# SYNTHESIS OF AN AERONAUTICAL PANEL SUBMITTED TO DYNAMIC LOADS BY MEANS OF EQUIVALENT STATIC LOADS

**Hélio Guerrini Filho, guerrini@ita.br**
**José Antônio Hernandes, hernandes@ita.br**
Instituto Tecnológico de Aeronáutica - Pça. Mal. Eduardo Gomes,50 - 12228-900 São José dos Campos /SP, Brazil

**Eliseu Lucena Neto, Eliseu@ita.br**
Instituto Tecnológico de Aeronáutica - Pça. Mal. Eduardo Gomes,50 - 12228-900 São José dos Campos /SP, Brazil

***Abstract.****Structural synthesis with dynamic response is usually very expensive due to the cost of a typical dynamic response finite element analysis. Considering the high number of required structural analysis cycles during the structural synthesis the final computational cost can grow very much and perhaps become prohibitive. A considerable reduction of the optimization cost can be made by using the transformation of the dynamic applied load into equivalent static loads, appropriately obtained for the problem at hands. The present work aims at the evaluation of doing the structural synthesis of an aeronautical panel of isotropic material under dynamic load and submitted to constraints on displacement, stress and dynamic buckling, by transforming the dynamical loads into equivalent static loads to obtain optimal design at a smaller computational cost.*

## 1. INTRODUCTION

The most existing structures are subjected to loads of dynamic nature. Buildings, bridges, cars, trains and airplanes have somehow in their mechanical structures dynamic forces that can be both random as well as periodical. The optimization applied to these types of structures are naturally of great interest. However, optimizing them using directly dynamic response analysis is very computationally expensive with regard to integration equations of motion. This effort becomes prohibitive when allied to the iterative optimization process, where usually an excessive number of structural analyses are required.

The possibility of reducing these problems by the use of equivalent static loads allows a significant reduction in computational effort. Furthermore, because problems of static nature are simplier, the structural optimization becomes easier.

The method used in this research is the same proposed by B S Kang and Park (2001) and W S Choi and Park (2005), that defines the equivalent static load (ESL) as the load capable of reproducing the same displacement field of the dynamic load in an arbitrary time instant. This instant should belong to the more critical structural condition at certain location and interval of application of the dynamic load. For different loads is possible to select several critical instants related to several critical structural conditions. This allows the composition of an optimization problem based on multiple static load conditions whose constraints are simultaneously evaluated during the optimization process.

The consideration of dynamic buckling demands a non linear geometric analysis and the formulation of an optimization problem with non-linearity is very hard and usually prohibitive in terms of computational cost. Therefore, the strategy of using equivalent static loads allow great simplification of the optimization, since it will be necessary only to deal with eigenvalue problems of linear buckling, demanding a much smaller computational burden.

The constraints present in the optimization are of displacement, von Mises stress and critical buckling load factor.

The computational implementation is performed by means of structural optimization modules codes written in PYTHON for ABAQUS language integrated to the ABAQUS solver environment.

## 2. THE CALCULATION OF EQUIVALENT STATIC LOADS

The finite element equations that describes the dynamic behavior of a structure are given by

$$[M]\frac{\partial^2\{d\}}{\partial t^2} + [C]\frac{\partial\{d\}}{\partial t} + [K]\{d\} = \{f(t)\} \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix, $[K]$ the stiffness matrix and $\{f(t)\}$ the dynamic load vector.

The equivalent static load (ESL) can be defined an the result of the multiplication between the global stiffness matrix and the dynamic displacement vector $\{d(t_a)\}$ at the instant $\{t_a\}$ W S Choi and Park (2005). Solving Eq. (1) to any dynamic load $\{f(t)\}$, it is easy to verify that the product $[K]\{d(t_a)\}$ is a static force that would produce the same displacement field of the dynamic load at an instant arbitrary time $t_a$ W S Choi and Park (2005). Therefore, the product

$[K]\{d(t_a)\}$ is itself defined as the equivalent static load:

$$\{s\} = [K(b_n)]\{d(t_a)\} \tag{2}$$

The Eq. (1) shows that $\{s\}$ is depends on the structural features given by stiffness matrix $[K(b_n)]$, which is a function of the design variables $b_n$.

Combining Eqs. (1) and (2) one obtains:

$$\{s\} = \{f(t)\} - [M]\frac{\partial^2\{d\}}{\partial t^2} - [C]\frac{\partial\{d\}}{\partial t} \tag{3}$$

Examining Eq. (3) it can be seen that the equivalent static load $\{s\}$ also can be defined as the result of the summation between the dynamic load $\{f(t)\}$ and the components $[M]\frac{\partial^2\{d\}}{\partial t^2}$ and $[C]\frac{\partial\{d\}}{\partial t}$ of the equilibrium equation. This shows that the ESL is the result of the dynamic load subtracted from the inertia effects present in response behavior in terms of the inertia and damping effects B S Kang and Park (2001) and W S Choi and Park (2005).

The Fig. (1) illustrates the equivalent static load vector calculated for an instant arbitrary $t_a$.

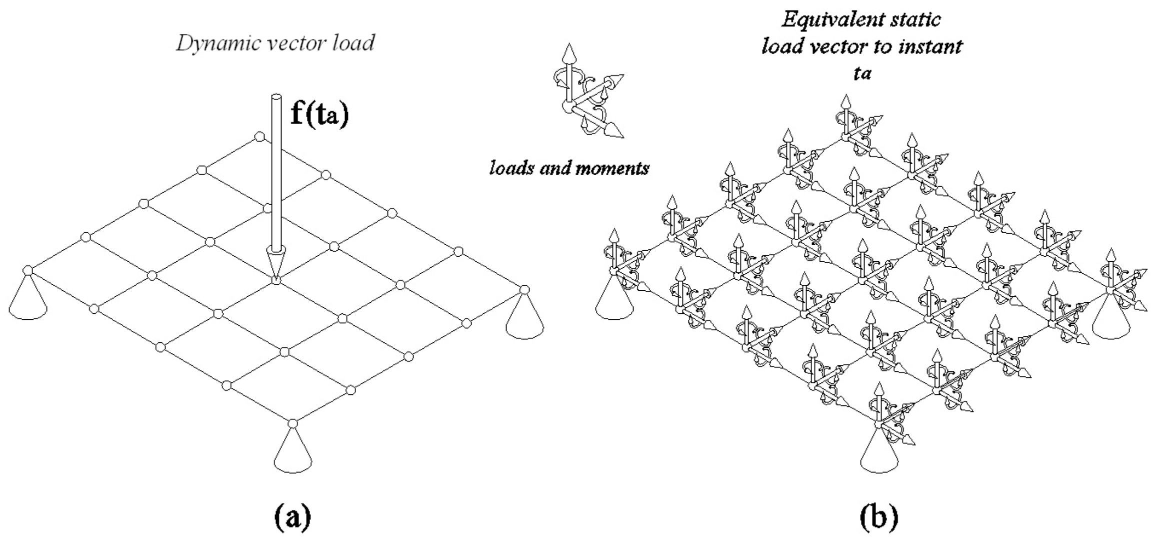


Figure 1. (a) Plate subject to dynamic load vector $f(t_a)$. (b) Plate subject to equivalent static load vector at instant $t_a$.

For the process of optimization the instant $t_a$ is not an arbitrary instant, but the instant more significant in the time domain, i.e., when the structure is in a critical condition, such as peak displacement or stress. If several critical instants occur, several ESL's must be calculated to create multiple equivalent static loading cases. When more ESL's are included into analysis the better are the results, although the optimization problem grows in size. Therefore, there is a compromises between the number of ESL's and the efficiency of the optimization and the best approach would be to use the smaller possible number of ESL in order to get a meaningful optimal solution.

The advantage in use of the ESL's into the process of optimization, is the possibility of reduction of computational cost by the replacing the real dynamic problem by one or more equivalent static problems. Therefore, where normally it would be demanded the integration of many dynamic equations in the time domain, the solution becomes one of some simpler static problems.

The equivalent static optimization problem solved in this work is the following:

$$\begin{aligned} \text{Minimize:} \quad & W(b_n) \quad n = (1, ..., p) \\ \text{Subject to:} \quad & [K(b_n)]\{d(t_i)\} = \{s\}_i \quad i = (1, ..., m) \\ & g(b_n, \{d(t_i)\})_{i,j} \leq 0 \quad j = (1, ..., k) \end{aligned} \tag{4}$$

$W(b_n)$ is the structural weight, $\{s\}_i$ the multiple ESL's and $g(b_n, \{d\})_j$ the behavior constraints. The indices n, i e j are respectively *n*-th design variables, *i*-th ESL and *j*-th constraint.

It is important to note that the changes of variables values during optimization directly influence the intensity load vector. These influences suggest that for any change in those values, the ESL's must be computed again. However, this would require many structural dynamic analysis, one for each design change, which of course would destroy the efficiency of the method. However, considering the hypothesis that a ESL suffers a small change for small dimensional variations imposed on the structure, in practice one assumes that the ESL's are constant and independent of design variables during each cycle of optimization B S Kang and Park (2001).

## 3. AERONAUTICAL PANEL

The model adopted in this research of the aeronautical panel is shown in Fig. 2.

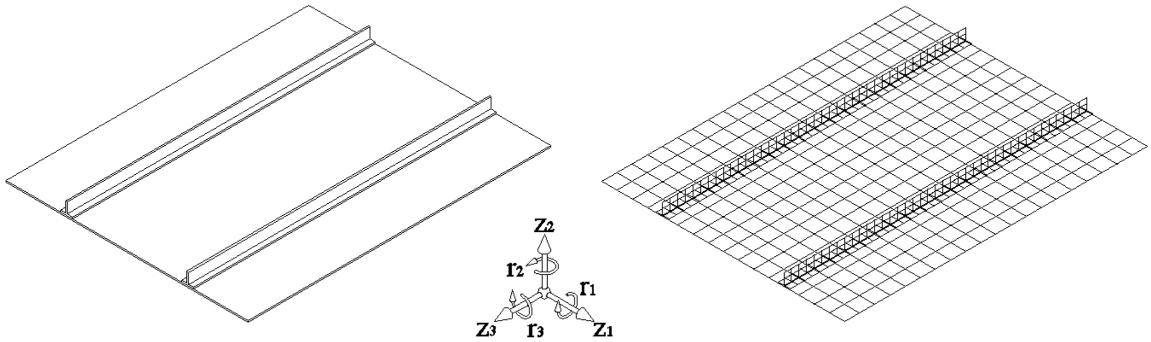


Figure 2. Aeronautical panel and model panel discretized in FE.

The dimensions and design variables $b_1$, $b_2$, $b_3$, $b_4$ and $b_5$ are detailed in Fig. 3.

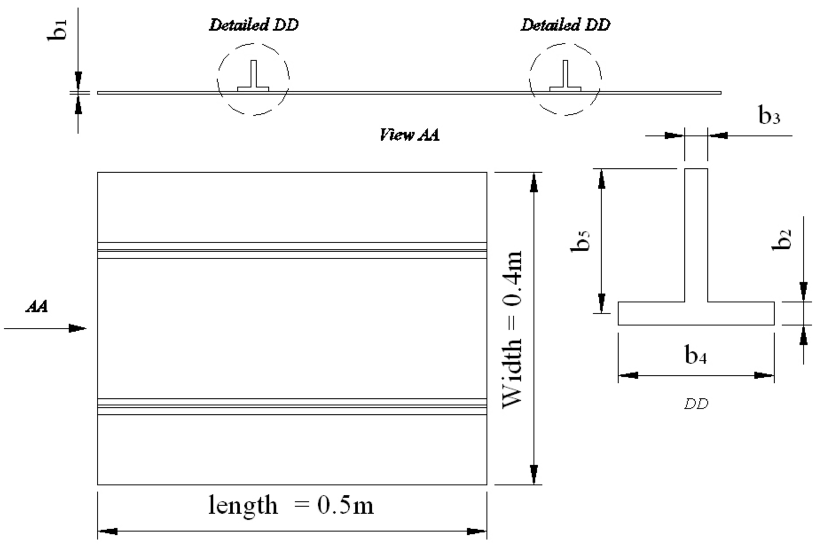


Figure 3. Detailed view

Because the entire panel is discretized with shell elements S4R of the library of ABAQUS solver, the variables $b_4$ and $b_5$ related to height and width of the stringer, are parameterized together with the nodal coordinates of the mesh. Therefore, the problem has two kinds of design variables; the *size variables* $b_1$, $b_2$ and $b_3$, that control the thickness of the finite elements and the *shape variables* $b_4$ and $b_5$, that control the width and the depth of stiffeners in terms of the coordinates of the associated grid. However, the distance between stiffeners is kept constant in the stringer-stiffened panel used in the example.

The parametrization uses the same idea of H F Guerrini and Ferreira (2009) for positioning of control points. Considering the Fig. 4, lets imagine a point $k$ defined over a segment between points A and B at position $Lp_1$ in the line $r$ of length $L_1$, which represents the original body. In the perturbed body, the point $q$ will assume a proportional location and in the direction of the variation $\Delta L_1$, as in the Fig. 4.

The new position of $q$ is given by $L_{p2} = L_{p1}\frac{L_2}{L_1}$. If points $q$ and $A$ have coordinates $x_1$ and $x_A$, then the new coordinate $x_2$ of point $q$ is given by:

$$x_2 = x_1 + \left( (x_1 - x_A)\frac{L_2}{L_1} - (x_1 - x_A) \right) \tag{5}$$

where, $L_{p1} = (x_1 - x_A)$, as illustrated in Fig. 5.

Applying the Eq. (5) to the structure of Fig. 5, if the nodes $B_1$, $B_2$, $B_3$ e $q$ at edge $\alpha$, $\beta$, $\gamma$ and $\delta$, have coordinates $z_1^\alpha$,

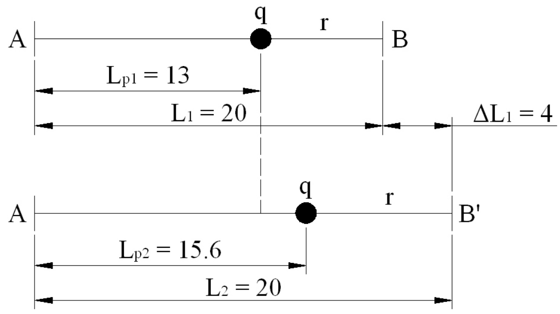


Figure 4. Displacement of the point $q$.

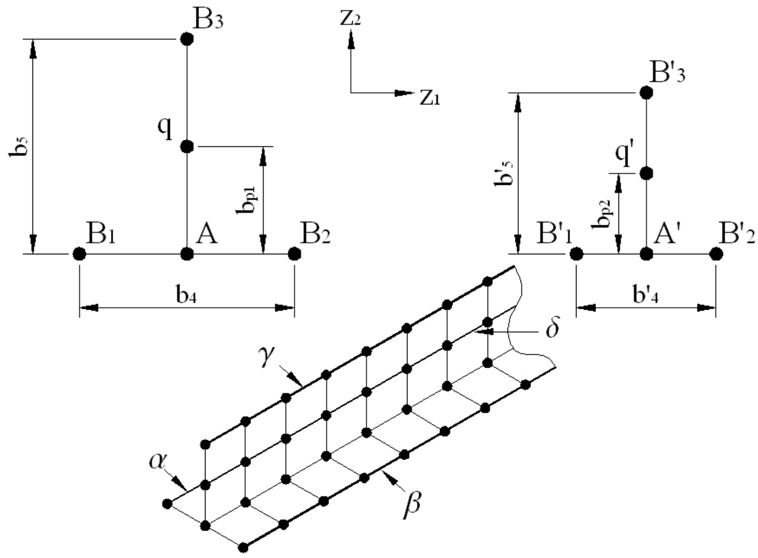


Figure 5. Displacement of the nodes of stringer.

$z_1^\beta$, $z_2^\gamma$ and $z_2^\delta$ respectively, then the new coordinates $z'^\alpha_1$, $z'^\beta_1$, $z'^\gamma_2$ and $z'^\delta_2$ of nodes $B_1$, $B_2$, $B_3$ and $q$ are given by:

$$\begin{aligned}
z'^\alpha_1 &= z_1^\alpha + \left[(z_1^\alpha - z_{2A})\frac{b'_4}{b_4} - (z_1^\alpha - z_{2A})\right] \\
z'^\beta_1 &= z_1^\beta + \left[(z_1^\beta - z_{2A})\frac{b'_4}{b_4} - (z_1^\beta - z_{2A})\right] \\
z'^\gamma_2 &= z_2^\gamma + \left[(z_2^\gamma - z_{2A})\frac{b'_5}{b_5} - (z_2^\gamma - z_{2A})\right] \\
z'^\delta_2 &= z_2^\delta + \left[(z_2^\delta - z_{2A})\frac{b'_5}{b_5} - (z_2^\delta - z_{2A})\right]
\end{aligned} \tag{6}$$

where, $-\frac{b_4}{2} = (z_1^\alpha - z_{2A}))$, $\frac{b_4}{2} = (z_1^\beta - z_{2A})$, $b_5 = (z_2^\gamma - z_{2A})$ and $b_{p1} = (z_2^\delta - z_{2A})$, being $z_{2A}$ the reference coordinates.

## 4. OPTIMIZATION OF AERONAUTICAL PANEL

The structural optimization problem to be solved consists in minimizing the mass of aeronautical panel under dynamic load $f(t)$ and boundary conditions of the Fig. 6, subject to maximum displacement, stress and buckling constraints.

The inplane external applied force is illustrated in Fig. 6 and its diagram is shown in Fig. 7. Two optimization cases were considered. The first $(1)$ optimization case makes use of the ESL method as given by Eq. (2). The second case $(2)$ does not use ESL approach, instead use a static loading whose intensity is the maximum peak value of the dynamic load diagram of Fig. 7. By the way, this would be the load used in normal design practice.

In both cases the optimal solution was obtained with sequential approximate optimization (SAO) using Woo (1987) generalized hybrid constraint approximation (GHC) and the Powell's method, whose algorithm was written in PYTHON language from the book of Kiusalaas (2005). The structural analysis required has been carried out in the ABAQUS solver. The sensitive analysis was carried out by finite differences.

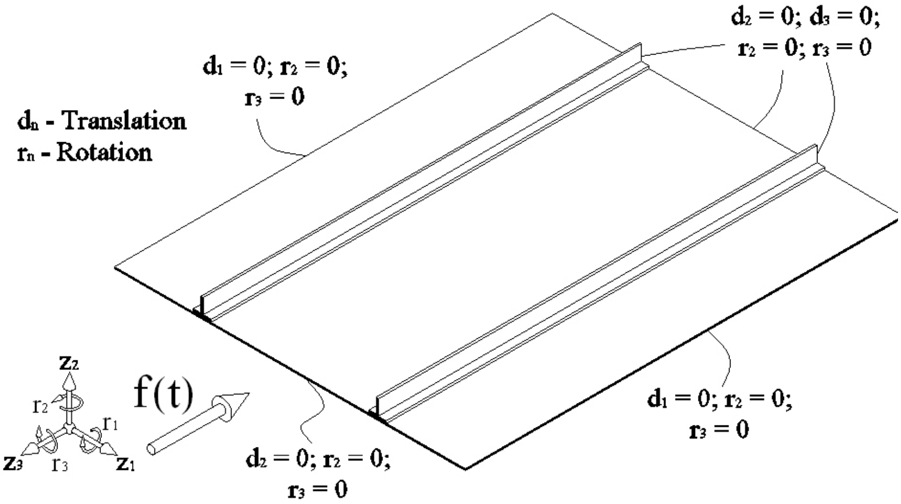


Figure 6. Aeronautical panel under dynamic load $f(t)$.

The Fig. 7 shows the dynamic load $f(t)$ which has a high frequency noise and to which the panel is subjected.

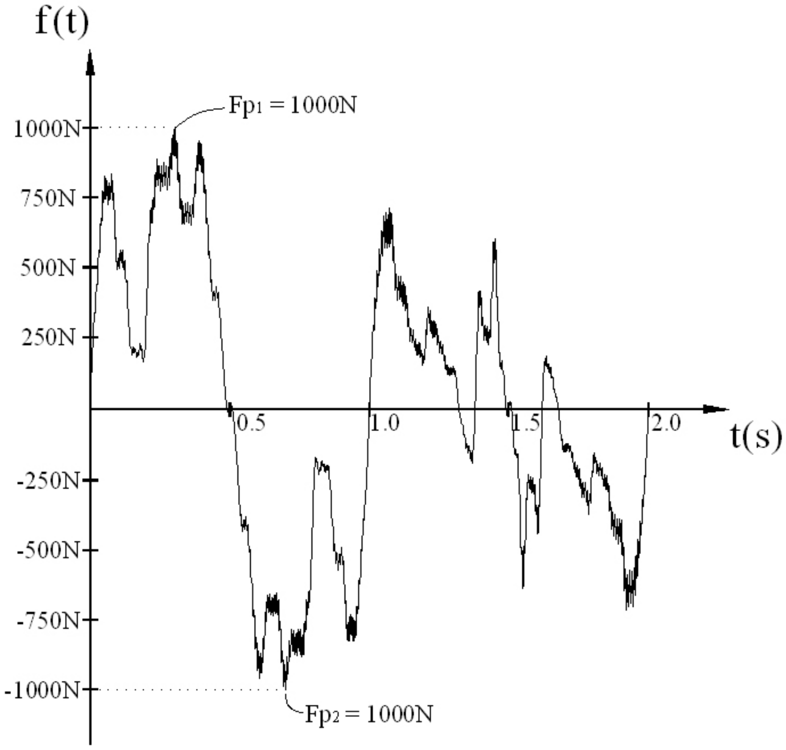


Figure 7. Dynamic load $f(t)$.

The original formulation of optimization problem is given by:

$$
\begin{aligned}
\text{Minimize:} \quad & M(b_n) \quad n = (1, ..., 5) \\
\text{Subject to:} \quad & [M]\frac{\partial^2\{d\}}{\partial t^2} + [C]\frac{\partial\{d\}}{\partial t} + [K]\{d\} = \{f(t)\} \\
& -2.5 \cdot 10^{-6} \leq d_2 \leq 2.5 \cdot 10^{-6} m \\
& \sigma_V \leq 6 \cdot 10^6 MPa \\
& ([K] - \lambda\,[K_G])\,\varphi = 0 \\
& \lambda \geq 1
\end{aligned}
\tag{7}
$$

where $M$, $d_2$, $\sigma_V$ and $\lambda$ are respectively the panel mass, displacement in direction $z_2$ , maximum von Mises stress and buckling load factor. The eigenvalue problem in Eq. (7) is supposed to exist for every time instant $t$.

### 4.1 (1) Dynamic optimization by use ESL's

The dynamic load optimization problem in Eq. (7) becomes by using ESL's, the following:

$$\begin{aligned}
\text{Minimize:} \quad & M(b_n) \quad n = (1, ..., 5) \\
\text{Subject to:} \quad & [K(b_n)]\{d(t_i)\} = \{s(b_n)\}_i \quad i = (1, ..., m) \\
& (K(b_n) - \lambda_i K_G^i(b_n, \{s\}_i))\varphi_i = 0 \\
& -2.5 \cdot 10^{-6} \leq d_{2i} \leq 2.5 \cdot 10^{-6} \\
& \sigma_{Vi} \leq 6 \cdot 10^6 MPa \\
& \lambda_i \geq 1
\end{aligned} \tag{8}$$

The time instants in which the ESL's were calculated are those corresponding to the: a) maximum von Mises stress over the structure, b) maximum longitudinal displacement and c) maximum transversal displacement. The search for these peak times must be done in the initial design and after each optimization cycle done with a fixed ESL, for new values of the design variables.

Table 1 has the initial design variables values and the optimum results for the aeronautical panel.

Table 1. Results of optimization under ESL's

| | $Mass(Kg)$ | Design Variables (m) | | | | | $Iteration$ |
|---|---|---|---|---|---|---|---|
| | $W(bn)$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | 5 |
| $Initial$ | 0.810 | 0.001200 | 0.001500 | 0.001500 | 0.0200 | 0.0200 | $Time(s)$ |
| $Final$ | 0.453 | 0.000733 | 0.000883 | 0.000883 | 0.0118 | 0.0120 | 20623 |

Table 2. Final results of constraints in critical times (s, MPa, m)

| $Instant(s)$ | $\sigma_V$ | $g_\sigma$ | $-d_2$ | $g_{dL}$ | $d_2$ | $g_{dU}$ | $\lambda$ | $g_\lambda$ |
|---|---|---|---|---|---|---|---|---|
| 0.302 | $2.82 \cdot 10^6$ | $-0.530$ | $-3.91 \cdot 10^{-7}$ | $-0.844$ | $6.44 \cdot 10^{-7}$ | $-0.742$ | 1.011 | $-0.011$ |
| 0.304 | $2.73 \cdot 10^6$ | $-0.545$ | $-6.86 \cdot 10^{-7}$ | $-0.726$ | $2.48 \cdot 10^{-7}$ | $-0.900$ | 1.031 | $-0.031$ |
| 1.062 | $1.59 \cdot 10^6$ | $-0.735$ | $-2.5 \cdot 10^{-6}$ | 0.000 | $1.26 \cdot 10^{-7}$ | $-0.950$ | 1.743 | $-0.743$ |
| 1.070 | $1.60 \cdot 10^6$ | $-0.733$ | $-2.48 \cdot 10^{-6}$ | $-0.008$ | $1.27 \cdot 10^{-7}$ | $-0.949$ | 1.737 | $-0.737$ |

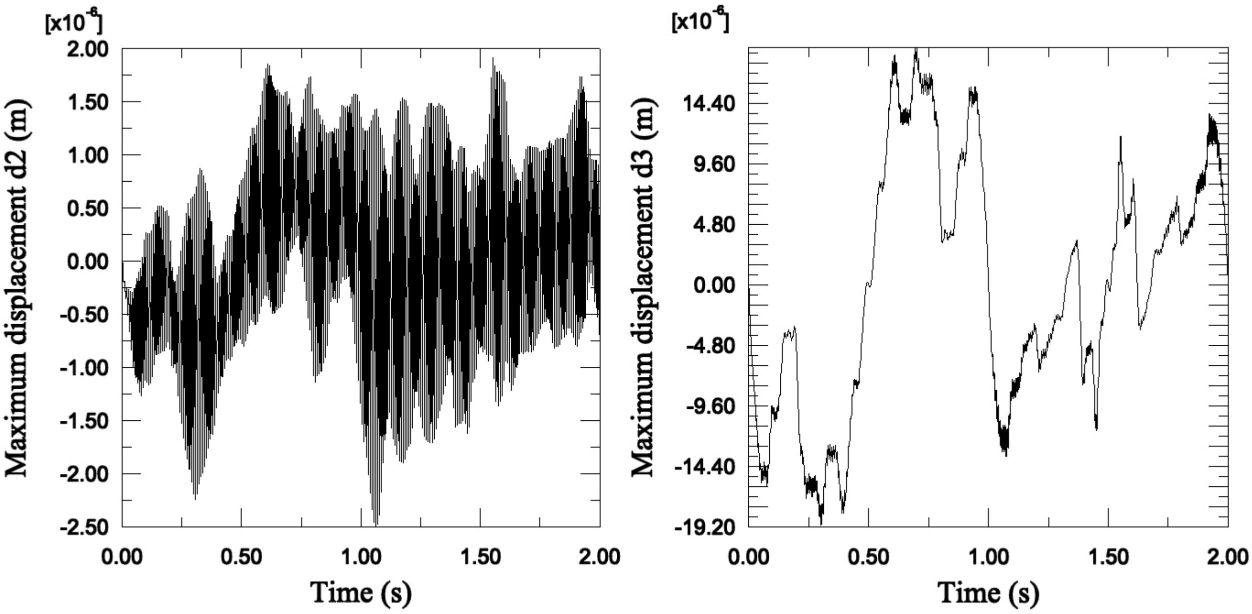


Figure 8. Maximum displacements in the directions $z_2$ and $z_3$. Results of the optimization by the use ESL.

The Tab. 2 has the constraints values for the optimal design. It can be observed that the constraints are satisfied. The symbols $g_{DL}$ and $g_{DU}$ used represent respectively the lower and upper displacement constraints. The critical constraints are the $g_{DL}$ and the buckling constraint $g_\lambda$. There The final optimal design was able to limit the effects of resonance of the panel, as is possible to see the restrictions on the values of $g_{dL}$ and $g_{dU}$. For the most critical instants of the table were computed ESL's $s_1$, $s_2$, $s_3$ and $s_4$.

### 4.2 $(2)$ Case. Static optimization by use $Fp_i$

The optimization problem for the panel under the peak values of load $F_p$ is given by:

$$
\begin{aligned}
\text{Minimize:} \quad & M(b_n) \quad n = (1, ..., 5) \\
\text{Subject to:} \quad & [K(b_n)]\{d(t_i)\} = \{F_p\}_i \quad i = (1, 2) \\
& (K(b_n) - \lambda_1 K_G^1(b_n, \{F_p\}_1))\varphi_1 = 0 \\
& -2.5 \cdot 10^{-6} \le d_{2i} \le 2.5 \cdot 10^{-6} \\
& \sigma_{Vi} \le 6 \cdot 10^6 MPa \\
& \lambda_i \ge 1
\end{aligned}
\tag{9}
$$

Only two peak values were selected, corresponding to the values marked in Fig. 7. The $Fp_1$ and $Fp_2$ are forces of peak referring to the instants $0.302s$ and $0.700s$. Table 3 has the initial values and the optimum results of the minimization.

Table 3. Results of optimization under loads $Fp_i$

| | $Mass(Kg)$ | $DesingVariables(m)$ | | | | | $Iteration$ |
|---|---|---|---|---|---|---|---|
| | $W(bn)$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | 7 |
| $Initial$ | 0.810 | 0.001200 | 0.001500 | 0.001500 | 0.0200 | 0.0200 | $Time(s)$ |
| $Final$ | 0.449 | 0.000719 | 0.000921 | 0.000921 | 0.01227 | 0.01227 | 4926 |

Table 4. Final results in critical times (s, MPa, m)

| $Instant(s)$ | $\sigma_V$ | $g_\sigma$ | $-d_2$ | $g_{dL}$ | $d_2$ | $g_{dU}$ | $\lambda$ | $g_\lambda$ |
|---|---|---|---|---|---|---|---|---|
| 0.302 | $2.88 \cdot 10^6$ | $-0.520$ | $-8.91 \cdot 10^{-7}$ | $-0.644$ | $2.56 \cdot 10^{-7}$ | $-0.898$ | 1.000 | 0.000 |
| 0.700 | $2.93 \cdot 10^6$ | $-0.512$ | $-3.98 \cdot 10^{-6}$ | 0.592 | $1.81 \cdot 10^{-7}$ | $-0.276$ | $-$ | $-$ |
| 0.783 | $1.91 \cdot 10^6$ | $-0.682$ | $-1.17 \cdot 10^{-7}$ | $-0.953$ | $6.38 \cdot 10^{-6}$ | 1.552 | $-$ | $-$ |
| 1.070 | $1.60 \cdot 10^6$ | $-0.733$ | $-6.53 \cdot 10^{-6}$ | 1.612 | $9.56 \cdot 10^{-8}$ | $-0.962$ | 1.737 | $-0.737$ |

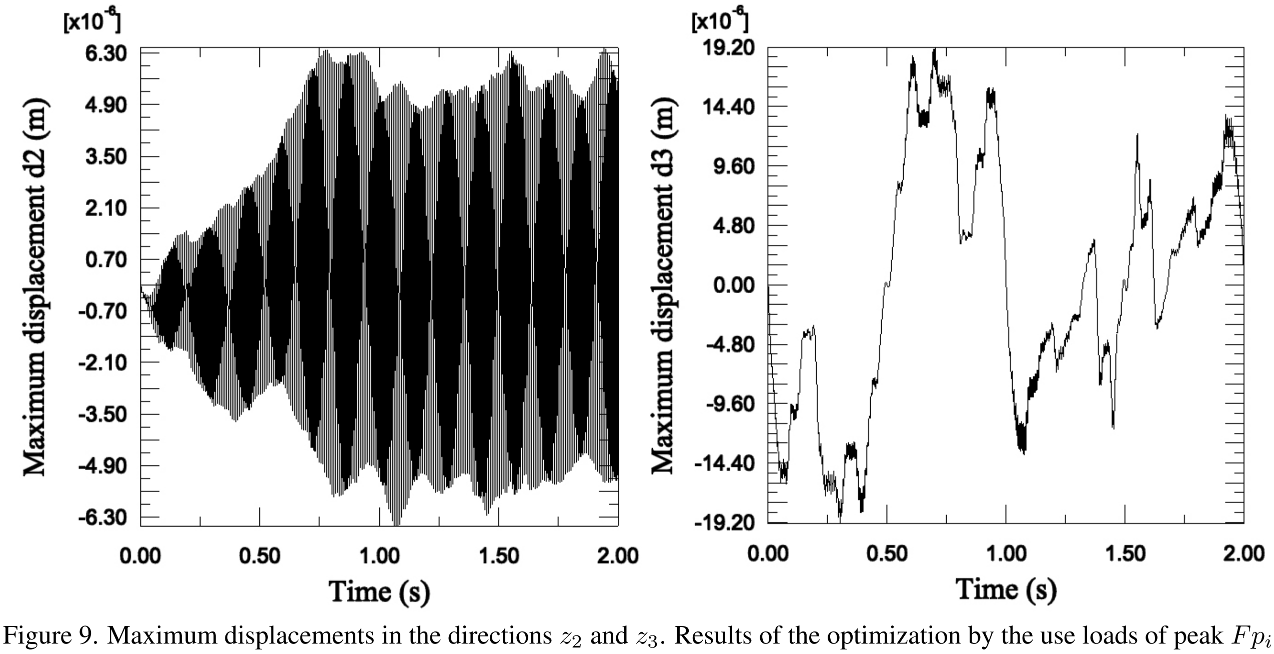


Figure 9. Maximum displacements in the directions $z_2$ and $z_3$. Results of the optimization by the use loads of peak $Fp_i$.

Table 4 contains the results after a new dynamic analysis with the optimal design for this case, where the buckling constraint is critical and is also satisfied, however it is seen that constraints $g_{dL}$ and $g_{dU}$ for the three time instants $0.700$, $0.783$ and $1.070$ are not satisfied, in fact the maximum violation is $162\%$.

Comparing the solutions of both cases it is apparent that in case 2 the part of transversal displacements due to resonance effects, caused by the high frequency noise from load $f(t)$, are not detected. Meanwhile, the ESL approach was able to perceive the resonance effects, leading to an optimal design with satisfied dynamic displacement constraints.

Note that maximum and minimum longitudinal displacement $d_3$ in Fig. 8 and Fig. 9 are practically the same in both cases, meaning that the resonance does not affect the displacements in this direction $z_3$.

## 5. CONCLUSION

It was shown that the use of ESL for the design optimization of an aeronautical panel under inplane dynamical load was very successful, producing an optimal panel obeying dynamical displacement and buckling constraints. The method was able to produce very good results with few samples of ESL's corresponding to selected displacement and stress peak times. The computer effort was kept very low, in the range of five complete dynamical analysis. The final optimum design of the aeronautical panel produced with ESL could deal with resonant displacements caused by the noise of high frequency present in dynamic load. However, the simplification made in case 2, usual in common practice, led to an inadequate optimal solution, where constraints are violated.

## 6. ACKNOWLEDGEMENTS

CAPES - Coordenação de Aperfeiçoamento de Pessoal de Nível Superior and Instituto Tecnológico de Aeronáutica.

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## 8. Responsibility notice