THREE-AXIS ATTITUDE CONTROL SYSTEM WITH REACTION WHEELS FOR LOW-COST UNIVERSITY SATELLITE

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Abstract. *Performance of a low-cost attitude control system for a small 3-axis-stabilized satellite is investigated to ascertain the feasibility of conjugating non-recursive attitude estimation based on vector measurements of the geomagnetic field and Sun direction with attitude control by means of reaction wheels. The configuration has been inspired and motivated by the small satellite series TUBSAT, which uses solar panels on various sides of the satellite and an actuation system involving magnetotorquers and a triad of reaction wheels for exchanging angular momentum. The satellite tumbles most of the time, with its attitude control system and other subsystems powered off to save battery power. The reaction wheels provide 3-axis attitude control. In controlled mode, the wheels absorb the angular momentum to halt tumbling, and provide control torques to point the antennas to Earth. Due to tumbling being interspersed by brief episodes in controlled mode, attitude estimation should provide fast, accurate estimates without any prior knowledge, neither recursive computations, nor waiting for estimation convergence. Two instantaneous methods for attitude estimation have been investigated. The first one employs the direction cosine matrix (DCM) attitude parametrization in the TRIAD method, and the second one is a quaternion estimation approach – the well-established QUEST method. In each case, a different control law commands the reaction wheels: one control approach deals with the DCM error whereas the other handles the quaternion error. Unlike TUBSAT, rate sensors are not available in the small, low-cost satellite and hence fast angular momentum determination has been attained by angular rate estimation based on the available vector measurements combined with a derivative approach. Simulation results for an axisymmetric satellite in a polar orbit are analyzed, and operational characteristics of both combinations of attitude estimator with control law are pointed out.*

Keywords*: 3-axis attitude control, instantaneous attitude estimation, reaction wheel.*

1. INTRODUCTION

Small satellite development has been a promising area of recent research on satellite technology (Kim *et al.*, 1996; Triharjanto *et al.*, 2005). Small satellites have lower development costs and may be used in several small missions aiming at imaging, surveillance and signal retransmission. A successful example of development is the TUBSAT program (Renner, 1999), with seven satellites already launched and operating nowadays.

In general, small satellites have reaction wheels and magnetotorquers for 3-axis attitude control. Reaction wheels are angular momentum exchange devices that provide the main actuation means whereas magnetotorquers produce far less torque, and are generally enabled when a failure in the wheels is detected.

The accuracy requirements for a satellite system heavily depend on its mission description. The accuracy of the attitude control system is directly related to the sensor suite available on board and its quality (Sidi, 1997). In small missions such as TUBSAT, the available sensors are Sun sensors, magnetometer, rate-gyros, and star trackers, whose existence on board depends on mission characteristics. In general, rate-gyros are common sensors on board satellites to measure the inertial angular rate. However, the absence of rate-gyros in low-cost satellites has often demanded the development of angular rate estimation-based attitude control laws (Lizarralde, et al., 1996). Thus, angular rate estimation is here proposed as a substitute for the use of rate-gyros. The objective here is not to develop a control system that complies with a given set of requirements, but to analyze the performance of a proposed low-cost system and to draw conclusions about its feasibility and applicability to a small satellite system.

The paper is organized as follows. Section 1 presents a brief review of the satellite mission and the proposed control system. Section 2 introduces the motion equations. A description of attitude control, and attitude and angular rate estimation is found in Sections 4 and 5, respectively. Finally, Section 6 shows the simulation parameters that have been used and the main results, and in Section 7 the results are discussed. The concluding remarks are given in Section 8.

2. SYSTEM DESCRIPTION

The mission operational characteristics are summarized in Figure 1. Similarly to TUBSAT, the mission can be divided in two operation modes: control and tumbling. When the satellite is not within the line of sight relative to the base station, all satellite systems but ground signal reception are turned off to save on-board battery power. Once the control system is disabled, the satellite starts tumbling until a turn-on command is received from the base station, after which the satellite enters control mode. In controlled mode, the satellite attitude is adjusted according to a commanded reference attitude. The controlled mode enters into effect only when there is line-of-sight visibility from the satellite to both the Sun and the ground control station. The following analysis assumes that the commanded attitude is a constant reference when in control mode.

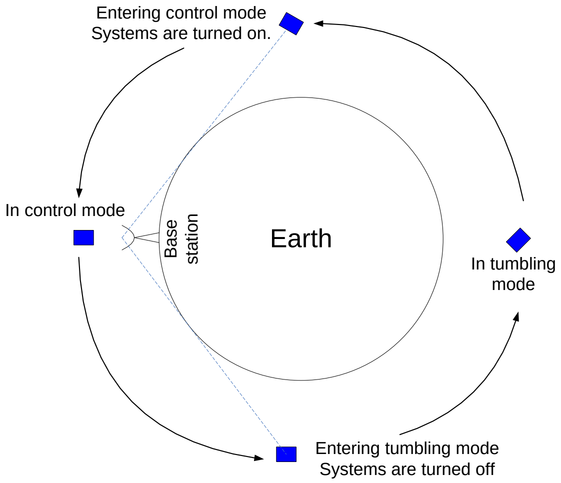

Figure 1. Operation modes for mission analysis.

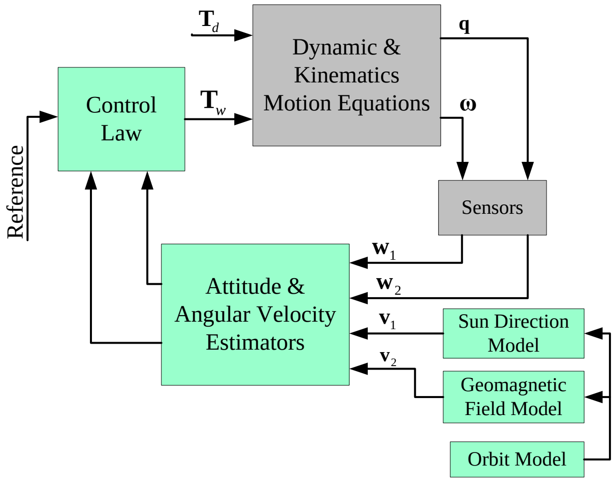

Figure 2. Overview of the proposed control system.

An overview of the structure for the proposed control system is presented in Figure 2.

Due to the low-cost constraint, the only sensors available are a tri-axial magnetometer and a couple of Sun sensors, which provides vector measurements of, respectively, the geomagnetic field and the Sun direction, both represented in the satellite body coordinate frame (S_b). The geomagnetic field has been modeled by a dipole whilst the Sun direction by a simplified algebraic model for the computation of both attitude and angular velocity reference vectors in a geocentric inertial frame (S_i). Both models have been fed by a Keplerian celestial mechanics model on board that provides the satellite position in its orbital flight (Wertz, 1978). The Simplified General Perturbations Satellite Orbit Model 4 (SGP4) has been employed to synthesize sensors data and yield vector measurements in the body coordinate frame S_b . The vector measurements from the sensors, and the vector references from the dipole and Keplerian models have been processed by estimators that output estimates of the satellite attitude and angular velocity for control purposes. The control law generates torque commands to a triad of reaction wheels aligned with the principal axes.

3. SATELLITE MODELING

Despite not being a rigid body due to the reaction wheels, the assumption that the wheels are symmetric about their spin axes and mounted on a rigid structure allows considering the satellite as a rigid body. That way, the attitude dynamics can be described with the well-known Euler equation:

$$\mathbf{I}\dot{\boldsymbol{\omega}}^{bi} = \mathbf{T}_d + \mathbf{T}_m - \mathbf{T}_w - \boldsymbol{\omega}^{bi} \times \left(\mathbf{I}\boldsymbol{\omega}^{bi} + \mathbf{h}_w\right) \tag{1}$$

with vector quantities expressed in S_b . $\mathbf{I}$ is the inertia matrix of the satellite body, $\boldsymbol{\omega}^{bi} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$ is the satellite angular rate vector with respect to the inertial reference frame, $\mathbf{h}_w$ is the reaction wheel angular momentum vector, $\mathbf{T}_d$ is the external disturbance torque vector, $\mathbf{T}_m$ is an auxiliary magnetic control torque vector induced by magnetotorquers, and $\mathbf{T}_w = \begin{bmatrix} T_{wx} & T_{wy} & T_{wz} \end{bmatrix}^T$ is the applied reaction wheel torque vector (Sidi, 1997). Using quaternion parametrization, the kinematic equations of motion describing the satellite attitude are given by:

$$\dot{\breve{q}} = \frac{1}{2} \underbrace{\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}}_{\boldsymbol{\Omega}(\boldsymbol{\omega}^{bi})} \breve{q} = \frac{1}{2} \underbrace{\begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}}_{\mathbf{Q}(\breve{q})} \boldsymbol{\omega}^{bi} \tag{2}$$

where $\breve{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = q_4 + \mathbf{q}$ is the quaternion representation of the satellite attitude (Wertz, 1978). The relation between quaternion and DCM attitude representations is given by:

$$\mathbf{D}_b^i = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{3}$$

4. ATTITUDE CONTROL LAWS

Depending on the attitude parametrization, either DCM or quaternion, there are different ways of representing the error between the satellite attitude (denoted by subscript *S*) and the reference target attitude (denoted by subscript *T*). This paper focuses on two candidate attitude control laws for driving exclusively the reaction wheels: one based on DCM feedback and the other on quaternion feedback, both using proportional derivative (PD) control. Auxiliary magnetic control has not been considered. Below is presented a brief description of these control laws (Sidi, 1997).

4.1 DCM Error-Based Control Law

Given a DCM representation of the satellite attitude $\mathbf{D}_{b,S}^i$ relative to the inertial coordinate frame S_i and a target attitude DCM denoted by $\mathbf{D}_{b,T}^i$, the attitude error is defined as

$$\mathbf{D}_S^T = \mathbf{D}_{b,S}^i \left(\mathbf{D}_{b,T}^i\right)^T \tag{4}$$

where $\mathbf{D}_S^T$ represents a residual rotation to be applied to the target DCM to coincide with the current satellite DCM $\mathbf{D}_{b,S}^i$. Hence, $\mathbf{D}_S^T$ elements a_{ijE} i,j=1,2,3 related to the attitude error should be used by the control law. Whenever $\mathbf{D}_S^T$ matrix becomes identity, $\mathbf{D}_{b,S}^i = \mathbf{D}_{b,T}^i$ and the satellite is in alignment with the desired attitude in space. Thus, a DCM estimate feedback combined with PD control (Sidi, 1997) is defined as

$$\begin{aligned} T_{wx} &= -\frac{1}{2}K_x\left(a_{32E} - a_{23E}\right) - K_{xd}\omega_{Ex} \\ T_{wy} &= -\frac{1}{2}K_y\left(a_{13E} - a_{31E}\right) - K_{yd}\omega_{Ey} \\ T_{wz} &= -\frac{1}{2}K_z\left(a_{21E} - a_{12E}\right) - K_{zd}\omega_{Ez} \end{aligned} \tag{5}$$

where $\mathbf{K}_P = \begin{bmatrix} K_x & K_y & K_z \end{bmatrix}^T$ and $\mathbf{K}_D = \begin{bmatrix} K_{xd} & K_{yd} & K_{zd} \end{bmatrix}^T$ are proportional and derivative control gain vectors, respectively, and $\boldsymbol{\omega}_E = \begin{bmatrix} \omega_{Ex} & \omega_{Ey} & \omega_{Ez} \end{bmatrix}^T$ is the angular error vector defined as

$$\boldsymbol{\omega}_E = \boldsymbol{\omega}_T^{bi} - \boldsymbol{\omega}_S^{bi} \tag{6}$$

with $\boldsymbol{\omega}_T^{bi}$ the target angular velocity, which in the case of an inertially stabilized spacecraft is zero in all axes.

4.2 Quaternion Error-Based Control Law

In the case of quaternion attitude parametrization, the attitude error may be represented by the following definition of quaternion error vector

$$\breve{q}_E = \breve{q}_T^{-1} \otimes \breve{q}_S \tag{7}$$

where $\otimes$ denotes quaternion multiplication operation and $\breve{q}_E$ represents a rotation from target to satellite's attitude.

There is an equivalence between the DCM elements and the elements of the quaternion error (Wertz, 1978). Thus, from Eq.(5), the quaternion estimate feedback combined with PD control (Kim, et al., 1996) is defined as

$$\begin{aligned} T_{wx} &= -2K_x q_{1E} - K_{xd}\omega_{Ex} \\ T_{wy} &= -2K_y q_{2E} - K_{yd}\omega_{Ey} \\ T_{wz} &= -2K_z q_{3E} - K_{zd}\omega_{Ez} \end{aligned} \tag{8}$$

5. ATTITUDE AND ANGULAR VELOCITY ESTIMATION

Attitude and angular velocity estimators are basically of two types: recursive and instantaneous. The former operates continuously using a recursive relation that aims at improving estimation accuracy as time goes by. The latter yields instantaneous estimates regardless of the previous estimator output history.

Given the operational characteristics of the mission, in which the satellite remains hibernating for a large portion of its orbital flight, recursive estimators cannot be used because the satellite should be periodically turned off to save battery power. Therefore, use of instantaneous estimation for both attitude and angular rate estimation is proposed here. For attitude estimation, two algorithms have been investigated, TRIAD and QUEST. Furthermore, angular velocity estimation is the result of a derivative approach.

5.1 TRIAD (Three-axis Attitude Determination) (Shuster and Oh, 1981)

Given two nonparallel reference unit vectors $\hat{\mathbf{v}}_1$ and $\hat{\mathbf{v}}_2$ and the corresponding observation unit vectors $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$ the main idea supporting TRIAD is to find an orthogonal matrix $\mathbf{D}_b^i$ which satisfies

$$\mathbf{D}_b^i \hat{\mathbf{v}}_1 = \hat{\mathbf{w}}_1 \qquad \mathbf{D}_b^i \hat{\mathbf{v}}_2 = \hat{\mathbf{w}}_2$$

Since $\mathbf{D}_b^i$ is overdetermined by the above equations, orthonormal vector bases are constructed as reference and observation coordinate frames, respectively $\{\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3\}$ in (Eq.(9)) and $\{\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2, \hat{\mathbf{s}}_3\}$ in (Eq.(10)), according to

$$\hat{\mathbf{r}}_1 = \hat{\mathbf{v}}_1 \qquad \hat{\mathbf{r}}_2 = \frac{\hat{\mathbf{v}}_1 \times \hat{\mathbf{v}}_2}{|\hat{\mathbf{v}}_1 \times \hat{\mathbf{v}}_2|} \qquad \hat{\mathbf{r}}_3 = \frac{\hat{\mathbf{v}}_1 \times (\hat{\mathbf{v}}_1 \times \hat{\mathbf{v}}_2)}{|\hat{\mathbf{v}}_1 \times \hat{\mathbf{v}}_2|} \tag{9}$$

$$\hat{\mathbf{s}}_1 = \hat{\mathbf{w}}_1 \qquad \hat{\mathbf{s}}_2 = \frac{\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2}{|\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2|} \qquad \hat{\mathbf{s}}_3 = \frac{\hat{\mathbf{w}}_1 \times (\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2)}{|\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2|} \tag{10}$$

Thus, there is a unique orthogonal matrix $\mathbf{D}_b^i$ which satisfies

$$\mathbf{D}_b^i \hat{\mathbf{r}}_i = \hat{\mathbf{s}}_i \quad (i = 1, 2, 3)$$

that is given by

$$\hat{\mathbf{D}}_b^i = \mathbf{M}_{obs} \mathbf{M}_{ref}{}^T = [\hat{\mathbf{s}}_1 \vdots \hat{\mathbf{s}}_2 \vdots \hat{\mathbf{s}}_3][\hat{\mathbf{r}}_1 \vdots \hat{\mathbf{r}}_2 \vdots \hat{\mathbf{r}}_3]^T \tag{11}$$

The estimation error is expressed in terms of a covariance matrix. It should be remarked that although estimation error covariance matrices may be expressed by different forms, the Cartesian error covariance matrix $\mathbf{P}_{\theta\theta}$ is chosen in the present analysis. This choice is convenient because the trace of $\mathbf{P}_{\theta\theta}$ provides a scalar quantity for judging the root-sum-square (RSS) accuracy of the attitude solution, which is independent of the choice of representation and the attitude. The Cartesian attitude covariance matrix for the TRIAD estimate is given by

$$\mathbf{P}_{\theta\theta} = \sigma_1{}^2 \mathbf{I} + \frac{(\sigma_2{}^2 - \sigma_1{}^2)\hat{\mathbf{w}}_1 \hat{\mathbf{w}}_1{}^T + \sigma_1{}^2 (\hat{\mathbf{w}}_1 . \hat{\mathbf{w}}_2)(\hat{\mathbf{w}}_1 . \hat{\mathbf{w}}_2{}^T + \hat{\mathbf{w}}_2 . \hat{\mathbf{w}}_1{}^T)}{|\hat{\mathbf{w}}_1 \times \hat{\mathbf{w}}_2|^2} \tag{12}$$

where σ_1 and σ_2 are the variances of the measurement vectors $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$, respectively (Shuster and Oh, 1981).

5.2 QUEST (QUaternion ESTimator) (Shuster and Oh, 1981)

QUEST is an algorithm for optimal attitude estimation based on finding the DCM $\mathbf{D}_{b,opt}^{i}$ that minimizes the loss function

$$L\left(\mathbf{D}_b^i\right)=\frac{1}{2}\sum_{i=1}^{n}a_i\left|\hat{\mathbf{w}}_i-\mathbf{D}_b^i\hat{\mathbf{v}}_i\right|^2$$

where $a_i, i=1,...,n$ are nonnegative weights. Using quaternion parametrization for attitude representation, the gain function defined as $g\left(\mathbf{D}_b^i\right)=1-L\left(\mathbf{D}_b^i\right)$ can be rewritten as

$$g\left(\breve{q}\right)=\breve{q}^T\mathbf{K}\breve{q} \tag{13}$$

with $\mathbf{K}$ being the 4×4 matrix constructed using the reference and observation unit vectors (Shuster and Oh, 1981). Thus, the optimal attitude quaternion $\breve{q}_{opt}$ is obtained maximizing $g\left(\breve{q}\right)$, which occurs when

$$\mathbf{K}\breve{q}_{opt}=\lambda_{\max}\breve{q}_{opt} \tag{14}$$

where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{K}$. This is the general solution for the optimal problem, which in the q-method resorts to Household transformations to solve for the maximum eigenvalue and the corresponding eigenvector (Shuster, 2006; Wertz, 1978). The proposed solution in the QUEST algorithm is to solve this problem without the need to solve the eigenvalue and eigenvector problem, which is computationally heavy for real-time implementation. Defining $\mathbf{X}$ and γ as quantities obtained by combining the reference and observation vectors (Shuster and Oh, 1981), the QUEST solution for the optimal quaternion is given directly by:

$$\breve{q}_{opt}=\frac{1}{\sqrt{\gamma^2+\left|\mathbf{X}\right|^2}}\begin{bmatrix}\mathbf{X}\\ \gamma\end{bmatrix} \tag{15}$$

where $\mathbf{X}$ and γ depend on $\lambda_{\max}$, and the latter is exactly obtained in closed-form for n=2 vector observations by

$$\lambda_{\max}=\sqrt{a_1^{\,2}+2a_1a_2\cos\left(\theta_V-\theta_W\right)+a_2^{\,2}} \tag{16}$$

with

$$\cos\left(\theta_V-\theta_W\right)=\left(\hat{\mathbf{v}}_1.\hat{\mathbf{v}}_2\right)\left(\hat{\mathbf{w}}_1.\hat{\mathbf{w}}_2\right)+\left|\hat{\mathbf{v}}_1\times\hat{\mathbf{v}}_2\right|\left|\hat{\mathbf{w}}_1\times\hat{\mathbf{w}}_2\right|$$

It is important to emphasize that the above solution is not an approximation. As with the TRIAD algorithm, the estimation error produced by QUEST can also be expressed in terms of the Cartesian covariance matrix, which for the special case of only two observation vectors is given by

$$\mathbf{P}_{\theta\theta}=\sigma_{tot}^{\,2}\mathbf{I}+\frac{\left(\sigma_2^{\,2}-\sigma_{tot}^{\,2}\right)\hat{\mathbf{w}}_1\hat{\mathbf{w}}_1^{\,T}+\left(\sigma_1^{\,2}-\sigma_{tot}^{\,2}\right)\hat{\mathbf{w}}_2\hat{\mathbf{w}}_2^{\,T}+\sigma_{tot}^{\,2}\left(\hat{\mathbf{w}}_1.\hat{\mathbf{w}}_2\right)\left(\hat{\mathbf{w}}_1\hat{\mathbf{w}}_2^{\,T}+\hat{\mathbf{w}}_2\hat{\mathbf{w}}_1^{\,T}\right)}{\left|\hat{\mathbf{w}}_1\times\hat{\mathbf{w}}_2\right|^2} \tag{17}$$

where $\sigma_{tot}^{\,-2}=\sigma_1^{\,-2}+\sigma_2^{\,-2}$.

5.3 Angular Velocity Estimation Using a Derivative Approach

Angular velocity must be estimated since there are no angular velocity measurements available in the proposed satellite sensors suite. As an expeditious solution to this problem, an angular rate estimator is here proposed based on a derivative approach (Bar-Itzhack, 2001; Sidi, 1997). Solving the kinematics in Eq.(2) for the angular velocity vector:

$$\boldsymbol{\omega}^{bi} = 2\mathbf{Q}_L \dot{\breve{q}} \tag{18}$$

where $\mathbf{Q}_L$ is the pseudo-inverse of matrix $\mathbf{Q}(\breve{q})$ (Wertz, 1978). As a result, the computation of the angular velocity vector calls for numerically computing the time-derivative of the quaternion estimates and applying Eq.(18). Due to the numerical differentiation, the computed angular rate vector is contaminated with high-frequency noise that has to be filtered out by a digital filter.

A first order low-pass digital filter has been used for the sake of simplicity, and is given by

$$\hat{\boldsymbol{\omega}}_f^{bi}(k) = \left(1 - \frac{\Delta}{\Delta + \tau}\right)\hat{\boldsymbol{\omega}}_f^{bi}(k-1) + \frac{\Delta}{\Delta + \tau}\hat{\boldsymbol{\omega}}^{bi}(k) \tag{19}$$

where Δ, τ and k represent, respectively, the sample period, the filter time constant and a discrete instant of time. The time constant must be chosen according to the desired control loop bandwidth and the expected satellite motion.

6. SIMULATION AND RESULTS

Simulations have been performed using the *MatLab/Simulink®* environment to investigate the feasibility of the proposed control system. For analysis purposes, a reference mission has been assumed with circular orbit parameters as presented in Table 1.

Mass unbalance yielding an imperfect axisymmetric satellite has been considered. The simulation initial conditions represent the satellite initially aligned with the reference coordinate frame and in tumbling mode with all systems, control system included, turned off. The ground-truth inertia matrix and the initialization parameters are presented in Table 2. The rotation sequence from S_i to S_b has been set to $\psi(3)-\theta(2)-\phi(1)$.

Table 1. Reference mission circular orbit parameters.

Parameter	Value
Launch date/hour	1/1/2012 at 0h:00min:00s
Altitude (*Km*)	600
Orbit Inclination (º)	98.5
Initial Right Ascension of the Ascending Node (RAAN) (º)	320
Initial Argument of Latitude (º) (with respect to the line pointing towards the ascending node)	0

Table 2. Satellite inertia and mass, initial conditions and commanded attitude.

Parameter	Value
Inertia Matrix ($Kg.m^2$)	$\begin{bmatrix} 3.2580 & -0.008 & -0.008 \\ -0.008 & 3.2420 & 0.008 \\ -0.008 & 0.008 & 4.008 \end{bmatrix}$
Mass (Kg)	85
Initial Angular Velocity (rad/s)	$\begin{bmatrix} 8.73\times10^{-2} & 8.73\times10^{-2} & 8.73\times10^{-2} \end{bmatrix}^T$
Commanded attitude	$\psi = 60^o \quad \theta = -5^o \quad \phi = 17^o$

The sensors parameters are presented in Table 3. The angular velocity estimator time constant (τ) in Eq.(19) has been adjusted to 1 s.

For battery charge consumption analysis it has been considered that the torque constant of the reaction wheels' motors are given by 0.023N.m/A (Carrara and Milani, 2007) and the wheel inertias are 8.8×10^{-4} Kg.m^2 (Hardhienata *et al.*, 2005).

Table 3. Sensors parameters.

Parameter		Value
Magnetometer	Bias (*T*)	2×10^{-7}
	Noise variance (T^2)	4×10^{-14}
Sun sensor	Noise variance (rad^2)	$(0.5\pi/180)^2$

The control gains in Eq. (5) are adjusted according to (Ismail *et al.*, 2010):

$$\begin{matrix} K_{pi} = \omega_n^{\;2} I_{ii} \\ K_{di} = 2\xi\omega_n I_{ii} \end{matrix}, \quad i = x, y, z \tag{20}$$

where ω_n and ξ are the desired natural frequency and damping ratio, respectively, for the closed-loop response.

Supposing an overshoot of 10% and a rise time (10-90%) of 5 seconds, the control gains are obtained as presented in Table 4. This is not the optimal tuning because an approximation has been used instead of more sophisticated tuning algorithms (Fan, et al., 2002; Kim, et al., 1996). Use of a more complex tuning is not fundamental to the present analysis though.

Table 4. Control gains adjustment.

Control Gain	Value
$\mathbf{K}_P\,(N.m/s)$	$[0.457 \quad 0.457 \quad 0.562]^T$
$\mathbf{K}_D\,(N.ms/rad)$	$[1.438 \quad 1.438 \quad 1.770]^T$

Using the aforementioned configuration, the next Sections are going to explore the performance of the attitude control system and of the estimators to determine the performance of the control system for the following combinations of control law with attitude estimator:

- **Case I**: DCM error based control law and TRIAD attitude estimator;
- **Case II**: Quaternion error based control law with QUEST attitude estimator.

Because of the random signals from the sensors and the uncertainty in the initial attitude and tumbling, the performance analysis has been carried out with a Monte Carlo simulation with 50 realizations. The initial attitude in terms of Euler angles may vary randomly between -90º and 90º according to a uniform density probability. The final result is given in terms of means (identified by the bar upon the variable) and standard deviations (identified by σ). In the following, the mean value added to 3σ has been referred to as the "worst condition".

6.1. Attitude Control Performance

The use of different attitude parameterizations in the attitude control laws leads to different control error definitions that preclude a direct comparative analysis (Sidi, 1997). Thus, a common error metric must be chosen. Thus, the performance of each attitude control system has been evaluated in terms of the error angle α about the Euler axis of rotation (Wertz, 1978) as defined in Eq.(21).

$$\alpha = \cos^{-1}\{0.5\cdot(\mathrm{trace}(\mathbf{D}_S^T) - 1\} \tag{21}$$

The evolution of the error angle α is an indication of the accuracy of the control laws, because it shows the satellite misalignment with respect to the commanded attitude during the maneuver. Figure 3 presents the evolution of $|\alpha|$ for cases I and II. These figures demonstrate the correct operation of the proposed control loops.

The integral of $|\alpha|$ is another important criterion for comparing the different attitude control laws (Sidi, 1997). The lower the value of this integral, the lower is the angular path performed by the satellite to achieve the commanded attitude and, consequently, the more effective is the control system. Figure 4 presents the evolution of this integral for the worst condition of each case.

Figure 5 shows the angular momentum stored in the reaction wheels for the worst condition of each case. During operation, it is important to guarantee that the angular momentum that is absorbed by the wheels does not yield wheel saturation to avoid performance deterioration of the attitude control system.

Figure 6 presents the overall battery charge required to drive the three motors acting on the reaction wheels triad in the worst condition of each case.

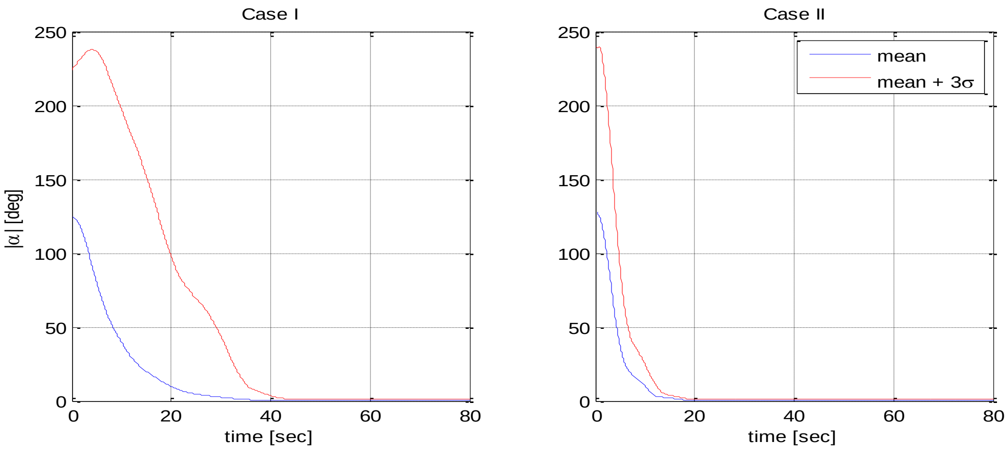

Figure 3. Euler angle evolution for combined control laws with attitude estimators.

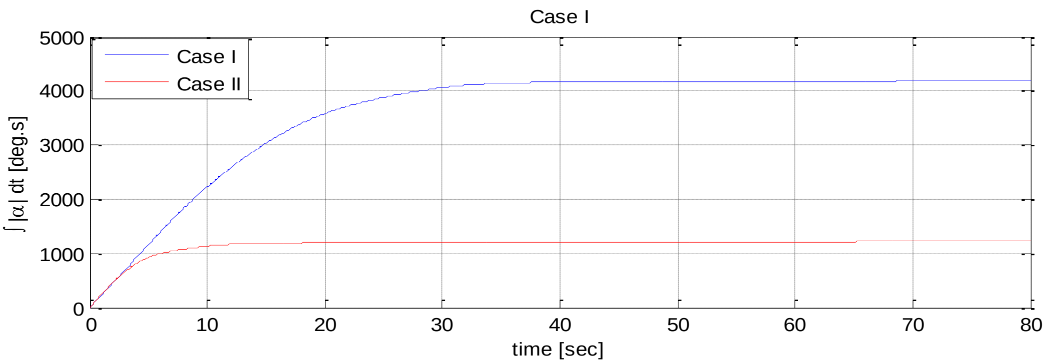

Figure 4. Integral of error angle |α| about the Euler axis of rotation (worst condition).

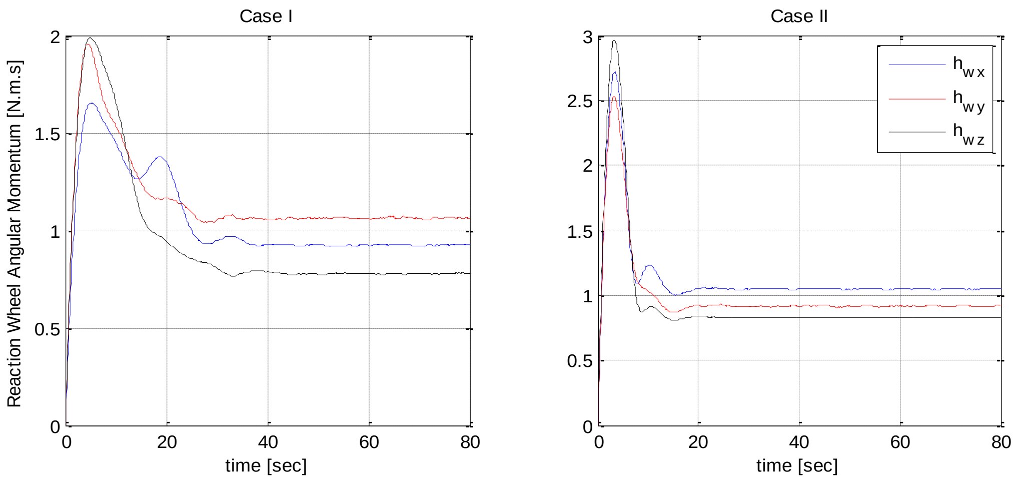

Figure 5. Angular momentum accumulated in reaction wheels (worst condition).

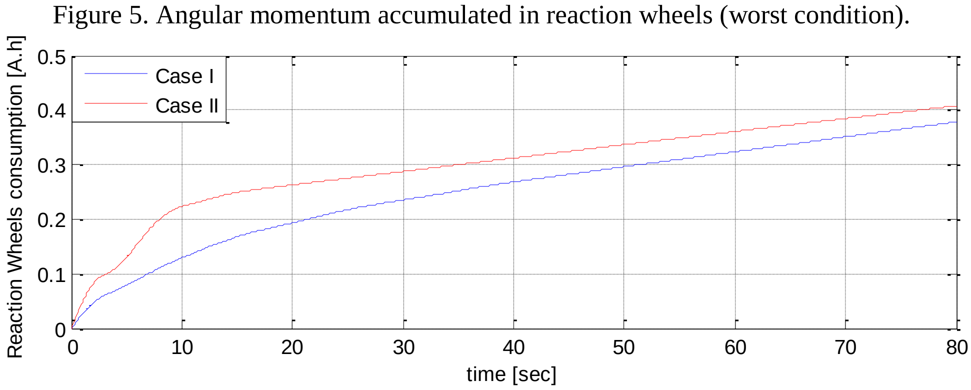

Figure 6. Reaction wheels battery charge consumption (worst condition).

6.2 Estimation Performance

The accuracy of the estimators is displayed in Figure 7, evaluated in terms of root-sum-square (RSS) as expressed by Eq.(22) (Shuster and Oh, 1981).

$$RSS = trace\left(\mathbf{P}_{\theta\theta}\right) \tag{22}$$

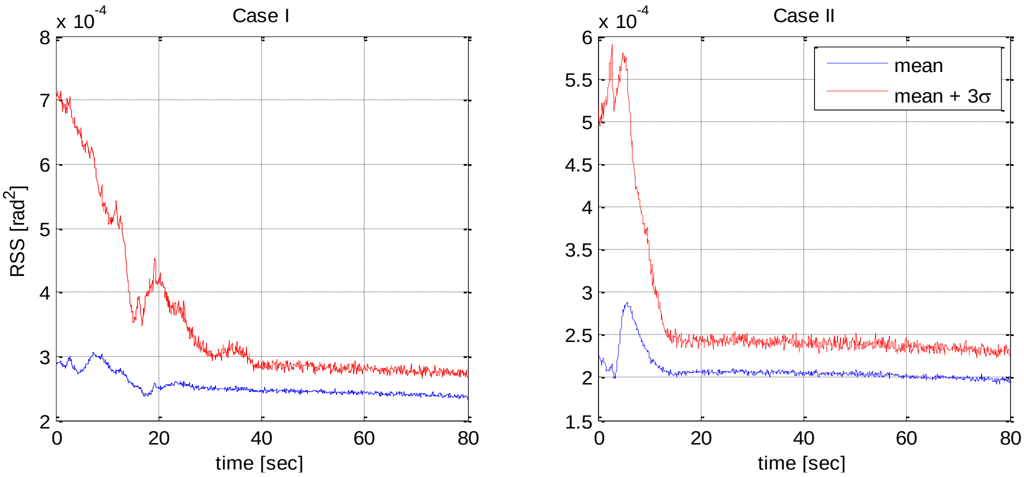

Figure 7. RSS accuracy for attitude estimators in cases I and II.

7. ANALYSIS AND DISCUSSION

The results indicate that 3-axis attitude control is effective in both cases, in which α converges to a mean pointing error of 0.5^o and 1.4^o in the worst condition ($\bar{\alpha} + 3\sigma_\alpha$).

Although both cases present similar steady-state performance, as mentioned above, a significant difference is noted in the transient response. Case I has a settling time of 45 seconds whilst Case II is only 20 seconds.

It can also be noted in Figure 3 that the maneuvering for Case I demands more time than for Case II. The satellite angular motion in Case I presents oscillations with less damping that end up increasing the settling time and the angular path traversed by the satellite prior to adequate attitude convergence. The latter is emphasized in Figure 4 where the enormous difference between angular paths obtained in each case can be seen, and confirms the superiority of the quaternion-based control law and the optimal QUEST estimator approach over the DCM-based control law and TRIAD attitude estimator.

One possible explanation for the better performance in Case II is given again by Figure 7. It has been claimed (Shuster, 2006) that QUEST renders a more accurate estimation than TRIAD and this investigation reinforces that claim. Also, given that attitude estimates are the main information for angular velocity estimation with a derivative approach, it is straightforward that more accurate angular velocity estimates will be obtained in Case II due to better attitude estimation, and therefore the reaction wheels are commanded more effectively.

The satellite is initially in the tumbling operation mode. When in controlled mode, the reaction wheels succeed in attitude stabilization by absorbing the angular momentum of the tumbling satellite. Therefore, the reaction wheels remain spinning while attitude stabilization of the rest of the satellite body is achieved, and then attitude adjustments for alignment with the target attitude can proceed. Figure 5 shows the stored angular momentum in the reaction wheels, which is quite useful for the specification of proper reaction wheels.

A disadvantage in Case II is the peak angular momentum stored in the wheels during maneuvers that is about 50% greater than in Case I. This can be a decisive factor when choosing the control approach, mainly in projects with a tight budget. For the mission that has been assumed here, the reaction wheels should be chosen such that its saturation occurs with an angular momentum magnitude higher than 3N.m.s for Case II, and 2N.m.s for Case I. Another point in favor of Case I approach is the slightly lower battery charge consumption when compared with the results of Case II seen in Figure 6. Considering the specific motor chosen and a simulation window of 80 seconds, the battery must be capable of providing around 0.38A.h and 0.43A.h for the reaction wheels, respectively, in Cases I and II.

An identical motor has been used by Viegas (2010) with a much heavier, $2.144\text{x}10^{-1}$ Kg.m^2 inertia wheel in a dual-spin configuration for the same satellite parameters seen in Table 2 to yield an alternative three-axis, gyroscopic stiffness-based attitude control system. According to Viegas (2010), that single momentum wheel demanded approximately 2A.h during 36 hours of operation. One should recall that a dual-spinner satellite is powered on all throughout the orbit, whereas both TUBSAT and its derivative here investigated need to be powered on only for a small portion of the orbit – when controlled mode is in effect. Given the constrained power generation on board small, low-cost satellites, the results presented herein strongly indicate that hibernating through most of the mission and awakening for a brief period when overflying the desired site can be far more advantageous for low-cost, small satellites.

From the previous analysis, the approach in Case II is the best in performance, whilst Case I is the best in power consumption and reaction wheel sizing, which may be interesting characteristics for low-cost, small size systems. Thus, if tight limitations on the size, mass, and cost of the reaction wheels are not hard constraints, then the approach in Case II is preferable. Otherwise, the designer should choose instead the approach in Case I.

8. CONCLUSIONS

This paper presents a performance analysis of a proposed attitude control system for a 3-axis stabilized satellite using only reaction wheels as attitude actuators. The low cost approach has been reached by usage of Sun sensors and a magnetometer, and resorting to an angular velocity estimator instead of rate-gyros. TRIAD and QUEST, which are instantaneous, non-recursive attitude estimation algorithms have been probed for attitude estimation. Different control laws based on PD logic have been used, one based on the quaternion error and the other on DCM error. The simulation has been conducted considering reference mission parameters, and the results showed that although both attitude control strategies yield similar steady-state performances, the combination of the quaternion error-based control law with the QUEST estimation algorithm yields an improved transient response. The maximum angular moment to be stored in the reaction wheels has not exceeded 3N.m.s in magnitude in this approach, and 2N.m.s in the alternative DCM-based control law combined with the TRIAD attitude estimator. Battery charge consumption has not exceeded 0.43A.h in the worst case (Case II), not passing 0.38A.h for Case I. In general, the investigation of both combinations of control law with estimator has indicated the feasibility of the overall attitude control system for application to a low-cost small satellite with a mean pointing error of 0.5^o and a settling time less than 20 seconds for the quaternion-based control law/QUEST attitude estimator control system and 45 seconds for the alternative approach.

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