A STOCHASTIC COLLOCATION METHOD APPLIED TO UNCERTAINTY QUANTIFICATION IN HYDRAULIC FRACTURE MODELING

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Abstract.
It has been recognized that there exists significative variability in geologic formations such as oil and gas reservoirs. However, only limited measurements are usually available. The theory of stochastic processes provides a natural method for evaluating these variability. In this study, we present a stochastic analysis of hydraulic fractures. There, some of the reservoir properties, such as the Elasticity Modulus are treated as random spatial functions and, therefore, the equations governing of fractures become stochastic partial differential equations SPDEs. We use the Implicit Level set algorithm and Stochastic Collocation method to solve this SPDEs. The probability distribution of the Elasticity Modulus was constructed using two approaches: the first is to assume that the Elasticity Modulus follow a log-normal distribution. The second approach is to build the probability distribution of the Elasticity Modulus using the maximum entropy method and build a generator of this probability distribution by the Metropolis-Hastings algorithm. The study demonstrates the importance of taking parameter uncertainty into account for hydraulic fractures modeling in order to assess the viability of numerical modeling.

Keywords: *Uncertainty, Implicit Level set algorithm, Stochastic Collocation, Metropolis-Hastings*

1. INTRODUCTION

Hydraulic fracturing is the process by which fracture initiates and propagate due to the fracturing fluid injected in the well with a high pressure (R.Carbonell, Minneapolis USA 1998). Today, hydraulic fracturing is used extensively in the petroleum industry to create additional passageways in the oil reservoirs that can improve the productivity of the well. Experimental test of this process is expensive and give limited information (Yew and Rosolen, 2008). In this case, numerical simulation is used to predict the fracture response (Adachi, 2007). In many case, a numerical simulation don't take into account the reservoirs uncertainty (Varela *et al.*, 2006; Amadei B, 1997). However it has been recognized that there exist significant variabilities in geologic formation such as oil and gas reservoirs and only limited measurements are available at a few locations of reservoirs. This incomplete information its leads to uncertainty about the values of reservoirs properties. Uncertainties in the input parameters(reservoirs properties) propagates uncertainties in estimates of production potential. So is very importante to quantify uncertainty in the hydraulic fracture process. The theory of uncertainty quantification provides natural methods for assessing impact of uncertainties (input) on the solution of problem (output) and provides important statistics information that can be help to take decision.

In this paper, we consider the Elasticity Modulus as uncertainty parameter(D.Ask, 2005). Using this information, the original governing equation can be reformulated as stochastic partial differential equations(SPDFs). To solve this SPDEs, we presente two methods such as Monte Carlo simulation (R.E.Caflish., 1998) and stochastic collocation method (Xiang and Zabaras, 2009). Monte Carlo simulation involves generating various realizations of the input parameter according to the underlying probability distribution and repeatedly resolve the deterministic solver for each realization. In stochastic collocation method we construct the interpolation of the output function using the lagrange polynomial and compute their value at each point of the stochastic space.

The rest of the paper is organized as follows: In the next section we present the governing equation of hydraulic fracture. Section 3 ilustre the stochastic formulation of hydraulic fracture modeling. Section 4 we present the result and discussion and the section 5 we draw the conclusion.

2. PROBLEM DEFINITION

The fracturing fluid is injected into the fracture at a constant volumetric flow rate Q_0, we seek to determine the fracture half length ℓ as a function of time t, the crack opening w(x,t), the fluid pressure $p_f(x,t)$(the net pressure p= $p_f - \sigma_0$) (Carbonell, 1999), see Fig 1.

The hydraulic fracture is governing by the elasticity equation, fluid flow equation and the initial and boundary conditions.

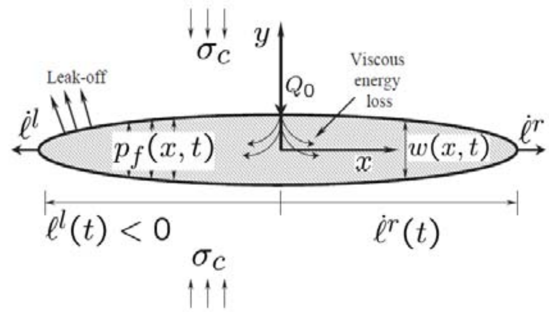

Figure 1. Geometria de uma fratura unidimensional.Rochinha and Peirce (2009)

- **Elasticity equation**

$$p(x,t) = \frac{-E'}{4\pi} \int_{\ell^l}^{\ell^r} \frac{w(x',t)}{(x-x')^2} dx', \tag{1}$$

- **Fluid flow equation**

$$\frac{\partial w}{\partial t} + \frac{\partial q}{\partial t} = \frac{C' H(t - t_0(x))}{\sqrt(t - t_0(x))} + Q(t)\delta(x). \tag{2}$$

$$q = \frac{-w^3}{\mu'} \frac{\partial p_f}{\partial x}, \ell^l(t) < x < \ell^r(t), \tag{3}$$

- **Initial and boundary conditions**

$$\pm q\Big|_{x=0^+}^{x=0^-} = \frac{Q_0}{2}. \tag{4}$$

$$w = 0, x = \ell^r; x = \ell^\ell. \tag{5}$$

$$w^3 \frac{dp}{dx}, x = \ell^r; x = \ell^\ell. \tag{6}$$

We use the implicit Level set algorithm (Peirce and Detournay, 2008) to solve the deterministic solution of hydraulic fracture. The stochastic solution are compute using the Stochastic collocation method.

3. STOCHASTIC FORMULATION AND QUANTIFICATION OF UNCERTAINTY

In this work, the Elasticity Modulus E' of rock reservoirs is treated as uncertainty parameter (Amadei B, 1997). To construct the probability distribution of this parameter we used two approaches. The first consist to consider that the Elasticity Modulus at a log-normal distribution (Eckhard limper, 2001; Sachs, 1997) and the parameter uncertainty $\widetilde{E'}$=$\overline{E'}(1+\kappa\varepsilon)$. Where κ is related to the variance and ε is the log-normal distribution. In the second approach $\widetilde{E'}$=$\overline{E'}(1+\varepsilon)$, we construct the probability distribution of Elasticity Modulus using the maximum entropy method (Soize, 2008; Sankaran and Zabaras., 2006) and build a generator of this PDF using the metropolis Hastings algorithm (Campillo, 2009; Johansen, 2010). To construct the PDF in the second approach, we used two statistics information: the mean $\overline{E'}$=1 like the first approach and the variability interval as [0.4 1.6]. the variability interval was chosen so that 96% of the samples obtained using the first approach to be contained. To construct the generator of the PDF construct, we used the uniforme distribution as *a priori* information see Fig 2-3.

Considering the uncertainties in the input parameter and writing the governing equation at the non-dimensional form we have:

$$\Pi_f(\chi,\tau,\xi) - \Sigma_0\Phi(\chi,\xi) = \frac{-\widetilde{g}_e}{4\pi} \int_{\gamma^l}^{\gamma^r} \frac{\Omega(\chi',\tau,\xi)}{(\chi-\chi')^2} d\chi', \tag{7}$$

$$\begin{aligned} \frac{\partial \Omega(\chi,\tau,\xi)}{\partial \tau} \quad = \quad & \frac{1}{g_m} \frac{\partial}{\partial \chi} (\Omega(\chi,\tau,\xi)^3 \frac{\partial \Pi_f(\chi,\tau,\xi)}{\partial \chi}) + \\ & \frac{g_c H(\tau - \tau_0(\chi))}{\sqrt(\tau - \tau_0(x))} + \Psi(\tau,\xi) g_v \delta(\chi), \gamma^\ell(\tau,\xi) < \chi < \gamma^r(\tau,\xi). \end{aligned} \tag{8}$$

$$\Omega(\gamma^{\ell,r},\tau,\xi)=0, \lim_{\chi\to\gamma}\Omega^3(\chi,\tau,\xi)\frac{\partial\Pi_f(\chi,\tau,\xi)}{\partial\chi}=0. \tag{9}$$

Where stochastic system and stochastic solutions of hydraulic fracture are:

$$\mathcal{L}(\Omega,\Pi_f,\ell:\chi,\tau,\xi)=0. \tag{10}$$

$$\begin{aligned}\Omega(\chi,\tau,\xi) &= \Omega(\chi,\tau,\xi),\\ \Pi_f(\chi,\tau,\theta) &= \Pi_f(\chi,\tau,\xi),\\ \ell(\tau,\theta) &= \ell(\tau,\xi),\end{aligned}$$

Ω, Π, ℓ are non-dimensional crack opening, fluid pressure and length.

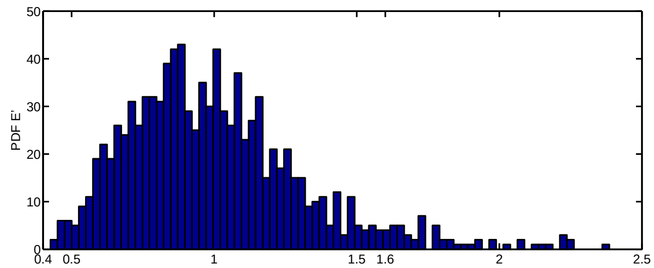

Figure 2. PDF E' for the first approach.

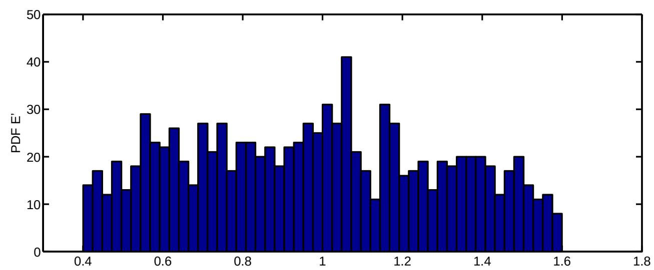

Figure 3. PDF E' for the second approach.

To solve the stochastic system, we used the Stochastic Collocation method (Ganapathysubramanian and Zabaras, 2008). The basic idea of the collocation approach is to build an interpolation function for dependent variables using their values at particular points in the stochastic space. Denote by ξ any point in the random space $\Gamma\subset\Re$, by $\prod_N$, the space of all N-variate polynomials and by $\prod_N^P$, the subspace of polynomials of total degree P. the problem of interpolation can be stated as follows:

Given a set of nodes $\theta_N=\{\xi_i\}_{i=1}^M$ in the N-dimensional random space Γ and the smooth function $f:\Re^N\longrightarrow\Re$ The polynomial approximation can be expressed using the Lagrange interpolation polynomials as follows:

$$\Im f(\xi)=\sum_{k=1}^{M}f(\xi_k)L_k(\xi), \tag{11}$$

where

$$L_i(\xi_j)=\delta_{ij}. \tag{12}$$

Consider now the Lagrange interpolated of the hydraulic fracture values (Ω,Π_f,ℓ), denoted by $(\widehat{\Omega},\widehat{\Pi_f},\widehat{\ell})$ are follows:

$$\begin{aligned}(\widehat{\Omega},\widehat{\Pi_f},\widehat{\ell}) &= (\Im\Omega(\xi),\Im\Pi_f(\xi),\Im\ell(\xi))\\ &= \sum_{k=1}^{M}(\Omega(\xi_k),\Pi_f(\xi_k),\ell(\xi_k))L_i(\xi).\end{aligned} \tag{13}$$

Substituting this into the governing equation Eq 10, gives

$$\mathcal{L}(\sum_{i=1}^{M}\Omega(\xi_i)L_i(\xi),\sum_{i=1}^{M}\Pi_f(\xi_i)L_i(\xi),\sum_{i=1}^{M}\ell(\xi_i)L_i(\xi):\chi,\tau,\xi)=0. \tag{14}$$

This equation is M decoupled deterministic systems like Monte Carlo simulation. To construct the stochastic collocation method we used the subroutine proposed by (Klimke, 2008). For the next section, we study the convergence of the Stochastic Collocation method compared to the Monte Carlo simulation.

4. RESULT AND DISCUSSION

In this section we present the solution of hydraulic fracture considering the uncertainty about the values of reservoir properties. This uncertainty is related to the Elasticity Modulus E'. In the first page of this section we analise the performance of Monte Carlo simulation and Stochastic Collocation method after this, we used the Stochastic Collocation method to quantified uncertainties in the hydraulic fracture process.

4.1 Performance analysis

To analyse the performance of the Monte Carlo simulation and Stochastic Collocation method, we consider $\Delta\chi$=2, $\Delta\tau$=10 and the fracture opening is started at $\tau = 38.54$. The convergence of Monte Carlo simulation and Stochastic Collocation method are analyse using the fracture opening result Ω at the times $\tau = 53$, $\tau = 105$, $\tau = 158$.

The convergence of Monte Carlo simulation is evaluated for an increasing number of realization to 50 until 3500. For the each realization, we calculated the stochastic solution mean and standard deviation. The convergence test is evaluated by the calculate of the absolute error. We consider the solution of Monte Carlo simulation at 3500 realizations as reference solution. So, the absolute error is equal to the euclidean norm of the reference solution menos the stochastic solution at each realization(<3500 realizations) i.e, $\|\overline{\Omega(\chi,\tau,\xi=3500)}-\overline{\Omega(\chi,\tau,\xi_i)}\|_2^2 = \frac{1}{N}\sum_{k=1}^{N}(\overline{\Omega_k(\chi,\tau,\xi_{3500})}-\overline{\Omega_k(\chi,\tau,\xi_i)})^2$. Where N is the number of total point fractured at each time τ and $\overline{\Omega}$ represent the mean of Ω. In the Monte Carlo simulation ξ_i represent realizations to 50 until 3500. While in the Stochastic Collocation method ξ_i represent the increasing level of interpolation until to get the same order of absolute error, see Fig 4-5. Observed that for the same absolute error 0.002, the Monte Carlo simulation used 3000 points(sample of E'). While the Stochastic Collocation method used 9 points(sample of E').

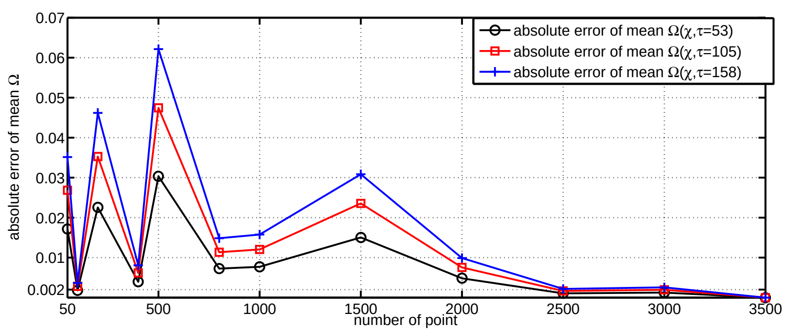

Figure 4. Convergence rate obtained by Monte Carlo simulation

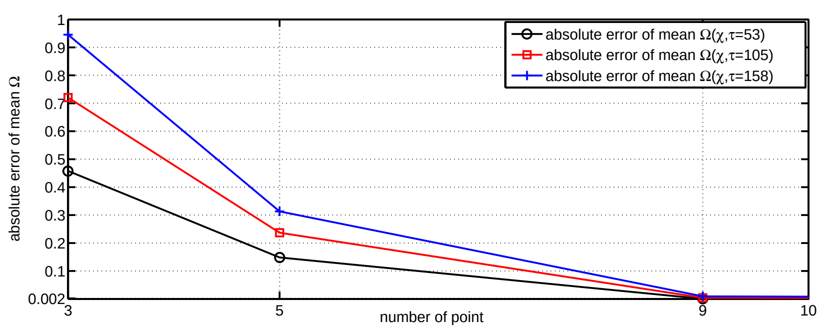

Figure 5. Convergence rate obtained by Stochastic Collocation method

In Fig 6, we compared convergence of $\overline{\Omega}(\chi, \tau = 158)$ obtained through Stochastic Collocation method to the reference solution(Monte Carlo 3500 points). We observed that at 9 points the stochastic Collocation solution is close to the reference solution and the solution at 9, 17, 33 is the same, this result shows the convergence of Stochastic Collocation method.

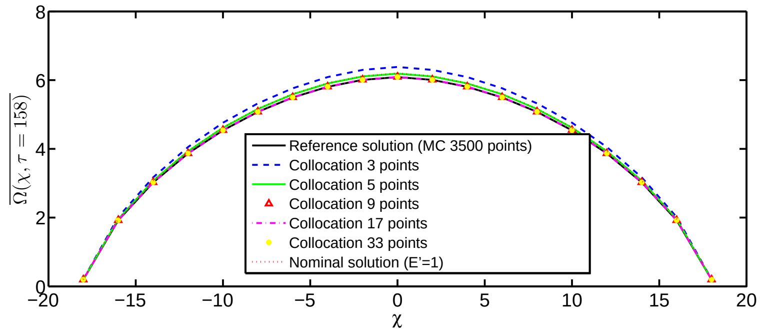

Figure 6. Stochastic Collocation solution compared to the reference solution

The we PDF convergence is study using the Stochastic Collocation method. We observed that the PDF convergence calculated using Stochastic Collocation method depend of two parameters N and P. N is the level of interpolation and P the number of sample used to calculate interpolated function. In Fig 7 , we have the PDF convergence with respect to the level of interpolation(points) but the Stochastic Collocation solution don't close to the reference solution. In Fig 8 when the number of sample P increases the Stochastic Collocation solution is close to reference solution.

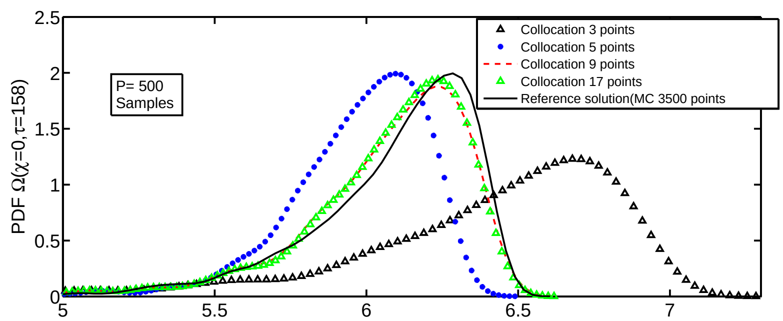

Figure 7. PDF $\Omega(\chi = 0, \tau = 158)$ convergence using stochastic collocation method for P=500 samples

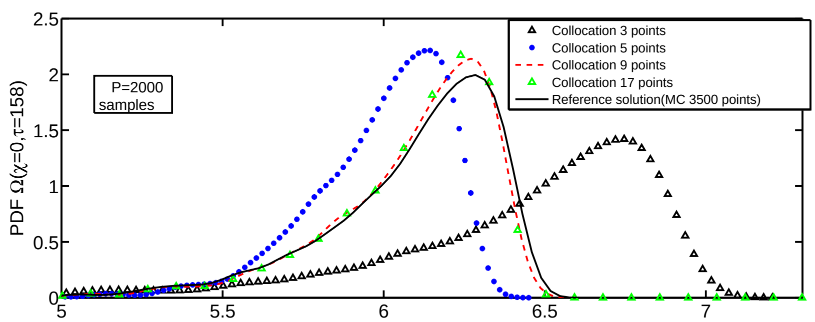

Figure 8. PDF $\Omega(\chi = 0, \tau = 158)$ convergence using stochastic collocation method for P=2000 samples

After this study, we observed that the Stochastic Collocation method is efficient to propagate uncertainties related E in the hydraulic fracture process.

5. Propagation and Quantification of uncertainties

In this section we propagate the uncertainty in the hydraulic fracture process through the Elasticity Modulus E' and analyse their effect on the output crack opening Ω. We calculated the uncertainty in the output parameter using the mean and standard deviation of Ω. The error bar is used to present uncertainties in the output parameter.

In Fig 9 we present the mean and uncertainty of the crack opening at the injected time τ= 53, τ=105, τ=159. We observed that uncertainties on the crack opening increases with the time and are large at the near of the well.

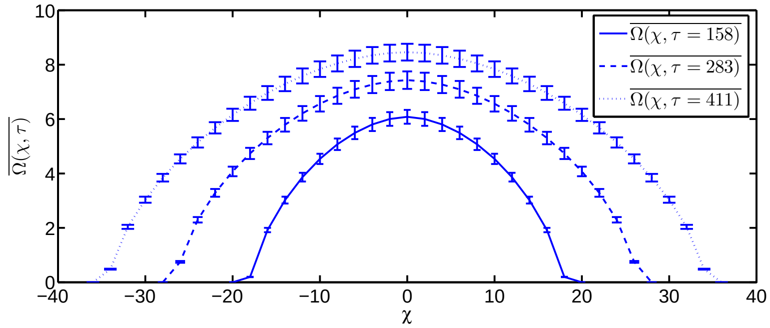

Figure 9. mean and uncertainties of crack opening.

In Fig 10 we present we present the result of crack opening at the source $\Omega(\chi = 0, \tau)$ for 3 values of κ. We see that when κ increases uncertainties on $\Omega(\chi = 0, \tau)$ increases.

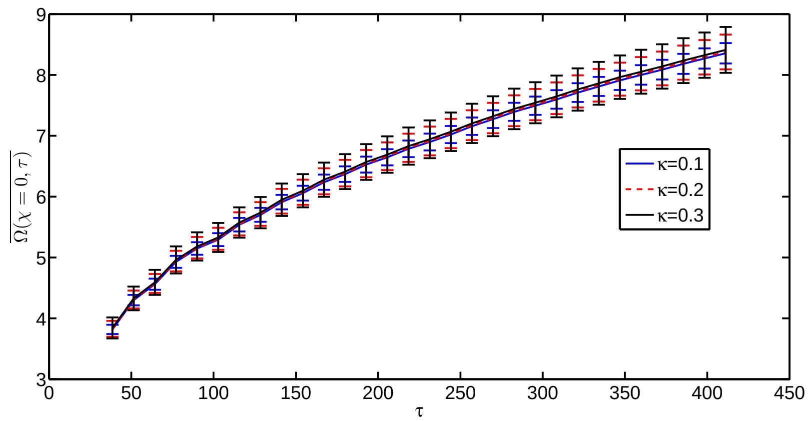

Figure 10. $\Omega(\chi = 0, \tau)$ for 3 values of κ.

In Fig we present the PDF of crack opening at the spatial ξ=0 and the injection time τ=411 and we compared the PDF obtained at a lognormal distribution constructed using the mean and variance of $\Omega(\chi = 0, \tau = 411)$. The idea of this comparison is to check if the PDF of a output $\Omega(\chi = 0, \tau = 411)$ is the same as the input PDF E'. We can observed that the both PDF are very diferente and the probability to get the crack opening $8.5 \leq \Omega(\chi = 0, \tau = 411) \leq 9$ is 49%.

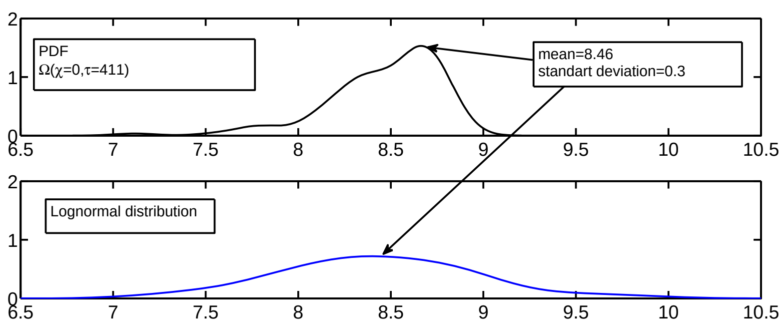

Figure 11. PDF of $\Omega(\chi = 0, \tau = 411)$.

Now we compare the result of crack opening obtained to the two approaches of modeling uncertainty about the elasticity modulus. We observed in the Fig 12-13 that the result of the crack opening are very diferente. Uncertainties on $\Omega(\chi = 0, \tau)$ are greater in the second approach than the first. Also we see that the PDF $\Omega(\chi = 0, \tau = 411)$ of both approach are very diferente.

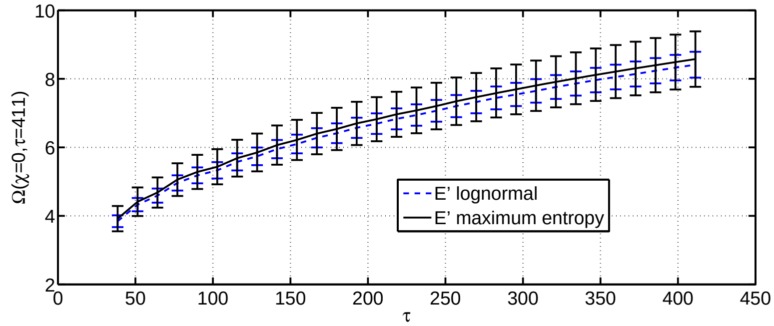

Figure 12. $\Omega(\chi = 0, \tau)$ with both approach.

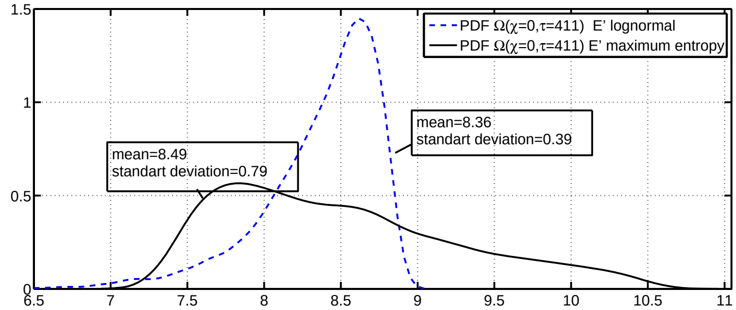

Figure 13. PDF $\Omega(\chi = 0, \tau = 411)$ with both approach.

In the last part of this section we present the result of the inclinometer (Rochinha and Peirce, 2009) coupled to uncertainty concept. This combination can used to construct a confidence interval of crack opening deformation. We use the mean and standard deviation to construct this confidence interval see Fig 14.

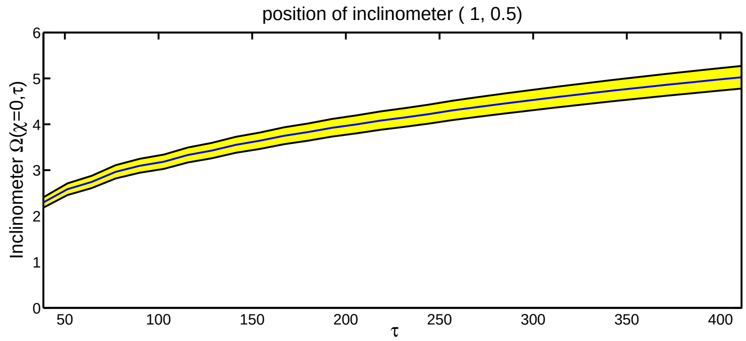

Figure 14. Deformation observed for an inclinometer at the position (1,0.5).

6. CONCLUSION

In this work we presented the importance of taking into account uncertainties in the resolution of complex problem like hydraulic fracture. We observed that the Stochastic Collocation method is efficient to propagate uncertainty in hydraulic fracture. We saw also the effect of Elasticity Modulus and we observed that uncertainty quantification can be very interesting to take decision through the statistic information like the PDF.

The future study is to consider that the regime of propagation of fracture is uncertainty.

7. ACKNOWLEDGEMENTS

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9. Responsibility notice

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