ON A MATHEMATICAL MODEL OF THE NON-IDEAL ENERGY HARVESTER VIBRATING SYSTEM

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Abstract. *In this paper, we proposed a Non-Ideal Energy Harvester (**NIEH**), where the excitation force is an electric motor, with eccentricity and limited power supply, called non-ideal system. In the passage through resonance region, when the frequency of the excitation force it is near the natural frequency of the supporting, the Sommerfeld effect occurs and we found a chaotic behavior. We showed by numerical simulations, the presence of high energy orbits in chaotic region where the maximum power harvested was reached.*

Keywords*: Energy Harvesting, Sommerfeld effect, Chaos*

1. INTRODUCTION

In the last years, we have been seeing a large development of miniaturized devices as sensors, actuators, flexible electronics circuits and implantable biosensors. Nowadays, the technology Microelectromechanical Systems (**MEMS**) and Nanoelectromechanical Systems (**NEMS**) have permitted the development of so called smart devices. Also today, components developed in micro and nano scale are yet incorporated in all kinds of electronic devices, in accordance with (Cottone, 2007).

Because of necessity of an energy source smaller and more efficient, for design of systems based in these new technology, the research on Energy Harvesting, has increasing substantially as an example, see: (Priya and Inman, 2009).

In works of (Chtiba *et* al., 2010 and De Marqui Jr., 2009) among others authors, they concluded that the best model to build energy harvesting devices is by using piezoelectric materials. So, many researchers have concentrated their efforts on finding the best configuration for these systems and to optimize its power output.

In agreement with (Sodano *et* al., 2004; Anton and Sodano, 2007) in the process of Energy harvesting, the electrical energy is obtained through of conversion of mechanical energy, created by an ambient vibration source by a type of transductor, as a piezoceramic thin film.

Several different electromechanical coupling mechanisms have been developed for harvesting devices in according to (Triplett and Quinn, 2009).

In works of (Triplett and Quinn, 2009), the authors introduced in their model of energy harvesting system, the role of nonlinearities in the electromechanical coupling, during the design process, because the constitutive laws of piezoelectric materials, which exhibits a strong dependence, between the applied strain and electric field in the piezoceramic material as shown by (du Toit and Wardle, 2007; Twiefel *et* al., 2008). Those nonlinearities have considerable importance in the system response. So, the inclusion the role of nonlinearities in the electromechanical coupling has permitted that the power the output prediction can be improved and system performance can be optimized, as well.

Figure 1 represents the model of Non-Ideal Energy Harvester (**NIEH**) proposed by us, which consists in a cantilevered beam with applied piezoelectric layers, in the free end is attached an electric motor with of unbalanced mass and in clamped end the piezoelectric elements are connected to an electrical load.

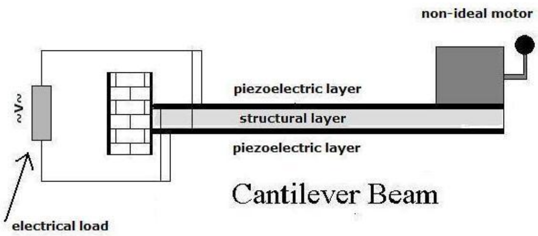


Figure 1. Idealization of the non-ideal energy harvester

This research considered the energy source with limited power supply, like in real motors.

We say that the energy source is non-ideal, against an ideal source, in which the amplitude and frequency are independent of the movement and response of the structure.

Our experimental research detected the Sommerfeld Effect: as the motor accelerates to reach near resonant conditions, a considerable part of its output energy is consumed to generate large amplitude motions of the structure and not to increase its own angular speed. For certain parameters of the system, the motor can get stuck at resonance not having enough power to reach higher rotation regimes. If more power is available, jump phenomena may occur to considerably regimes of higher motor speed near resonance, no stable motions being possible between these two.

The non-ideal systems theories can seen details in: (Kononenko, 1969), (Nayfeh and Mook, 1979), (Balthazar *et* al., 2003), (Dantas and Balthazar, 2007) and (Felix *et* al, 2009), without undeserved.

The main goal of this paper was to analyze the model of a non-ideal energy harvester proposed by us using numerical simulations and shows results obtained.

2. NON-IDEAL ENERGY HARVESTER MODEL

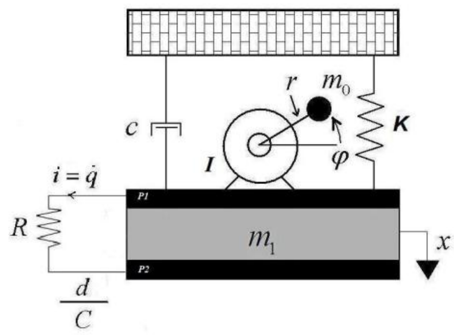


Figure 2. Mathematical model of the non-ideal system

2.1. Non-ideal structural mathematical modeling

The mathematical model of the (**NIEH**) as shown in Fig. 2 is represented by the following governing equations of motion:

$$M\ddot{x} + f(x,\dot{x}) + \frac{\partial U(x)}{\partial x} - \frac{d(x)}{C}q = F(\dot{\varphi},\ddot{\varphi},r,m_0)$$

$$I\ddot{\varphi} + H(\dot{\varphi}) = L(\dot{\varphi}) + R(\varphi,\dot{\varphi},\ddot{x},r,m_0) \quad (1)$$

Where the quantity $M = m_1 + m_0$ is the total mass of the (**NIEH**), x is displacement of the (**NIEH**), φ is angular displacement of the rotor, $F(\varphi,\dot{\varphi},\ddot{x},r)$ expresses the action of the source of energy on the oscillating system (angular velocity of motor, that is not constant), parameters r is the eccentricity and m_0 the mass of unbalanced shaft of the electric motor, $I = J + m_0 r^2$ is the moment of inertia of the rotor. The function $R(\varphi,\dot{\varphi},\ddot{x},r)$ represents the action of the oscillating system on the source of energy. The function $H(\dot{\varphi})$ is the resistive torque applied to the motor and the function $L(\dot{\varphi})$ is the driving torque of the source of energy (motor). Note that, usually, the inductance is much smaller than the mechanical constant time of the system and, then in stationary regime, we can take $L(\dot{\varphi})$ as (linear) $L(\dot{\varphi}) - H(\dot{\varphi}) = u_1 - u_2\dot{\varphi}$, where are u_1 related to voltage applied across to the armature of the DC motor, that is, a possible control parameter of the problem and u_2 is a constant for each model of DC motor considered. The function $f(x,\dot{x})$ is the linear and non-conservative part of the restoring force, while $\frac{\partial U(x)}{\partial x}$ is its conservative part (U is the potential, or strain energy).

The function $f(x,\dot{x})$ and the potential U are defined as:

$$f(x,\dot{x}) = c\dot{x}\,,\; U(x) = \frac{1}{2}k_1 x^2 + \frac{1}{4}k_2 x^4 \tag{2}$$

P1 and *P2* are de thin film piezoelectric applied layers. In agreement with (Triplett and Quinn, 2009), the electrical charge developed in the coupled circuit given by *q*. The term $\frac{d(x)}{C}q$ represents the piezoelectric coupling to the mechanical component, with a strain-dependent coupling coefficient $d(x)$. The voltage V across the piezoelectric material has the form:

$$V = -\frac{d(x)}{C}x + \frac{q}{C} \tag{3}$$

Here the *C* represents the piezoelectric capacitance and with $V = -R\dot{q}$ the **(NIEH)** coupled governing equations of motion are:

$$M\ddot{x} + c\dot{x} + k_1 x + k_2 x^3 = m_0 r\left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right) + \frac{d(x)}{C}q \tag{4}$$

$$I\ddot{\varphi} = \Gamma\left(\dot{\varphi}\right) + m_0 r\ddot{x}_1\cos\varphi \tag{5}$$

$$R\dot{q} - \frac{d(x)}{C} + \frac{q}{C} = 0 \tag{6}$$

Where c is the linear damping coefficient and k_1 is the linear stiffness coefficient k_2 is the non-linear stiffness coefficient.

2.2 *Non-dimensionalization*

Considering the following non-dimensional parameters:

$$\omega_0 = \sqrt{k_1/M}\,,\;\; \tau = \omega_0 t\;\;, u = \frac{x}{r}\,,\;\; v = \frac{q}{q_0}\,,\;\; \mu_1 = \frac{u_1}{I\omega_0^2}\,,\; \mu_2 = \frac{u_2}{I\omega_0}\,,\;\; \Gamma(\varphi') = \mu_1 - \mu_2\varphi'$$

$$\alpha = \frac{c}{M\omega_0}\,, \beta = \frac{k_1}{M\omega_0^2}\,, \beta_1 = \frac{k_2 r^2}{M\omega_0^2}\,, \delta_1 = \frac{m_0}{M}\,, \delta_2 = \frac{m_0\omega_0^2}{M}\,,\;\; \gamma = \frac{m_0 r^2}{I}\,,\;\; \varepsilon = \frac{m_0}{M}\,,$$

$$\rho \equiv RC\sqrt{k_1/M}\ ,\ \hat{d}(u)=\theta(1+\Theta|u|),\quad \hat{d}(u)\equiv\left(\frac{r}{q_0}\right)d(r),\quad \theta\equiv\left(\frac{r}{q_0}\right)d_{lin},\quad \Theta\equiv rd_{nlin}, \tag{7}$$

The dimensionless piezoelectric coupling coefficient used by us was approximate by (Triplett and Quinn, 2009), as $\hat{d}(u)=\theta(1+\Theta|u|),$ where the piezoelectric coefficient divide in linear part**,** represented by θ and non-linear part defined by Θ, so we may reduce the equations of motion to:

$$\begin{aligned}
&u''+\varepsilon\alpha u'+\beta u+\varepsilon\beta_1 u^3-\varepsilon\theta(1+\Theta\,|\,u\,|)v=\varepsilon\delta_1\varphi''\cos\varphi-\varepsilon\delta_2\varphi'^2\sin\varphi\\
&\varphi''=\varepsilon\Gamma(\varphi')+\varepsilon\gamma u''\cos\varphi\\
&\rho v'-\theta(1+\Theta\,|\,u\,|)u+v=0
\end{aligned}\tag{8}$$

After defined the variables as: $x_1=u$, $x_2=u'$, $x_3=\varphi$, $x_4=\varphi'$, $x_5=v$, the (**NIEH**), may be rewritten in state space representation as:

$$\begin{aligned}
&\dot{x}_1=x_2\\
&\dot{x}_2=-\varepsilon\alpha x_2-\beta x_1-\varepsilon\beta_1 x_1^{\,3}+\varepsilon\theta(1+\Theta\,|\,x_1\,|)x_5-\varepsilon\delta_1\dot{x}_4\cos x_3+\varepsilon\delta_2 x_4^{\,2}\sin x_3\\
&\dot{x}_3=x_4\\
&\dot{x}_4=\varepsilon\Gamma(x_4)+\varepsilon\gamma\dot{x}_2\cos x_3\\
&\dot{x}_5=\frac{(\theta(1+\Theta\,|\,x_1\,|)x_1-x_5)}{\rho}
\end{aligned}\tag{9}$$

We will obtain the following first order differential equations to the (**NIEH**), making some algebraic manipulations:

$$\begin{aligned}
&\dot{x}_1=x_2\\
&\dot{x}_2=\frac{1}{\Delta}\Big[-\varepsilon\alpha x_2-\beta x_1-\varepsilon\beta_1 x_1^3+\varepsilon\left(\theta x_5+\theta\Theta|x_1|x_5\right)-\varepsilon\left(\delta_2 x_4^2\sin x_3\right)+\varepsilon\left(\delta_1\cos x_3\right)\varepsilon\left(\mu_1-\mu_2 x_4\right)\Big]\\
&\dot{x}_3=x_4\\
&\dot{x}_4=\frac{1}{\Delta}\Big[\left(\varepsilon\left(\mu_1-\mu_2 x_4\right)+\varepsilon\gamma\cos x_3\right)\left(-\varepsilon\alpha x_2-\beta x_1-\varepsilon\beta_1 x_1^3+\varepsilon\left(\theta x_5+\theta\Theta|x_1|x_5\right)-\varepsilon\left(\delta_2 x_4^2\sin x_3\right)\right)\Big]\\
&\dot{x}_5=\frac{\left(\theta x_1+\theta\Theta|x_1|x_1\right)-x_5}{\rho}
\end{aligned}\tag{10}$$

Where:

$$\Delta=1-\varepsilon^2\delta_1\gamma\cos^2 x_3 \tag{11}$$

With the voltage previously defined as $V=-R\dot{q}$ the power harvested from the mechanical component is V^2/R, so the non-dimensional electrical power harvested from the system is identified by (Triplett and Quinn, 2009) as:

$$P=\rho v'^2 \tag{12}$$

3. NUMERICAL SIMULATIONS RESULTS

In this work, numerical simulations were carried out in Matlab™ using as the numerical integrator ode113, Adams-Bashforth-Moulton PECE solver algorithm with variable step-length. For all numerical simulations performed, were considered the initial conditions equal to zero. And all numeric simulations were done over the dimensionless time range $0 \le \tau \le 5000$.

We analyzed the **(NIEH**), proposed by us, excited by a non-ideal source using the same values to damping, stiffness linear and non-linear, linear and non-linear piezoelectric coupling coefficients, non-dimensional electrical coefficient and smaller parameter ε , in accordance com the work of (Triplett and Quinn, 2009).

$$\alpha = 0.50;\ \beta = 1.00;\ \beta_1 = 0.25;\ \theta = 1.00;\ \Theta = 1.00;\ \rho = 1.00;\ \varepsilon = 0.10;$$

The specific parameters of excitation to (**NIEH**) were defined in agreement with work of (Warminski and Balthazar, 2003).

$$\delta_1 = 0.40;\ \delta_2 = 0.40;\ \gamma = 0.60;$$

In this work, the characteristic parameters of DC motor are:

$$\mu_1 = 0 \le 30;\ \mu_2 = 15;$$

Figure 3 shows the phase portraits of the (**NIEH**), with control parameter μ_1 assuming the values 15 and 23, chaotic behavior is found and with μ_1 assuming the values 10 and 26, periodic solutions were encountered in the steady state motion.

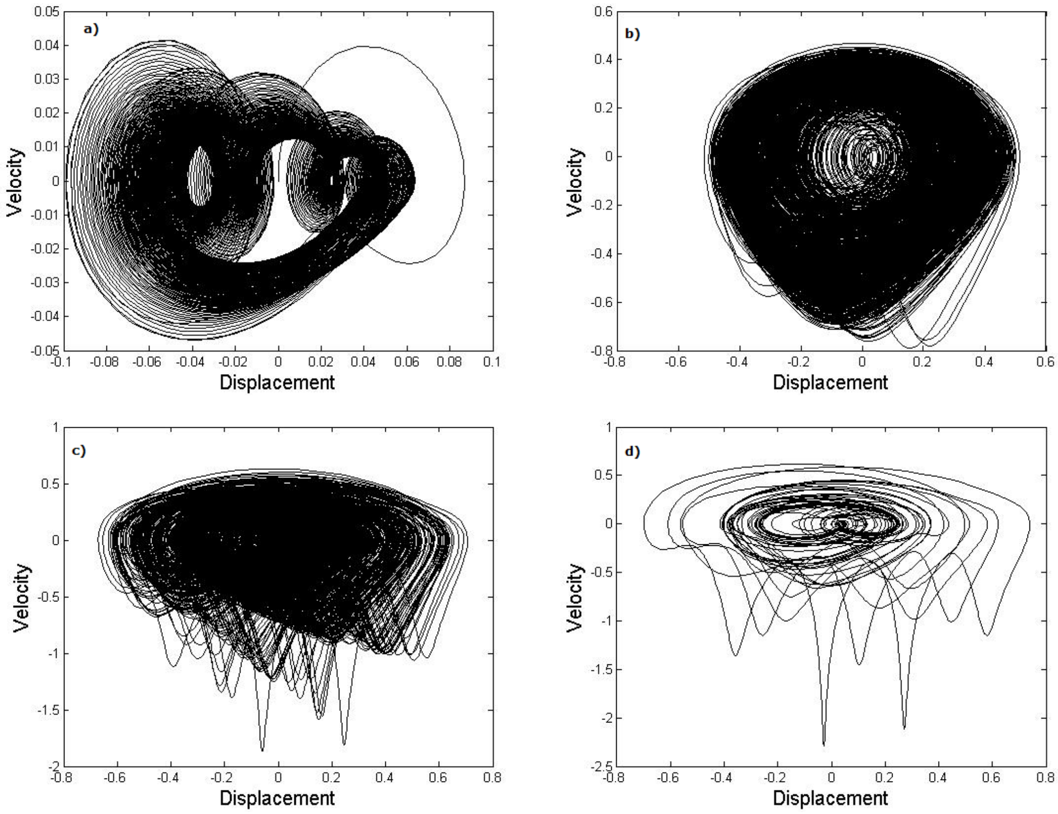


Figure 3. Phase Portraits of the (NIEH): with (a) μ_1 =10; (b) μ_1 =15; (c) μ_1 =23; (d) μ_1 =26;

With the phase portraits in third dimension depicted in the Fig. 4 over time, can be noticed the existence of high energy orbits when the (**NIEH**) is in a chaotic regime of vibration with the control parameter μ_1 **= 23**. Such orbits are desirable because in them one can achieve maximum power output of the system.

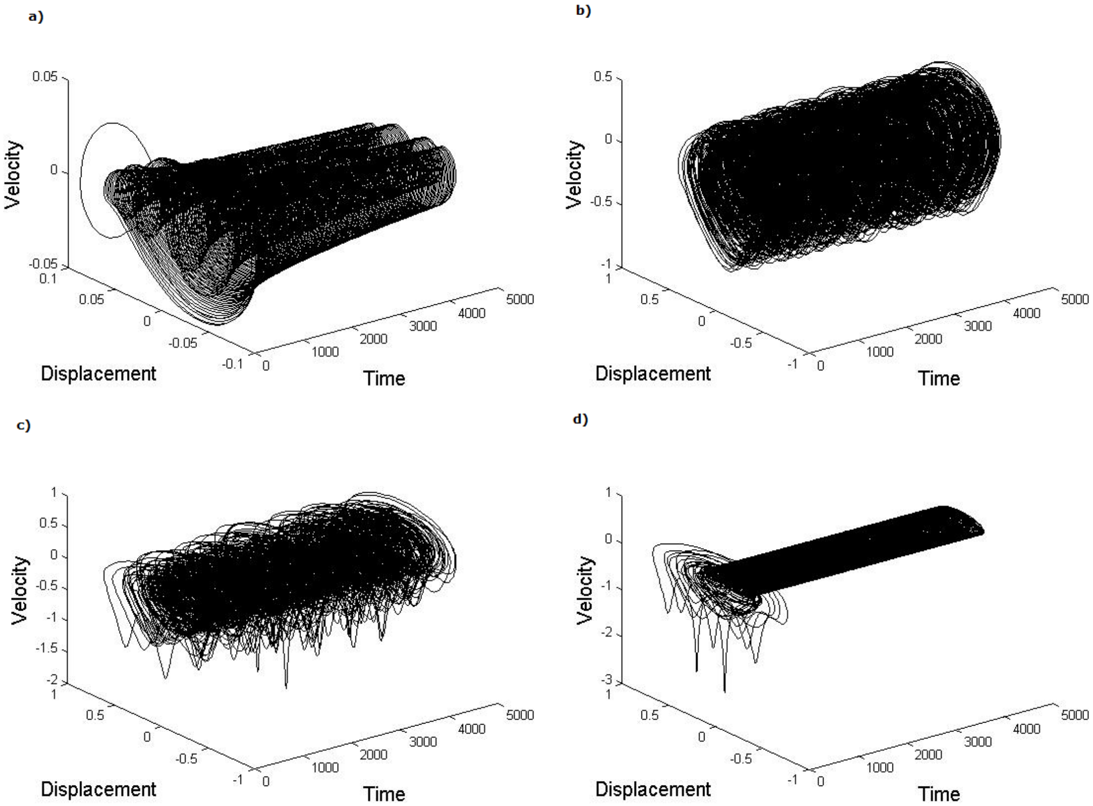


Figure 4. Phase Portraits in Third Dimension of the (NIEH): with (a) μ_1 =10; (b) μ_1 =15; (c) μ_1 =23; (d) μ_1 =26;

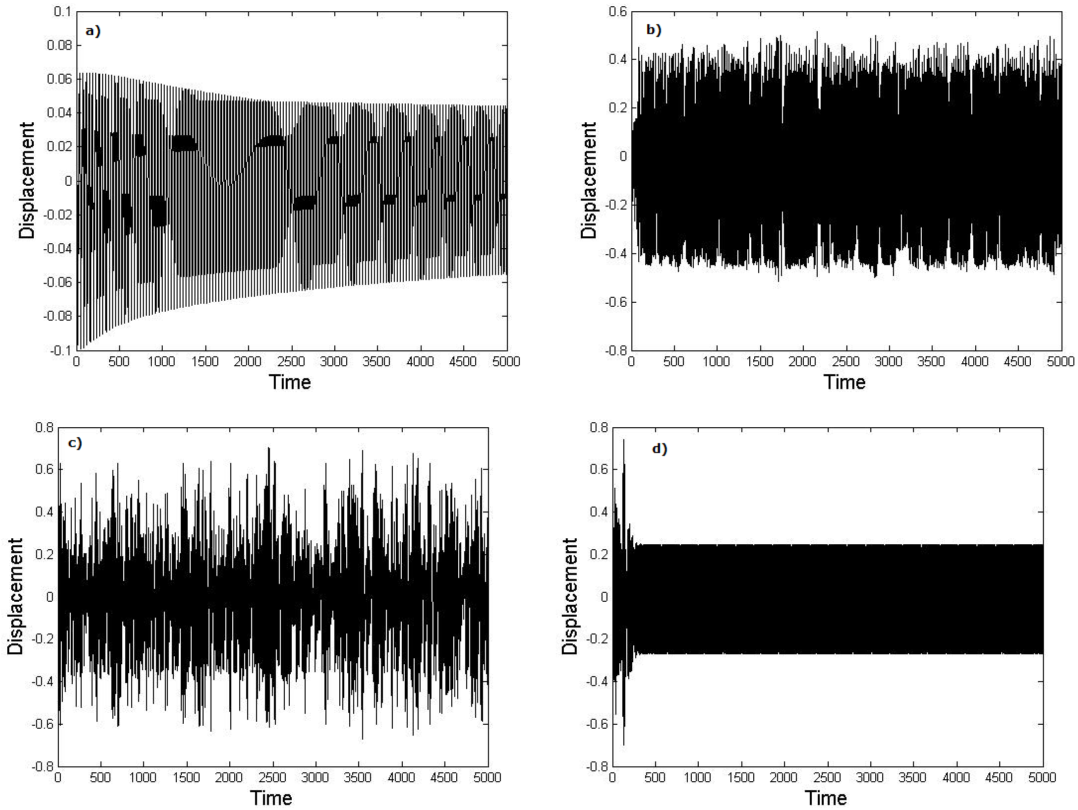


Figure 5. Time Histories of the (NIEH) displacement: with (a) μ_1 =10; (b) μ_1 =15; (c) μ_1 =23; (d) μ_1 =26;

Displacements of the cantilevered beam are depicted in the Fig. 5 where it can be seen the large amplitude vibration, with increasing of control parameter μ_1, and quasi-periodic, chaotic and periodic motions of the model were encountered. In chaotic regimes the maximum peaks of displacement are reached.

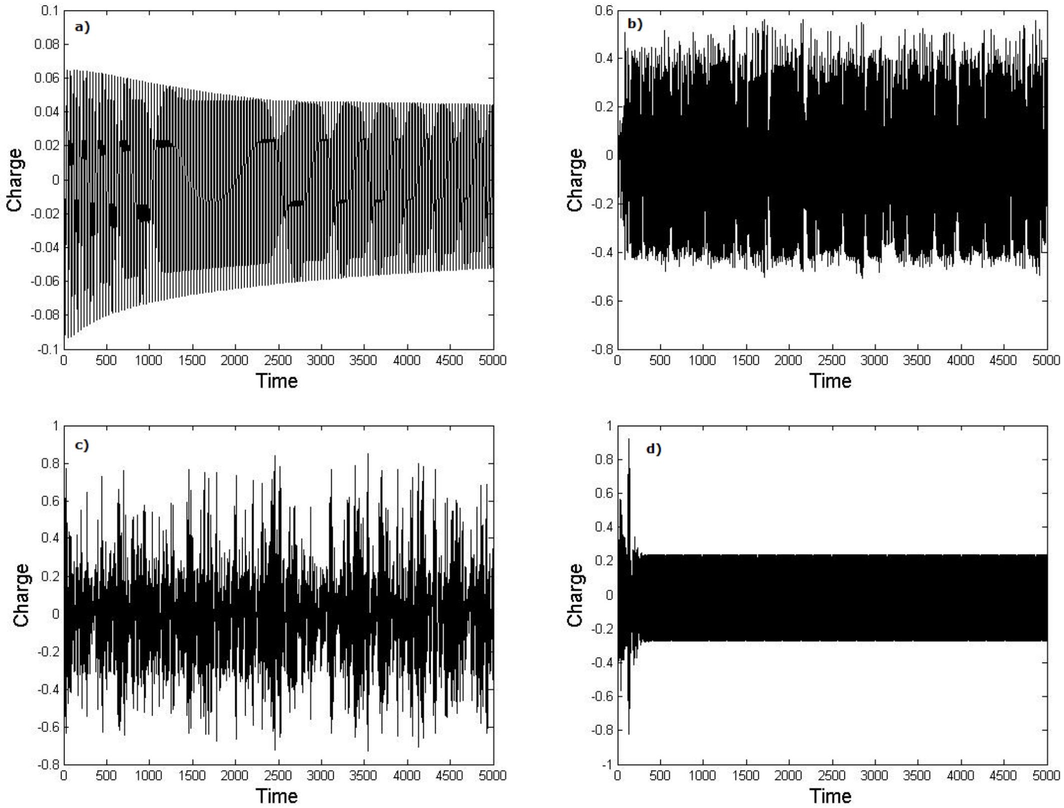
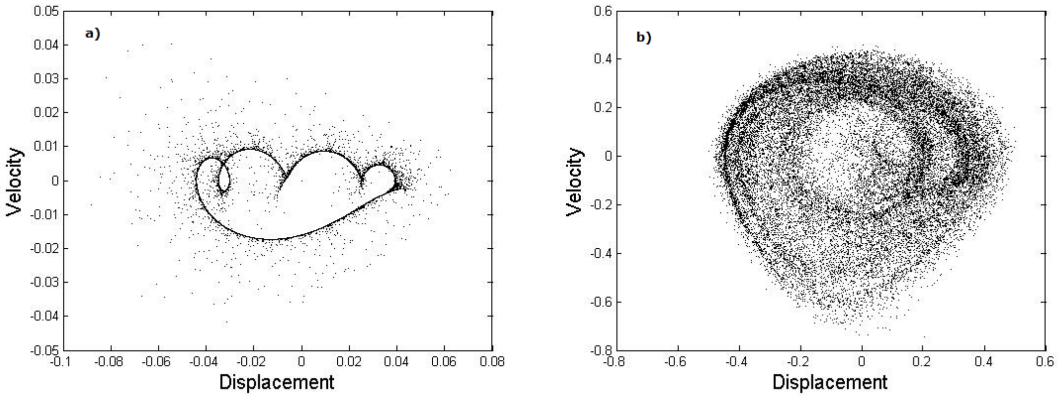


Figure 6. Time Histories of the (NIEH) charge: with (a) μ_1 =10; (b) μ_1 =15; (c) μ_1 =23; (d) μ_1 =26;

The electrical charge also suffers the influence of vibration of the structure because it is strongly coupled with the beam. The responses by the motions of charge with increasing of the value of control parameter are depicted in Fig. 6.



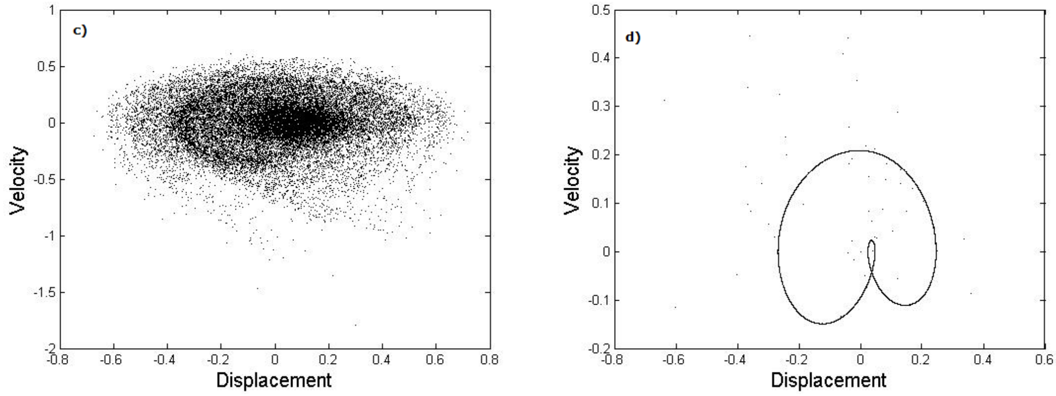


Figure 7. Poincare Sections of the (NIEH): with (a) μ_1 =10; (b) μ_1 =15; (c) μ_1 =23; (d) μ_1 =26;

The Poincare sections are displayed in Fig. 7 where as the control parameter is increased, it can be noted that the system passes through a quasi-periodic attractor before the resonance region, with more voltage applied to the motor in an attempt to pass through resonance, the system gets stuck in a chaotic attractor, and with a further increase in the value of control parameter, the system response is taken to a periodic attractor.

Figure 8, exhibits the existence of the Sommerfeld effect in the displacement of the (**NIEH**) due to increasing of voltage of DC motor (control parameter μ_1). This occurs at the moment, where the rotation frequency of the DC motor (rotational velocity φ') is close of the natural frequency of the system $(\varphi' \approx 1)$ and thus it can be captured by the resonance, or by increasing the voltage to the motor passage through resonance jumping to another point of equilibrium.

After the passage through resonance region, increases in the motor speed with increasing the control parameter does not increase the displacement of the structure satisfactorily.

In the figure the arrows indicate the point where occurs the jump phenomena and the point where it appears chaotic behavior in the system due to the fact that the energy source is interacting, with the structure.

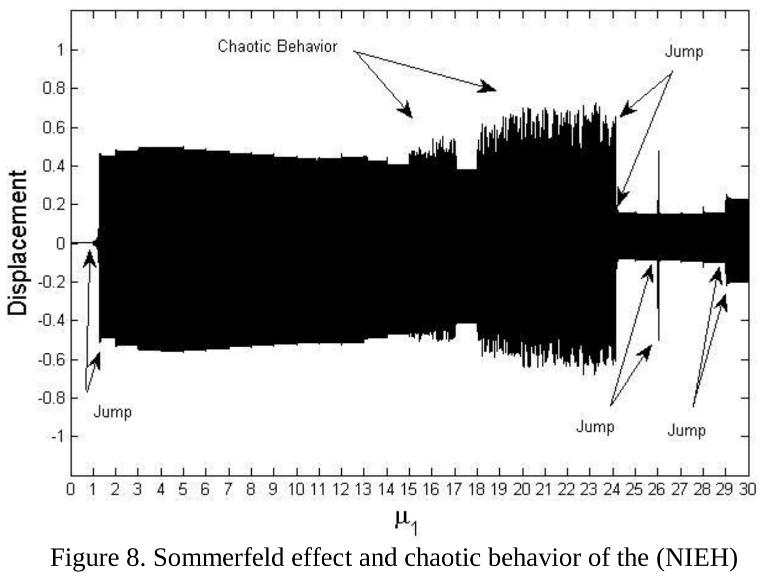


Figure 8. Sommerfeld effect and chaotic behavior of the (NIEH)

The maximum non-dimensional power harvested, was found over the high energy orbits, in the resonance region and inserted into the chaotic motion as shown in Fig. 9.

With the increasing of the value of the control parameter μ_1 , the maximum power is then reached, but as the energy source interacting with the structure is limited, the maximum power harvested is not constant along the time.

The numerical simulations shown that, the maximum power harvested in the (**NIEH**) occurs in the peaks over the chaotic motion.

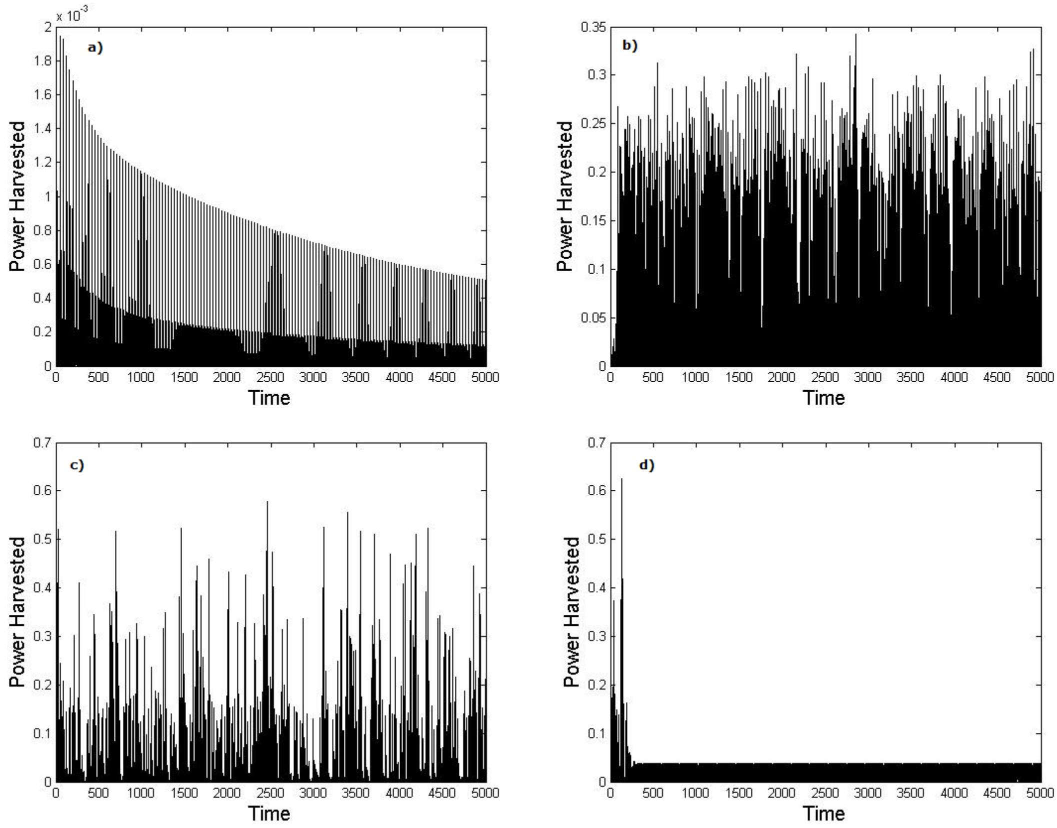


Figure 9. Maximum non-dimensional power harvested of the (NIEH):
With (a) μ_1=10; (b) μ_1=15; (c) μ_1=23; (d) μ_1=26;

Figure 10, in left side shown the high energy orbits to the (**NIEH**) with increment of the control parameter μ_1 and in right side a zoom in region of maximum peaks, where the best value achieved for the maximum power harvested is demonstrated.

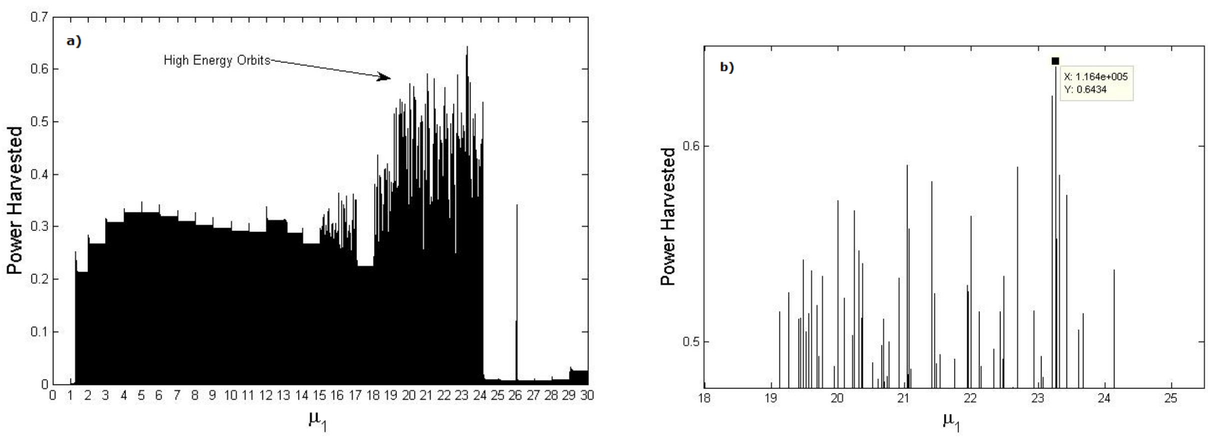


Figure 10. High energy orbits of the (NIEH): in (a) along increments of control parameter μ_1,
in (b) a zoom of High energy orbits

4. CONCLUSION

The Non-Ideal Energy Harvester proposed by us in this paper, shows that the energy source of limited power tends to influence in the response of the structure of the electromechanical device, leading to energy losses, instable motions where a chaotic behavior occurs.

Due to the existence of the Sommerfeld effect, the maximum power harvested, is not constant in the regions near where the jump phenomena happened.

We show that not only in the region of resonance are the best orbits for energy harvesting. And it is possible to take advantage of the chaotic dynamics of the model and find high-energy orbits in the middle of chaotic motion.

In future works, a strategy of control and reducing of Sommerfeld effect, can be implemented in order to keep the motions of the (**NIEH**) in a desirable orbit, so stabilizing the maximum power harvested in output of system.

5. ACKNOWLEDGEMENTS

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