lower triangle matrix L times its transpose, i.e. $AA^T = L^T L$. With this substitution, Eqn. (7) can be rewritten as

$$R_{AB}^2 = R_{AB}^T R_{AB} = \left(L^{-1}W\right)^T \left(L^{-1}W\right). \tag{8}$$

Since L is a lower triangle matrix, its inverse L^{-1} is also a lower triangle matrix. Defining the quantities *L*, *B*, *C* and *D* as

$$L^{-1} = \begin{pmatrix} l_{11} & & \\ l_{21} & l_{22} & \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \tag{9}$$

$$B = l_{11} w_1 \tag{10}$$

$$C = l_{21} w_1 + l_{22} w_2 \tag{11}$$

$$D = l_{31} w_1 + l_{32} w_2 + l_{33} w_3 \tag{12}$$

Eqn. (8) then becomes

$$R_{AB}^2 = B^2 + C^2 + D^2 . \tag{13}$$

Based on Eqn. (13), it can be immediately concluded that the following inequalities have to hold

$$|B| \le d^2 \tag{14}$$

$$|C| \le \sqrt{d^2 - B^2} \,. \tag{15}$$

This is because the baseline length can not be longer than *d* and all the solutions are constrained on a sphere of radius *d*. From Eqn. (14), the ambiguity search range for the first ambiguity parameter $(N_0)_{AB}^{21}$ can be obtained as

$$\frac{\left(-d/l_{11} + DD_{AB}^{21}\right)}{\lambda} \le (N_0)_{AB}^{21} \le \frac{\left(d/l_{11} + DD_{AB}^{21}\right)}{\lambda} \,. \tag{16}$$

For each integer ambiguity $(N_0)_{AB}^{21}$ within the range (16), w_1 can be computed and therefore B is known. By using Eqn. (15), the ambiguity search range for the second ambiguity parameter $(N_0)_{AB}^{31}$ under $(N_0)_{AB}^{21}$ fixed is

$$\frac{\left(\left(-\sqrt{d^2 - B^2} - l_{21}w_1\right)/l_{22} + DD_{AB}^{31}\right)}{\lambda} \le (N_0)_{AB}^{31} \le \frac{\left(\left(\sqrt{d^2 - B^2} - l_{21}w_1\right)/l_{22} + DD_{AB}^{31}\right)}{\lambda} \,. \tag{17}$$

Once $(N_0)_{AB}^{21}$ and $(N_0)_{AB}^{31}$ are set to integer numbers, w_1 and w_2 can be computed and hence the quantities *B* and *C* are known. The third ambiguity $(N_0)_{AB}^{41}$ can then be solved from Eqn. (13) as

$$(N_0)_{AB}^{41} = \left(\pm\sqrt{d^2 - B^2 - C^2} - E\right)/(\lambda l_{33}) \tag{18}$$

with $E = l_{31}w_1 + l_{32}w_2 + l_{33}(-DD_{AB}^{41})$.

Based on Eqn. (18), there are only two trial values for $(N_0)_{AB}^{41}$ (rounded to the nearest integers) to be tested for each integer trial set $((N_0)_{AB}^{21}, (N_0)_{AB}^{31})$.

From a statistical point of view, the agreement between the measured and adjusted observations to a chosen correct ambiguity can be qualified by the quadratic form residuals, $\hat{v}^T C_{obs}^{-1} \hat{v}$, where $\hat{v}$ is the vector of least squares adjusted observation residuals and C_{obs} is the covariance of the observations matrix. If the errors in observations are Gaussian and the tested ambiguity set is the correct, $\hat{v}^T C_{obs}^{-1} \hat{v}$ will have a Chi-square distribution (Koch, 1989). Therefore, the testing for the correct ambiguity, known as ambiguity "acceptance" test, can be formulated as

$$\hat{v}^T C_{obs}^{-1} \hat{v} \le \mathrm{X}_{f,1-\alpha}^2 \tag{19}$$

where $\mathrm{X}_{f,1-\alpha}^2$ is the Chi-square percentile corresponding to the degrees of freedom *f* and confidence level *1-α*. Usually, *f=n-4*, with *n* being the number of satellites in view.

Due to the insufficient geometry information and error effects, more than one potential ambiguity set may pass the Chi-square test at a certain epoch. In this case, each passed ambiguity set is saved and further tested using the observations from the following epochs. The quadratic form of residuals related to an ambiguity set that passed the test is also saved and accumulated with those from the following epochs. The test is then performed on the accumulated quadratic forms of residuals, which is called the "discrimination" test. As more epochs of observations are used, all the false ambiguity sets of the primary satellites will gradually be rejected except the correct one.

In order to accelerate the convergence time and reduce the effect of the *a priori* carrier phase variance the ambiguity discrimination test is used. Such test is based on test statistic which is constructed by the difference between the minimum and second minimum quadratic form of the residuals in ambiguity identification. When the number of potential ambiguity sets is reduced to a relatively low number after the acceptance testing, the discrimination test is computed, as

$$\frac{\left(\sum \hat{v}^T C_{obs}{}^{-1} \hat{v}\right)_{second\min}}{\left(\sum \hat{v}^T C_{obs}{}^{-1} \hat{v}\right)_{\min}} > threshold \tag{20}$$

If the left hand side of Eqn. (20) is greater than a preset threshold, the potential ambiguity set with the smaller quadratic form of residuals is selected as the correct ambiguity set. The determination of the threshold value usually depends on the error magnitudes and multipath effects on carrier phase and 2 to 3 is often used in practice (Wei, 1986; Landau and Euler, 1992; Lachapelle *et al*, 1993).

2.2. MILES

The MILES method solve the Mixed Integer Least Squares (MILS) in a way similar to the LAMBDA method (Teunissen, 1994), (de Jonge *et al*, 1996), but can be faster, as suggested by (Chang *et al*, 2007). Both methods return as solution the ambiguity value which presents smallest square residuals.

For the MILES implementation, let the sets of all real integer m x n matrices be denoted by R^{mxn} and Z^{mxn}, respectively, and the sets of real and integer n-vectors by R^n and Z^n, respectively. Let ‖•‖ denote the 2-norm of a vector, i.e., if $\boldsymbol{a}=(a_i) \in R^n$, then

$$\|a\| = \sqrt{\sum_{i=1}^{n} a_i^2} \tag{21}$$

Given $\boldsymbol{A} \in R^{mxk}$, $\boldsymbol{B} \in R^{mxn}$ and $\boldsymbol{y} \in R^m$, suppose that $[\boldsymbol{A},\boldsymbol{B}]$ has full column rank. A function produce p optimal solutions to the mixed integer least squares (MILS) problem

$$\min_{x \in R^k, z \in Z^n} \|y - Ax - Bz\|^2 \tag{22}$$

in the sense that a pair $\{x^{(j)}, z^{(j)}\} \in R^k \ x \ Z^n$ is the jth optimal solution if its corresponding residual norm $\left\|y - Ax^{(j)} - Bz^{(j)}\right\|$ is the jth smallest (note that some of these p residual norms can be equal), i.e.,

$$\left\|y - Ax^{(1)} - Bz^{(1)}\right\| \leq \ldots \leq \left\|y - Ax^{(j)} - Bz^{(j)}\right\| \leq \ldots \leq \left\|y - Ax^{(p)} - Bz^{(p)}\right\|. \tag{23}$$

where p is a parameter chosen at random.

If the matrix $\boldsymbol{A}$ is nonexistent, Eqn. (21) becomes an ordinary integer least squares (ILS) problem

$$\min_{z \in Z^n} \|y - Bz\|^2. \tag{24}$$

To solve the MILS problem, it is transformed into an ILS problem and a real upper triangular linear system of equations. By solving these two sub-problems sequentially, the MILS solution is obtained. Suppose $\boldsymbol{A}$ has the QR factorization

$$A = \left[Q_A, \overline{Q_A}\right] \begin{bmatrix} R_A \\ 0 \end{bmatrix} \tag{25}$$

where $\left[\underset{k}{Q_A}, \underset{m-k}{\overline{Q_A}}\right] \in R^{mxm}$ is orthogonal and $\boldsymbol{R_A} \in R^{kxk}$ is nonsingular upper triangular. This factorization can be computed by Householder transformations (Björck, 1996). Then

$$\|y - Ax - Bz\|^2 = \left\| \begin{bmatrix} Q_A^T \\ \overline{Q_A^T} \end{bmatrix} y - \begin{bmatrix} R_A \\ 0 \end{bmatrix} x - \begin{bmatrix} Q_A^T B \\ \overline{Q_A^T B} \end{bmatrix} z \right\|^2 = \left\|Q_A^T y - Q_A^T Bz - R_A x\right\|^2 + \left\|\overline{Q_A^T} y - \overline{Q_A^T} Bz\right\|^2. \tag{26}$$

Note that for any fixed z, we can chose $x \in R^k$ such that the first term on the right hand side of Eqn. (25) is equal to zero. Therefore, to solve de MILS problem, we first solve the ordinary ILS problem

$$\min_{z \in Z^n} \left\| \overline{Q_A^T} y - \overline{Q_A^T} z \right\|^2 \quad (27)$$

to obtain the solution $\hat{z} \in Z^n$, and then solve the upper triangular system

$$R_A x = Q_A^T y - Q_A^T B\hat{z} \quad (28)$$

to obtain the real solution $\hat{x} \in R^k$. If we find p optimal integer solution to Eqn. (26), then we can obtain the corresponding p real solutions by solving Eqn. (27). Thus the key is to solve the ILS problem and for details see (Chang, 2006).

3. SIMULATION RESULTS

The data used to test the algorithms were obtained from Dai *et al* (2008), where the ambiguity problem is supposed to be solved from the outset. More precisely, the problem addressed by Dai is not the one considered here. The GNSS measurements have been acquired by using a NovAtel® DL-4 receiver and IFEN NavX® RF GNSS simulator. The simulator generates the RF signals according to the antenna position in ECEF frame specified by the user. The signals are then transferred to the GNSS receiver and the measurements are then converted to RINEX format. The antenna's positions are configured according to Fig. 2.

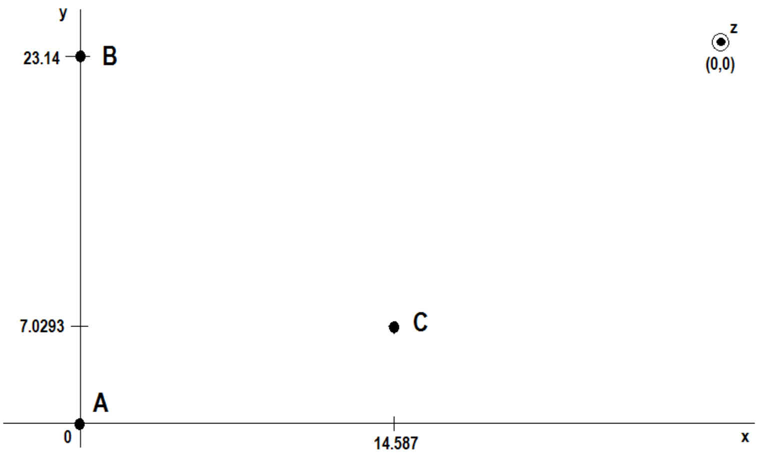

Figure 2 - Antenna's positions in the platform frame, measured in meters

The antenna C has been used in the simulation and test of the implemented algorithms. The results are presented in sections 3.1 and 3.2.

3.1. LEAST SQUARES BY CHOLESKY DECOMPOSITION

This method has three main design parameters: preset time period, threshold of the ratio test and chi-square value. These parameters determine the performance of the method and a poorly made choice can lead to unsatisfactory results. The algorithm was executed about two hundred epochs with a preset time period equal to ten epochs. There were nine satellites in view and the acceptance test was considered with a confidence interval equal to 95%. The threshold value of the discrimination test was chosen as 2.7 and results are shown in Fig. 3.

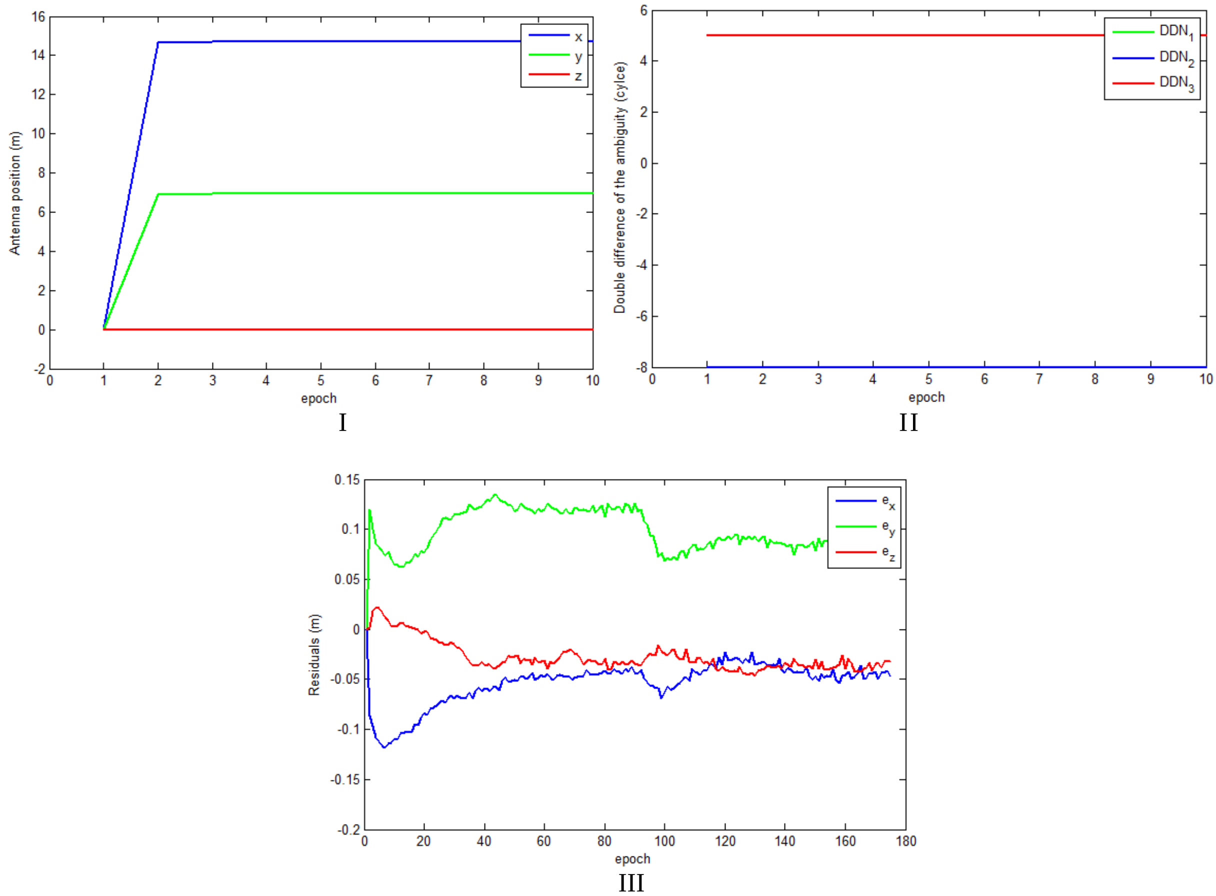

Figure 3 - I) Estimated positions of the antenna C; II) Estimated Ambiguity; III) Residuals between true and estimated antenna C coordinates

The estimated coordinates, $R_{AC} = [14.633 \quad 6.9422 \quad 0.034]^T m$, correspond approximately to coordinate of the antenna $C = [14.587 \quad 7.0293 \quad 0]^T m$. The value obtained for the ambiguities corresponds to real values, namely $N_{AC} = [-8 \quad -8 \quad 5]^T$. To measure the quality of the estimate, Fig. 3.III shows the graph with the residuals of the estimated coordinates with few meters of errors.

3.2. MILES

In the MILES, in contrast to the LAMBDA method, it is not necessary to provide the cofactor matrix at the beginning of the integer search process. By comparing these 2 methods, the obtained results were similar. Basically, their success depends on the true integer ambiguity belonging to the potential set, i.e., the candidate ambiguity set obtained by these 2 methods. The number of elements in this set is controlled by the user, i.e., the user selects the number of integer solutions among which the true integer ambiguity is to be searched for. If the measurement noise is large, or there are few visible satellites and/or epochs or few candidates number is selected, then the true ambiguity may be missed, which implies failure in the baseline estimation, regardless of the validation procedure.

Once the set containing the possible integer ambiguities is obtained, via MILES or LAMBDA, there is the problem associated to the selection of the best integer estimate. The usual procedure is the one already adopted by the Lu's method: statistic and ratio test. This procedure is prone to failure, and a more direct and a less user dependent solution has been proposed by (Li *et al*, 2008), which can be summarized as follows: by using SVD, separate the integer ambiguity estimates in 2 parts: N_i, where *"i"* stands for independent, containing the 3 components which produces the smallest condition number for the remaining $(m-3)x(m-3)$ normal matrix, where *m* is the number of ambiguities, and N_d, where *"d"* stands for dependent, containing the remaining $(m-3)$ components. A float estimate for N_i is obtained by using the available data and the searching is performed for N_i, by using either MILES or LAMBDA. For each integer estimate, the corresponding N_d components are estimated, as float. If the N_d estimate is far from an integer vector, the N_d estimate is immediately dropped out. This approach has 2 main benefits when compared to the usual procedure for integer ambiguity validation: 1) the search space is always 3 dimensional, regardless of the number of visible satellites, and 2) the test for deciding which is the correct N_d is carried out by using a very well conditioned normal equation.

We now consider the performance of both MILES and LAMBDA for estimating the baseline, with the data from (Dai *et al*, 2008). A potential set of integer solutions with 1000 candidates, for 10 epochs, is generated by the MILES and LAMBDA in around 0.27s, by using MATLAB. With the (Li *et al*, 2008) method this time is reduced to 0.052s. The presence of the true ambiguity in the potential set, which is critical to the correct baseline estimation, is now verified. By using the potential set with 1000 candidates, the MILES and LAMBDA methods produced similar results, but the true ambiguity vector belongs to the potential set only between samplings 78 and 106, as shown in Fig. 4, where the behavior of the last component of the integer ambiguity vector is shown.

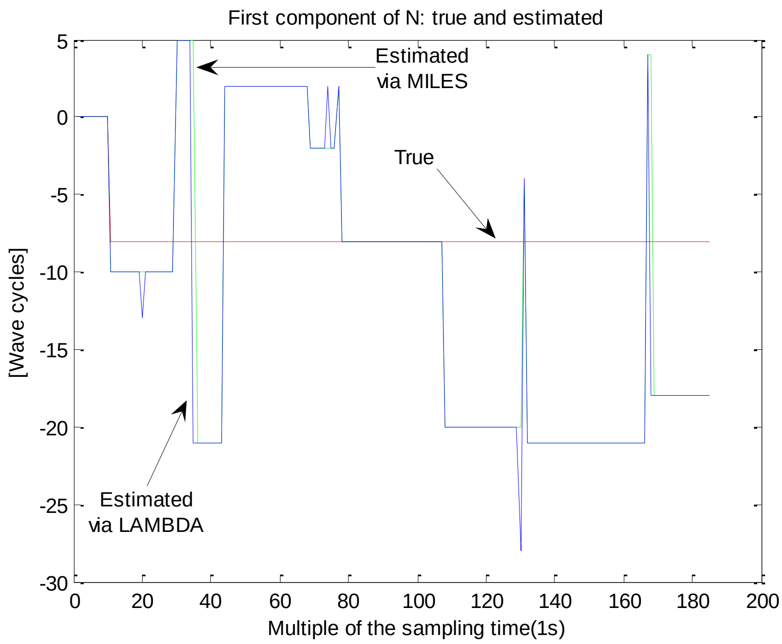

Figure 4 - Last component of the integer ambiguity vector. Estimation starts at t=10s, since 10 epochs are used.

This means that any validation method would have to indicate failure outside this interval. The conclusion here is: the data is quite noisy, hence 10 epochs are not enough for providing the correct ambiguity, even with a potential set with 1000 candidates. The behavior of the method proposed by (Li *et al*, 2008) is similar, but has the advantage of indicating failure of the validation procedure straightway, simply by looking at the float values of N_d as was verified in the simulations.

In order to smooth the noise, the number of epochs could be increased, but this has the inconvenient of reducing the recovery time in the event of cycle slip. Therefore, the only option left, due to the high noise, is to increase the number of candidates in the potential set. This difficulty was already expected, since the original least squares problem is badly conditioned because only double-differenced carrier-phases are used.

4. CONCLUSIONS

From the results it is possible to conclude that, with the present data, the Least Square by Cholesky Decomposition performed better than the MILES, fixing correctly the ambiguities and thereby providing correct antenna positions estimates. This is because the method makes use of a more efficient technique, for short baselines, in the building of the potential set. The MILES provides a number of candidates which can be selected by the user, packed accordingly to the residues amplitude. In a favorable scenario, i.e., small noise, the true integer ambiguity belongs to the potential set even if few candidates are considered. For large measurement noise the number of necessary candidates can be too high, which implies unacceptable computational load for real time application. This also holds true for the LAMBDA method. However, the MILES performance does not depend on the baseline length, since the search procedure is not an enumeration, but an integer optimization. In this sense, it can provide an initial estimate for the Least Square by Cholesky Decomposition in applications where the baseline is not very short. Since in this work the main reason for estimating the baseline is in devising an attitude IMU/GNSS fusion by using GNSS attitude reading, the length of the baseline is known. In this work this information is explored by the Cholesky Decomposition method, but not by MILES. Therefore, its performance could be improved by solving a mixed integer least squares problem with the baseline length constraint. Regarding the LAMBDA method, the first approach along this line seems to be Teunissen *et al.* (2011).

5. ACKNOWLEDGEMENTS

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