# TOPOLOGY OPTIMIZATION AND OPTIMAL CONTROL IN STRUCTURAL DESIGN

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***Abstract.*** *This work present a structural design methodology considering control effects, the change of the topology by a control force action, and design modal control for suitable fixed actuator locations. The actuators are composed by piezoelectric material. The topology optimization in this work uses homogenization design method, based on the concept of optimizing the material distribution, through a density distribution, while the control force is obtained by the optimal control design for transient response and performed in the modal space. A Continuum finite elements modeling is applied to simulate the dynamic characteristics of the structure. The cost functional is the strain energy of the structure and the control energy. Results of numerical simulations for a cantilever beam model are presented and discussed.*

***Keywords****: Topologic Optimization, actuators, dynamics, vibration control.*

## 1. INTRODUCTION

Structural vibration control is a particularly important consideration in the design of dynamic systems. The main idea of the structural optimization is to obtain an optimal material layout of a load-bearing structure. Usually, continuum topology optimization problems are formulated to minimize the structural material volume or to optimize the structural performance. A typical example is to raise the first fundamental frequency of a structure while obeying a volume constraint (Zhan, Xiaoming and Rui, 2009). Meanwhile, structural dynamics control, considering piezoelectric actuators, is used to minimize suppress vibration effects. The design of these structures takes into account the interaction of the applied forces, the elastic modes and the actuator placement.

There are always fundamental interest in designs with efficient structural control system from both structural and control engineers. However, these groups have been working independently. Traditionally, the structural designer develops his design based on strength and stiffness requirements, and the control designer creates the control algorithm to reduce the dynamic response of a structure (Ou and Kikuchi, 1996). In this work we are designing the structure and controls simultaneously, meaning that the cost function includes not only the strain energy, but also the control energy.

The reason why topology optimization is becoming a very important research field is the necessity of efficient methodologies to design structures, thus saving material and time. The main objective of the topology optimization problem is to find a material distribution that minimizes a given objective functional, subjected to a set of constraints, achieved by a consistent parameterization of the material properties in each part of the design domain. A natural question is whether there exists or not material in a given point, which leads to a discrete problem. It is well-known that this integer parameterization leads to numerical difficulties, associated with the integer problem convergence (Cardoso and Fonseca, 2003; Bendsøe and Kikuchi, 1988; Bendsøe and Sigmund, 1999). Minimizing the vibration effects of the dynamic response is an important goal for the structural vibration control, and the effectivity of the control depends on the weighting matrices.

The objective of this paper is to present a structural design methodology considering the control effects, the change of the topology by a control force action, and design modal control for suitable fixed actuator locations. The structural optimization design is completed through a density design method, while the control force is obtained by the optimal control design for transient response and performed in the modal space.

The efficient structural control design needs a careful selection of actuator positions (Ou and Kikuchi, 1996). However, in this work the actuators locations are chosen arbitrarily prior to the structural design. In fact, it is well known that a good location for an actuator in a cantilever structure is close the fixed size of the structure, since it acts upon the first and most significant mode (Sun et al., 2004, Donoso, A. and Sigmund, O., 2009; Molter, A. et al, 2010). The lower fundamental modes are responsible for the most of the tip displacement of the beam; therefore, the first two eigenfunctions are computed and considered in this work. The dynamics and control design were included in a topology optimization code. Simulations were conducted to assess the effectiveness and control model efficiency.

## 2. FORMULATION OF STRUCTURAL TOPOLOGY OPTIMIZATION CONSIDE-RING CONTROL ACTION

The homogenization design method (Bendsøe and Sigmund, 2003) is choosing the tool for the topology optimization considering a control action. This method is based on the concept of optimizing the material distribution, through a density distribution. A finite element mesh is defined to perform the structural modal analysis (Bathe, 1996). As a simplification, we assume that the density is constant in each finite element. An optimality criteria (OC) is derived from the necessary minimization conditions, and it is solved to update the density distribution. A number of simplifications are introduced to the implementation, as a regular mesh.

We now consider that the objective function is the sum of the strain energy and the control energy. Then, the topologic optimization problem in steady state has the form

$$\begin{aligned} &\min_{\mathbf{x}} J,\ J(\mathbf{x}) = \mathbf{f}^{\mathrm{T}}\mathbf{R}\mathbf{f} + \mathbf{U}^{\mathrm{T}}\mathbf{Q}\mathbf{U} \\ &\text{subject to} \quad \frac{V(\mathbf{x})}{V_0} = V_{min} \\ &\qquad \mathbf{K}\mathbf{U} = \mathbf{H}\mathbf{f} + \mathbf{F} \\ &\qquad 0 < \mathbf{x}_{min} \le \mathbf{x} \le \mathbf{1} \end{aligned} \tag{1}$$

where $\mathbf{U}_{n\mathrm{x}1}$ is the displacement vector, $\mathbf{H}_{nxm}$ is a location matrix for the control force, $m$ is the number of action control forces and $\mathbf{F}_{n\mathrm{x}1}$ is the applied external force vector, $\mathbf{f}_{mx1}$ is an applied control force in terms of electric load. The magnitudes of the matrices $\mathbf{Q}_{nxn}$ and $\mathbf{R}_{m\mathrm{x}m}$ are assigned according to the relative importance of the state variables and the control force in the minimization procedure. $\mathbf{K}_{nxn}$ is the finite element global stiffness matrix. $\mathbf{x}$ is the vector of design variables, $\mathbf{x}_{min}$ is a vector of minimum relative densities. $V(\mathbf{x})$ and $V_0$ is the material volume and design domain volume, respectively and $V_{min}$ is the prescribed volume function. Considering the discretization,

$$\mathbf{U}^{\mathrm{T}}\mathbf{Q}\mathbf{U} = \sum_{e=1}^{N} \left(x_e\right)^p \mathbf{u}_e^T \mathbf{q}_e \mathbf{u}_e , \tag{2}$$

where $N$ is the number of elements, $p$ is the penalization exponent, $\mathbf{u}_e$ and $\mathbf{q}_e$ are the element displacement vector and weighting matrix, respectively.

The optimization problem is solved using the Optimality criterion (OC), and this criterion is derived from the Karush-Kuhn-Tucker conditions (Bendsøe and Kikuchi, 1988). The Lagrangian function of the minimization problem is

$$L(\mathbf{x}) = J(\mathbf{x}) + \lambda_0 (V(\mathbf{x}) - V_{min} V_0) + \boldsymbol{\lambda}_1^T (\mathbf{K}\mathbf{U} - (\mathbf{H}\mathbf{f} + \mathbf{F})) + \sum_{e=1}^{N} \lambda_{2e} (x_{\min} - x_e) + \sum_{e=1}^{N} \lambda_{3e} (x_e - x_{\max}) . \tag{3}$$

where the scalar $\lambda_0$ and the vector $\boldsymbol{\lambda}_1$ are the global Lagrangian multipliers, and the scalars $\lambda_{2e}$ and $\lambda_{3e}$ are Lagrangian multipliers for lower and upper side constraints.

To locate a stationary point, it is necessary that $\partial L / \partial x_e = 0$, then

$$\frac{\partial L}{\partial x_e} = \frac{\partial J}{\partial x_e} + \lambda_0 \frac{\partial V}{\partial x_e} + \boldsymbol{\lambda}_1^T \frac{\partial(\mathbf{K}\mathbf{U} - (\mathbf{H}\mathbf{f} + \mathbf{F}))}{\partial x_e} - \lambda_{2e} + \lambda_{3e} = 0. \tag{4}$$

Here, we assume that constrains of the design variables are not active, $\lambda_{2e} = \lambda_{3e} = 0$ and that the load and forces are design independent, $\frac{\partial(\mathbf{H}\mathbf{f} + \mathbf{F})}{\partial \mathbf{x}} = 0$.

The feedback requires a full knowledge of states. By using the displacement closed-loop feedback control we can assume

$$\mathbf{f} = -\mathbf{R}^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{U} , \tag{5}$$

then the equilibrium constraint from Eq. (1) becomes

$$\mathbf{K}_c \mathbf{U} = \mathbf{F} , \tag{6}$$

where

$$\mathbf{K}_c = \mathbf{K} + \mathbf{H}\mathbf{R}^{-1}\mathbf{H}^T . \tag{7}$$

We can note that this $\mathbf{K_c}$ is the modified matrix under control effect and the modification appears where the force control is applied, which affects also the eigenvalues and displacement of the structure. The problem can be solved as the conventional static finite element method in standard form $\mathbf{K}_c\mathbf{U} = \mathbf{F}$.

The influence of the weighting matrix **R** is an important aspect to consider. To have significant effect on the topology of the structure, the matrix $\mathbf{R^{-1}}$ need an equivalent magnitude compatible with the stiffness matrix. Since the stiffness is modified on each iteration, then **R** is chosen as $\mathbf{R} = G diag\left(P^a / \mu\right)$, where $\mu$ are the eigenvalues (the smallest to the largest), *G* is a weighting constant is and $\mathbf{P}^a_{mxm}$ is computed as the energetic equivalent generalized force (Yang et al., 2005; Kumar and Narayanan, 2008; Molter et al, 2010), that is, as a moment.

### 2.1. Sensitivity Analysis

Sensitivities are defined as the derivatives of the objective function and the constraints with respect to the design variables, and is often the major computational cost of the optimization. In this model, the objective function sensitivity requires differentiating displacements (which implies stiffness differentiation) and eigenvalues. Substituting Eq. (5) in *J* from Eq. (1), this yield

$$J = \mathbf{U}^T\mathbf{\Gamma}\mathbf{U} + \mathbf{U}^T\mathbf{Q}\mathbf{U} = \mathbf{U}^T\left(\mathbf{\Gamma} + \mathbf{Q}\right)\mathbf{U}, \tag{8}$$

where $\mathbf{\Gamma} = \mathbf{H}\mathbf{R}^T\mathbf{R}\mathbf{H}^T$. The derivatives of *J* can be computing by

$$\frac{\partial J}{\partial \mathbf{x_e}} = \frac{\partial \mathbf{U}}{\partial \mathbf{x_e}}\left(\mathbf{\Gamma} + \mathbf{Q}\right)\mathbf{U} + \mathbf{U}\left(\mathbf{\Gamma} + \mathbf{Q}\right)\frac{\partial \mathbf{U}}{\partial \mathbf{x_e}} . \tag{9}$$

It is possible obtain a simplification for the derivatives of *J*, Eq (9), adjusting the matrix **Q** by **Q**=**K**. Then, the objective function change, substituting Eq. (7) into Eq. (6) and then Eq. (6) into Eq. (1), this yield

$$J = \mathbf{f}^T\mathbf{R}\mathbf{f} + \mathbf{U}^T\left(\mathbf{H}\mathbf{f} + \mathbf{F}\right). \tag{10}$$

Using Eq. (5) into Eq. (10), we obtain the simplified objective function

$$J = \mathbf{F^T U}. \tag{11}$$

Taking the derivative of the objective function, on each element, one can obtain

$$\frac{\partial J}{\partial \mathbf{x_e}} = \mathbf{F}^T\frac{\partial \mathbf{U}}{\partial \mathbf{x_e}} , \tag{12}$$

and substituting

$$\frac{\partial \mathbf{U}}{\partial \mathbf{x_e}} = -\mathbf{K}_c^{-1}\frac{\partial \mathbf{K}_c}{\partial \mathbf{x_e}}\mathbf{U} \tag{13}$$

and

$$\frac{\partial \mathbf{K}_c}{\partial \mathbf{x_e}} = \frac{\partial \mathbf{K}}{\partial \mathbf{x_e}} + \frac{1}{GP^a}\mathbf{H}\frac{\partial \boldsymbol{\mu}}{\partial \mathbf{x_e}}\mathbf{H}^T \tag{14}$$

into (12), we have

$$\frac{\partial J}{\partial \mathbf{x_e}} = -\left( \mathbf{U}^{\mathrm{T}} \frac{\partial \mathbf{K}}{\partial \mathbf{x_e}} \mathbf{U} + \frac{1}{GP^{a}} \mathbf{U}^{\mathrm{T}} \mathbf{H} \frac{\partial \boldsymbol{\mu}}{\partial \mathbf{x_e}} \mathbf{H}^{\mathrm{T}} \mathbf{U} \right). \tag{15}$$

The sensitivity of the each eigenvalue $\mu$ is computed by (Haftka *et al.*, 1990)

$$(\mathbf{K} - \mu \mathbf{M})\phi = 0 \tag{16}$$

and

$$\frac{\partial \mu}{\partial \mathbf{x_e}} = \phi^{\mathrm{T}} \left( \frac{\partial \mathbf{K}}{\partial \mathbf{x_e}} - \mu \frac{\partial \mathbf{M}}{\partial \mathbf{x_e}} \right) \phi = \phi_{\mathbf{e}}^{\mathrm{T}} \left( \frac{\partial \mathbf{k_e}}{\partial \mathbf{x_e}} - \mu \frac{\partial \mathbf{m_e}}{\partial \mathbf{x_e}} \right) \phi_{\mathbf{e}}, \tag{17}$$

where $\phi$ is the mass-normalized eigenvector and $\mathbf{M}$ is the mass matrix, on each element $\phi_{\mathbf{e}}$ , $\mathbf{k_e}$ and $\mathbf{m_e}$.

With some expanding of the terms, also simplification of the equations and heuristics scheme for the design variables (Sigmund, 2001), we can obtain the new $x_e$ for each iteration:

$$x_e^{(k+1)} = \begin{cases} \max\left(x_{\min}, x_e^{(k)} - \delta\right) & \text{if } x_e^{(k)} B_e^{\alpha} \leq \max\left(x_{\min}, x_e^{(k)} - \delta\right), \\ x_e^{(k)} B_e^{\alpha} & \text{if } \max\left(x_{\min}, x_e^{(k)} - \delta\right) \leq x_e^{(k)} B_e^{\alpha} \leq \min\left(1, x_e^{(k)} + \delta\right), \\ \min\left(1, x_e^{(k)} + \delta\right) & \text{if } \min\left(1, x_e^{(k)} + \delta\right) \leq x_e^{(k)} B_e^{\alpha}, \end{cases} \tag{18}$$

where $\delta$ and $\alpha$ are respectively, the prescribed move limit and the prescribed numerical damping coefficient. The term $B_e$ is defined from the optimality condition as

$$B_e = \frac{p\left(x_e\right)^{p-1} \mathbf{u}_e \mathbf{k}_e \mathbf{u}_e}{\lambda_0 \rho^e V_e}, \tag{19}$$

where $\rho^e$ is the mass density of the material, $V_e$ is the elemental volume. The mesh-independent filter is provided from Sigmund (2001).

## 3. CONTROL EFFECTS ON STRUCURAL TOPOLOGY

We can imagine *a priori* that control forces acting in different locations on the structure should influence the optimized design. To exemplify this fact, we consider a design domain as a cantilever beam shows in Fig. 1.

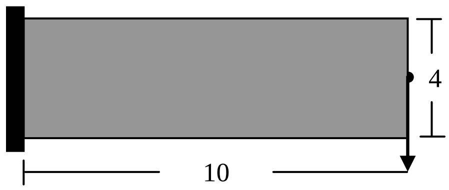
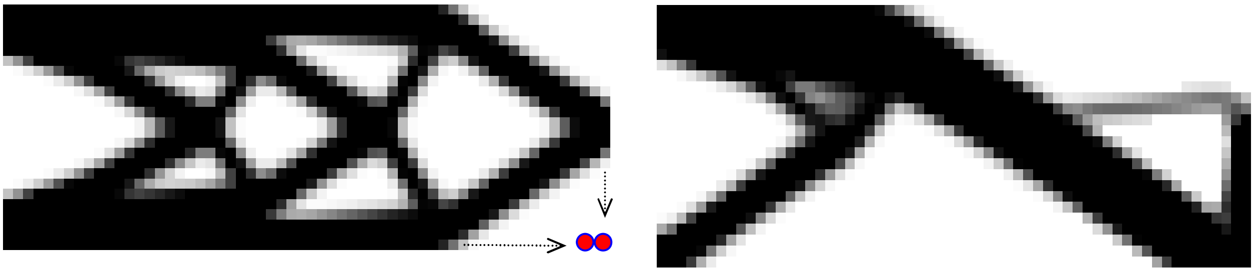



Figure 1. Design domain cantilever beam.

For a structural only design of this domain, we use the compliance as the objective function, and obtain the topology, for the cantilever beam, shown in Fig. 2a. Then we try to introduce a control forces on this design layout. It is possible that on the desired location for the actuator there is no material. If the optimization is performed without considering the control forces, then we need either to change the actuator location or to redesign the structure. In Fig. 2a we indicated with points (small circles) the actuator location and designed the structure again, this time considering the control force. The value of the parameters are: $p = 3$, $\alpha = 0.5$, $\delta = 0.2$ , $\mathbf{P}^a = diag\left(\text{-0.002, -0.01}\right)$ and $G$ is adjusted with similar magnitude of the stiffness inverse values. The new topology for this problem is shown in Fig. 2b, corresponding to the design domain, Fig. 1. The mesh domain in the simulation uses 1440 finite elements.



(2a) (2b)

Figure 2. a-Topology optimization without control force action. b-Topology optimization with control force action.

In this simulations it can be noted that the structure design change completely with the control action effects. Additionally some attention for the actuator location is required to assure the controllability of the system and the value of $G$ and the actuator location has considerable influence of control efficiency and the final topology.

## 4. OPTIMAL CONTROL DESIGN IN MODAL SPACE

After computing the optimal structure we search for the vibration suppression for a transient response of the system. It is possible to design the control for the displacement of a particular point of the structure. In this work we derive the control in independent modal space, computing the behavior of the system on the nodes where the control forces are applied .

The formulation of independent modal space control, derived by the classical optimal theory (Naidu, 2003), associated with the distributed-parameter system can be written briefly as follows. The modal formulation for the system is

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{\omega}^2 \boldsymbol{\eta} = \phi^{\mathbf{T}} \bar{\mathbf{G}} \bar{\mathbf{P}}^{a} \mathbf{V}_a \,, \tag{20}$$

where $\omega$ are the frequencies and $\mathbf{V}_a$ the voltage applied to actuator.

Let assume that $l$ = *(m + number of modes)*. Then, $\bar{\mathbf{G}}_{lxl}$ is a weighting matrix and $\bar{\mathbf{P}}^{a}_{l\,\mathrm{x}\,l}$ is the energetic equivalent generalized force matrix. The dynamic system defined by Eq. (20) can be parameterized in first order equations and written in the state-coefficient form

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\mathbf{V}_a; \quad \mathbf{z} = \mathbf{C}\mathbf{y}, \tag{21}$$

where $\mathbf{y}_{2lx1}$ is a state, time dependent variable, $\dot{\mathbf{y}}_{2l\,\mathrm{x}\,1}$ is the vector of the first order time derivates of the states in modal space, $\mathbf{V}_a \in S \in \Re^{l}$ is the control vector, $S$ is the control constraint set. $\mathbf{z}_{2lx1}$ is sensor output and $\mathbf{C}_{2lx2l}$ is the sensor output matrix in modal space. This system represents the constrains from the nonlinear regulator problem, together with $\mathbf{y}(\mathbf{t_0}) = \mathbf{y_0}$, $\mathbf{y}(\infty) = \mathbf{0}$ , respectively the initial and final conditions.

The coefficient matrices, in modal space, without considering damping, are given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{l\,\mathrm{x}\,l} & \mathbf{I}_{l\,\mathrm{x}\,l} \\ -\boldsymbol{\omega}^2_{l\,\mathrm{x}\,l} & \mathbf{0}_{l\,\mathrm{x}\,l} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{l\,\mathrm{x}\,l} \\ \phi^{\mathbf{T}} \bar{\mathbf{G}} \bar{\mathbf{P}}^{a} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{0}_{l\,\mathrm{x}\,l} & \bar{\mathbf{C}}_{l\,\mathrm{x}\,l} \\ \mathbf{0}_{l\,\mathrm{x}\,l} & \mathbf{0}_{l\,\mathrm{x}\,l} \end{bmatrix} \tag{22}$$

A state feedback rather than output feedback is adopted to enhance the control performance. The quadratic cost function for the regulator problem is given by

$$J_c = \frac{1}{2} \int_{t_0}^{\infty} \left[ \mathbf{y}^{\mathbf{T}} \bar{\mathbf{Q}} \mathbf{y} + \mathbf{V}_a^{T} \bar{\mathbf{R}} \mathbf{V}_a \right] dt \,, \tag{23}$$

where $\bar{\mathbf{Q}}$ is semi-positive-definite weighting matrix on the outputs and $\bar{\mathbf{R}}$ positive definite weighting matrix on the control inputs. Assuming full state feedback, the control law is given by

$$\mathbf{V}_a = -\bar{\mathbf{R}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{z} = -\bar{\mathbf{K}}\mathbf{z}\,, \tag{24}$$

where **P** satisfies the algebraic Riccati equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\bar{\mathbf{R}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \bar{\mathbf{Q}} = \mathbf{0}\,. \tag{25}$$

The computational cost is high if all modes are considered. But it can be dramatically reduced if only a few modes are dominant and their control is sufficient for the whole structure.

The closed loop dynamics of the system is given by

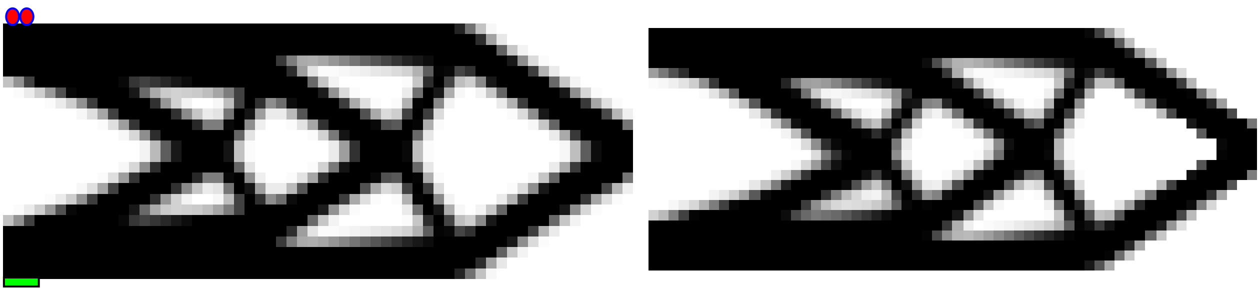
$$\dot{\mathbf{y}} = \left(\mathbf{A} - \mathbf{B}\bar{\mathbf{K}}\mathbf{C}\right)\mathbf{y}\,, \tag{26}$$

The stability of the feedback matrix $\bar{\mathbf{A}} = \left(\mathbf{A} - \mathbf{B}\bar{\mathbf{K}}\mathbf{C}\right)$ is an important condition for the existence of the feedback control. It can be show that for our problem the stability for $\bar{\mathbf{A}}$ is assured.

## 5. RESULTS

The physical system considered in the simulations is composed by cantilevered steel beam shown in Fig. 2a and the piezoelectric actuator bonded on the upper surface, at the beginning of the beam. The sensor is considered a piezofilm bonded on the bottom surface, also at the beginning of the beam. The resulting topology for this problem is show in Fig. 3a and Fig. 3b, where the locations of the horizontal control forces are indicate by points (small circles) and the sensor by a small rectangle. This location for the actuator was chosen because it is a known fact that the best place for one actuator, bonded on a cantilever beam, is as close as possible to the fixed size of the structure, which bears the maximum stress induced by the first and most significant mode.

Some simplifications are introduced to the problem and its response analysis. We assume that the two control points can have different forces. This means that there are two external actuators. Only one actuator would generate equal magnitude opposing forces and need to be explicitly included in the model.

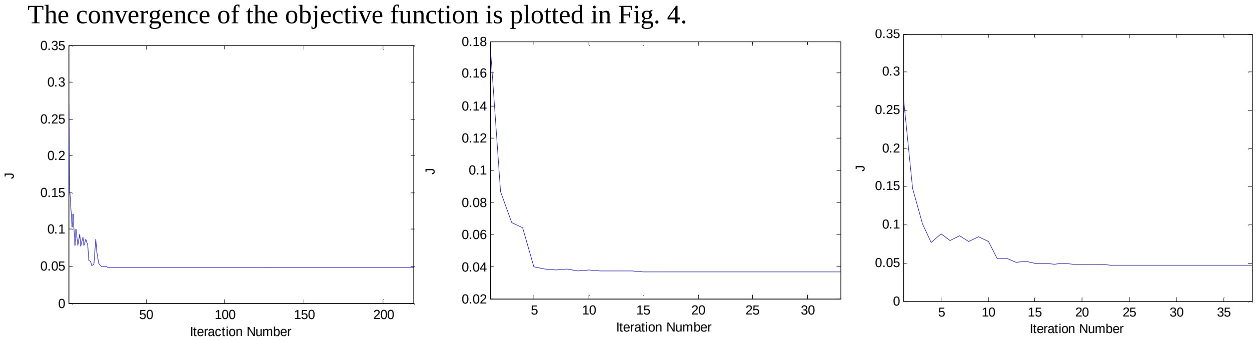


(a) (b)

Figure 3. a-Topology optimization without control force action. b-Topology optimization with control force action.

It can be note in Fig. 3b that the topology has not changed as much as in Fig. 2b. This can be attributed to location and magnitude of the force control have been less incisive in the topology than in the previous cases, but with enhanced control efficiency.

The convergence of the objective function is plotted in Fig. 4.

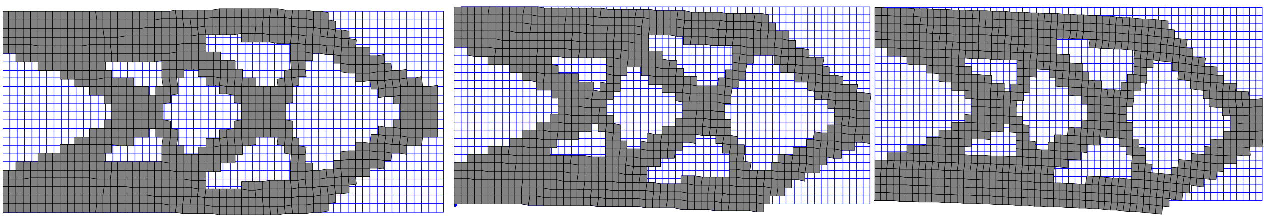



(a) (b) (c)

Figure 4. Objective function convergence of the cantilever beam case a- without control; b- with control force at the end of the beam ; c- with control forces at the beginning of the beam.

We can observe in Fig. 4 that the convergence is faster in the initial 30 iterations, after there is a smaller change of the objective function value at each iteration.

The topology shown in Figure 2a subject to transient forces produces initial deformation and so active the natural vibrations. The three free vibration modes of the model, which finite element discretization are shown in Fig. 5, whose frequencies are 0.15*Hz*, 0.15*Hz* and 0.24*Hz*, respectively.



mode1 mode2 mode3

Figure 5. Deflections of the modes in modal space.

In Fig. 5, it can be note that the first mode is not a vertical and the second and third modes are vertical modes.

The results of the optimal control simulation in Matlab are shown in Fig. 6. The weighting matrices and control matrices are: $\bar{\mathbf{G}} = diag(10^9, 10^9, 10^9, 10^9)$, $\bar{\mathbf{Q}} = diag(10,10,10,10)$. Here are considered the two first modes of the optimized structure. The position 1 is on the left point (small circle) and position 2 on the right, shown in Fig. 3. The fourth-order Runge-Kutta method was used to integrate the equations for a thirty seconds simulation.

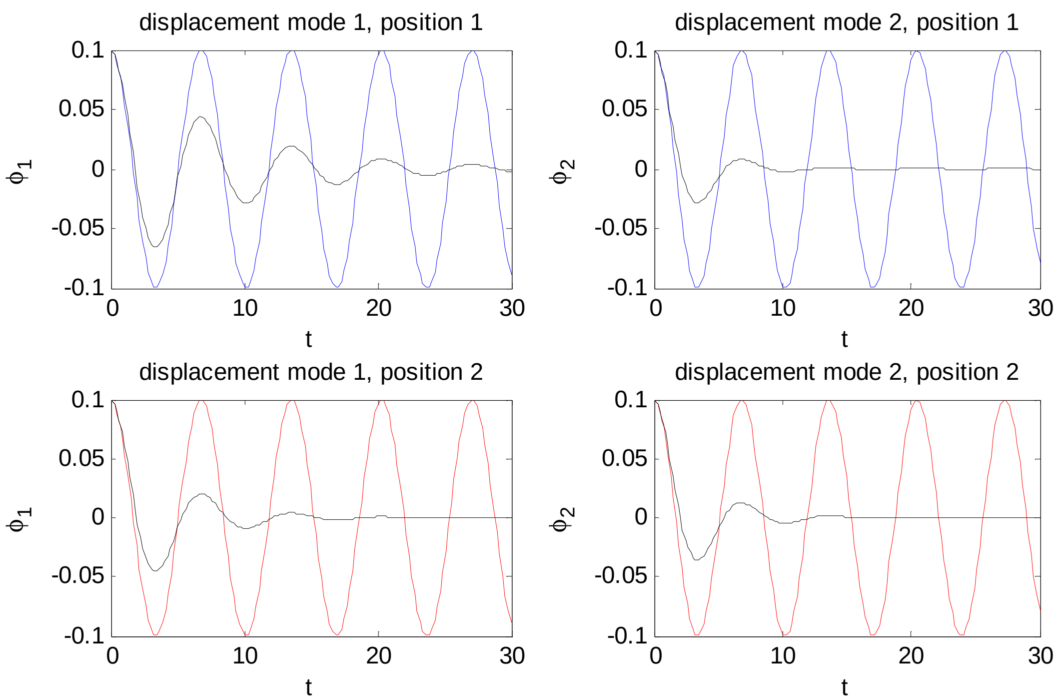



Figure 6. Deflections of first and second modes without independent modal control (blue and red) and with independent modal control (black).

It is possible observe that the modal displacement go quickly to zero, even without natural damping. The choice of the best values for the state and control weighting matrices $\bar{\mathbf{Q}}$ and $\bar{\mathbf{R}}$ is important. A good choice can improve the efficiency of the controllers. In this paper we have tested some weighting matrices and concluded that, for our control design, the good results are obtained around the values chooses above. Smaller or greater values affects the control efficiency.

## 6. ACKNOWLEDGEMENTS

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## 8. RESPONSIBILITY NOTICE

The authors Alexandre Molter, Valdecir Bottega Jun S. O. Fonseca and Otávio A. Alves da Silveira are the only responsible for the printed material included in this paper.