THERMAL PROPERTIES SIMULTANEOUS ESTIMATION AND INVERSE HEAT TRANSFER ANALYSIS

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Abstract. *The accurate knowledge of thermal conductivity and volumetric heat capacity is very important for the optimization of engineering design and development of new materials. Nowadays, there are many methods with this purpose, but most of them present problems when these parameters have to be determined simultaneously. Then, a method for the estimation of these parameters in AISI 304 and AISI 316 Stainless Steels and ASTM B265 Grade 2 Titanium samples is presented in this paper. The thermal model used is based on a transient one-dimensional diffusion equation. This model uses a uniform heat flux on the top surface and insulation condition on the bottom surface where the temperature is measured with the use of a thermocouple. In order to ensure the one-dimensional condition, the samples present much smaller thickness than their other dimensions besides being totally isolated by expanded polystyrene plates; in addition, the experiments are carried out very fast. Thus, the properties estimation was based on two parts: the analysis of the normalized sensitivity coefficients defined by the first partial derivative of the temperature in relation to the parameter to be analyzed times the analyzed parameter; and the minimization of the error function, which must present a minimum global value for each property. Based on these analyses, two different intensities of heat flux were used with the purpose to achieve the best condition for estimation. To estimate these properties, an error function defined by the square difference between the experimental temperature and numerical temperature is minimized by applying the optimization technique BFGS (Broyden-Fletcher-Goldfarb-Shanno). The numerical temperature is obtained through the resolution of the proposed thermal model by using the Finite Difference Method with an implicit formulation. Good results were obtained for the estimated properties. In addition, the analysis of inverse heat transfer is another part of this work and is made separately of the thermal properties estimation. This analysis is based on the comparison between inverse problem techniques like Function Specification, Tikhonov's Regularization, BFGS and Brent. This comparison is accomplished by using the estimated thermal properties of one experiment. All these techniques use the three-dimensional model to solve the heat diffusion equation. Good results were obtained for all cases. A filter is used along with the optimization techniques (BFGS and Brent) to obtain better results and to propose a different methodology.*

Keywords: *thermal properties, optimization, heat transfer, inverse problems, experimental technique.*

1. INTRODUCTION

The technique proposed in this paper can be used, for example, to correctly choose, under the point of view thermal properties, the materials to be used in the manufacture of a heat exchanger. This choice is made by taking into account the values of thermophysical properties, which should be ideal to yield a saving that is directly linked to energy and environmental issues, widely discussed in the current global circumstances.

Another example can be a machining process which great part of the heat generated by friction between the workpiece and the cutting tool must be transferred to the tool holder, as the tool wear is directly linked to temperature increase. Thus, the right tool for the process can be chosen through the knowledge of its thermal conductivity, since this property determines the range of the working temperature of the material. From these needs, researchers have developed many techniques which are being improved continuously (Carvalho *et al.*, 2006 and Brito *et al.*, 2009). These techniques can estimate the properties simultaneously and non-simultaneously.

There are three frequently used methods among these techniques like: the Guarded Hot Plate, Hot Wire Technique, and the Flash Method. The Guarded Hot Plate Method (ASTM C177, 1997) which is widely used to determine the thermal conductivity, λ, of insulating materials, is considered by many researchers as Wulf *et al.* (2005) and Xamán *et al.* (2009), among others, the most accurate and reliable. The Hot Wire Technique presented by Blackwell (1954) became widely used to determine the thermal conductivity. Several researchers have improved this technique in order to determine the properties of other materials (Nahor *et al.*, 2003 and Adjali and Laurent 2007). The former optimized the position of the hot wire to find food conductivity, and the latter proposed a change in methodology to determine the conductivity of a water-agar gel mixture by varying the temperature. The Flash Method developed by Parker *et al.* (1961) is used to determine the thermal diffusivity. A limitation to determine the thermal conductivity in this technique is the need to know the amount of energy absorbed on the front face of the sample. Since this is a widely researched topic, new methods have been developed to eliminate the limitations of the above techniques (Santos *et al.*, 2005 and Coquard and Panel 2008).

Taktak *et al.* (1993) determined λ and volumetric heat capacity, ρc_p, simultaneously for a carbon fiber and epoxy compound. The assembly consisted of a square-shaped sample with prescribed heat flux condition on the top surface, and prescribed temperature on the opposite surface. The temperatures were monitored on both sides. This study aimed to demonstrate the ideal conditions to perform the experiment in order to achieve reliable and accurate results.

Dowding *et al.* (1995) used a sequential technique in transient experiments to determine λ and ρc_p, simultaneously, for a carbon-carbon compound. The symmetrical assembly consisted of a heater placed between two samples isolated by a non-conductor ceramic plate.

Blackwell *et al.* (2000) proposed the determination of λ in the transient state. To achieve this goal, the sensitivity coefficients were analyzed to guide the design of an experiment to estimate the thermal conductivity for the steel AISI 304. The conductivity was determined by an experimental setup, where the heat conduction was considered axial on the walls of a hollow cylinder.

Borges *et al.* (2006) presented a method to obtain simultaneously and independently α and λ for conductive and non-conductive materials. A disadvantage of this study is the small number of points to estimate α and how it is estimated first, since this may influence the results of λ.

Jannot *et al.* (2006) developed a Transient Hot Plate Method to determine simultaneously the thermal effusivity, b, and the thermal conductivity of metallic materials such as aluminum, titanium and steel. The proposed device uses a simple heating element inserted between a plane face sample of the material to be characterized and a sample of an insulation material. One disadvantage of this study is the large thickness of the samples, which increases the cost.

Ghrib *et al.* (2007) developed a method based on the Mirage Effect, which is possible to estimate simultaneously α and λ of metallic materials like aluminum, steel, titanium, among others. The disadvantaged of this method is the high cost of the experimental apparatus.

So far, many methods in order to determine the thermal properties were presented. From this point, some papers on the analyses of inverse techniques are shown. The intention of this study is to present a background with the purpose of developing a comparison among inverse problems techniques.

Inverse Problems in heat transfer makes use of measured temperatures and the heat diffusion equation to estimate some unknown thermal parameters, which may be the thermal properties of a material, the coefficient of heat transfer by convection, or heat flux. One of the first Inverse Problems work in heat transfer was shown by Stolz (1960), where the Stolz's method was introduced to estimate the heat flux prescribed on the surface of spheres during the quenching process with the inside temperature of the sample as data input. This method can also be extended to cylinders and plates. Stolz assumed that the thermophysical properties were constant, no internal heat generation. The system was treated as linear, so Duhamel's theorem was used in the elaboration of this methodology. The Stolz's method showed good results for a large range of situations, facility to implement, and did not require large processing power. There were problems with noise in the input data as well as small time interval of sampling.

Beck (1962 and 1979), Beck *et al.* (1982) started improvements to fix these limitations, and resulted in the publication of a book by Beck *et al.* (1985). These improvements were based on bringing information from future times to the present time by reducing the noise effects in the output data. The authors used the Duhamel's theorem, Taylor's series expansion, and least squares optimization. In this method, the present heat flux is calculated using the previous heat fluxes; also, the subsequent heat fluxes are considered null and a fictitious heat flux is assumed, which can be constant, linear or quadratic, applied over a number of future times. This method shows resistance to noise in the input data, is easily implemented, and does not require large processing power.

Another technique widely used in Inverse Problems solution is called Tikhonov's Regularization, which was proposed by Tikhonov and Arsenin (1977), and used in several works, such as Jarny *et al.* (1991) and Postnov (2010). The Tikhonov's Regularization uses the Duhamel's theorem as well as least squares optimization, moreover there is an additional factor whose function is to attenuate noise in the temperature signal. This additive term has different degrees of filtering, popularly called orders; nevertheless this technique requires the inversion of matrices whose dimensions are related to the number of sampling points. Furthermore this inversion requires a substantial processing power to perform calculations.

The Function Specification and the Tikhonov's Regularization methods use a functional to be minimized. This functional is the sum of the square of the differences between the experimental and calculated temperatures. But there are other techniques for Inverse Problems using other functional minimization forms. There are techniques that use derivatives, such as Conjugate Gradient, which was used in Alifanov and Artyukhin (1975) and Andrei (2010), and Variable Metric Methods used by Colaço *et al.* (2006). There are also techniques that do not use derivatives for the optimization process, such as the Brent minimization method presented by Brent (1973) and used by Rodriguez *et al.* (2010), and the Golden Section used by Carvalho *et al.* (2006).

As it can be seen, in some papers aforementioned, the properties can be estimated; however, with some restrictions. Thus, in the present work, a method is proposed to determine the thermal conductivity and the volumetric heat capacity for metallic materials. In addition, an analysis of the applied heat flux by using inverse techniques in the same experiment is also done. This method is based on a one-dimensional transient heat conduction model. The properties are estimated by minimizing the quadratic error function based on the difference between the experimental and numerical temperatures. To minimize this function, the sequential optimization technique BFGS is used. The temperature is obtained numerically by using the Finite Difference Method with implicit formulation. Furthermore, analyses of the sensitivity coefficients are performed to find the best setting and region to obtain the properties.

Therefore, the objective of this work is to develop a methodology, seeking to eliminate the unfeasibility found in other studies to determine the thermal conductivity and the volumetric heat capacity for metallic materials and to perform a comparison among inverse problems techniques considering the estimated heat flux, by using the estimated thermal properties, and the processing time spent for each technique.

2. THEORETICAL ASPECTS

2.1. Thermal Model

Figure 1 shows the proposed one-dimensional thermal model, which consists of a sample located between a resistive heater and an insulator. The sample has much smaller thickness than its others dimensions and all the surfaces, except the heated ($x = 0$), were isolated to ensure the unidirectional heat flux.

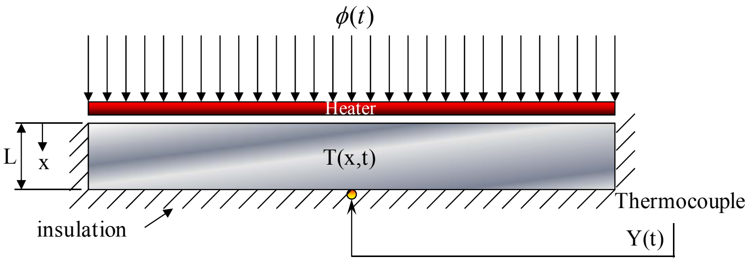

Figure 1 – One-dimensional thermal model.

The heat diffusion equation for the problem presented in Figure 1 can be written as:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c_p}{\lambda}\frac{\partial T(x,t)}{\partial t} \tag{1}$$

subject to the boundary conditions:

$$-\lambda \frac{\partial T(x,t)}{\partial x} = \phi(t) \text{ at } x = 0 \tag{2}$$

$$\frac{\partial T(x,t)}{\partial x} = 0 \text{ at } x = L \tag{3}$$

and the initial condition:

$$T(x,t) = T_0 \text{ at } t = 0 \tag{4}$$

where x is the Cartesian coordinate, t the time, ϕ the prescribed heat flux, T_0 the initial temperature of the sample and L the thickness.

The numerical temperature is obtained through the solution of the one-dimensional diffusion equation using the finite difference method with an implicit formulation.

2.2. Analyses of the best region to determine the properties λ and ρc_p

Studies of the sensitivity coefficient for each sample are performed in this work in order to determine the ideal region to estimate the properties and the best configuration of the experimental setup. This study provides information such as: the correct positioning of the thermocouples, the experimental time, and the time interval of the applied heat flux incidence. The higher the coefficients value, the better the chance of obtaining the properties reliably.

The normalized sensitivity coefficient is defined by the first partial derivative of the temperature in relation to the parameter to be analyzed (λ or ρc_p), being written as follows:

$$X_{ij} = P_i \frac{\partial T_j}{\partial P_i} \quad (5)$$

where T is the numerical temperature, P the parameter to be analysisd (λ or ρc_p), i the index of parameter, and j the index of points. As in this work, only two properties will be analysisd, $i = 1$ for λ and $i = 2$ for ρc_p.

Besides this, analyses of the error function were done in order to guarantee that in the analyzed region there is enough information to estimate the properties simultaneously. One can verify this information if a minimum value of the function error is found when there are changes of the properties values. This error function is represented by Eq. (6) in the next section.

2.3. Thermal conductivity and volumetric heat capacity simultaneous estimation and heat flux analysis

To estimate the two properties it is necessary to use an error function based on the square difference between the experimental and numerical temperatures. This equation can be written as:

$$F = \sum_{j=1}^{m} \left(Y_j - T_j\right)^2 \quad (6)$$

where, m is the total number of points, and Y the experimental temperature with random errors.

Thus, it is known that the optimal value for λ and ρc_p that is, the value that minimizes the error function, is the value of the properties to be estimated. For the estimation of the two properties, the BFGS technique was used.

Analyzing the heat flux estimation, it can be seen that there is a difference due to the variation of the heat flux with time and position. In addition, each kind of inverse technique needs to be considered separately. Function Specification and Tikhonov's Regularization do not need an error function because in their formulation has been incorporate a minimization process for the heat flux estimation. However, for the optimization techniques (BFGS and Brent) it is necessary the use of a new error function. This new error function is described for the Eq. (7):

$$F = \left(Y_j - T_j\right)^2 \quad (7)$$

3. INVERSE PROBLEMS TECHNIQUES

3.1. Function specification method

In the Function Specification Method, the previous heat flux is used in the estimation of the present heat flux. The method consists of assigning a temporary form of a transient heat flux on the sample surface for instants higher than the current time in the estimation. In this case the form of the heat flux may be constant, parabolic, exponential or cubic. A simpler method of Function Specification is using a sequence of constant line segments as a form to describe the behavior of surface heat flux for future times (Beck *et al.*, 1985).

3.2. Tikhonov's regularization

In this method, an additional term is added to the functional. This term is the square of integral of heat flux, or its derivative, multiplied by a scalar. This scalar is called regularization parameter. Soon after, the optimization is done by using least squares for this new functional. This method is known as the Tikhonov's Regularization. The order of regularization depends on the order of the derivative used in the heat fluxes (Beck *et al.*, 1985).

3.3. Brent

The Brent minimization algorithm combines a parabolic interpolation with the Golden Section algorithm. It produces a fast algorithm which is still robust. The outline of the algorithm can be summarized as follows: Brent's method approximates the function using an interpolating parabola through three existing points at each iteration. The minimum of the parabola is taken as a guess for the minimum. If it lies within the bounds of the current interval then the interpolating point is accepted, and used to generate a smaller interval. If the interpolating point is not accepted then the algorithm falls back to an ordinary Golden Section step (Brent, 1973).

3.4. BFGS

The BFGS (Broyden Fletcher Goldfarb Shanno) sequential optimization technique used in this work and presented in Vanderplaats (2005) may be applied to obtain the values of the properties. This technique is a particularity of the

Variable Metric Methods. The advantages of this method are the fast convergence and the ease to work with many design variables.

4. EXPERIMENTAL PROCEDURE

The experimental apparatus used to determine the properties of AISI 316 and 304 Stainless Steels and ASTM B265 Grade 2 Titanium is shown in Fig. 2. The AISI 304 Stainless Steel plate has the dimensions of 49.9 x 49.9 x 10.9 mm the AISI 316 Stainless Steel plate 49.9 x 49.9 x 10.1 mm and the ASTM B265 Grade 2 Titanium plate 49.9 x 49.9 x 10.1 mm. The resistive kapton heater has a resistance of 15 Ω and the dimensions of 50.0 x 50.0 x 0.2 mm. The resistive kapton heater was used because it is very thin, allowing faster overall warming. This heater was connected to a digital power supply Instrutemp ST – 305D-II to provide the necessary heat flux. In this work, different intensities of heat flux were used in the same experiment as an attempt to achieve the best condition to estimate the properties simultaneously in accordance to the analyses of the sensitivity coefficients. To achieve this heat flux condition, the digital power supply has a configuration that allows to work at parallel or series connection. Then, we used the series condition to provide the highest heat flux for the first period of the experiment, and the parallel condition to supply the lowest heat flux for the second part. A symmetrical assembly was used to minimize the errors in the measured of the heat flux to be generated on the sample surface. In addition, the applied current and voltage values were measured by the calibrated multimeters Instrutherm MD-380 and Minipa ET-2042C. Once the contact between the resistive heater and the sample is not perfect, the silver thermal compound Arctic Silver 5 was used to avoid the air interstices present in the assembly. The great advantage of this compound refers to its high thermal conductivity. In addition, weights were used on top of the isolated set samples-heater to improve the contact between the components. To ensure a unidirectional flux and minimize the effect of convection caused by the air circulating in the environment, the set samples-heater was isolated with polystyrene plates. Temperatures were measured using thermocouples type K (30AWG) welded by capacitor discharge and calibrated using a bath temperature calibrator Marconi MA 184 with a resolution of ± 0.01 °C. This thermocouple was connected to a data acquisition Agilent 34980A controlled by a microcomputer. In order to obtain better results, all experiments were performed in controlled room temperature.

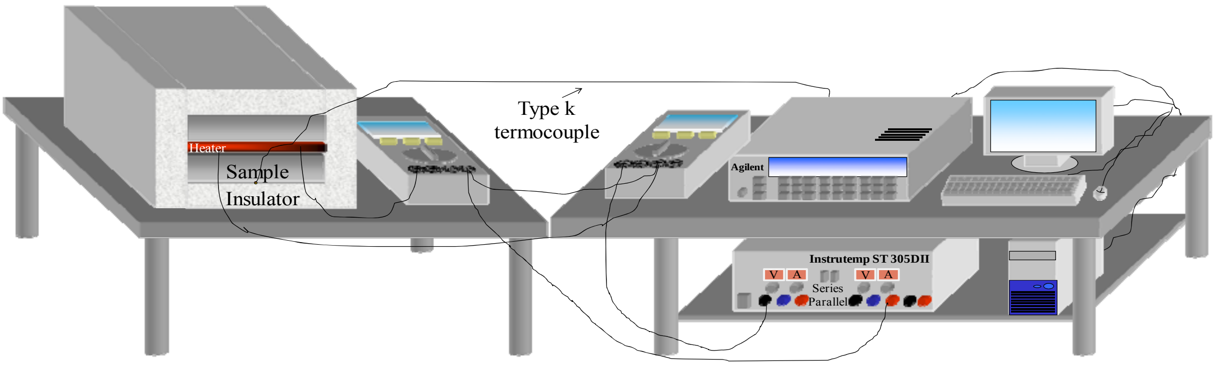

Figure 2 – Sketch of experimental apparatus used to determine the properties.

5. RESULTS ANALYSES

5.1. Thermal Properties

5.1.1. AISI 304 stainless steel

Forty experiments were performed to simultaneous estimate the thermal conductivity and the volumetric heat capacity of AISI 304 Stainless Steel. Each experiment lasted 160 s, but the heat flux was imposed from 0 to 140 s. In the first part, that consist in the interval of 0 to 20 s, the applied heat flux was approximately 2640 W/m². For the second part, the time between 20 to 140 s, the imposed heat flux was around 660 W/m². The time interval used to monitor the temperature was 0.1 s. This configuration for the heat fluxes was chosen with the purpose to keep the temperature difference lower than 5 ºC in order to guarantee the hypotheses of thermal properties constants adopted.

The sensitivity analysis was performed to determine the best region to estimate the properties. This analysis was performed by using the values of λ and ρc_p obtained from Borges *et al.* (2006). Analyses of the error function were done allied to sensitivity analysis in order to guarantee that there was enough influence to determine these properties in the selected region. Figure 3 shows the sensitivity coefficients at $x = L$ for λ and ρc_p, and Fig. 4 presents the values of the error function.

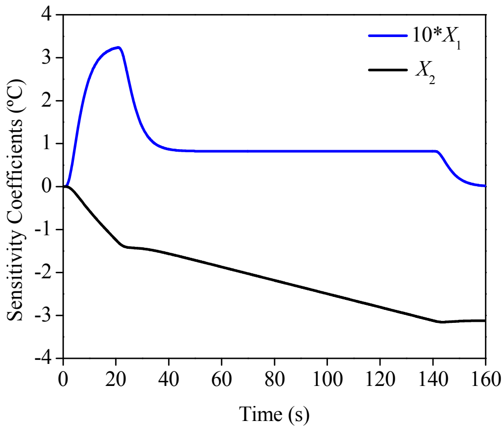

Figure 3. Sensitivity Coefficients.

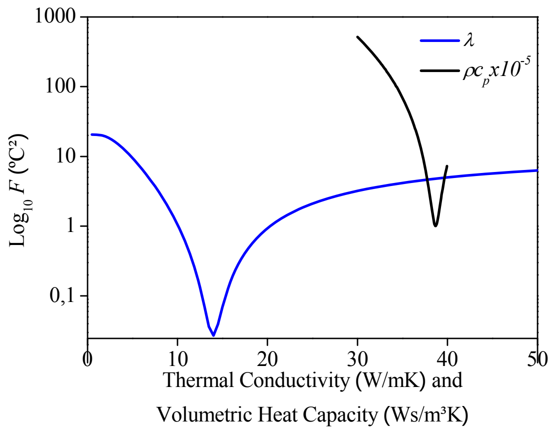

Figure 4. Error Function Values (F).

X_1 represents the sensitivity coefficient for λ and X_2 represents the sensitivity coefficient for ρc_p, both on the isolated surface. The former is multiplied by a factor in order to improve the visualization of the curve. By analyzing Figure 3, one can see that X_1 increases during the first 20 s, and after this, it keeps constant up to the change of the heat flux, and X_2 increases at the same proportion that the temperature increases. Because of this behavior, the highest heat flux was applied in the first period of time, resulting in a high sensitivity for λ; and the lowest heat flux was applied on the second part in order to increase the sensitivity for ρc_p and keeps the sensitivity for λ. Besides to keep the temperature difference lower than 5 ºC, this procedure was done, because it is necessary to control the magnitude relation between X_1 and X_2, in order to guarantee that the estimation will happen for the two properties; in other words, if one coefficient is much larger than the other, the estimation, by using minimization, will occur only for the property which presents the higher coefficient. So, Figure 4 shows that there is enough influence to determine the properties simultaneously at the region analyzed, because a minimum value was found for each property. Another objective of sensitivity analysis is to determine the number of points in the curve which should be used to estimate the properties. These points to be considered should not have derivative equal to zero. The sets of points that do not fit this description should be disregarded in the estimation of properties. In this work, the points chosen to estimate the properties corresponds the points where there is applied heat flux, in other words, the interval between 0 to 140 s.

In order to check if these conditions resulted in good experiments, another analysis was done. This analysis was based in Dowding *et al.* (2005) that said: when the sum of the sensitivity coefficient of λ and ρcp, considering the boundary conditions of prescribed heat flux on the top surface and insulation on the bottom surface, plus the temperature gradient is equal zero ($X_1 + X_2 + Y - Y_0 = 0$), the best condition and design for the experiment was achieved. Then, Fig. 5 shows the results of this analysis. One can see that the result for this analysis is very good, because the highest difference was around 0.08 ºC. Thus, this proves the well done experiment.

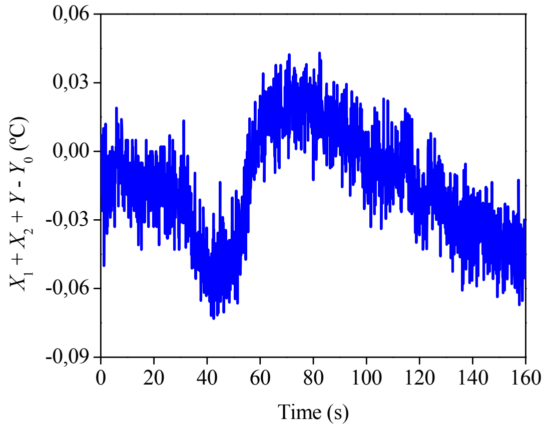

Figure 5. Analysis for the best condition and design.

Figure 6 presents the distribution for experimental and numerical temperatures for the plate, at $x = L$ and the imposed heat flux at $x = 0$. The numerical temperature is achieved by employing the properties values λ and ρc_p estimated for one of the accomplished experiments. These temperatures present good concordance that one can be proved by analysis of the temperature residuals. The temperature residuals are shown in Figure 7, in other words, it is the percentage difference between the experimental and the numerical temperatures. These residuals are calculated by doing the difference between the experimental and numerical temperatures, and these differences are divided by the numerical temperature. One observes the good agreement of the results for the AISI 316 Stainless Steel. For the thermocouple located on the opposite surface, a difference of up to 0.5 % was sensed. These deviations may be due to contact resistance between the resistive heater and the sample, and the difficulty of isolating the experiment.

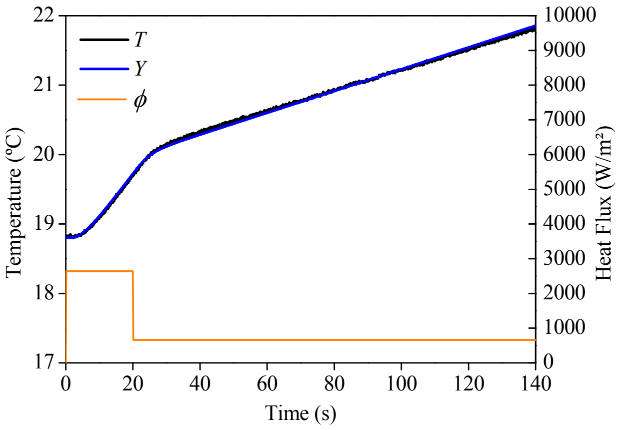

Figure 6. Numerical (*T*) and Experimental (*Y*) Temperatures with Heat Flux (ϕ).

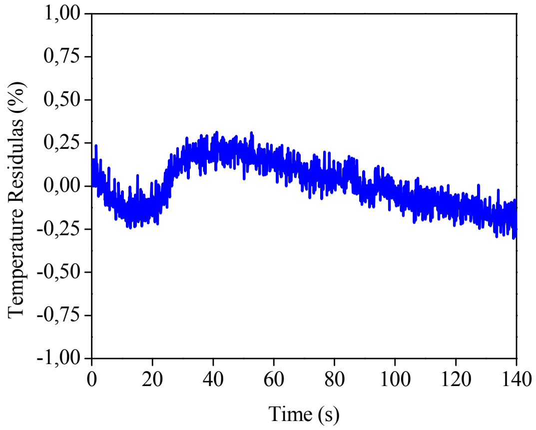

Figure 7. Temperature residuals.

Table 1 presents the mean value of this work, the standard deviation and the error (the percentage difference between the mean and the literature value) for λ and ρc_p of AISI 304 Stainless Steel.

Table 1. Results obtained for the AISI 304 Stainless Steel.

Property	Present work	Borges *et al.* (2006)	S. D.	Difference (%)
λ (W/mK)	14.61	14.64	± 0,16	0,21
ρc_px10^{-6} (Ws/m^{3}K)	3.91	3.84	± 0,04	0,51

The estimated values of λ and ρc_p, when compared with the literature values, are in good agreement. Thus, the error found in estimating the thermal conductivity is consistent when compared with the values found in the literature.

5.1.2. AISI 316 stainless steel

This part presents an analysis of the results obtained for the determination of λ and ρc_p of an AISI 316 Stainless Steel sample. The same procedure adopted for the AISI 304 Stainless Steel was used for this material. Forty experiments were carried out, and 1500 points were collected in each one, but the heat flux was applied during the 130 s. The increment of time used to get the temperatures was the same used for the AISI 304 Stainless Steel (0.1 s). The applied heat flux was about 2640 W/m^2 for the first part (0 to 30 s), and 660 W/m^2 for the second part (30 to 130 s).

Table 2 presents the mean value of the present work, the standard deviation (S. D.), and the comparison with the reference value obtained from literature, for the λ and ρc_p of AISI 316 Stainless Steel. Similar to AISI 304 Stainless Steel, the results presented good agreement with the literature value.

Table 2. Results obtained for the AISI 316 Stainless Steel.

Property	Present work	Incropera *et al.* (2007)	S. D.	Difference (%)
λ (W/mK)	13.52	13.40	± 0.20	0.89
ρc_px10^{-6} (Ws/m³K)	3.93	3.86	± 0.04	1.78

5.1.3. ASTM B265 grade 2 titanium

This part presents an analysis of the results obtained for the determination of λ and ρc_p of an ASTM B265 Grade 2 Titanium sample. The procedure used was similar to the others materials. Forty experiments were performed and each one lasted 150 s, but the heat flux was imposed from 0 to 120 s. In the first part, that consist in the interval of 0 to 20 s, the applied heat flux was approximately 2680 W/m². For the second part, the time between 20 to 120 s, the imposed heat flux was around 675 W/m². The time interval used to monitor the temperature was 0.1 s. The analyses were done similar to the others materials based on the thermal properties extracted from GMTTitanium (2010).

Table 3 presents the mean value of the present work, the standard deviation (S. D.), and the comparison with the reference value obtained from literature, for the λ and ρc_p of ASTM B265 Grade 2 Titanium.

Table 3. Results obtained for the ASTM B265 Grade 2 Titanium.

Property	Present work	GMTTitanium (2010)	S. D.	Difference (%)
λ (W/mK)	17.88	18.06	± 0.27	1.00
ρc_px10^{-6} (Ws/m³K)	2.71	2.66	± 0.05	1.88

Similar to the Stainless Steels, the results presented good agreement with the literature value.

5.2. Heat flux

5.2.1. AISI 304 stainless steel

This part shows the analyses of the applied heat flux. The values of thermal conductivity and volumetric heat capacity used for these analyses were obtained from the experiments.

Figure 13 presents a comparison between experimental heat flux and estimated heat flux for one experiment. In this figure, it is observed that all techniques showed satisfactory results for the heat flux estimation. It can be seen that the Tikhonov's Regularization and the Function Specification presented better results than the optimization techniques. This affirmation can be proved by analyzing the behavior of the heat flux at the beginning of the experiment, where there is a little deviation between the estimated and experimental heat flux. In this analysis, seventy-five future times (r = 75) with a sequential constant heat flux functional form were used in the Function Specification method, and for the Tikhonov's Regularization, the regularization parameter was equal to 0.05 considering a second-order regularization . A FFT filter was used in the results from the optimization techniques with the purpose of minimizing the deviation presented in the estimated heat flux. This deviation was found only in the optimization techniques because in the BFGS and Brent the heat flux is optimized point by point (see Eq. 7) according to the temperature signal while in the Tikhonov's Regularization and Function Specification techniques the heat flux is optimized in the whole domain for the lowest result of the objective function. Figure 14 shows a comparison between experimental and numerical temperatures. The numerical temperatures were obtained by using the estimated heat flux. It was observed that these techniques had a good correlation with the experimental temperature.

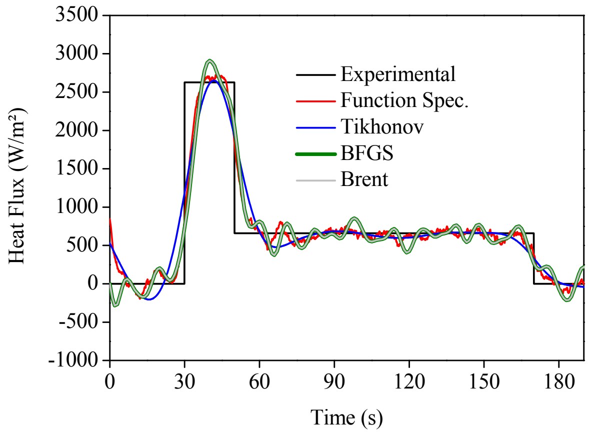

Figure 13 – Comparison between experimental and estimated heat fluxes.

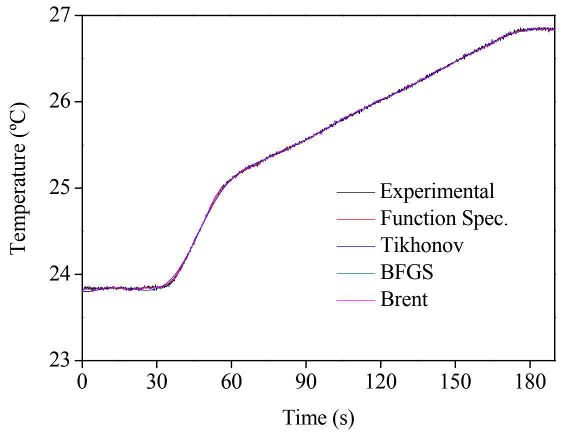

Figure 14 – Comparison between experimental and numerical temperatures.

Figure 15 shows the residuals between the experimental and the calculated temperatures for the Function Specification and Tikhonov's Regularization techniques. The results presented differences of less than 0.5 % in relation to the experimental temperature. Figure 16 presents the residuals for the BFGS and Brent techniques. It can be seen that the maximum residual is within the interval from -0.0001 % to 0.0001 %. The explanation for these small deviations is due to the optimization for these techniques occur point by point.

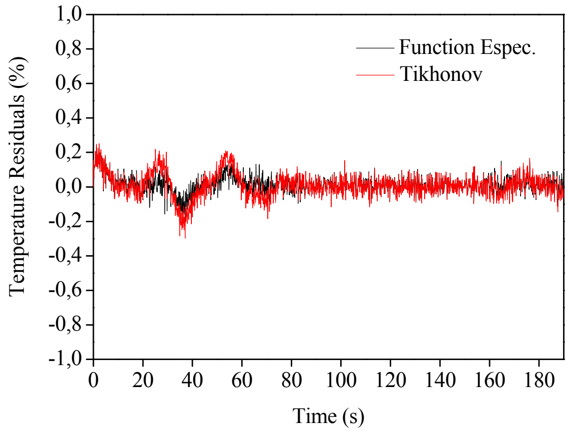

Figure 15 – Residuals between experimental and numerical temperatures.

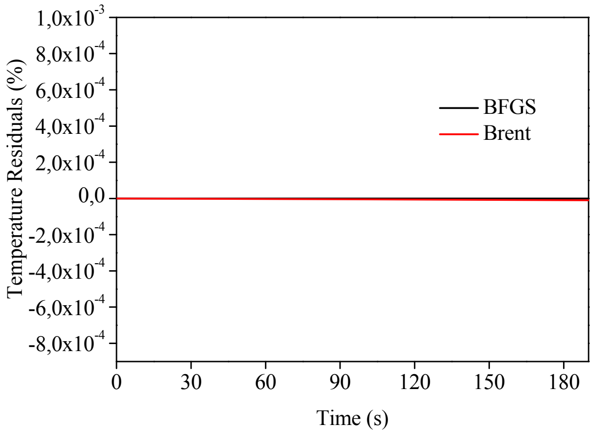

Figure 16 – Residuals between experimental and numerical temperatures.

All inverse techniques were programed using computer language c/c++ in an Windows platform. For each technique there were five executions and the shorter time spent in the calculations was considered. The Brent technique spent 0.03 seconds to evaluate the estimated heat fluxes. Function Specification Method spent 0.47 seconds doing this same task. The optimization method BFGS completed the estimation of the heat fluxes with 0.59 seconds. And last, the Tikhonov's Regularization spent 45.40 seconds for accomplish the calculate of estimated heat fluxes.

According to these results, it can be seen that all techniques present good results, but only Tikhonov's Regularization spent more than 1 second to estimate the heat flux. However, for the three-dimensional case, these results are different. In this case, the optimization techniques spent much longer than the others studied methods.

6. CONCLUSIONS

This study presents a technique to estimate the thermal conductivity and volumetric heat capacity of metallic materials using the same experiment, besides a comparison of inverse problems techniques. Three materials were analyzed: AISI 304 Stainless Steel, AISI 316 Stainless Steel and ASTM B265 Grade 2 Titanium. The estimated properties are in good agreement with the literature for all materials. For the inverse problems, all techniques showed good results for the estimation of heat flux and for the estimation of temperature of the AISI 304 Stainless Steel. An advantage of this work is the use of a filter to obtain better results for the optimization techniques by minimizing the high deviation in the heat flux estimation. For future work, some improvements are proposed: the development of a methodology to estimate the thermal properties and heat flux simultaneously and application of this methodology in the resolution of practical problems, for example, welding processes.

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