# STRUCTURAL AND ECONOMICAL OPTIMIZATION OF WELDED STEEL STORAGE TANKS 

Leonardo Roncetti, leonardo@techcon.eng.br

TechCon Engenharia e Consultoria - Avenida Nossa Senhora da Penha, 699 - B, sl. 913-29056-250, Vitória, ES, Brasil.
Abstract. With the growing demand for storage of bulk liquids in the industries of oil and gas, petrochemical, chemical, steel, cellulose, environmental control, food and others, the storage facilities increase their influence in the overall cost of projects. Assessment is made for factors such as structural, physical, geometrical and economical that affects the optimization for lower weight of atmospheric above ground welded steel tanks, made from carbon steel, for the storage of liquids.
Through a software, it is determined, for several volumes stored, how the optimization leads to lower cost of tanks considering the diameter, height, thicknesses of shell plates along the height, structure for roof support, fabrication, erection and other factors. It is also analyzed the support for the decision on the optimal quantity of tanks for a given volume, specifying if it is preferable more tanks with less volumes or less tanks with higher volumes.
The analyzed tanks are designed according to API 650 Standard $11^{\text {th }}$ Edition.
Keywords: Storage tanks, optimization, tanks, storage, API 650.

## 1. INTRODUCTION

In recent years, with the world economic growth, there was an increase of the demand for storage of bulk liquids in various economic activities: oil and gas, biofuels, chemical, steel mills, food, and others. There was also the increase of the demand of tanks for intermediate activities: sanitation, waste treatment, refrigeration, and other industrial processes where it is necessary to store or process liquids.

With the growth of the strategic importance of storage, the cost of the equipment used, directly and significantly affects the final cost of the projects of storage facilities.

The most used means are the storage tanks, among which are highlighted those fabricated in steel by welding, mostly aboveground and cylindrical, with various diameters and heights.

It is necessary hence to study the optimization of storage tanks, defining geometric relations that lead to more economic equipment for the same volume stored.

This optimization is done by determining the ratio $(D / H)$ of diameter of the tank $(D)$ over its height $(H)$ that leads to the minimum steel consumption, which leads directly or indirectly to a more economical tank. The calculations are done by a software developed by the Author.

Even obtaining the optimal relations, other technical and economical requirements must be considered, for example, the area available for the tankage installation, which could limit the diameter of the tanks, requirements of the industrial process that may require tanks with great heights, split of the total volume in many areas of the plant and others.

All tanks considered in this paper are atmospheric, aboveground, cylindrical, with fixed cone roof or open-top. The joint efficiency is 1,0 , the specific gravity of liquid is 1,0 and there is no corrosion allowance.

## 2. OPTIMIZATION OF UNIFORM THICKNESS TANKS

Considering the theoretical case, which purpose is to determine what geometry leads to the lowest weight, and considering all the plates with the same thickness to be used in tanks, there are basically the situations of open-top tanks and fixed cone roof tanks.

In both of them, the goal is to determine the $D / H$ ratio that leads to the lowest area of the plates. Both solutions are well defined and explicit, since the roof is not inclined more than 9 degrees.

These two cases, in practice, cover the tanks of small volume, which can be shop-assembled, where the constructive requirement rules and practice for thickness of the plates exceeds the strength criteria.

For a fixed cone roof tank, with a given volume $V$, this volume is expressed by Eq. (1), where $D$ is the diameter of the tank.

$$
\begin{equation*}
V=\frac{\pi D^{2} H}{4} \tag{1}
\end{equation*}
$$

The total height of the shell of the tank, $H$, in function of the volume and diameter is:

$$
\begin{equation*}
H=\frac{4 V}{\pi D^{2}} \tag{2}
\end{equation*}
$$

The total external area of plates of the fixed cone roof tank, $A_{l}$, in function of the diameter can be defined by Eq. (3), with the parts relating to the bottom, shell courses and roof, respectively, and $\theta$ is the angle of the roof with the horizontal.

$$
\begin{equation*}
A_{l}(D)=\frac{\pi D^{2}}{4}+\frac{4 V}{D}+\frac{\pi D^{2}}{4 \cos (\theta)} \tag{3}
\end{equation*}
$$

Taking $\cos (\theta)=1$, which is valid for inclinations of less than $9^{\circ}$, differentiating $A_{l}$ with respect to $D$, it is possible to determine an optimal diameter, $D_{l}$ that leads to a minimum lateral area $A_{l}$.

$$
\begin{align*}
& A_{I}^{\prime}(D)=\pi D-\frac{4 V}{D^{2}}=0  \tag{4}\\
& D_{I}=\sqrt[3]{\frac{4 V}{\pi}} \tag{5}
\end{align*}
$$

Thus, for a volume $V$, the diameter $D_{l}$ leads to the lighter tank with fixed cone roof. By substituting Eq. (5) into Eq. (2) we obtain the optimal $D / H$ ratio:

$$
\begin{equation*}
\frac{D_{1}}{H}=1 \tag{6}
\end{equation*}
$$

It follows that for a given volume $V$, tanks with homogeneous thickness have less weight when the diameter / height ratio is equal to 1 .

For open-top (no roof) tanks, eliminating the last part of Eq. (3) and adopting the same procedure of differentiation, we have for the optimal diameter, $D_{2}$.

$$
\begin{equation*}
D_{2}=2^{3} \sqrt{\frac{V}{\pi}} \tag{7}
\end{equation*}
$$



Figure 1.Weight of a tank with respect to $D / H$ ratio for constant volume, open-top or with fixed cone roof, for bottom, shell and roof with the uniform plate thickness.

By substituting Eq. (7) into Eq. (2) we obtain the optimal $D / H$ ratio for an open-top tank:

$$
\begin{equation*}
\frac{D_{2}}{H}=2 \tag{8}
\end{equation*}
$$

The relations of Eq. (6) and Eq. (8) were confirmed by the Author, using his software, for tanks with elements of the same plate thickness, as well as studies made by Brownell and Young (1959).

From Fig. 1, it is concluded that the adoption of $D / H$ ratios smaller than the optimum, will lead faster to heavier tanks.

## 3. METHODOLOGY FOR DETERMINING THE WEIGHT OF API 650 TANKS

Tanks with all the plates with the same thickness correspond to a small portion of the universe of dimensions used in the design practice. Therefore, it is necessary to analyze and determine the optimal $D / H$ ratios for the cases where the bottom thickness varies, considering there may be even two different bottom thicknesses in the case of annular plates, thicknesses that vary along the height of the shell, as well as the roof and its supporting structure.

The following is the method used for the calculation of the thicknesses and weights of the tanks, according to API 650 (2007) criteria.

### 3.1. Bottoms without annular plates

To determine the weight of the bottom without annular plates, the plates were considered to be cut as shown in Fig. 2, and overlapped 5 times the plate thickness. Although API 650 (2007) states that the lap need not to be greater than 25 mm , this paper considers the lap to have at least 5 times the plate thickness.

To consider the overweight due to the plate overlap, situations for commercial plate thicknesses generally used in bottoms were simulated, determining the percentage of overlapped area with respect to the total area of the bottom $\mathrm{fsc} \%$. The results are expressed in Tab. 1.

Table 1. Percentage of overlapped area with respect to the total area of the bottom of the tank (fsc\%).

| Thickness of the <br> bottom plate <br> $(\mathrm{mm})$ | Overlap <br> $(\mathrm{mm})$ | fsc\% for plates <br> 1.800 mm wide <br> $(\%)$ | fsc\% for plates <br> 2.400 mm wide <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
| 4,75 | 25 | 1,71 | 1,36 |
| 6,3 | 35 | 2,26 | 1,90 |
| 8,0 | 40 | 2,76 | 2,18 |
| 9,5 | 50 | 3,47 | 2,73 |
| 12,5 | 65 | 4,56 | 3,59 |

In this paper, all the elements of the tanks are made from plates with 2.400 mm wide.
The values from Tab. 1 can also be used for the calculation of the weight of the roof, considering the plate overlap.


Figure 2. Typical arrangement of bottom plates with annular plates (left) and for bottoms without annular plates or roofs (right), adapted from PETROBRAS (2007).

### 3.2. Bottom with annular plates

To determine the weight of the bottom with annular plates, these were considered to have circular shape in the outer part of the first course and polygonal shape in the inner region of the first course, as shown in Fig. 2. For the central region, the plates were considered to be overlapped with each other, as shown in Tab. 1, and also overlapped with the annular plate. The minimum width of the annular plate is 750 mm .

The dimensions and hence the number of annular plates that compose the bottom were determined so that it is possible to cut 2 annular plates using one single plate with 2.400 mm wide, and it considers that the total number of annular plates is always an even number. These aim to reduce the leftover material, to provide a limited amount of butt weld joints between the annular plates and to always lead to the symmetry of the quadrants, as shown in Fig. 2.

The software developed by the Author was used for the automatic detailing of the bottoms of the tanks in a CAD system, creating bill of materials, detailing bottoms with diameters from 15 to 50 m in the above conditions. It was determined that the fraction of the annular plates with respect to the total area of the bottom, $f c a$ can be expressed by Eq. (9).

$$
\begin{equation*}
f_{c a}=3,2 D^{-0,93} \tag{9}
\end{equation*}
$$

Where $D$ is the internal diameter of the tank, in the region of the first shell course. Eq. (9) can be used for tanks with diameters from 10 m to 80 m , overestimating the $f c a$ for diameters smaller than 10 m .

For example, for a tank with 30 m of diameter, we have $f_{c} a$ equal to 0,135 , that is, $13,5 \%$ of the area of the bottom of the tank is composed by annular plates.

The thicknesses of the annular plates were determined according to API 650 (2007), in function of the stresses acting in the first course, as stated in the item 5.5.3 of this Code.

For tanks with diameter up to $6,0 \mathrm{~m}$, a flat bottom is adopted and a bottom with slope of $1 / 120$ in the other cases.

### 3.3. Roof

For diameters up to $4,50 \mathrm{~m}$, a self-supporting fixed cone roof with a plate $4,75 \mathrm{~mm}$ thick is adopted, and for diameters up to $6,0 \mathrm{~m}$, a self-supporting fixed cone roof with a plate $6,3 \mathrm{~mm}$ thick is adopted. For larger diameters, a supported cone roof with a plate $4,75 \mathrm{~mm}$ thick is adopted. In all cases, it is considered steel with yield strength of 205 MPa without any corrosion allowance for the roof plate.

The inclination of the roofs are always the minimum allowed by API 650 (2007) for each type: $9,5^{\circ}(1 / 6)$ for selfsupporting cone roof, and $3,6^{\circ}(1 / 16)$ for supported cone roof. In any case, a live load of $1,0 \mathrm{kN} / \mathrm{m}^{2}$ is adopted on the horizontal projected area of the roof.

The roof plate weight is calculated in function of the thickness and by applying the factor due to the overlap as stated in Tab. 1.

The supported roofs use columns of circular pipe whose sizes vary in function of the length of the column. Generally, the maximum slenderness ratio established by API 650 (2007) rules the design, that is, $L / r \leq 180$, where $L$ is the length of the column and $r$ is the minimum radius of gyration of the section. Table 2 presents columns that can be used for the pre-design, in function of their lengths. The sections presented are classified as "compact" according to the AISC (1989) criteria.

Table 2. Circular pipes for structural columns supporting roofs frames.

| Column length (L) | Pipe size to be used |
| :---: | :---: |
| $\mathrm{L} \leq 10,0 \mathrm{~m}$ | Ø8" SCH40 |
| $10,0 \mathrm{~m}<\mathrm{L} \leq 13,5 \mathrm{~m}$ | Ø8" SCH20 |
| $13,5 \mathrm{~m}<\mathrm{L} \leq 16,5 \mathrm{~m}$ | Ø10" SCH20 |
| $16,5 \mathrm{~m}<\mathrm{L} \leq 20,0 \mathrm{~m}$ | Ø12" STD |
| $20,0 \mathrm{~m}<\mathrm{L} \leq 25,0 \mathrm{~m}$ | Ø16" SCH20 |

The roof supporting structure is composed of secondary beams, or rafters, that support the plates, girders that support the rafters and columns that support the girders. A typical arrangement of these profiles is shown in Fig. 3.


Figure 3. Typical arrangement of a supported fixed cone roof of a tank, adapted from Brownell and Young (1959) and Long and Garner (2004).

For the calculation of the weight of the roof structure, the design of these structures for tanks with diameter from 7 to 50 m has been done, without including the weight of the columns, which are chosen using Tab. 2. It is verified that the weight of the supporting structure of the roof is between 0,14 and $0,15 \mathrm{kN} / \mathrm{m}^{2}$, considering the projected area of the tank, that is, the area equivalent to the bottom, and the highest values are for the tanks with smaller diameters.

### 3.4. Shell courses

Through the same software developed by the Author, the thickness of each plate that composes the courses was determined for each $D / H$ ratio, considering all the normative and constructive aspects required by API 650 (2007). The software is also responsible for the calculation of the bottom, roof and its structure, when appropriate, doing the summation to determine the total weight.

The "1-foot method" was used for the calculation of the thicknesses, without any corrosion allowance and joint efficiency of 1,0 .

The thicknesses of the steel plates used for all the simulations were, in millimeters: 4,75, 6,30, $8,00,9,50,12,50$, $16,00,19,00,22,40,25,00,31,50,37,50$, all 2.400 mm wide.

## 4. OPTIMIZATION RESULTS FOR API 650 TANKS

Next, Tab. 3, Tab. 4 and Tab. 5 show the results of the optimization to determine the $\mathrm{D} / H$ ratio that leads to the lesser weight, for the indicated situations.

The tables can be used to show what would be the minimum weight for determined tanks, since the indicated values are the minimum weights for technically feasible tanks.

Table 3. Optimization of tanks with fixed cone roof, steel with $f y=250 \mathrm{MPa}$, no corrosion allowance, specific gravity of 1,0 , joint efficiency of 1,0 .

| $\begin{aligned} & \text { Volume } \\ & \left(\mathrm{m}^{3}\right) \end{aligned}$ | Optimal D/H | $\begin{gathered} \mathrm{D} \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} \mathrm{H} \\ (\mathrm{~m}) \end{gathered}$ | Optimal total weight (kN) | Shell courses (kN) | Bottom plate (kN) | Roof plate (kN) | Structure of the roof (kN) | Optimal range for D/H ${ }^{(1)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 0,70 | 3,55 | 5,06 | 32,4 | 23,8 | 4,9 | 3,7 | 0,0 | 0,35 to 1,35 |
| 75 | 0,75 | 4,15 | 5,54 | 41,9 | 30,1 | 6,7 | 5,1 | 0,0 | 0,40 to 0,95 |
| 100 | 0,70 | 4,47 | 6,38 | 50,4 | 36,7 | 7,7 | 5,9 | 0,0 | 0,40 to 0,85 |
| 125 | 0,55 | 4,44 | 8,07 | 58,6 | 45,1 | 7,7 | 5,8 | 0,0 | 0,35 to 0,95 |
| 150 | 0,45 | 4,41 | 9,81 | 66,9 | 53,6 | 7,6 | 5,7 | 0,0 | 0,35 to 1,10 |
| 160 | 0,40 | 4,34 | 10,84 | 70,7 | 57,8 | 7,3 | 5,5 | 0,0 | 0,30 to 1,05 |
| 200 | 0,65 | 5,49 | 8,45 | 81,7 | 58,2 | 11,7 | 11,9 | 0,0 | 0,30 to 1,00 |
| 250 | 0,65 | 5,91 | 9,10 | 94,5 | 67,1 | 13,6 | 13,8 | 0,0 | 0,35 to 1,00 |
| 300 | 0,55 | 5,94 | 10,81 | 106,7 | 79,1 | 13,7 | 13,9 | 0,0 | 0,35 to 1,10 |
| 400 | 0,40 | 5,88 | 14,71 | 131,7 | 104,6 | 13,4 | 13,6 | 0,0 | 0,30 to 1,35 |
| 500 | 0,85 | 8,15 | 9,59 | 152,8 | 97,0 | 25,8 | 19,4 | 10,6 | 0,40 to 1,35 |
| 600 | 0,70 | 8,12 | 11,60 | 171,8 | 115,3 | 25,6 | 19,2 | 11,7 | 0,45 to 1,35 |
| 750 | 0,70 | 8,74 | 12,49 | 198,5 | 133,2 | 29,7 | 22,3 | 13,3 | 0,50 to 1,35 |
| 1.000 | 0,75 | 9,85 | 13,13 | 239,1 | 157,3 | 37,6 | 28,3 | 15,9 | 0,50 to 1,35 |
| 1.250 | 0,85 | 11,06 | 13,01 | 277,1 | 175,1 | 47,5 | 35,7 | 18,9 | 0,50 to 1,40 |
| 1.500 | 0,75 | 11,27 | 15,03 | 312,4 | 204,6 | 49,3 | 37,1 | 21,4 | 0,60 to 1,45 |
| 2.000 | 0,95 | 13,42 | 14,13 | 379,7 | 229,8 | 70,0 | 52,6 | 27,3 | 0,75 to 1,30 |
| 2.500 | 0,90 | 14,20 | 15,78 | 462,9 | 296,8 | 78,3 | 58,8 | 29,0 | 0,75 to 1,05 |
| 3.000 | 0,80 | 14,51 | 18,14 | 550,0 | 369,9 | 81,8 | 61,4 | 36,9 | 0,70 to 0,85 |
| 4.000 | 1,55 | 19,91 | 12,85 | 743,6 | 409,0 | 153,4 | 115,7 | 65,6 | 1,15 to 1,80 |
| 4.500 | 1,20 | 19,02 | 15,85 | 831,7 | 512,8 | 139,8 | 105,5 | 73,6 | 1,20 to 1,35 |
| 5.000 | 2,10 | 23,73 | 11,30 | 918,9 | 431,8 | 218,0 | 164,3 | 104,8 | 1,30 to 2,70 |
| 6.000 | 1,95 | 24,61 | 12,62 | 1.067,9 | 542,6 | 234,3 | 176,6 | 114,3 | 1,75 to 2,55 |
| 7.500 | 1,70 | 25,32 | 14,89 | 1.313,3 | 737,2 | 248,2 | 187,0 | 140,9 | 1,60 to 2,90 |
| 8.000 | 1,90 | 26,85 | 14,13 | 1.386,2 | 750,6 | 279,1 | 210,3 | 146,3 | 1,65 to 2,65 |
| 10.000 | 2,60 | 32,11 | 12,35 | 1.726,9 | 810,5 | 399,3 | 300,8 | 216,3 | 2,25 to 3,45 |
| 12.500 | 2,30 | 33,20 | 14,44 | 2.150,5 | 1.130,5 | 427,0 | 321,6 | 271,3 | 2,10 to 3,00 |
| 15.000 | 2,90 | 38,12 | 13,14 | 2.562,9 | 1.278,3 | 563,0 | 423,9 | 297,7 | 2,30 to 3,80 |
| 20.000 | 4,25 | 47,66 | 11,21 | 3.344,6 | 1.420,3 | 880,3 | 662,5 | 381,4 | 4,00 to 5,20 |
| 30.000 | 3,55 | 51,38 | 14,47 | 4.874,5 | 2.567,5 | 1.023,2 | 770,0 | 513,8 | 3,40 to 5,40 |
| $40.000{ }^{(2)}$ | 4,50 | 61,20 | 13,60 | 6.517,7 | 3.257,0 | 1.452,1 | 1.092,6 | 716,1 | 4,05 to 6,80 |

${ }^{(1)}$ : The optimal range indicates the $D / H$ ratio where the variation of the total weight is up to $5 \%$ for more or less compared to the optimal weight of the tank.
${ }^{(2)}$ : The diameter of the tank that exceeds the limit of application of the " 1 -foot" method.

Table 4. Optimization of tanks with fixed cone roof, steel with $f y=205 \mathrm{MPa}$, no corrosion allowance, specific gravity of 1,0 , joint efficiency of 1,0 .

| Volume ( $\mathrm{m}^{3}$ ) | $\begin{gathered} \text { Optimal } \\ \text { D/H } \end{gathered}$ | $\underset{(\mathrm{m})}{\mathrm{D}}$ | $\begin{gathered} \mathrm{H} \\ (\mathrm{~m}) \end{gathered}$ | Optimal total weight (kN) | Shell courses (kN) | Bottom plate (kN) | Roof plate (kN) | Structure of the roof (kN) | Optimal range for D/H ${ }^{(1)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 0,70 | 3,55 | 5,06 | 32,4 | 23,8 | 4,9 | 3,7 | 0,0 | 0,35 to 1,35 |
| 75 | 0,75 | 4,15 | 5,54 | 41,9 | 30,1 | 6,7 | 5,1 | 0,0 | 0,40 to 0,95 |
| 100 | 0,70 | 4,47 | 6,38 | 50,4 | 36,7 | 7,7 | 5,9 | 0,0 | 0,40 to 0,85 |
| 125 | 0,55 | 4,44 | 8,07 | 58,6 | 45,1 | 7,7 | 5,8 | 0,0 | 0,35 to 0,95 |
| 150 | 0,45 | 4,41 | 9,81 | 66,9 | 53,6 | 7,6 | 5,7 | 0,0 | 0,35 to 1,10 |
| 160 | 0,40 | 4,34 | 10,84 | 70,7 | 57,8 | 7,3 | 5,5 | 0,0 | 0,30 to 1,05 |
| 200 | 0,65 | 5,49 | 8,45 | 81,7 | 58,2 | 11,7 | 11,9 | 0,0 | 0,30 to 1,00 |
| 250 | 0,65 | 5,91 | 9,10 | 94,5 | 67,1 | 13,6 | 13,8 | 0,0 | 0,35 to 1,00 |
| 300 | 0,55 | 5,94 | 10,81 | 106,7 | 79,1 | 13,7 | 13,9 | 0,0 | 0,35 to 1,10 |
| 400 | 0,40 | 5,88 | 14,71 | 131,7 | 104,6 | 13,4 | 13,6 | 0,0 | 0,30 to 1,35 |
| 500 | 0,85 | 8,15 | 9,59 | 152,8 | 97,0 | 25,8 | 19,4 | 10,6 | 0,40 to 1,35 |
| 600 | 0,70 | 8,12 | 11,60 | 171,8 | 115,3 | 25,6 | 19,2 | 11,7 | 0,45 to 1,35 |
| 750 | 0,70 | 8,74 | 12,49 | 198,5 | 133,2 | 29,7 | 22,3 | 13,3 | 0,50 to 1,35 |
| 1.000 | 0,75 | 9,85 | 13,13 | 239,1 | 157,3 | 37,6 | 28,3 | 15,9 | 0,50 to 1,35 |
| 1.250 | 0,85 | 11,06 | 13,01 | 277,1 | 175,1 | 47,5 | 35,7 | 18,9 | 0,65 to 1,40 |
| 1.500 | 0,85 | 11,75 | 13,83 | 313,2 | 197,1 | 53,6 | 40,3 | 22,2 | 0,65 to 1,45 |
| 2.000 | 1,30 | 14,90 | 11,46 | 389,0 | 209,5 | 86,2 | 64,8 | 28,4 | 1,10 to 1,30 |
| 2.500 | 1,05 | 14,95 | 14,24 | 482,2 | 299,5 | 86,8 | 65,2 | 30,7 | 0,90 to 1,05 |
| 3.000 | 0,85 | 14,81 | 17,42 | 592,5 | 406,1 | 85,1 | 64,0 | 37,3 | 0,75 to 1,05 |
| 4.000 | 1,30 | 18,78 | 14,44 | 775,1 | 466,2 | 136,4 | 102,9 | 69,7 | 1,30 to 1,65 |
| 4.500 | 1,65 | 21,14 | 12,81 | 873,5 | 472,6 | 173,0 | 130,4 | 97,4 | 1,65 to 2,30 |
| 5.000 | 2,05 | 23,54 | 11,48 | 959,4 | 478,6 | 214,5 | 161,7 | 104,5 | 1,70 to 2,65 |
| 6.000 | 2,30 | 26,00 | 11,30 | 1.142,8 | 566,6 | 261,6 | 197,2 | 117,4 | 1,45 to 3,40 |
| 7.500 | 2,30 | 28,00 | 12,18 | 1.364,6 | 699,5 | 303,7 | 228,8 | 132,6 | 2,00 to 2,80 |
| 8.000 | 2,55 | 29,62 | 11,61 | 1.450,8 | 714,5 | 339,6 | 255,9 | 140,8 | 2,30 to 2,70 |
| 10.000 | 3,55 | 35,62 | 10,03 | 1.824,6 | 738,5 | 491,6 | 370,2 | 224,4 | 3,50 to 3,65 |
| 12.500 | 2,90 | 35,87 | 12,37 | 2.192,0 | 1.073,1 | 498,5 | 375,4 | 245,0 | 2,90 to 3,10 |
| 15.000 | 3,80 | 41,71 | 10,98 | 2.725,4 | 1.224,1 | 674,3 | 507,6 | 319,4 | 3,75 to 4,75 |
| 20.000 | 4,05 | 46,90 | 11,58 | 3.561,0 | 1.689,6 | 852,4 | 641,6 | 377,4 | 3,45 to 5,15 |
| 30.000 | 4,00 | 53,46 | 13,37 | 5.225,4 | 2.743,7 | 1.108,0 | 833,8 | 539,9 | 3,65 to 6,00 |
| 40.000 | 4,15 | 59,57 | 14,35 | 6.981,4 | 3.880,0 | 1.375,8 | 1.035,2 | 690,5 | 4,05 to 6,35 |

${ }^{(1)}$ : The optimal range indicates the $D / H$ ratio where the variation of the total weight is up to $5 \%$ more or less compared to the optimal weight of the tank.

Table 5. Optimization of open-top tanks, steel with $f y=250 \mathrm{MPa}$, no corrosion allowance, specific gravity of 1,0 , joint efficiency of 1,0 .

| Volume <br> $\left(\mathrm{m}^{3}\right)$ | Optimal <br> D/H | D <br> $(\mathrm{m})$ | H <br> $(\mathrm{m})$ | Optimal <br> total <br> weight <br> $(\mathrm{kN})$ | Shell <br> courses <br> (kN) | Bottom <br> plate <br> (kN) | Optimal <br> range for <br> D/H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 1,15 | 4,18 | 3,64 | 28,0 | 21,2 | 6,8 | 0,60 to 2,65 |
| 75 | 1,20 | 4,86 | 4,05 | 36,1 | 26,9 | 9,2 | 0,60 to 2,35 |
| 100 | 1,20 | 5,35 | 4,46 | 43,3 | 32,2 | 11,1 | 0,60 to 2,40 |
| 125 | 1,25 | 5,84 | 4,67 | 49,8 | 36,6 | 13,2 | 0,60 to 2,40 |
| 150 | 1,25 | 6,20 | 4,96 | 55,9 | 40,9 | 14,9 | 0,65 to 2,45 |
| 160 | 1,25 | 6,34 | 5,07 | 58,2 | 42,6 | 15,6 | 0,65 to 2,45 |
| 200 | 1,25 | 6,83 | 5,46 | 67,1 | 49,0 | 18,1 | 0,65 to 2,45 |
| 250 | 1,30 | 7,45 | 5,73 | 77,3 | 55,7 | 21,6 | 0,65 to 2,50 |
| 300 | 1,30 | 7,92 | 6,09 | 86,8 | 62,5 | 24,3 | 0,65 to 2,50 |
| 400 | 1,30 | 8,72 | 6,70 | 104,4 | 74,9 | 29,5 | 0,65 to 2,55 |
| 500 | 1,30 | 9,39 | 7,22 | 120,5 | 86,3 | 34,2 | 0,65 to 2,55 |
| 600 | 1,35 | 10,10 | 7,48 | 135,5 | 95,9 | 39,6 | 0,65 to 2,55 |
| 750 | 1,35 | 10,88 | 8,06 | 156,5 | 110,5 | 46,0 | 0,70 to 2,55 |
| 1.000 | 1,35 | 11,98 | 8,87 | 188,5 | 132,8 | 55,7 | 0,70 to 2,60 |
| 1.250 | 1,35 | 12,90 | 9,56 | 217,8 | 153,2 | 64,6 | 0,70 to 2,10 |
| 1.500 | 1,35 | 13,71 | 10,16 | 245,2 | 172,2 | 73,0 | 0,70 to 1,75 |
| 2.000 | 1,30 | 14,90 | 11,46 | 295,8 | 209,5 | 86,2 | 0,95 to 1,30 |
| 2.500 | 1,05 | 14,95 | 14,24 | 372,8 | 286,0 | 86,8 | 0,80 to 1,05 |
| 3.000 | 1,95 | 19,53 | 10,02 | 445,2 | 297,7 | 147,5 | 1,45 to 3,70 |
| 4.000 | 2,70 | 23,96 | 8,87 | 545,8 | 323,6 | 222,2 | 1,50 to 4,00 |
| 4.500 | 3,40 | 26,91 | 7,91 | 604,5 | 324,1 | 280,3 | 1,75 to 4,50 |
| 5.000 | 2,10 | 23,73 | 11,30 | 649,8 | 431,8 | 218,0 | 2,10 to 3,60 |
| 6.000 | 3,00 | 28,40 | 9,47 | 749,9 | 437,5 | 312,4 | 2,70 to 4,15 |
| 7.500 | 3,65 | 32,67 | 8,95 | 919,1 | 505,8 | 413,3 | 2,85 to 4,85 |
| 8.000 | 3,95 | 34,27 | 8,67 | 971,1 | 516,2 | 454,9 | 3,20 to 4,55 |
| 10.000 | 3,05 | 33,86 | 11,10 | $1.201,4$ | 757,2 | 444,2 | 2,60 to 3,65 |
| 12.500 | 3,60 | 38,55 | 10,71 | $1.508,2$ | 932,3 | 575,9 | 3,45 to 5,65 |
| 15.000 | 4,90 | 45,40 | 9,27 | $1.770,3$ | 971,4 | 798,9 | 4,90 to 6,00 |
| 20.000 | 4,25 | 47,66 | 11,21 | $2.300,6$ | $1.420,3$ | 880,3 | 4,20 to 5,95 |
| 30.000 | 4,75 | 56,61 | 11,92 | $3.431,1$ | $2.188,5$ | $1.242,6$ | 4,45 to 7,25 |
| $40.000^{(2)}$ | 6,15 | 67,91 | 11,04 | $4.511,5$ | $2.723,0$ | $1.788,5$ | 5,20 to 8,00 |

${ }^{(1)}$ : The optimal range indicates the $D / H$ ratio where the variation of the total weight is up to $5 \%$ more or less compared to the optimal weight of the tank.
${ }^{(2)}$ : The diameter of the tank exceeds the limit of application of the " 1 -foot" method.
The table for optimization of open-top tanks using steel with yield strength of 205 MPa was omitted but the total weights of these tanks can be seen in Fig. 5.


Figure 4. Typical curve of weight of tank with respect to $D / H$ ratio.


Figure 5. Optimal weight of the tanks with respect to volume.
The difference in the total weight of the tanks with plates made of steel with yield strength of 205 MPa or 250 MPa can only be noticed for volumes above $1.500 \mathrm{~m}^{3}$ for tanks with cone roof and above $2.000 \mathrm{~m}^{3}$ for open-top tanks. This difference, which can be noticed in Fig. 5, varies from $1 \%$ to $9 \%$ with average of $5 \%$, and the tanks made of steel with 250 MPa are lighter. The small difference is explained by the fact that the optimum $D / H$ ratio is different for the tanks, which indicates that the resistance of the steel of the shell influences this ratio.

It is concluded from Fig. 5 that the weight of tanks with optimum $D / H$ ratio is directly proportional to their volumes.
The optimization is used to indicate, for tanks with other $D / H$ ratio and a given volume, how far it is from the minimum weight that is possible for this volume.

## 5. OPTIMUM NUMBER OF TANKS

Based on the values from the previous tables, it is possible to establish the number of tanks required to store a certain volume that would lead to a minimum consumption of steel.

Table 6, which uses the weights of Tab. 3, indicates that in the majority of the cases, a smaller number of tanks leads to a small consumption of steel. This fact is also observed using the data of the other tables (4 and 5).

Table 6. Weight of the set of tanks needed for each volume, in kN , using the values from Tab. 3.

| Volume <br> $\left(\mathrm{m}^{3}\right)$ | Number of tanks |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 10 | 20 |
| 100 | 50 | 65 | - | - | - | - | - |
| 160 | 71 | 87 | - | - | - | - | - |
| 200 | 82 | 101 | 117 | 130 | - | - | - |
| 300 | 107 | 134 | 151 | 168 | - | - | - |
| 400 | 132 | 163 | 184 | 202 | 218 | - | - |
| 500 | 153 | 189 | 217 | 234 | 252 | 324 | - |
| 1.000 | 239 | 306 | 345 | 378 | 409 | 504 | 648 |
| 2.000 | 410 | 478 | 552 | 611 | 658 | 817 | 1.008 |
| 3.000 | 643 | 644 | 717 | 794 | 859 | 1.067 | 1.339 |
| 4.000 | 846 | 820 | 879 | 956 | 1.035 | 1.317 | 1.635 |
| 5.000 | 1.031 | 1.014 | 1.053 | 1.115 | 1.195 | 1.528 | 1.889 |
| 10.000 | 1.954 | 2.063 | 2.102 | 2.029 | 2.051 | 2.391 | 3.055 |
| 20.000 | 3.834 | 3.908 | 3.927 | 4.125 | 4.228 | 4.101 | 4.782 |
| 30.000 | 5.741 | 5.936 | 5.861 | 5.924 | 5.978 | 6.428 | 6.445 |
| 40.000 | 7.622 | 7.667 | 7.899 | 7.815 | 7.762 | 8.455 | 8.202 |

If, on one hand, a smaller amount of tanks leads to lower total weights, it is necessary to remind that there are other factors that may rule this decision, such as restrictions of the industrial process, layout of facilities, operation and others.

Myers (1997) indicates other issues to be considered, such as greater flexibility in the maintenance schedule with more tanks and the need of periodic inspection of the tanks for quality control. Moreover, a risk analysis can indicate that it may be better to store the same product in different places, due to fire, deterioration, contamination and other reasons.

## 6. CONCLUSIONS

By the results presented, it follows that it is possible to establish optimum $D / H$ ratios that lead to a minimum weight of the tank for a fixed volume, and that these ratios deviate more from the solution of tanks with uniform plates as the volume increases. This fact happens due to the thickness variations in the bottom and shell, besides the inclusion of the structure that supports the roof. It is also concluded that the use of steels with yield strength of 250 MPa for the shell reduces the total weight of the tanks in $1 \%$ to $9 \%$ related to the steels with yield strength of 205 MPa , for volumes over $1.500 \mathrm{~m}^{3}$ for tanks with fixed cone roof and $2.000 \mathrm{~m}^{3}$ for open-top tanks. It is also possible to conclude that in the absolute majority of cases, the smaller the number of tanks to the storage of a total volume, lower will be the total weight of steel needed to the fabrication of these tanks. By Fig. 4, it is concluded that the curve of total weight with respect to $D / H$ ratio is similar to the curve for uniform tanks with roofs (Fig. 1). The discontinuity is explained by the discretization of the thickness of the many plates of the tank.

In future work, the Author will consider in the optimization of tanks the influence of the variable-design-point method for great volumes, a possible optimization of the supporting structure of the roof and the specific gravity of the stored fluid.

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## 8. RESPONSIBILITY NOTICE

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