HERTZ THEORY x EXPERIMENT: CONTACT PARAMETERS IN SPHERE-PLAN CONFIGURATION FOR COMPOSITES OF POLYESTER/GLASS FIBER AGAINST A STEEL SPHERE IN QUASISTATICS TESTS

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Abstract. *In order to weaken the environmental impacts generated by the world discharge of thermoset matrix composites like Polyester/Glass Fiber (Po/GF), have developed composites Po / GF loaded with waste obtained through the grinding of knives and balls mill of their own composite Po / GF and compared in terms of tribological with composites Po / GF using* $CaCO_3$ *as filler, in their internal structure, to identify the possibility of reuse of composites and total or partial replacement of the residues by* $CaCO_3$*. The aim of this work is to compare the theoretical footprints obtained by the Hertzian model of a set of sphere-plan contacts and the experimental footprints obtained due to the quasistatic contacts of a metal and viscoelastic composite materials. A steel sphere of diameter 1/4" was used to promote indentations in the plan surfaces of nine composites. To promote the indentations was used a durometer with a normal load of 100kgf, carrying out seven indentations for each composite that were geometrically characterized. To measure the contact scars was used a profile projector. The theoretical values of the Hertz contact parameters were obtained using an Excel® spreadsheet. The elastic properties were obtained from the DMA testings (Young's modulus of composites) and literature (Poisson ratios). The experimental and theoretical mean values of the contact diameter, 2a were compared after the application of this normal load. The experimental measurements were larger than the expected theoretical values for all the composites. This phenomenon was attributed to viscoelasticity features of these polymeric composites.*

Key-words: Thermoset composites, Hertz model, Contact parameters.

1. INTRODUCTION

Actually, there are thousands of types of polymers in the entire world with different characteristics to the most several applications. One of the reasons which the use of these materials have increased significantly in the society and industry, beside their good properties that differ them from other materials, is associated with the creation of new and rigorous norms of environmental protection in the world. This subject has been of large importance and very commented in this century due to the occurrences and events in consequence of the answer from the nature to the exploration and human interference.

Thermoset materials based in epoxy resins, polyester, among others are the principal support of many technical applications due to their high resistance to water, chemical products and ultraviolet rays (when used with absorbs of UV), quick cure, high cycle of life, stiffness, dimensional stability, superficial hardness, lightness, design free, easiness in the cleaning and maintenance, low cost, low viscosity, good mechanical properties. In many cases, the association of properties offered by thermoset materials cannot be obtained by engineering thermoplastics (Crawford, 1998 and Ram, 1997). Nascimento *et al.* (2009), compared the theoretical footprints obtained by the Hertzian model of a set of sphere-plan contacts and the experimental footprints obtained due to the quasistatic contacts of a metal and viscoelastic materials.

Thinking about it, this work purposes an evaluation of thermoset polymeric composites materials through the analysis of experimental and Hertz contact parameters in order to verify the surface behavior (reaction to the external force) and the applicability of these new materials in many areas of engineering, beginning a large study that can contribute to the preservation of environmental and creation of materials with good properties and low cost to apply in the industry.

Since this study is possible to evaluate the viscous component of these polymeric composite materials through the results of the Hertz contact mechanics and comparing with the experimental results, besides to observe the influence of production process.

1.1 Hertz Contact Mechanics Theory

In 1882, Hertz published the article "On the contact of elastic solids" which according to Johnson (1989) was the first study about contact mechanics presenting a relationship among geometric and elastic properties of bodies in contact with a normal force that result in calculate of contact parameters and so, he is considered the pioneer in this subject. The question appeared when was questioned if the elastic deformation of glass lenses under the action of the force that kept them in contact could have a significant influence on the standard of the interference fringes.

Hertz studied the contact between two elastic solids with profiles of smooth surface (without roughness) which could be approached as a parabola near the contact area (see fig. 1). This theory predicts that the contact area increases non-linearly with the squeezing force **F**, but as $\mathbf{A} \sim \mathbf{F}^{2/3}$ (Persson, 2006).

According to Medeiros (2002), the contribution accomplishes of Hertz was to demonstrate mathematically that, in contact between static solids non-conformal squeezed itself and without friction, *geometric* and *elastic* parameters at the material are necessary and enough to define the *contact area* and the *states of operating stress and deformation*.

However, it is necessary to emphasize some assumptions of the Hertz model application. These are the following: (i) the materials in contact under a Hertzian pressure ***p*** are homogeneous and the yield shear stress, ***k***, is not exceeded, i.e. the relationship $\mathbf{p}/\boldsymbol{k} < \mathbf{1}$ (contact is kept in the elastic zone) (Johnson, 1989); (ii) contact stress is caused by the load which is normal to the contact tangent plan which effectively means that there are no tangential forces acting between the solids; (iii) the contact area is very small compared with the dimensions of the contacting solids; (iv) the contacting solids are at rest and in equilibrium; (v) surfaces without roughness.

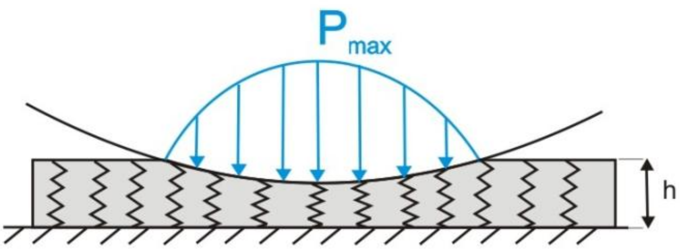


Figure 1. Foundation of the elastic model with a rigid base of depth ***h*** that is squeezed by a rigid indentator and the contact pressure distribution ***p*** on the contact area (based in JOHNSON, 1989).

When two bodies with plain, concave or convex surface enter in contact under a defined load, these two surfaces deform giving forms to a small contact area. The deformation may be either plastic or elastic depending on the magnitude of the applied load and the material's elastic properties. In many engineering applications, for example, rolling contact bearings, gears, cams, seals, etc., the contacting surfaces are non-conformal hence the resulting contact areas are very small and the resulting pressures are very high (Stachowiak e Batchelor, 2005).

These stresses can be determined from the analytical formulae, based on the theory of elasticity, developed by Hertz and they are simplified when the contact area is to circulate as for spheres or sphere-plan in contact. Figure 2 shows to the contact between a sphere and a plan, its respective dimension of contact area and the distribution of the pressure on the basis at the Hertz theory.

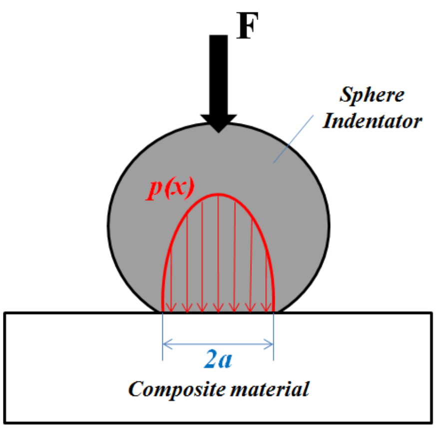


Figure 2. Contact between a sphere and a flat surface under the action of a normal force **F**, contact area dimension (***2a***) and pressure distribution ***p(x)***.

The distribution of pressure in the contact sphere-plan is the same for all the parallel plans the action of the load, because the contact area has a circular geometry and therefore, the half-axles in the plans of contact have the same contact radius that act on the axes of contact, i.e, are equal in module to ***a***. The contact pressure distribution is given by (Norton, 1996; Johnson, 1989):

$$p = p_{máx}\,(1 - (x^2/a^2))^{1/2} \quad (1a)$$

$$p_{máx} = 3/2\, p_{avg} = [3/2\,(F/\pi a^2)] \quad (1b)$$

Where ***a*** is the contact area radius with circular geometry and, on the basis the Hertz contact mechanics theory is given by the expression:

$$a = [(3FR/4E)^{1/3}] \quad (2)$$

R is the equivalent radius of curvature in contact, $1/R = (1/R_{sphere} + 1/R_{flat})$. The radius at the sphere and the plan are, respectively, R_{sphere} and R_{flat}. The plan radius R_{flat} is considered tending the infinite or very bigger than the sphere radius R_{sphere}, i.e., $R_{flat} \rightarrow \infty$ or $R_{flat} >> R_{sphere}$. E is the elasticity modulus equivalent of the bodies in contact, $1/E = [((1-\nu_{sphere}^2)/E_{sphere})+((1-\nu_{flat}^2)/E_{flat})]$, ν_{sphere} and ν_{flat} are respectively, the Poisson ratio the sphere and the plan; E_{sphere} and E_{flat} are, respectively, the elasticity modulus (or Young modulus) of the materials the sphere and the plan.

2. EXPERIMENTAL

2.1 Materials

Polyester/glass fibers (GFs) residues: The material used as a residue was obtained from polyester composites (density 1.09 g/cm^3) with 12 wt. % glass fibers which had been molded by a variant of the resin transfer molding (RTM) process called light RTM.

In this process a liquid resin, pre-catalyzed for curing later, is injected into a closed mold with the help of a vacuum pump, impregnating dry fibers. Once the fibers are well impregnated and the mold filled, curing is performed.

To mold new composites, the following materials were used: (i) A medium viscosity (90 to 120 cPoise) polyester resin (UCEFLEX UC 5518 from Elekeiroz); (ii) Glass Fiber (GF) mats with an aerial density of 300 g/m^2; (iii) calcium carbonate ($CaCO_3$) with an average density of 2.82 ± 0.01 g/cm^3; (iv) Butanox M-50 (methyl-ethyl-ketone peroxide, MEKP, 33 % dimethyl phthalate), 1.5% v/v, as the initiator.

2.2 Methodology Used

The polyester with glass fibers composite wastes were ground in a knife mill with a 8 × 8 mm screen, then in a ball mill (for 1 min), reaching a particle size 9 - 16 mesh. This material was incorporated into virgin polyester + glass fiber composites by distributing them in the center region (in-between glass mat layers), as shown by the schematic drawing in fig. 3.

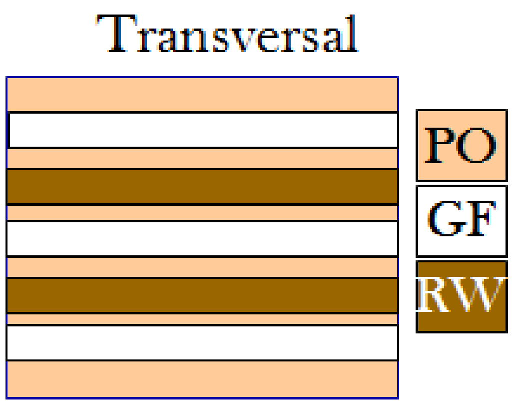


Figure 3. Transversal section of the composite.

The production of composites with $CaCO_3$ was performed in the same way as the composites molded with only waste, the difference is that the composites with $CaCO_3$ as filler were first mixed the resin with $CaCO_3$, after manual mixing, were mixed by a mechanical stirrer Brand Fisatom model 713D, with a speed of 340-360 rpm for 10 minutes to get a good homogeneity, after immediately was placed the initiator in the mixture and was performed the molding.

The composites were molded by hot compression (using 6 ton distributed on a 270 × 170 mm metallic mold at the temperature of 90°C) and the following formulations were used: two control groups (polyester + glass fibers and polyester + $CaCO_3$, 50 wt. % polyester in both cases), binary (polyester + residues) and two ternary families of composites: polyester + glass fibers + $CaCO_3$ (50/35/15, 50/25/25 and 50/15/35, weight basis) and polyester + glass fibers + residues (50/35/15, 50/25/25 and 50/15/35, weight basis).

Table 1 presents the elastic properties these materials, whose elasticity modulus were obtained through dynamic mechanical analysis (DMA) the ambient temperature at 30°C and their Poisson ratio removed of literature (Matweb.com). To apply the elastic properties of the steel sphere in the formulation of the contact area (Hertz theory) was considered **E = 2.10 GPa** and ***ν* = 0.29**, usual value for a carbon steel.

Table 1. Identification of composites, storage modules and Poisson ratios**.** Storage modules were obtained from the temperature of 25°C.

Identification	Composite	Storage Modulus (E) [Pa]	Poisson Ratio (ν)
C1	PO/CaCO3 (50:50)	2,10E+10	0.35
C2	PO/GF/CaCo3 (50:25:25)	4,37E+10	0.35
C3	PO/Residues (50:50)	1,94E+10	0.35
C4	PO/ GF (50:50)	3,65E+10	0.35
C5	PO/GF/CaCo3 (50:15:35)	3,60E+10	0.35
C6	PO/GF/CaCo3 (50:35:15)	2,77E+10	0.35
C7	PO/GF/Residues (50:25:25)	3,07E+10	0.35
C8	PO/GF/Residues (50:15:35)	3,70E+10	0.35
C9	PO/GF/Residues (50:35:15)	3,37E+10	0.35

A durometer Rockwell C was used to carried out the indentations in sphere-plan (Metal-Composite) configuration with 100 kgf of load, carbon papers of thickness 100µm, a metallic sphere of diameter 1/4" (indentator) (6,35mm), a profile projector was used to determinate the diameter of the scar after the indentations.

3. RESULTS AND DISCUSSION

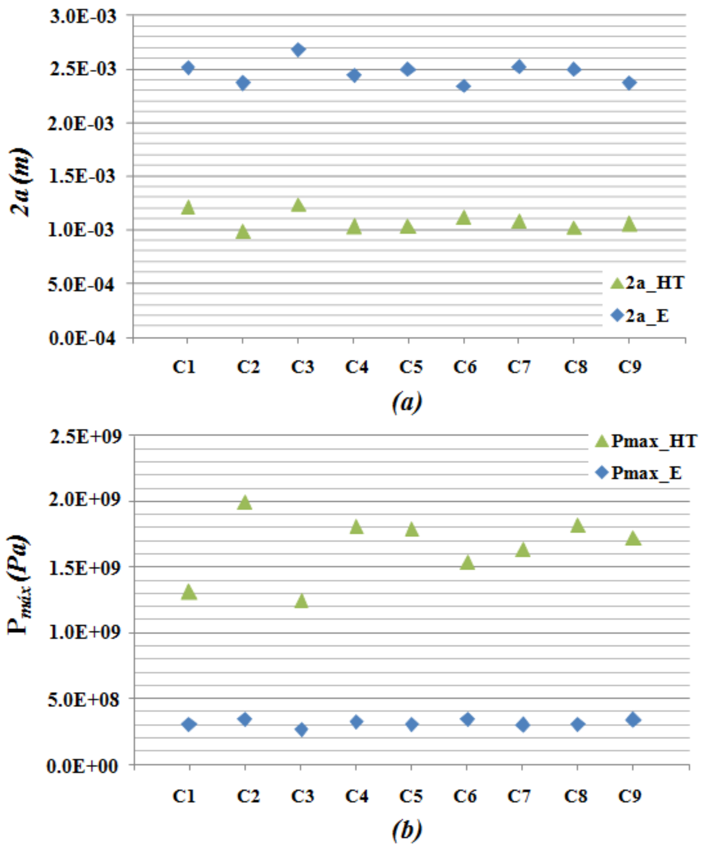
Table 2 shows the scar dimension results to a set of seven quasistatic contacts sphere-plan obtained experimentally for each one composite sample. It was calculated the mean ($2a_{md}$), standard deviation (SD), contact area (A) and maximum contact pressure ($p_{máx}$).

Table 2. Contact diameter or scar dimension (*2a*), standard deviation [SD] (experimental), maximum contact pressure, relationship between experimental and Hertz theory contact diameter and maximum contact pressure results.

Composites	Hertz Theory		Experimental			$2a_E / 2a_{HT}$	$P_{máxE} / P_{máxHT}$
	2a (m)	*$P_{máx}$ (Pa)*	*2a (m)*	*SD*	*$P_{máx}$ (Pa)*		
C1	*1.21E-03*	*1.31E+09*	*2.50E-03*	*3.88E-05*	*2.99E+08*	*2.07*	*0.229*
C2	*9.79E-04*	*1.99E+09*	*2.36E-03*	*6.79E-05*	*3.36E+08*	*2.41*	*0.169*
C3	*1.24E-03*	*1.25E+09*	*2.67E-03*	*8.70E-05*	*2.63E+08*	*2.16*	*0.211*
C4	*1.03E-03*	*1.80E+09*	*2.43E-03*	*1.19E-04*	*3.16E+08*	*2.37*	*0.175*
C5	*1.03E-03*	*1.79E+09*	*2.49E-03*	*5.39E-05*	*3.01E+08*	*2.42*	*0.168*
C6	*1.11E-03*	*1.54E+09*	*2.34E-03*	*3.60E-05*	*3.41E+08*	*2.10*	*0.222*
C7	*1.08E-03*	*1.64E+09*	*2.52E-03*	*4.07E-05*	*2.95E+08*	*2.33*	*0.180*
C8	*1.03E-03*	*1.82E+09*	*2.49E-03*	*7.32E-05*	*3.01E+08*	*2.43*	*0.165*
C9	*1.05E-03*	*1.72E+09*	*2.37E-03*	*3.51E-05*	*0,00E+00*	*2.25*	*0.194*

Using these results presented in the table 2 and those calculated by the Hertz theory contact mechanics (equations 1b and 2) through the Excel® spreadsheet, it is possible to print the graph 1 (*a*) and (*b*) comparing the experimental and Hertz theory results to the composites.

Figure 4 shows the Hertz theory and experimental results to the contact or mean scar diameter (experimental) ($2a$) and maximum contact pressure ($P_{máx}$). The contact area (a) was calculated by the equation 2 from the Hertz theory. Hertz theory and experimental results are represented, respectively, by _HT and _E in the graph.



Graph 1. Results of *(a)* contact diameter (Hertz theory) and dimension of scar diameter; *(b)* maximum experimental and Hertzian contact pressure.

Composites C6 and C2 were those that presented the smallest values of scar dimension, being the composite C6, the first. Both these composites have $CaCO_3$ in their internal structure on percentages of 15% and 25% respectively. Composites without glass fiber (C1 and C3) only with residues and $CaCO_3$ on proportion of 50:50 are among those that presented the highest results of scar diameter.

If we organize the composite storage modulus in a descending order, initially negligencing its inherent frequency dependence, we can see that the addiction of residues or $CaCO_3$ with proportion of 35% and 25%, respectively, in the composite polyester/glass fiber provide an increase of storage modulus **E** compared to Po/GF (50:50), being the second composite Po/GF/$CaCO_3$ (50:25:25) the highest modulus (**E** = 43,70 GPa). Pogotskaya *et al.* (2010) evaluating the elastic behavior of nanometric coatings, compared the results obtained using the Hertz theory with those obtained by the theory of contact deformation of laminar system also noticed neglect the effect of the substrate, in this case causing overestimation of the elastic properties of the specimen by 1,7 - 3 times. The great significance of the modulus of elasticity when the penetration is shallow is explained by the specifics of formation of the material structure in the surface layer. A gradient variation of the modulus of elasticity through depth can be assumed.

In other hand, these results express that a composite formed only with Po/Residues (50:50) and Po/$CaCO_3$ (50:50), i.e, without glass fiber, exhibit the smallest modulus. An increase on percentage of $CaCO_3$ from 15% to 35% in the composite Po/GF (with 50% of Po) resulted in an increase of the modulus from 27,70 GPa to 36,00 GPa, but both of them presented storage modulus smallest when the proportion was the same to the glass fiber and $CaCO_3$, with 25% each one.

Composites with residues on the proportion of 25% and 15% presented highest modulus than that with $CaCO_3$, on the proportion of 15%, being the sixth and fifth highest modulus.

According to the graph 1 (*b*) of contact pressure checks the results of experiments were all lower than the theoretical and followed a similar trend for all composites tested, always staying below 0.5 GPa, while theoretical results in a variation, not following the same trend.

Table 3 shows the expected, based on the Hertz contact mechanics theory given by equation 2 shown above, and obtained minor contact area experimentally to the composites according to the storage modulus E of composites. Printed in black (expected contact area), the sequence from the first minor contact area (1) to the largest contact area (9). Printed in red (obtained contact area), the experimental results of minor contact area calculated from the mean scar diameter.

Table 3. Composites with their Storage Modulus E and respective numbered minor expected and obtained contact area.

Composites		C1	C2	C3	C4	C5	C6	C7	C8	C9
E (GPa)		*21.0*	*43.7*	*19.4*	*36.5*	*36.0*	*27.7*	*30.7*	*37.0*	*33.7*
A	Expected	8	1	9	3	4	7	6	2	5
	Obtained	7	2	9	4	6	1	8	5	3

Although the storage modulus represents physically the elastic force of links in a internal structure of a material, commonly called of stiffness of body, keeping the geometric characteristics of a contact and the normal load applied, the Hertzian contact parameters are only modified by the storage modulus. To the contact radius, when increase the storage modulus, decrease the contact radius and consequently the contact area.

4. CONCLUSIONS

Thinking in applications where resistance to indentation and abrasion, such as pumping units, pneumatic and hydraulic farm machinery, is required, it can be concluded that the variation in composition and use of different reinforcement fillers in the composites polyester/glass fiber did not generate great influence on the results of contact pressure obtained. However, it was possible to identify the influence of loads on the modules of elasticity, where the composite without glass fiber stood out for having the smaller modules, confirming the influence of glass fiber in these composites. Moreover, these same composites without fiber glass showed the highest results of the scar diameter.

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