PERFORMANCE ASSESSMENT OF MODEL REFERENCE ADAPTIVE CONTROL – A BENCHMARK BASED COMPARISON AMONG TECHNIQUES

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Abstract. The main objective of adaptive control schemes is to produces reliable controllers under plant uncertainties, non-linearity and time varying parameters. Adaptation schemes are designed to cope with slow parameter changes; in addition, input signals are assumed to have frequency spectrum restricted to low frequency. In model reference adaptive control (MRAC) schemes, however, perfect model tracking depends on some conditions that usually are too restrictive for the class of plants under consideration. The derivations of control laws for MRAC are usually based on Lyapunov's and Popov's theorems, some basic assumptions appear in almost every variation of those approaches; the most restrictive one is that the plant must remain strictly positive real (SPR) for all time. Alsol, to avoid dimensionality problems, MRAC schemes are implemented based only on output measurements. The discussions here are based on some of the main contributions in the area of model reference adaptive control, such as, the command generator tracking (CGT) technique and on the dynamic model reduction adaptive control (DMR-AC) approach. This work considers adaptive control applied to plants with time varying parameters and time delays. The results are given for the case in which the plant, defined by [A, B, C, D] and the reference model [Am, Bm, Cm, Dm], do not share the same state space realization. Simulation results are presented for a time-varyng non-strictly positive real linear model with time delay. They show and compare the performance degradation of the adaptive controllers when the plant parameters drift from their nominal values.

Keywords: Direct adaptive control; model reference adaptive control; benchmark-based assessment.

1. INTRODUCTION

Adaptive control schemes have been delivered as the solution for model uncertainties and plant parameters variation. Among a constantly increasing number of adaptive control techniques, model reference adaptive control plays an important role in the area. However, several problems remain unsolved, among them: current theoretical results for MRAC yield asymptotic stability only for strictly positive real (SPR) plants and this condition is too restrictive for most of the industrial control problems, and usually, the controller performance is quite poor and few things can be done to improve it. The search for a solid theoretical result through the past decades has neglected the main issue in control engineering, which is the controller performance. The current results in this area, although based on consistent theorems, are also too conservatives delivering, in general, poor controller performance.

This paper challenges the current well-accepted ideas in model reference adaptive control. It follows an engineering approach and shows through simulation results the outstanding performance of a new adaptive control scheme. Starting from a nominal plant model, the plant parameters are modified through the simulated experiments keeping the control algorithm unchanged. Two adaptive control algorithms are tested and their performance compared: the command generator tracking (CGT) technique (Clarke, Mohtadi and Tuffs, Parts I and II, 1987) and the dynamic model reduction adaptive control (DMR-AC) approach (Galvez, 2010). The paper is organized as follows: Section2 presents a brief revision on some direct adaptive control squemes, Section 3, presents a time varying plant model used for comparing controller performance trough numerical simulations. Finally, Section 4 presents final comments and conclusions.

2. DIRECT MODEL REFERENCE ADAPTIVE CONTROL SCHEMES

The main objective of adaptive schemes is to produce a robust controller under plant uncertainties, nonlinearities, and time varying parameters. In direct model reference adaptive control schemes, however, perfect model tracking depends on some conditions that are not always valid for the class of plants under consideration. The derivation of the control law for these schemes is not unique. Several derivations based on Lyapunov and Popov's theorems have been proposed in the literature. However, some basic assumptions appear in almost every variation of them; the most restrictive is the one that the plant must remain strictly positive real (SPR) for all time.

THE CGT TECHNIQUE

The CGT technique was originally proposed by Clarke et al. (Clarke, Mohtadi and Tuffs, Parts I and II, 1987) and it has become a milestone in the direct adaptive control area. Several fine works have been presented based on the CGT algorithm.

However, most of them have been focused on releasing the SSR condition from the derivation of the control law, and few attempts have been made to improve the controller structure, neglecting in some way the controller performance. Fig. 1 shows the block diagram of the CGT technique.
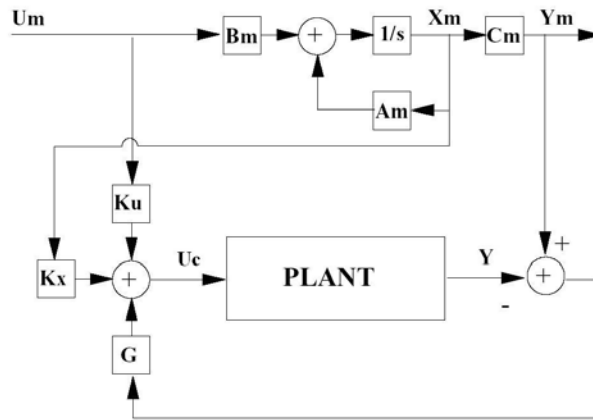

Figure 1. The CGT Adaptive Controller.

The CGT objective is to create an adaptive control signal, $u(t)$, that forces the plant output, $y(t)$, to track the reference model output, $y_m(t)$. To avoid dimensionality problems, the CGT control law is obtained and implemented based on output measurements. The most recent results are for the case in which the plant, defined by $[A\ B\ C\ D]$ and the model $[A_m\ B_m\ C_m\ D_m]$, do not share the same dimension. In the CGT case, the control law is usually defined as

$$u_c = G(t)\,e_y + K_x(t)\,x_m + K_u(t)\,u_m = K(t)\,z \tag{1}$$

where u_m is the model input, x_m is the model state and e_y is the output error. The gain matrices, $K(t)$, are updated by an adaptation law (only dependent on output measurements) of the form

$$K(t) = K(t_o) + \int_{t_o}^{t} e_y(\tau)\,z^T(\tau)\,d\tau \tag{2}$$

To obtain the adaptation law it is assumed the existence of a positive defined matrix P, which satisfies:

$$\begin{aligned} [A + BG(t)]^T P + P\,[A + BG(t)] &= -Q \qquad ; \qquad Q > 0 \\ B^T P &= C \end{aligned} \tag{3}$$

Notice that this is an especial case ($D = 0$) of the well-known Kalman-Yacubovich Lemma. Although the Lemma allows a partial solution for the stability proof (allowing the adaptation law from output measurements), it penalizes the method by constraining the plant to be strictly positive real (SPR). In addition, there is an implicit assumption that there exists a gain matrix such that $A_c = [A+BG(t)C]$ is stable (i.e., $[A\ B\ C]$ can be stabilized by output feedback). Finally, to validate the proof, $[A_c\ B\ C]$ must remain SPR for all time; this can not always be established since, at least, the matrix $G(t)$ is time varying (as part of the control law) and $A_c = [A+BG(t)C]$. These are too strong assumptions, since, in the adaptive control area, the plant is assumed being poorly known and usually time varying. It can be shown that the existence of ideal trajectories and, particularly, the existence of model matching conditions are intimately related to control performance and internal stability of the system (Balas and Johnson, 1980), (Balas, Kaufman and Wen, 1984). (Clarke, Mohtadi and Tuffs, Parts I and II, 1987).

THE DINAMIC MODEL REDUCTION - ADAPTIVE CONTROL (DMR-AC) SCHEME

The DMR-AC technique (Galvez, 2010) is a new structure of direct adaptive control. Its performance overcomes any other direct model reference adaptive type controller at the present stage of the technology (at least to this author knowledge). The DMR-AC algorithm is based on the concept of pole dominance in the frequency domain and conditions for asymptotic stability could be heuristically established without constraining the plant to be strictly positive real (SPR). It has been shown that the DMR-AC scheme is asymptotic stable inside of a relatively large neighborhood of the nominal plant dynamics (Galvez, 2010). The model reduction is performed in the frequency domain and is only meaningful in the context of the DMR-AC scheme. One can verify that through this approach, the possible closed-loop unstable dynamics can be confined to a frequency range, (Galvez, 2010) and easily compensated using residual mode

filters (RMF) (Balas, 1988). The derivation of the control law is based on Lyapunov's method. The results are given for the class of plants in which the dimension of the plant is much larger than the dimension of the reference model *[Am Bm Cm]*, i.e., *dim x >> dim xm*.

Let the plant be defined by

$$\dot{x} = A x + B u_c \quad ; \qquad y = C x \tag{4}$$

and let the dynamic projection model (DPM) be defined by some minimal realization *[Ap Bp Cp]* such that

$$\dot{x}_p = A_p x_p + B_p u_p + \varepsilon_y \quad ; \qquad y_p = C_p x_p \qquad \& \qquad \varepsilon_y = H_p (y - y_p) \tag{5}$$

In an ideal frame, perfect tracking means that $y_p(t) = y(t)$ $(\varepsilon_y = 0)$ or $y_p(\omega) = y(\omega)$ for all ω on the domain of the spectrum of $u(\omega)$. The particular case in which *dim x = dim xp* and *[A B C] = [Ap Bp Cp]* is just the state estimator case and for some properly designed the matrix H_p the state estimation error will asymptotically converge to zero. In the general case, however, *dim x >> dim xp* and an exact solution of the dynamic projection problem cannot be actually obtained. Nevertheless, it can be shown that the tracking error can be made as negligible as desired.

Usually, the projection model *[Ap Bp Cp]* is chosen such that *[Ap Bp Cp] = [Am Bm Cm]*. Then, the problem of creating a dynamic projection of the plant output, *y*, on the reference model *[Am Bm Cm]* coordinates is reduced to find a matrix H_p, such that, the transfer function in the frequency domain from *y* to y_p be as "flat" as possible over the assumed frequency range of the plant dynamics. For such H_p one has

$$\lim_{t \to \infty} \varepsilon_y = \lim_{t \to \infty} (y - y_p) = 0 \tag{6}$$

Let the reference model be defined as

$$\dot{x}_m = A_m x_m + B_m u_m \quad ; \qquad y_m = C_m x_m \tag{7}$$

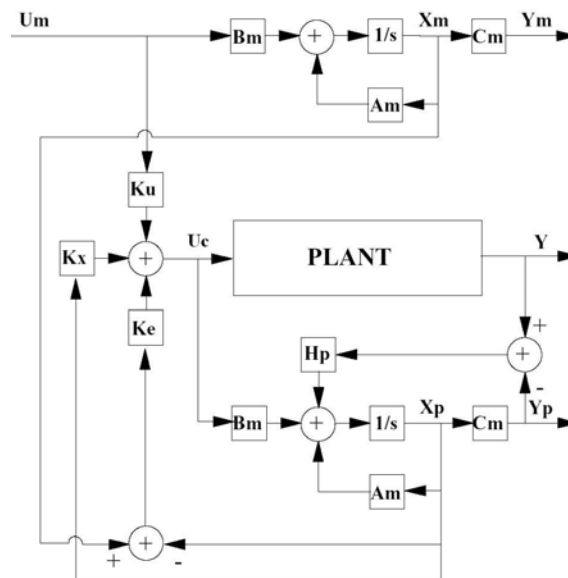Figure 2 shows the block diagram of the DMR-AC technique for *[Ap Bp Cp] = [Am Bm Cm]*.

Figure 2. The DMR-AC Block Diagram.

The error equation between the reference model states and the projection states can be written as

$$e_x = x_m - x_p \tag{8}$$

The control law is here defined as

$$u_c = K_u(t) u_m + K_x(t) x_p + K_e e_x = K(t) z + K_e e_x \tag{9}$$

where $K(t) = [K_u(t) \quad K_x(t)]$ and K_e is a fine-tuned constant matrix. The primary objective is to find an adaptation mechanism for $K(t)$ such that

$$\lim_{t \to \infty} e_x = \lim_{t \to \infty} (x_m - x_p) = \lim_{t \to \infty} C(x_m - x_p) = \lim_{t \to \infty} (y_m - y_p) = 0 \tag{10}$$

It can be observed that the adaptive control signal, u_c, (applied to the plant) drives the projection output, y_p, to track the model output, y_m. On the other hand, from Eq. (6), the projection output y_p is an accurate image of the plant output, y, (inside the working frequency range, by design) such that $\lim_{t \to \infty} y_p = y$ thus

$$\lim_{t \to \infty} e_x = 0 \qquad \Rightarrow \qquad \lim_{t \to \infty} (y_m - y) = 0 \tag{11}$$

A proper Lyapunov's function and its derivative can be written as

$$V = e_x{}^T P e_x + \text{trace} \left\{ \Delta K(t) \ \Delta K(t)^T \right\} \geq 0 \quad ; \qquad P > 0 \tag{12}$$

$$\dot{V} = e_x{}^T \left(A_c{}^T P + P A_c \right) e_x + 2 \text{trace} \left\{ (\Delta \dot{K}(t) - B_p{}^T P e_x z^T) \Delta K(t)^T \right\} \quad ; \qquad P > 0 \tag{13}$$

where $A_c = A_m - B_p K_e$

notice that $\Delta \dot{K}(t) = \dot{K}(t)$, therefore, $\dot{V} \leq 0$ requires that

$$\dot{V} = e_x{}^T \left(A_c{}^T P + P A_c \right) e_x \leq 0 \tag{14}$$

$$\dot{K}(t) = \Delta \dot{K}(t) = B_p{}^T P e_x z^T \tag{15}$$

Equation (14) is satisfied by the solution of the Lyapunov's Equation given by

$$A_c{}^T P + P A_c = -Q \qquad ; \qquad Q > 0 \tag{16}$$

Equation (15) yields the adaptation law us

$$\dot{K}(t) = B_p{}^T P e_x z^T \qquad \Rightarrow \qquad K(t) = K(t_o) + \int_{t_o}^{t} B_p{}^T P e_x(\tau) z^T(\tau) \, d\tau \tag{17}$$

for such adaptation law, Eq. (14) becomes

$$\dot{V} = -e_x{}^T Q e_x \leq 0 \qquad ; \qquad Q > 0 \tag{18}$$

Equations (6) and (18) are sufficient conditions for asymptotic stability.

3. SIMULATION RESULTS

The reference and the projection models can be chosen to have the desired plant dynamics and are kept unchanged through this experiment.

THE REFERENCE MODEL

For all Examples $\quad G_m(s) = \dfrac{1}{s^2 + 1.5s + 1}$

Eigenvalues	Damping	Freq.(rd/s)	Gain Margin (dB)	Phase Margin (deg)
$-0.75 \pm j0.66$	0.75	0.00	Inf	180

The parameters of the plant (a non minimal phase linear system with time delay) drift from their nominal values through the experiment to reflect possible parameters degradation, as shown below

THE TIME - VARYING PLANT

Example_1 $\quad G(s) = \dfrac{1\, e^{-0.1s}}{s^2 + 1.5s + 1}$

Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
$-0.75 \pm j0.66$	0.75	1.00	Inf	180

Example_2 $\quad G(s) = \dfrac{1.5\, e^{-0.25s}}{s^2 + 1.5s + 1}$

Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
$-0.75 \pm j0.66$	0.75	1.00	12.61	75.79

Example_3 $\quad G(s) = \dfrac{1.5\, e^{-1.0s}}{s^2 + 1.5s + 1}$

Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
$-0.75 \pm j0.66$	0.75	1.00	2.46	32.85

Example_4 $\quad G(s) = \dfrac{1.5\, e^{-1.0s}}{s^2 + 0.5s + 1}$

Eigenvalues	Damping	Frequency (rd/s)	Gain Margin (dB)	Phase Margin (deg)
$-0.25 \pm j0.97$	0.25	1.00	-7.49	-56.41

The Examples show a possible deterioration of the plant nominal dynamics. The static gain, time delay and damping ratio have been changed through the experiment as follows: Static Gain (K_{ss}) = [1, 1.5]; Time Delay (τ) = [0.1, 1] and Damping Ratio (ζ) = [0.75, 0.25] . Only the extreme values have been used for documentation purposes.

The CGT approach, as originally proposed, is a fully adaptive controller and it does not require any previous fine tunning. The DMR-AC scheme, on the other hand, requires a fine tuning as shown bellow:

The dynamic projection frequency response was designed to have a flat response up to 100 rd/s, which can be obtained by chosing

$$H_p = \begin{bmatrix} 125.225 \\ 50.000 \end{bmatrix}$$

Following the procedure presented in Galvez (2010) the constant parameters of the controller are defined as

The adaptation law was defined with

$$K_e = [\,100\ \ 150\,]$$

Solving Eq. (16) for a matrix Q > 0, it was found that

$$Q = \begin{bmatrix} 5.05 & 9.92 \\ 9.92 & 168.36 \end{bmatrix} \quad \Leftrightarrow \quad P = \begin{bmatrix} 50 & 0.025 \\ 0.025 & 0.558 \end{bmatrix}$$

Figures 3 through 6 show the simulation results for the CGT case. Figures 7 to 10 present simulation results for the DMR-AC scheme. In the following Examples: Figure *(1,1) shows the reference model output, Figure *(1,2) shows the plant output, Figure *(2,1) shows the model tracking performance and Figure *(2,2) shows the control signal.

The results for the CGT controller, as expected since the plant is not stricktly positive real (SPR), are unsatifactory. The CGT controller delivers a poor performance for Example 1 and unstable performances for Examples 2, 3 and 4. Quite the contrary, the DMR-AC shows outstanding performance through the experiment. It should be noted that the plant in Example 4 (Figs. 5 and 10, CGT and DMR-AC respectively) has unstable phase and gain margins, yet the DMR-AC still delivers a stable result.
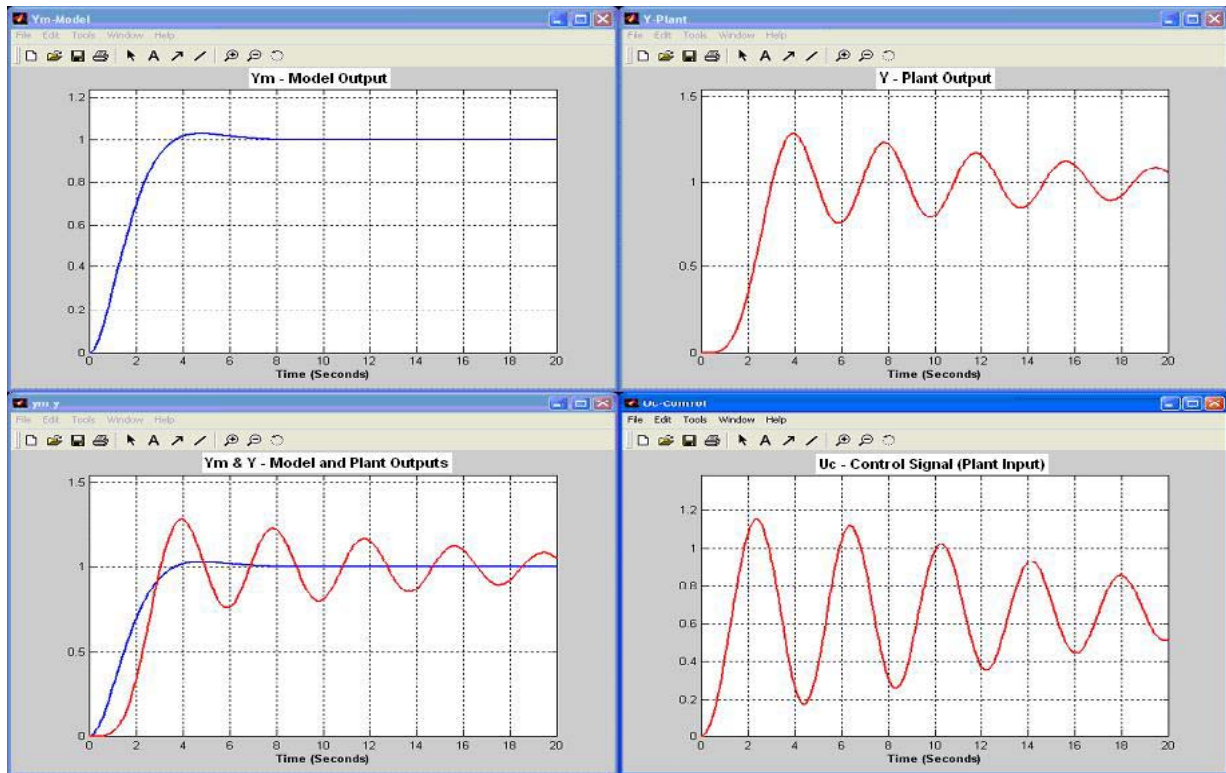
CGT EXAMPLE 1 – Results for the Nominal Plant.

Figure 3. CGT Controller Performance with Nominal Plant (Model Matching Conditions).

CGT EXAMPLE 2 – Results for the Nominal Plant with a Defective Sensor (Static Gain Drift+Time Delay).
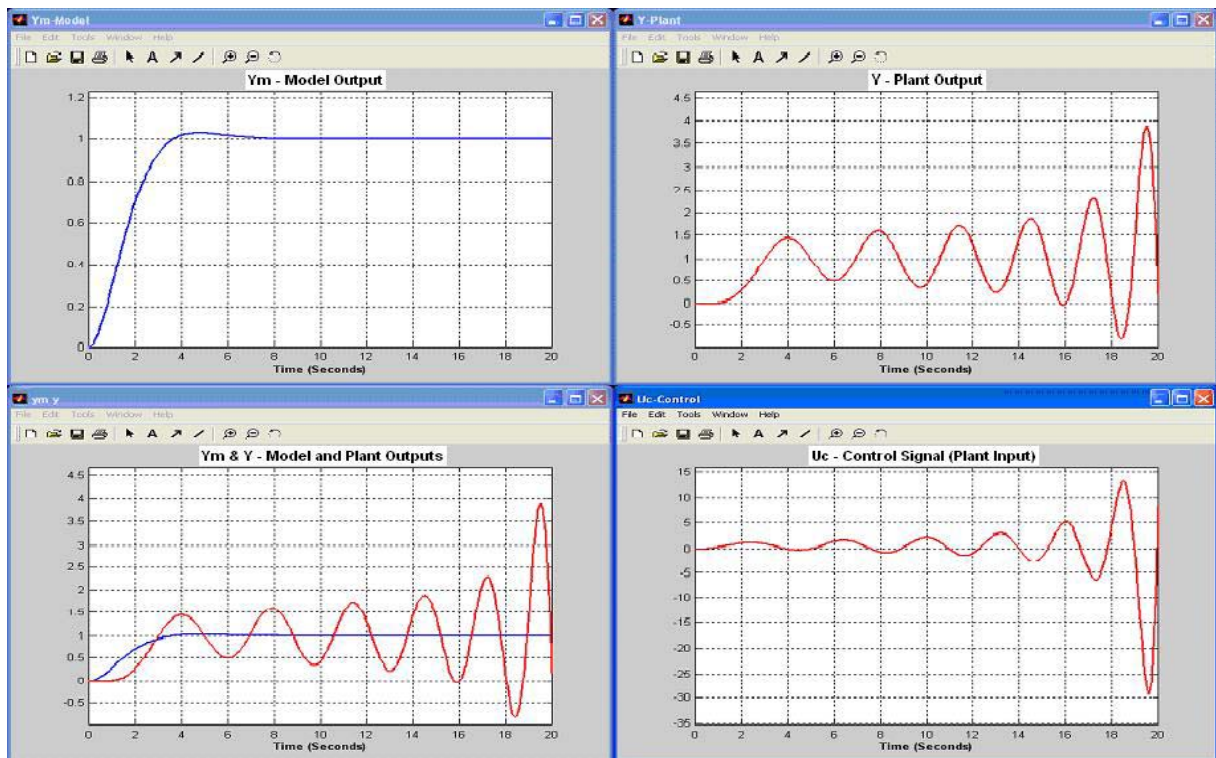

Figure 4. CGT Controller Performance with Nominal Plant with a Defective Sensor (Static Gain Drift+Time Delay).

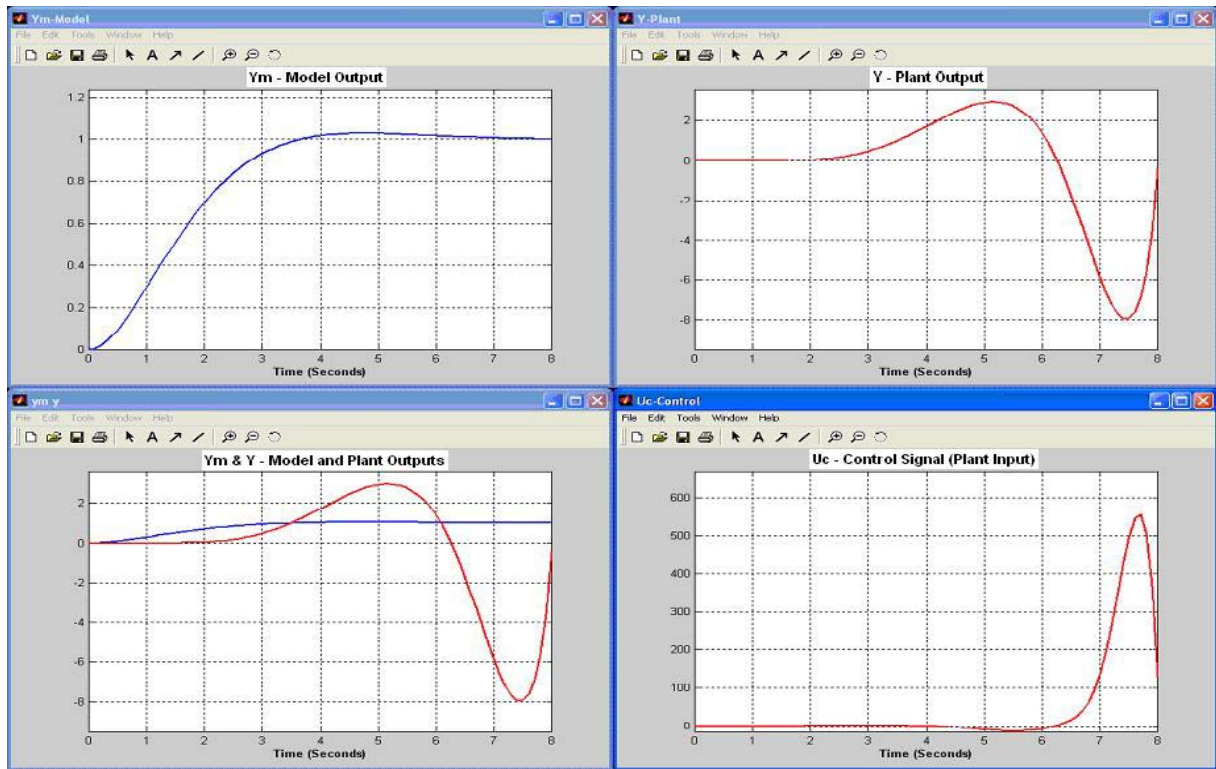CGT EXAMPLE 3 – Results for the Nominal Plant with a Severe Damage Sensor (Larger Time Delay).

Figure 5. CGT Controller Performance with a Severe Damage Sensor (Larger Time Delay).

CGT EXAMPLE 4 – Results for a Highly Perturbed Plant (Unstable Gain and Phase Margins)
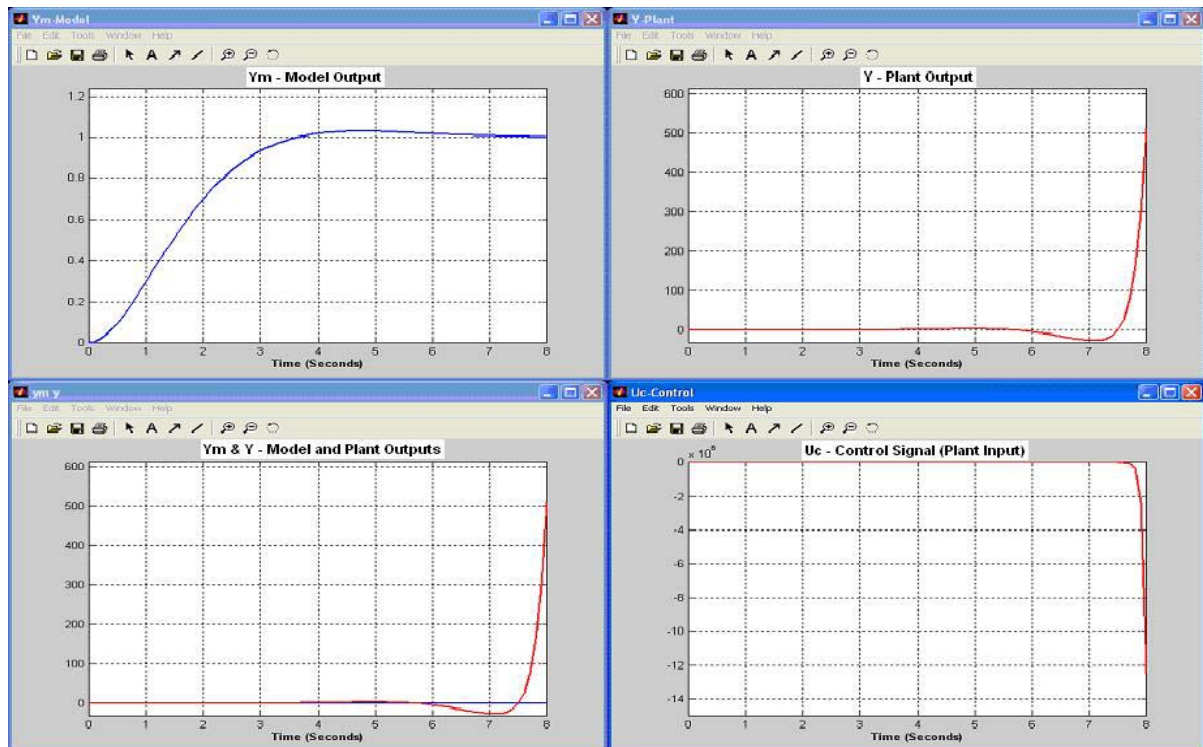

Figure 6. CGT Controller Performance with Highly Perturbed Plant (Unstable Gain and Phase Margins).

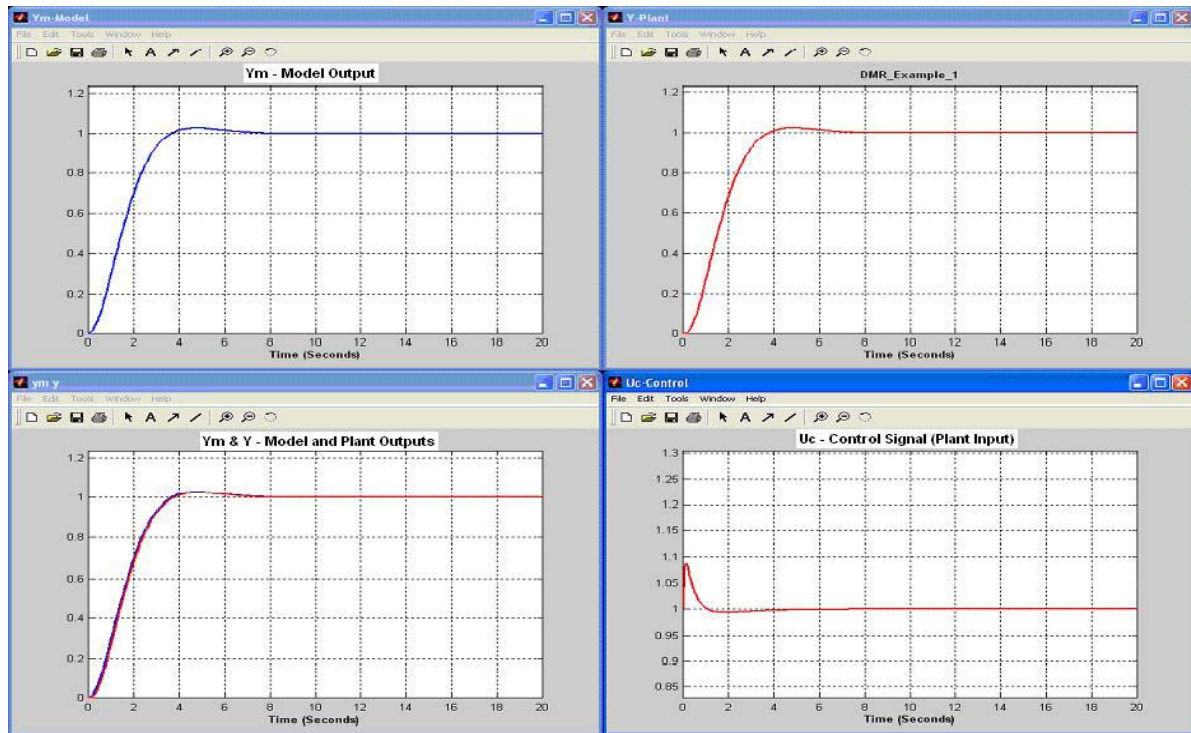DMR-AC EXAMPLE 1 – Results for the Nominal Plant.

Figure 7. DMR-AC Performance with Nominal Plant (Model Matching Conditions).

DMR-AC EXAMPLE 2 – Results for the Nominal Plant with a Defective Sensor (Static Gain Drift+Time Delay).
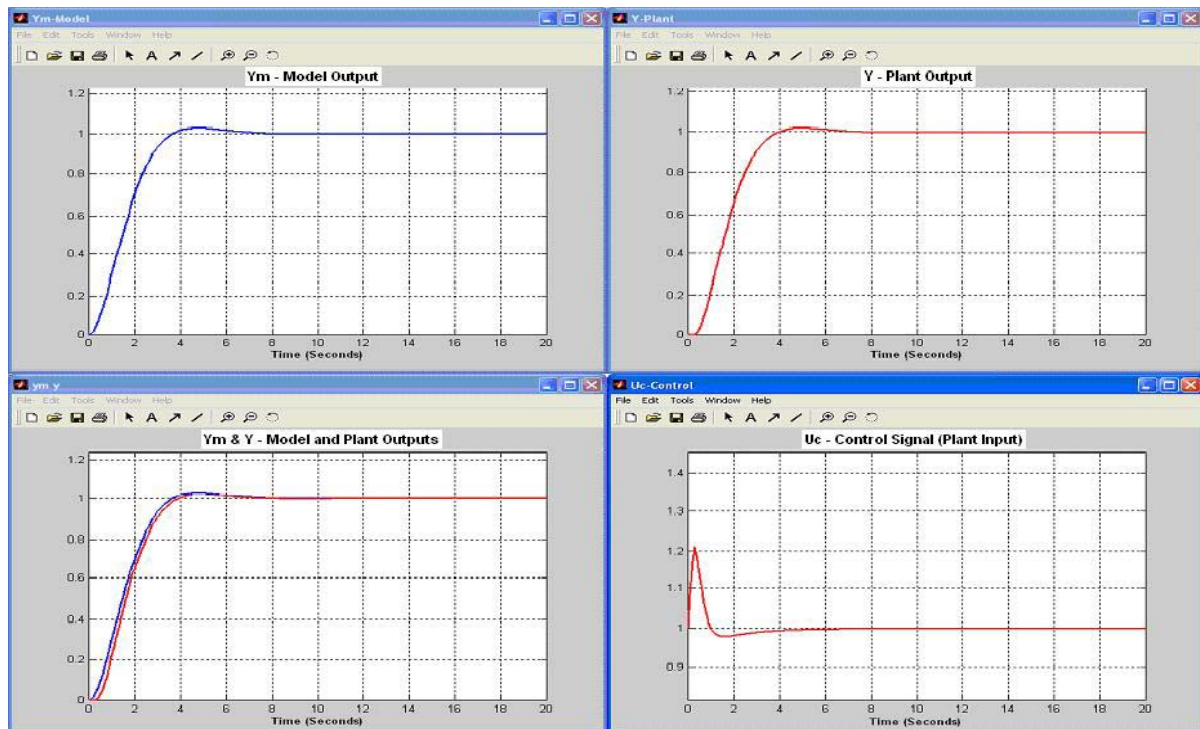

Figure 8. DMR-AC Performance with Nominal Plant with a Defective Sensor (Static Gain Drift+Time Delay).

DMR-AC EXAMPLE 3 – Results for the Nominal Plant with a Severe Damage Sensor (Larger Time Delay).
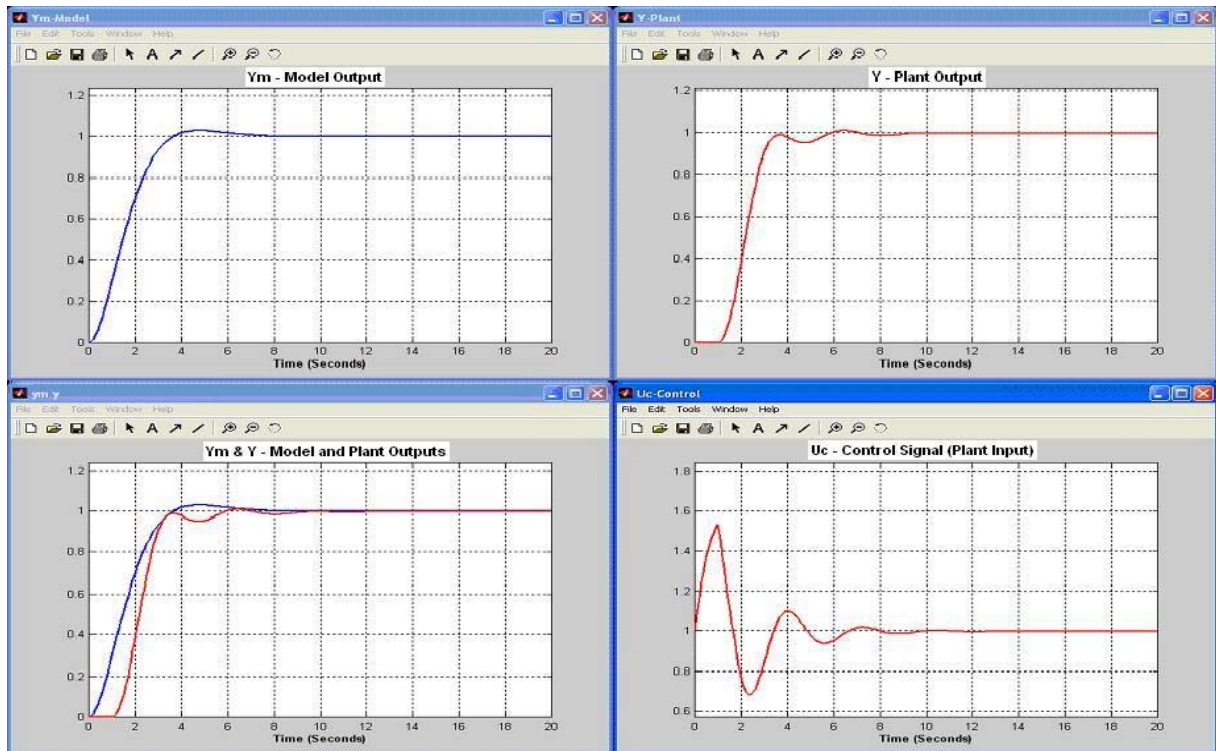

Figure 9. DMR-AC Performance with a Severe Damage Sensor (Larger Time Delay).

DMR-AC EXAMPLE 4 – Results for a Highly Perturbed Plant (Unstable Gain and Phase Margins)
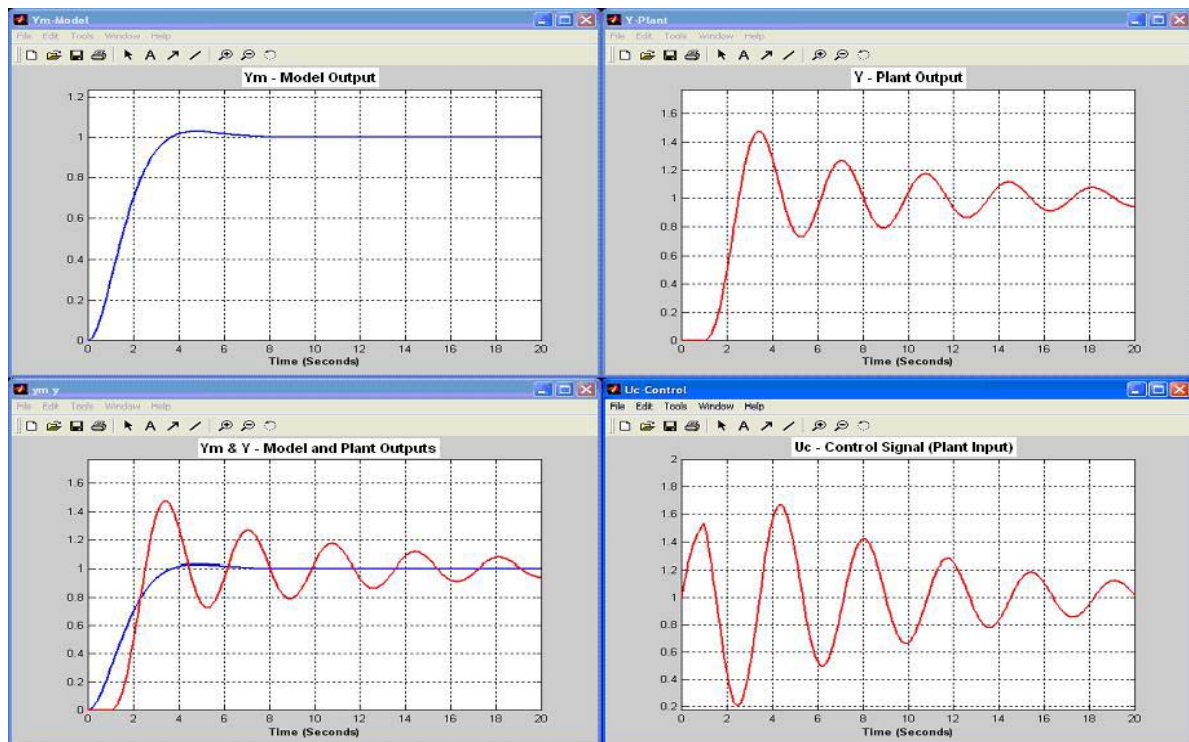

Figure 10. DMR-AC Performance with Highly Perturbed Plant (Unstable Gain and Phase Margins).

4. FINAL COMMENTS AND CONCLUSIONS

This paper presented an experimental procedure for performance assesment of adaptive controllers. A benchmark plant was used to perform an experimental comparative analysis of two direct model reference adaptive controllers, the Command Generator Tracking (CGT) approach and the Dinamic Model Reduction - Adaptive Controller (DMR-AC) scheme. The plant dynamics was changed from a stable model to an unstable one, through the experiment allowing a rightfull comparison between the adaptive controllers.

A new adaptive control scheme (DMR-AC) has been tested and the results shown it as an outstanding alternative for the control of large scale systems, specifically, plants with time delays, flexible structures and flexible arms. The derivation of the DMR-AC gains adaptation law has been performed based on Lyapunov's method without constraining the plant to be strictly positive real. The results are for the general case in which the dimension of the plant is larger than the dimension of the reference model. It has been verified that the dynamic projection state vector can be used to overcome the dimensionality problem in the derivation of adaptation laws for adaptive schemes. Conditions for asymptotic stability has been verified through numerical simulation.

Finally, it has experimentally verified that by slightly relaxing formality one can reach new controller structures and substantially improve the performance of direct adaptive controllers.

5. ACKNOWLEDGEMENTS

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