THE USE OF GENETIC ALGORITHMS AND THE MAXIMUM RECTANGLE HULL FOR A STRONG REDUCTION IN COMPUTATIONAL COST FOR CRITICAL PLANE APPROACHES IN MULTIAXIAL FATIGUE

ALBERTO PINHEIRO DANTAS, adantas@unb.br
JOSÉ ALEXANDER ARAÚJO, alex07@unb.br
FABIO COMES DE CASTRO, fabio.comes.castro@gmail.com
Universidade de Brasília, Departamento de Engenharia Mecânica, Brasília, DF 70910-900, Brazil.

TITO DIAS JUNIOR, titodiasjr@gmail.com
Departamento de Polícia Federal, SAS Quadra 6, lotes 09/10 - ED.SEDE/DPF, Brasília,DF 70037.900 , Brazil.

Abstract. *The goal of this work is to propose an alternative measure to compute the shear stress amplitude in critical plane multiaxial criteria. In the setting of High Cycle Multiaxial Fatigue the critical plane is usually defined as the material plane experiencing the largest amplitude of the shear stress vector path. Usually such amplitude is characterized by the radius of the minimum circle circumscribing the shear stress history in a material plane. Here, an alternative measure which considers the maximum circumscribing hull is considered.*
The computation of the shear stress amplitude by the Maximum Rectangular Hull (MRH) was conducted for available experimental data involving proportional and nonproportional stress paths . A genetic algorithm was used to search the critical plane. Such algorithm is based on evolutions concepts and is capable to find the critical plane without the need to determine the equivalent shear stress amplitude in "all" material planes. The estimates were improved for most data when the shear stress amplitude was computed in terms of the maximum rectangular hull. The Genetic Algorithm not only reduces the computational cost associated with the search of the critical plane but also provides accurate values of the maximum shear stress amplitude.

Keywords*: Genetic Algorithm, Multiaxial Fatigue, Shear Stress Amplitude, Critical Plane*

1. INTRODUCTION

Structural or mechanical components are usually subject to complex service loading, which produces a multiaxial state of stress that must be, in a number of cases sustained for a high number of fatigue cycles (HCF). Under such conditions the use of stress based models are appropriate to estimate the component durability. In this setting, critical plane models, as first proposed by Findley (1959) and later by others (Fatemi & Kurath (1988), McDiarmid (1991), Susmel & Lazzarin (2002), Carpinteri et al. (2009)) have gained significant attention from the academia and also from the industrial sector. Critical plane models are extremely attractive from a mechanical standpoint, as criteria developed within this framework do not only provide the fatigue strength estimates of the component, but also indicate the location and direction expected for early crack initiation. Although a precise definition for the critical plane will be provided in another section of this manuscript, here we should advance that a material plane is considered to be the critical one when an appropriate combination of shear and normal stresses reaches a maximum value. This means that an intensive search process is necessary to find such a plane in a material point. Considering that real components have been often designed with the aid of Finite Element Models, containing thousands or hundreds of thousands nodes, it is then possible to realize the computational cost associated with the determination of the critical plane in each of these nodes to find the global maximum. To reduce this cost numerical methodologies have been proposed elsewhere (Susmel, 2010), however they usually provide local maximum too. The aim of this work is to propose an optimization method based on Genetic Algorithms, which associated with an alternative measure for the equivalent shear stress amplitude in a material plane (Araújo et al., 2011), will provide a strong reduction in the computational cost required to the locate the critical plane.

2. GENETIC ALGORITHMS

Genetic algorithms (GA) are holistic optimum search algorithms based on the biological concepts of evolution of species and they were first proposed by John Holland (Goldberg, 1989). The basic principle of these algorithms is to operate on an initial random population of individuals evaluating their fitness, selecting the best fitted and allowing them to reproduce in order to form the next generation, (as showed in Fig.1).

The algorithm is initialized with generation of a set of individuals randomly in bitwise fashion. It follows that the algorithm is straightforward: in each generation all chromosomes are evaluated (using function f on the decoded sequences of variables); select a new population with respect to the probability distribution based on fitness values and alter the chromosomes in the new population by mutation and crossover operations; when no further enhancement is observed, the best chromosome represents an (possibly the global) optimal solution. Generally the program is stopped after a fixed number of iterations (generations).

The crossover operator, which, in this work, corresponds to mating, performs information exchange between the best fitted individuals to obtain the new generation. In practice, one crossover point is randomly chosen, the individual chromosomes are broken in this point and the recombination of the broken parts forms the two new individuals. The long term effect of this operator is to evolve the population to the best combination of the initial randomly generated individuals, which means that the best information about the objective function optimum present in the initial random population will be reached. However, if the initial random population does not contain information, or schema, about the global optimum, the evolving process may converge to a local optimum. To avoid this, it is necessary to add new random information during the evolving time. In this work, this is accomplished by the mutation operator, which randomly chooses an individual, a mutation point in its chromosome and changes the numerical value found there. Since this operator introduces new information in the population evolving process, it increases the probability of information about the global optimum to be included in the population, and consequently, to be selected.

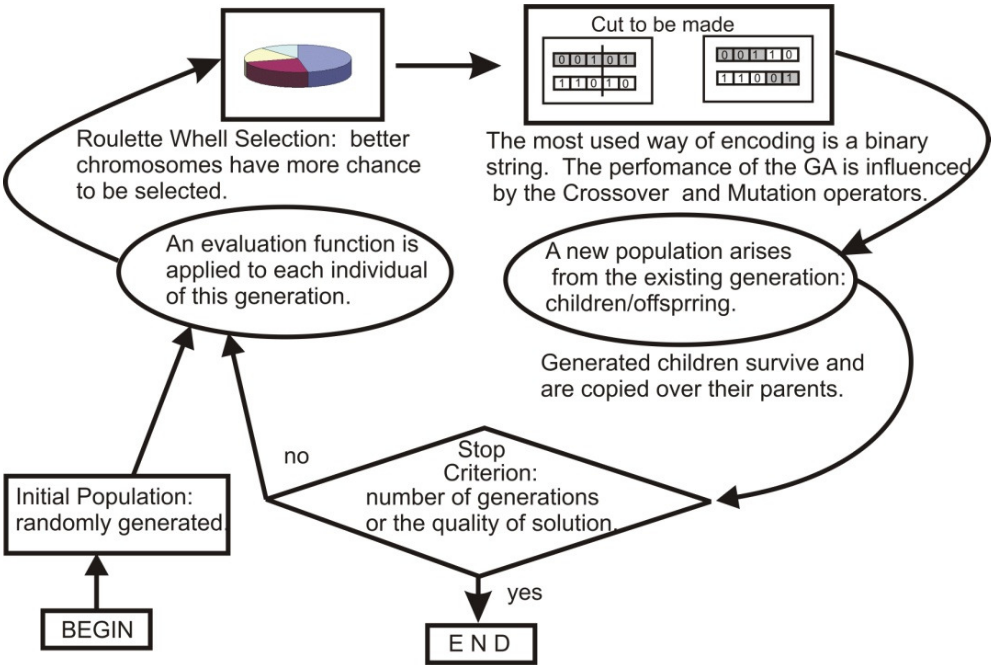


Figure 1. Flowchart of a genetic algorithm.

However, if mutation occurs too frequently, which means a high probability of mutation occurrence, the evolving process can be slowed or, even, prevented. Therefore it was adopted a low mutation probability and an elitism selection strategy, in which some of the best individual of a given generation will always be cloned to the next generation, in order to preserve the best solution from the mutation operator. Elitism can rapidly increase the performance of GA, because it prevents losing the best found solution.

3. BASIC CONCEPTS OF EQUIVALENT SHEAR STRESS AMPLITUDE

Consider a material plane Δ characterized by its unit normal vector **n**, passing through point O of a mechanical component submitted to a multiaxial cyclic loading (Fig. 2a).

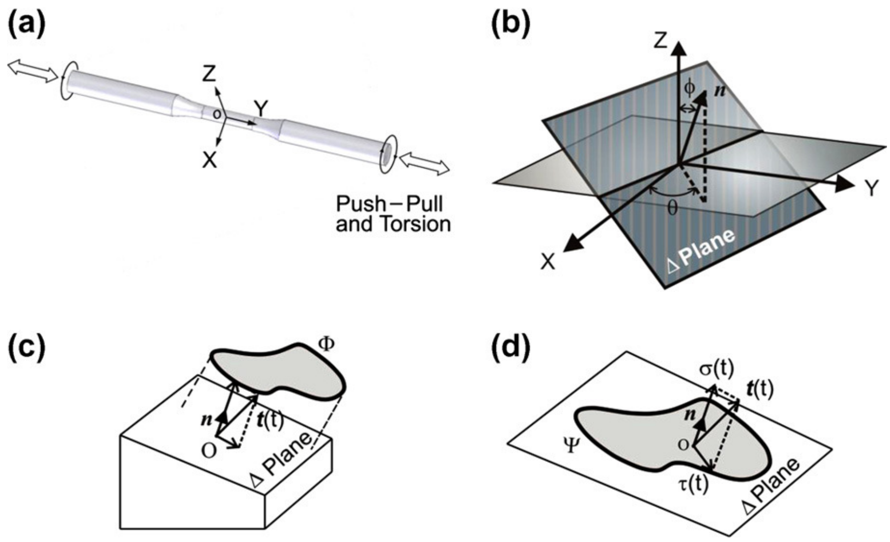


Figure 2. (a) Specimen under multiaxial loading, (b) spherical coordinates ϕ and θ which characterize the material plane, (c) stress vector $\mathbf{t}(t)$ acting on a material point O and describing a stress path Φ, and (d) its normal, σ, and shear, τ, stress components in a material plane Δ.

This vector is defined by its spherical coordinates ϕ and θ (Fig. 2b). Cauchy´s theorem states that the stress vector $\mathbf{t}(t)$ on Δ, which describes a stress vector path Φ during a time period (Fig. 2c) is given by

$$\mathbf{t}(t) = \mathbf{T}(t)\mathbf{n}, \tag{1}$$

where T(t) is the stress tensor at time instant t. The normal and shear stress components of the stress vector $\mathbf{t}(t)$ (Fig. 2d) are:

$$\boldsymbol{\sigma}(t) = \left(\mathbf{T}(t)\mathbf{n}\cdot\mathbf{n}\right)\mathbf{n}, \tag{2}$$

and

$$\boldsymbol{\tau}(t) = \mathbf{T}(t)\mathbf{n} - \boldsymbol{\sigma}(t), \tag{3}$$

where the dot in Eq. (2) stands for the scalar product. While the computation of the amplitude and mean values of $\boldsymbol{\sigma}(t)$ is a straight-forward task, as such a vector varies in magnitude but not in direction, the same cannot be said about the calculation of these quantities associated to the shear stress vector path, ψ, described by $\boldsymbol{\tau}(t)$ in the material plane Δ (Fig. 2d) during a time period.

4. NEW DEFINITION OF CRITICAL PLANE IN STRESS-BASED MULTIAXIAL FATIGUE MODELS AND ITS IDENTIFICATION

The critical plane related to stress-based multiaxial fatigue criteria has been classically defined by a number of authors as the material plane where the shear stress amplitude reaches its maximum value (Carpinteri *et al.*, 2009, Carpinteri and Spagnoli, 2001, Dang Van, 1973, McDiarmid, 1991 and Susmel & Lazzarin, 2002).

This is in line with the physical nature of fatigue damage and the mechanics of plastic flow, where, at least for ductile engineering materials, cracks are observed to initiate in slip band regions associated to shear motion between crystal planes. However, such a definition constitutes an ill-posed problem in numerical terms, since there are cyclic stress histories for which a number of material planes present maximum and identical values of τ_a. As an example, Fig. 3(a) shows a graph of τ_a for each material plane, identified by the coordinates (ϕ, θ) in a multiaxial state of stress given by test # 20, Tab. 1 ($\sigma_x(t)$ = 242 sin ωt [MPa] and $\tau_{xy}(t)$ = 121 sin ($2\omega t - \pi/2$) [MPa]). It can be clearly observed that there are four material planes with rigorously the same τ_a value.

To further illustrate such a characteristic, the graph of τ_a vs $\sigma_{n,max}$ in each material plane (planeincrements $\Delta\phi = \Delta\theta$ = 1º) for the above multiaxial state of stress is plotted in Fig. 3(b). Such a graph shows that there are a number of material planes experiencing essentially the same τ_a, with maximum stress normal to the plane varying from 70 MPa to 270 MPa.

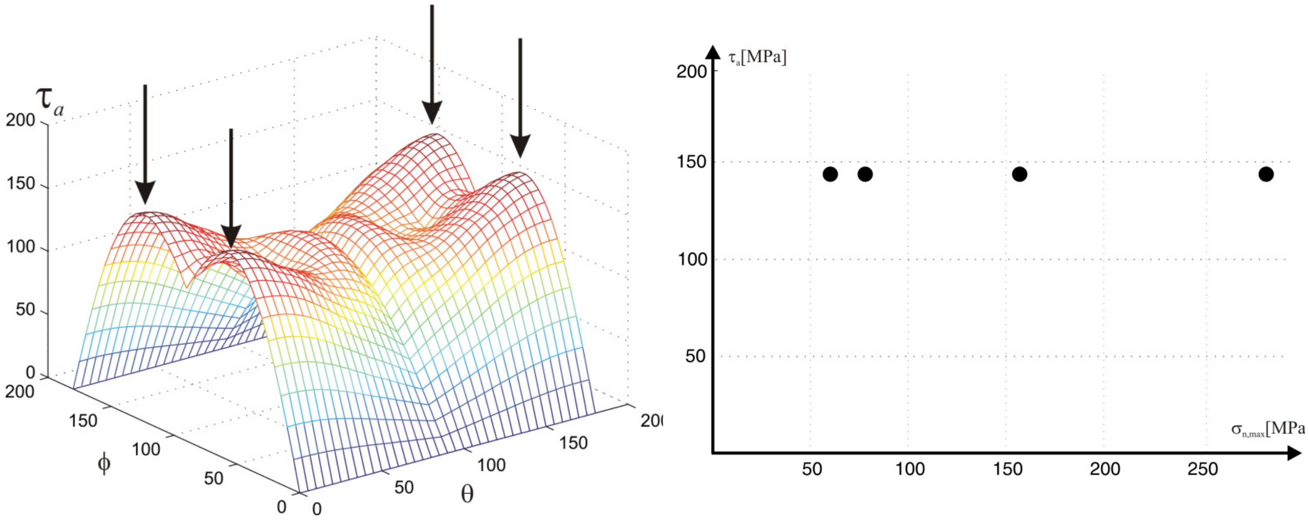


Figure 3(a). Equivalent shear stress amplitude in a number of material planes (ϕ,θ) for a multiaxial stress history (corresponding to test 20 in Table 1)

(b) τ_a v $\sigma_{n,max}$ for the material planes containing the highest shear stress amplitude for a multiaxial stress history (test # 20, Table 1, $\sigma_x(t) = 242 \sin \omega t$ [MPa] and $\tau_{xy}(t) = 121 \sin (2\ \omega t - \pi/2)$ [MPa]). The same τ_a for different values of $\sigma_{n,max}$

On the other hand, there may be cases where the multiaxial state of stress presents a plane containing a unique highest value for τ_a but there is a wide variation of normal stress values around the shear stress as is the case in the load history represented by $\sigma_x(t) = \sigma_y(t) = 250 + 205 \sin \omega t$ [MPa] and $\tau_{xy}(t) = 96 \sin \omega t$ [MPa], see Fig. 4).

Notice however that, as is displayed in such a figure, the differences in τ_a values for a number of other material planes are negligible, but not regarding the $\sigma_{n,max}$ value. Thus, in this case, would the crack prefer to nucleate and grow in the plane, where τ_a is the highest? The above examples illustrate the need for a better and well-posed definition for the damage parameter to be used to determine the critical plane. In this setting, it is claimed here that the fatigue damage is more severe in a plane, where (i) τ_a is close to its highest value (but not necessarily the maximum) and (ii) $\sigma_{n,max}$ is more significant.

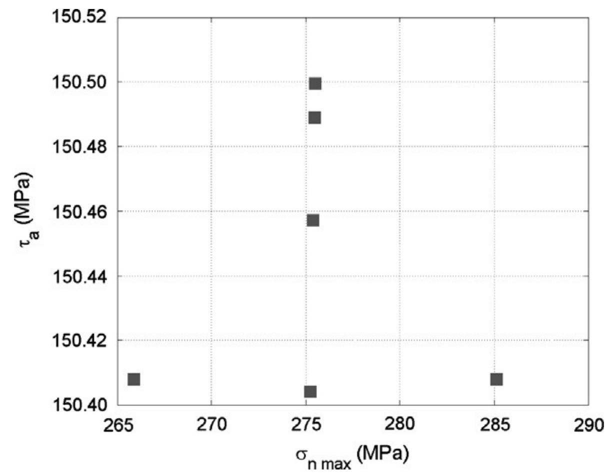


Figure 4. τ_a v $\sigma_{n,max}$ for some material planes containing the highest shear stress amplitude for a multiaxial stress history ($\sigma_x(t) = \sigma_y(t) = 250 + 205 \sin \omega t$ [MPa], and $\sigma_{xy}(t) = 96 \sin \omega t$ [MPa]). Differences in τ_a values (notice the scale in the τ_a axis) are neglible, but not in $\sigma_{n,max}$.

This essentially means that, to determine the critical plane, we first need to select a set of candidate planes, here defined by the ones, where τ_a reaches at least 99% of its maximum value $\tau_a^{\max}$ (this is the adopted tolerance, *tol*), being the critical one the plane (amongst the candidates), where the maximum value of $\sigma_{n,max}$ is attained. In mathematical terms, the identification of the critical plane is as follows:

- Step 1: Find the maximum equivalent shear stress amplitude among all material planes:

$$\tau_a^{\max} = \max_{\phi,\theta}\left\{\tau_a\left(\phi,\theta\right)\right\}, \tag{4}$$

- Step 2: Select the candidate planes within the established tolerance:

$$\text{Candidate Planes} = \left(\phi^*,\theta^*\right) = \left\{\left(\phi,\theta\right): \tau_a^{\max} - tol \le \tau_a\left(\phi,\theta\right) \le \tau_a^{\max}\right\} \tag{5}$$

- Step 3: Identify the critical plane among the candidate planes as the one, where the maximum normal stress is maximized:

$$\text{Critical Plane } \left(\phi^C,\theta^C\right) = \max_{\phi^*,\theta^*}\left\{\sigma_{n,\max}\left(\phi^*,\theta^*\right)\right\}. \tag{6}$$

5. ALTERNATIVE METHOD TO COMPUTE THE EQUIVALENT SHEAR STRESS AMPLITUDE IN A MATERIAL PLANE

There are a number of available techniques to compute in material plane such as Maximum Chord, Maximum Projection, Minimum Circumscribed Circle (MCC) and Minimum Ellipsoids (Bernasconi, 2002). The MCC is the most popular methodology to compute τ_a which was proposed by Dang Van (1973) and later by Papadopoulos (1994).

Recently, a new and an alternative method to compute the equivalent shear stress amplitude, τ_a, was proposed to characterize the fatigue damage under multiaxial loadings (Araújo *et al*. 2011). In such method the equivalent shear stress amplitude is given by the Maximum Rectangular Hull (MRH) of the shear stress vector path in a material plane as follows. The halves of the sides of a rectangular hull with orientation φ (with respect to τ_i) bounding the shear stress path, ψ, can be defined as follows (Fig. 5a):

$$a_i\left(\varphi\right) = \frac{1}{2}\left[\max_t \tau_i\left(\varphi,t\right) - \min_t \tau_i\left(\varphi,t\right)\right], \quad i = 1,2. \tag{7}$$

For each φ-oriented rectangular hull one can define its amplitude as

$$\tau_a\left(\varphi\right) = \sqrt{a_1^2\left(\varphi\right) + a_2^2\left(\varphi\right)}. \tag{8}$$

Then, the equivalent shear stress amplitude is the one which maximizes Eq. (4), as is illustrated in Fig. 5b:

$$\tau_a = \max_{\varphi}\sqrt{a_1^2\left(\varphi\right) + a_2^2\left(\varphi\right)}.$$

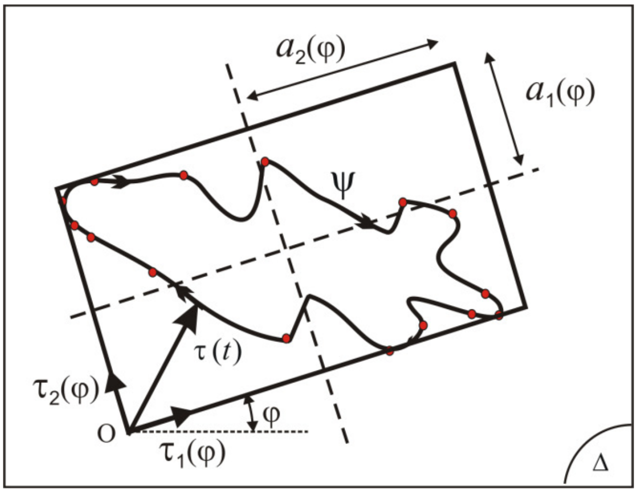
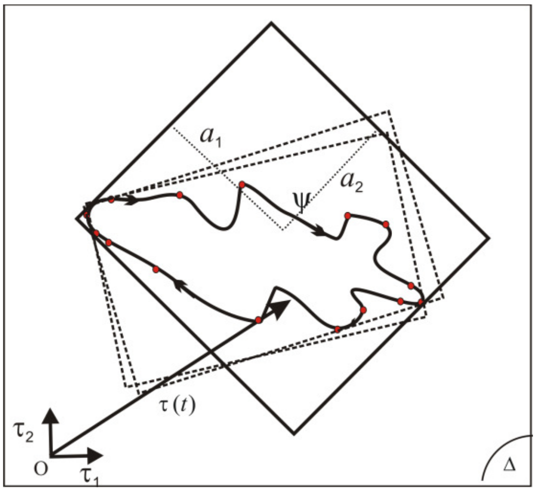


Figure 5. (a) Half sides $a_1(\varphi)$ and $a_2(\varphi)$ of a rectangular hull with orientation φ bounding the shear stress path ψ in a material plane.



(b) the Maximum Rectangular Hull (MRH) for ψ.

The Maximum Rectangular Hull of the shear stress path requires no more than simple axes rotation in two dimensions in order to calculate τ_a since the hull is rotated on the material plane.

6. THE MODIFIED WÖHLER CURVE METHOD FOR HIGH CICLE MULTIAXIAL FATIGUE CRITERION

In order to evaluate the impact of the equivalent shear stress amplitude, determined by the Maximum Rectangular Hull (MRH) method on the estimation of the multiaxial fatigue strength, it is necessary to invoke an appropriate model.

Carpinteri *et al.* (2009) observed that the multiaxial High-Cycle Fatigue behavior of metallic materials could successfully be estimated by using a simple τ_a vs $\sigma_{n,\max}/\tau_a$ relationship. The model has been described by Susmel and co-workers in a series of articles (Carpinteri *et al.*, 2009, Fatemi and Kurath, 1988 and Susmel, 2010). Briefly, the MWCM can be formalized as follows:

$$\tau_a\left(\phi^C,\theta^C\right)+\kappa\frac{\sigma_{n,\max}}{\tau_a}\left(\phi^C,\theta^C\right)\leq\lambda \tag{9}$$

In the above equation, τ_a (ϕ^C,θ^C) is the equivalent shear stress amplitude in the critical plane (ϕ^C,θ^C) (as is defined in the previous Section), $\sigma_{n,\max}$ is the maximum stress perpendicular to this plane, and the parameters κ and λ are material constants that can be obtained from two fatigue strengths generated under different loading conditions. For instance, if the uniaxial, f_{-1}, and the torsional, t_{-1}, fully-reversed plain fatigue strengths (at 2 x 10^6 loading cycles) are used to calibrate Eq. (9), such constants turn out be (Carpinteri *et al.*, 1989):

$$\kappa=t_{-1}-\frac{f_{-1}}{2}\ \text{and}\ \lambda=t_{-1}. \tag{10}$$

Still concerning Eq. (9), it is possible to define a variable

$$\rho=\frac{\sigma_{n,\max}}{\tau_a}\left(\phi^C,\theta^C\right), \tag{11}$$

that is a normalized measure of the influence of the mean normal stress on the multiaxial analysis

In order to evaluate the performance of such a multiaxial mode when values of τ_a computed by the MRH method are fed into, the following error index can be adopted:

$$EI(\%)=\frac{\tau_a\left(\phi^C,\theta^C\right)+\kappa\frac{\sigma_{n,\max}}{\tau_a}\left(\phi^C,\theta^C\right)-\lambda}{\lambda}\cdot 100 \tag{12}$$

A negative value of the above error index indicates that fatigue failure should not occur up to 2 x 10^6 loading cycles. It is interesting to observe also that, from an engineering point of view, a negative value of the index indicates the fact that the structural component dimensions could be reduced down to the limiting condition given by EI(%) = 0

7 . COMPUTATIONAL SEARCH OF THE CRITICAL PLANE

In this work, the numerical search for the critical plane was carried by two methods. The first one, named Plane Increment Method (PIM), is characterized by an exhaustive plane by plane search, through "all" material planes for a specific stress history in a material point. In this method the material plane search starts by fixing θ and varying ϕ in incremental steps $\Delta\phi$ = 1°. The process continues until the varying angle reaches 180°. Then θ is also incremented in a step $\Delta\theta$ =1° and the search process restarts until θ= 180°. This essentially means that 32.400 material planes need to be investigated.

The second method to find the critical plane considers the use of a Genetic Algorithm (GA). It requires that the variables are coded according to the concepts of chromosomes, genes, individuals and generations that make up an element or part of the solution in the search domain. The chromosome (as a potential solution) is represented by a binary string of length $m=\sum_1^2 m_i$ where m_1 bits map into a value from the range of $\phi \in [0, \pi]$ and m_2 bits map into a value from the range of $\theta \in [0,\pi]$. An individual will be the binary representation of a concatenated bit string consisting of both ϕ and θ representing the angle of a material plane.

The determination of the critical plane by the PIM method is a computationally expensive but rather trivial task. The code simply requires a comparison command to select the candidate planes and finally the critical one (Equations 4 to 6). In GA there are no structures for comparison, since the choice of the best individual (solution) is made by constant evaluation and selection of individuals who have the best features. Consequently, it was necessary to establish a function that simulates a tolerance around the maximum shear stress amplitude value and, at the same time, should consider the contribution of normal stress on the fatigue resistance. For this purpose, a fitness or simply Fit function was defined as:

$$Fit(i)=\tau_a(i)+\xi\sigma_{n,\max}(i), \tag{13}$$

where:

Fit(i) is objective function that prescribes the optimum solution;

$\tau_a(i)$ is the amplitude shear stress on the each material plane *i* ;

$\sigma_{n,\max}(i)$ is the maximum normal stress for $\tau_a(i)$ and;

ζ is a weight factor, here set as $\zeta = 0.01$.

The parameter ζ was calibrated in a previous work by Inácio (2008). This value proved appropriate to compute the critical plane without compromising the physical understanding that cracks usually start in Stage 1, i.e, on planes of high shear stress.

The chromosome length, l, is associated with the level of precision in the solution. To find, l, one can apply the following relation:

$$l = \log_2\left(1 + \frac{x_{\max} - x_{\min}}{tol}\right) \tag{14}$$

where $x_{\max} = \pi$ and $x_{\min}, = 0$

Adopting tol = 0.003° (9.55 x 10^{-4}rad), $x_{\max} = \pi$ and $x_{\min}, = 0$, a 10 bits string (phenotype) is obtained. Therefore, each individual (material plane) will be represented by a 20 bits chromosome, as to define a material plane one needs two variables, ϕ and θ.

The size of a population (number of individuals) can not be too small to compromise the quality of results neither extremely big to increase the computational cost and so invalidate its use as an optimization process. It was found that a population consisting of 40 individuals and 40 generations provided an appropriate tradeoff. The mutation rate was 1/15. All these values were set based on the studies conducted by Dantas (2009).

Although the hull provide an invariance in the case of proportional loading or when the ellipse is a good approximation to the stress paths, a study was conducted to identify a minimum number of rotations of the hull to calculate the amplitude of shear stress for a attractive computational cost. In this regard, $\Delta\varphi$ = 9° or 10 rotations to the hull is sufficient to assure better accuracy of results, regardless of load histories (not proportional and asynchronous) (Dantas, 2009). The load history was discretized in 32 time intervals for each 2π period.

8. EXPERIMENTAL DATA FROM THE LITERATURE AND RESULTS

To compare the performance of the GA with the PIM in estimating fatigue strength biaxial fatigue experimental data for six different steel alloys have been collected from the literature. These data were produced by Nishihara and Kawamoto (1945), Mielke (1980), Kaniut (1983), Heidenreich *et al.* (1984), Froustey and Lasserre S. (1989) and are reported in Tab. 1. They correspond to tests on hard metals, as defined by Papadopoulos (1994) under synchronous and asynchronous sinusoidal combined loadings. The following nomenclature is adopted in Tab. 1: the subscript a stands for the amplitude of stresses; σ_x and σ_{xy} are the normal and the shear stresses, respectively, whereas λ_{xy} is the frequency ratio between the signal of τ_{xy} and that of σ_x, and β_{xy} is the phase difference. The stress values reported in Tab. 1 correspond to the maximum combination of stresses that the specimen can stand without failing (up to a limit of 2 x 10^6 loading cycles). Fatigue strength under fully-reversed bending f_{-1} and torsion t_{-1} in correspondence to 2 x 10^6 loading cycles are also provided in Tab. 1 for each material. Figures 6a and 6b show some typical stress paths and loading histories to synchronous and asynchronous data, respectively.

Genetic Algorithm is random, i.e., its use to optimize the same problem twice, in exactly the same way, will provide two different responses (unless the exact optimum is found). Notice that in GA one cannot provide the same initial population since it is randomly generated. To test the convergence and ability of the method to provide reliable results despite of its random nature we chose to run three consecutive times tests numbers 1 (proportional, synchronous), 3 (non-proportional, synchronous) and test 12 (non-proportional asynchronous). In all these cases results proved that the largest difference in τ_a and $\sigma_{n,max}$ for a same test was 0.1 MPa and 2 MPa respectively.

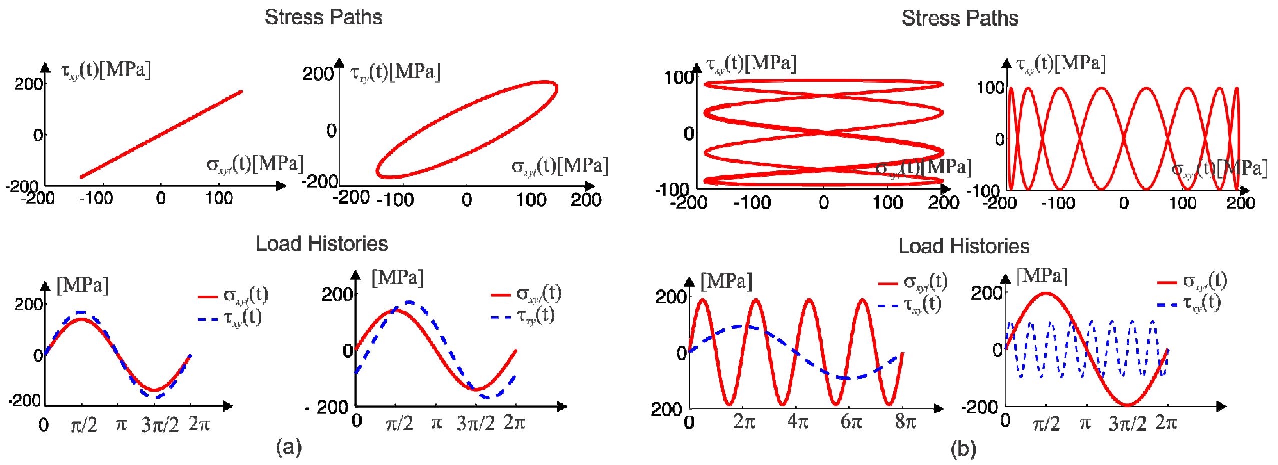


Figure 6(a). Some examples of stress paths and loading histories for synchronous (tests 1 and 2) and (b) asynchronous data (tests 12 and 21).

Table 1. Experimental data and fatigue strength properties (for different steels tested under combined loadings).

Test#	σ_{xa}	τ_{xya}	λ_{xy}	β_{xy}
Material: Hard steel; f_{-1}=319.9MPa; t_{-1}=196.2MPa				
1	138.1	167.1	1	0
2	140.4	169.9	1	30
3	145.7	176.3	1	60
4	150.2	181.7	1	90
5	245.3	122.6	1	0
6	249.7	124.8	1	30
7	252.4	126.2	1	60
8	258.0	129.0	1	90
9	299.1	62.8	1	0
10	304.5	63.9	1	90
Material: 34Cr4 steel; f_{-1}=415.0 MPa; t_{-1}=259.0 MPa				
11	263	132	4	0
Material: GGG60 steel; f_{-1}=275.0 MPa; t_{-1}=249.0 MPa				
12	186	93	0.25	0
13	185	93	4	0
Material: 30NCD16 steel; f_{-1}=585.0 MPa; t_{-1}=405.0 MPa				
14	285	285	0.25	0
15	290	290	4	0
Material: 39NiCrMo3 steel; f_{-1}=585.0 MPa; t_{-1}=405.0 MPa				
16	259.5	150.0	2	0
17	266.0	153.6	3	0
Material: 25CrMo4 steel; f_{-1}=340.0 MPa; t_{-1}=228.0 MPa				
18	210	105	0.25	0
19	220	110	2	0
20	242	121	2	90
21	196	98	8	0

Figures 7(a) and 7(b) show τ_a and $\sigma_{n,max}$ values determined the PIM and the GA. In both cases τ_a is obtained by the MRH. It is noticed that the GA provides values equal or slightly higher than those determined by the PIM. This can be explained by the fact that the GA is searching more planes than the PIM and hence can provide more accurate values for the these stresses in the critical plane.

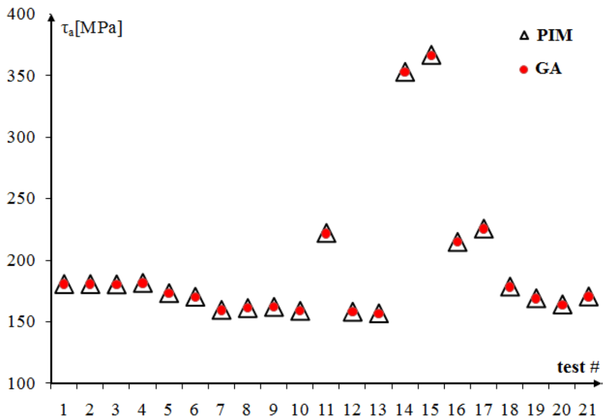
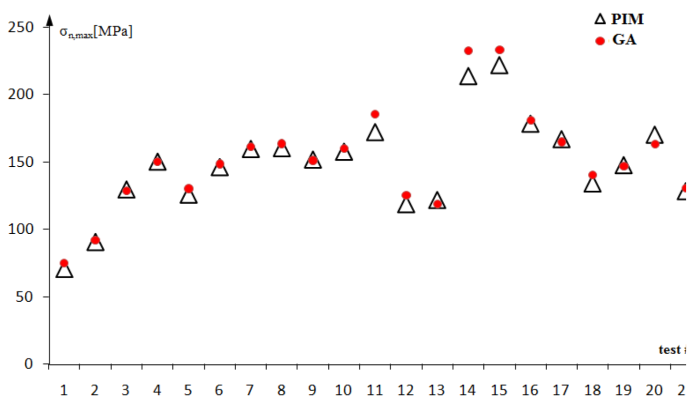


Figure 7(a). τ_a values determined by applying MRH method for PIM and GA process



(b) $\sigma_{n,max}$ values determined by applying MRH method for PIM and GA process

For the sake of comparison, the results obtained in terms of the EI(%) by the GA and PIM are plotted in the same graph for the data set being analyzed (Fig. 8). Note that, the computed EI values by both methods are quite similar. However, the computational costs are significantly different as depicted in Fig. 9. This graph shows the computational time necessary to determine the critical plane by the PIM and GA. The Analysis was divided in two groups depending on the frequency ratio between τ_{xy} and σ_x, λ_{xy}. For load stories where λ_{xy} is smaller than 1 it is necessary to divide the stress history in $2\pi/\lambda_{xy}$. This will increase a lot the computational cost for both methods when compared to tests where λ_{xy} is greater than or equal to 1. It can be seen that the calculation carried out by the GA provides results around 100 times faster than the ones by the PIM. In the cases where $\lambda \geq 1$ the processing time is so small that it can hardly be visualized in such scale.

Tests were carried out in a PC, 2MB of RAM, AMD Athlon ® 64 X2 Dual Core Processor 3600 +. The algorithms of PIM and GA were implemented in the commercial package Matlab ®.

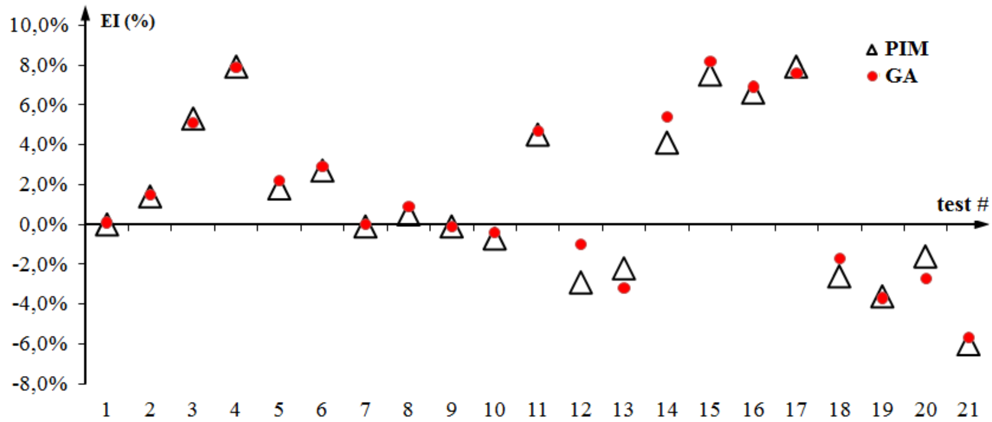


Figure 8. Variation of the Error Index (EI) for each test considering the GA and PIM.

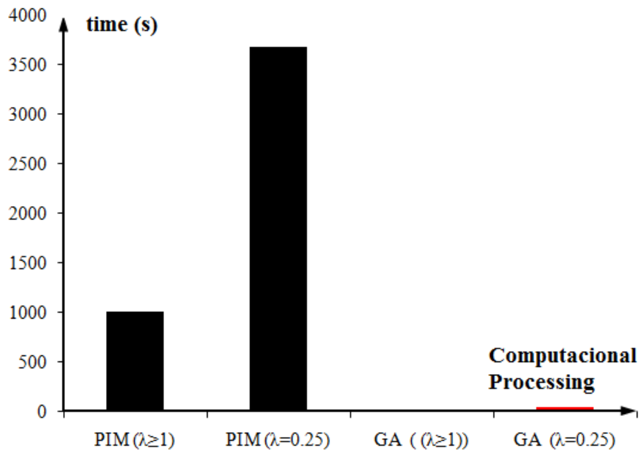


Figure 9. Computational cost to determine the Critical Plane by PIM and GA methods considering the λ_{xy} values.

7. DISCUSSION AND CONCLUSIONS

To find the critical plane using a classical PIM a great number of material planes are usually investigated, turning time processing prohibitively expensive, mainly for practical design of mechanical components based on Finite Element Analysis, where usually thousands of nodes will need to be considered. On the other, to conduct the search process in just few planes may lead to inaccurate results. Further, as discussed previously, it is usual do find stress histories where the definition of the critical plane lead to the existence of local maximum problems. The genetic algorithm proved to be an appropriate optimization tool in such cases. When compared to the PIM it showed the ability to consistently find the global optimum solution without the danger of getting stuck at some local maximum. In the PIM we adopted precision of 1° in determining the critical plane. Large plane increments could have been tried to investigate the accuracy of the method. However, as we sought to compare the accuracy of the results provided by the GA, a small plane increment was chosen but so that the analysis would not to extrapolate the processing time. Comparisons showed the GA is capable to find the critical plane not only with great accuracy but also 100 times faster than the PIM as described.

8. REFERENCES

Bernasconi A. 2002. Efficient algorithms for calculation of shear stress amplitude and amplitude of the second invariant of the stress deviator in fatigue criteria applications. Int J Fatigue. Vol.30, pp. 1430-1440.

Carpinteri A. Spagnoli A. Vantadori S. 2009. Multiaxial fatigue life estimation in welded joints using the critical plane approach. Int J Fatigue. Vol. 31, pp.188-196.

Carpinteri A. Spagnoli A. 2001. Multiaxial high cycle fatigue criterion for hard metals. Int J Fatigue. Vol.23, pp. 135-145.

Dang Van K. 1973. Sur la resistance a la fatigue des metaux. Sci Tech L'armement. Vol. 47, PP.429-453.

Dantas AP. 2009. Proposta alternativa para cálculo da amplitude da tensão cisalhante e uso de algoritmos genéticos para determinação do plano critico em fadiga multiaxial. Universidade de Brasília. Brazil PhD thesis.[in portuguese].

Fatemi A. Kurath P. 1988. Multiaxial fatigue life predictions under the influence of mean stresses. J Eng Mater Technol. Vol. 47, pp. 429-453.

Findley, W.N. 1959. A Theory for the Effect of Mean Stress on Fatigue Metals under Combined Torsion and Axial Load or Bending. Journal of Engineering for Industry. Vol. 81, pp.301-306.

Froustey C. Lasserre S. 1989. Multiaxial fatigue endurance of 30NCD16 steel. Int Fatigue. Vol. 11, pp.169–175.

Goldberg. D. E. 1989. Genetic Algorithms in Search Optimization and Machine Learning. Addison-Wesley Pub. Co.

Heidenreich R. Richter I. Zenner H. 1984. Schubspannungsintensitatshypothese – weitere experimentelle und theoretische Untersuchungen. Konstruktion. Vol. 36, pp.99–104.

Holland. J.H. 1975. Adaptation in Natural and Artificial Systems. University of Michigan Press. Ann Arbor.

Inácio. TB. 2008. Otimização de Modelo de Fadiga Multiaxial por meio de Algoritmo. Genético. Dissertação de Mestrado. Universidade de Brasília. Brazil [in Portuguese].

J.A. Araújo, A.P. Dantas, F.C. Castro, E.N. Mamiya, J.L.A. Ferreira 2011. On the characterization of the critical plane with a simple and fast alternative measure of the shear stress amplitude in multiaxial fatigue. Int Journal of Fatigue (In Press).

Kaniut C. 1983. Zur Betriebsfestigkeit metallischer Werkstoffe bei mehrachsiger Beanspruchung. PhD thesis. Aachen: RWTH Aachen.

MATLAB. The Language of technical computing. The MathsWorks. Inc.

McDiarmid DL. 1991. A general criterion for high cycle multiaxial fatigue failure. Fatigue Fract Eng Mater Struct. Vol. 14, pp.429–453.

Mielke S. 1980. Festigkeitsverhalten metallischer Werkstoffe unter zweiachsiger schwingender Beanspruchung mit verschiedenen Spannungszeitverlaufen. PhD thesis. Aachen: RWTH Aachen.

Mitchell. M.; 1996. " An Introduction to Genetic Algorithm". 1ª Edição. MIT Press. Cambridge. EUA.

Nishihara T. Kawamoto. 1945. M. The strength of metals under combined alternating bending and torsion with phase difference. Mem Coll Eng. Kyoto Imp Univ. Vol.11, pp. 85-112.

Papadopoulos IV. 1994. A new criterion of fatigue strength for out-of phase bending and torsion of hard metals. Int J Fatigue. Vol 16, pp. 377–384.

Susmel L. Lazzarin P. 2002. A bi-parametric Wohler curve for high cycle multiaxial fatigue assessment. Fatigue Fract Eng Mater Struct. Vol. 25, pp. 63–78.

Susmel L. Tovo R. Lazzarin P. 2005. The mean stress effect on the high-cycle fatigue strength from a multiaxial fatigue point of view. Int J Fatigue. ; Vol. 27, pp. 928–943.

Susmel L. 2010. A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems. Int J Fatigue. Vol. 32:1875–1883.