IDENTIFICATION AND VIBRATION CONTROL OF A FLEXIBLE STRUCTURE

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Abstract. This paper concerns the identification and vibration control of a flexible structure. A state-space model of the system is used in the implementation of a digital controller to control the first two modes of a cantilever beam. To determine the parameters for the model of such a system, the Eigensystem Realization Algorithm (ERA) is often used, However, this was developed for an impulsive input, and here white noise was used for system identification. Thus, a variation of this algorithm called Observer/Kalman Filter Identification (OKID) was used instead. This incorporates a state observer, which means that the system can be identified for any input. The system identification procedure is outlined in some detail. The control system was implemented in real time on an aluminium cantilever beam, which had a PZT patch bonded to it as an actuator and a co-located PVDF patch which was used as the sensor. It was shown experimentally that such a control system, coupled with the actuators and sensors, is capable of increasing the damping ratio of the first mode of vibration of a cantilever beam by an order of magnitude, when it is subject to impulsive excitation.

Keywords: Control, Linear Quadratic Regulator, ERA, OKID.

1. INTRODUCTION

The accuracy of the mathematical models of dynamic systems is essential for the success of modern control laws. Several methodologies are available to identify the mathematical representation of a physical structure or phenomenon. Among these methodologies, the Eigensystem Realization Algorithm (ERA), Juang *et al.* (1985), and its derivative ERA/OKID (Observer/Kalman Filter Identification), Juang *et al.* 1993, belong to the category of black box identification, where the model of a system is identified through experimental data.

The identification process, such as the least square method (Gauss, 1809), has been studied by many researchers. In the twentieth century (mostly since 1965), the focus has been on building models to be applied in control problems. In 1965, two influential papers were published, (Ho and Kalman, 1965) that led to the subspace identification method and (Åström-Bohlin, 1965) that led to the Prediction Error methods, (Gevers, 2006). The ERA belongs to the subspace identification class and is based on realization theory, that is, the computation of the triplet $[\mathbf{A}, \mathbf{B}, \mathbf{C}]$, which correspond to the state matrix, the input matrix, and the output matrix respectively. They are obtained from measured data, in this case, the impulse response function (IRF), from which the Markov parameters can be determined. The ERA can only be used if the IRF is known, and sometimes this is not the case. With the OKID method, the ERA can be used with other signals (Juang *et al.* 1993). This identification method uses a state observer in the process allowing the Markov parameters to be obtained through the response of a signal that is easy to generate, for example white noise, which is the case in this paper. Here, a state-space model of a cantilever beam is generated using the OKID method, which is then used in an Linear Quadratic Regulator (LQR) controller (Ogata, 2008) to experimentally control the vibration of the beam. The beam has piezoelectric elements bonded to it as the control actuator and the sensor, and the controller was implemented using dSpace. The paper is organized as follows. The system identification procedure is described in Section 2, the control of the beam in Section 3, and some general conclusions are given in Section 4.

2. SYSTEM IDENTIFICATION

2.1. Overview

System identification is the process of finding a mathematical model that represents the dynamical behaviour of the physical system under consideration. This model may be linear or nonlinear, continuous or discrete, and time variant or invariant. There are basically two important methodologies in system identification: parametric and non-parametric methods. Non-parametric methods are those that, through input/output data, obtain a graphical representation of a system. This representation may be the impulse response function or the frequency response function. Parametric methods, such as the one described in this paper, estimate the parameters that represent the system through experimental

data. Among the parametric methods are the ERA and a version of this which incorporates a state observer, the ERA/ OKID method. These methods are described in this section, and are applied to a cantilever beam, with piezoelectric actuators and sensors attached, The resulting state-space model is used in Section 3 in the design of the controller which is implemented experimentally on the beam.

2.2. Eigensystem Realization Algorithm

The ERA was developed in 1985 by Juang and Pappa to estimate the state matrices of a linear, discrete and time-invariant system. The state matrices are estimated through the discrete time Impulse Response Function (IRF) of the system. This is then built into the Hankel matrix and via Singular Value Decomposition (SVD), a set of matrices that describes the realization in state space is obtained. Consider the discrete space state realization of a generic system

$$\mathbf{x}(k{+}1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \tag{1a,b}$$

where $\mathbf{A}$ is the state matrix, $\mathbf{B}$ is the input matrix, $\mathbf{C}$ is the output matrix, $\mathbf{D}$ is the direct transmission matrix, $\mathbf{x}(k)$ is the state vector, $\mathbf{u}(k)$ is the input (control) vector and $\mathbf{y}(k)$ is the output vector.

Assuming zero initial conditions, $\mathbf{x}(0)=0$, and an unit impulse input at a given instant $\mathbf{u}(t)=1$, then the corresponding equations for $k=0,1,...,l-1$ is:

$$\mathbf{u}(k) = \begin{cases} 0, & k < t \\ 1, & t = k \\ 0, & k > t \end{cases} \tag{2}$$

Then,

$$\mathbf{x}(0) = 0 \qquad \Rightarrow \qquad \mathbf{y}(0) = \mathbf{D}\mathbf{u}(0)$$
$$\mathbf{x}(1) = \mathbf{B}\mathbf{u}(0) \qquad \Rightarrow \qquad \mathbf{y}(1) = \mathbf{C}\mathbf{B}\mathbf{u}(0) + \mathbf{D}\mathbf{u}(1)$$
$$\mathbf{x}(2) = \mathbf{A}\mathbf{B}\mathbf{u}(0) + \mathbf{B}\mathbf{u}(1) \qquad \Rightarrow \qquad \mathbf{y}(2) = \mathbf{C}\mathbf{A}\mathbf{B}\mathbf{u}(0) + \mathbf{C}\mathbf{B}\mathbf{u}(1) + \mathbf{D}\mathbf{u}(2)$$
$$\vdots$$
$$\mathbf{x}(l-1) = \sum_{i=1}^{l-1} \mathbf{A}^{i-1}\mathbf{B}\mathbf{u}(l-1-i) \quad \Rightarrow \quad \mathbf{y}(l-1) = \sum_{i=1}^{l-1} \mathbf{C}\mathbf{A}^{i-1}\mathbf{B}\mathbf{u}(l-1-i) + \mathbf{D}\mathbf{u}(l-1) \tag{3}$$

The Markov parameters of the system are given by $\mathbf{Z}_0=\mathbf{D}$, $\mathbf{Z}_1=\mathbf{CB}$, $\mathbf{Z}_2=\mathbf{CAB}$, $\mathbf{Z}_3=\mathbf{CA}^2\mathbf{B}$, ..., $\mathbf{Z}_k=\mathbf{CA}^{k-1}\mathbf{B}$,

2.3. Observer/Kalman Filter Identification

The relationship between the output $\mathbf{y}(k)$ and the input $\mathbf{u}(k)$ can be put in a matrix form as

$$\mathbf{Y} = \mathbf{Z}\mathbf{U} \tag{4}$$

where

$$\mathbf{Y} = [\mathbf{y}(0) \quad \mathbf{y}(1) \quad \mathbf{y}(2) \quad ... \quad \mathbf{y}(l\text{-}1)]$$
$$\mathbf{Z} = [\mathbf{D} \quad \mathbf{CB} \quad \mathbf{CAB} \quad ... \quad \mathbf{CA}^{l\text{-}2}\mathbf{B}]$$

and

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & ... & \mathbf{u}(l-1) \\ & \mathbf{u}(0) & \mathbf{u}(1) & ... & \mathbf{u}(l-2) \\ & & \mathbf{u}(0) & ... & \mathbf{u}(l-3) \\ & & & ... & \vdots \\ & & & & \mathbf{u}(0) \end{bmatrix}$$

Because of the low damping in many flexible structures the decay of free vibration takes a long time, as discussed by Alves, 2005. From the structural control point of view this means the delay time is very large. In this context, it is necessary to use a state observer, that introduces artificial damping in the system, decreasing the length of the vector of the data acquired.

Adding and subtracting $\mathbf{G}\mathbf{y}(k)$ to the right-hand side of the state Eq.(1) and combining this with Eq. (2) results in

$$\mathbf{x}(k+1) = \overline{\mathbf{A}}\mathbf{x}(k) + \overline{\mathbf{B}}\mathbf{v}(k) \tag{5}$$

where

$$\overline{\mathbf{A}} = \mathbf{A} + \mathbf{G}\mathbf{C}, \qquad \overline{\mathbf{B}} = [\mathbf{B} + \mathbf{G}\mathbf{D}, -\mathbf{G}], \qquad \mathbf{v} = \{\mathbf{u}(k) \ \mathbf{y}(k)\}^{T}$$

in which the superscript T denotes the transpose, and $\mathbf{G}$ is an arbitrary matrix of appropriate dimension chosen to ensure the system has the degree of stability required. The eigenvalues of the closed-loop state matrix for a stable system have negative real parts. The input-output matrix form for Eq. (5) which corresponds to Eq. (4) is:

$$\mathbf{Y} = \overline{\mathbf{Z}}\mathbf{V} \tag{6}$$

where

$$\overline{\mathbf{Z}} = [\mathbf{D} \ \ \mathbf{C}\overline{\mathbf{B}} \ \ \mathbf{C}\overline{\mathbf{A}}\overline{\mathbf{B}} \ \ ... \ \ \mathbf{C}\overline{\mathbf{A}}^{(p-2)}\overline{\mathbf{B}} \ ... \ \mathbf{C}\overline{\mathbf{A}}^{(l-2)}\overline{\mathbf{B}}]$$

is the matrix of observer Markov parameters, and

$$\mathbf{V} = \begin{bmatrix} \mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & \cdots & \mathbf{u}(p) & \cdots & \mathbf{u}(l-1) \\ & \mathbf{v}(0) & \mathbf{v}(1) & \cdots & \mathbf{v}(p-1) & \cdots & \mathbf{v}(l-2) \\ & & \mathbf{v}(0) & \cdots & \mathbf{v}(p-2) & \cdots & \mathbf{v}(l-3) \\ & & & \ddots & \vdots & \cdots & \vdots \\ & & & & \mathbf{v}(0) & \cdots & \mathbf{v}(l-p-1) \\ & & & & & \ddots & \vdots \\ & & & & & & \mathbf{v}(0) \end{bmatrix}$$

2.4. System Markov Parameters

Overall, there are system Markov parameters and observer gain parameters. The system Markov parameters are used to determine the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$, whereas the observer Markov parameters are used to determine the observer gain matrix $\mathbf{G}$. To determine the system Markov parameters in $\mathbf{Z}$ from the matrix of observer Markov parameters $\overline{\mathbf{Z}}$, it is partitioned as

$$\overline{\mathbf{Z}} = \begin{bmatrix} \overline{\mathbf{Y}}_0 & \overline{\mathbf{Z}}_1 & \overline{\mathbf{Z}}_2 & \cdots & \overline{\mathbf{Z}}_p \end{bmatrix} \tag{7}$$

where $\overline{\mathbf{Z}}_k = \mathbf{C}\overline{\mathbf{A}}^{(k-1)}\overline{\mathbf{B}}$. The elements of this matrix can be written as

$$\overline{\mathbf{Z}}_k = [\overline{\mathbf{Z}}_k^{(1)} \quad -\overline{\mathbf{Z}}_k^{(2)}] \tag{8}$$

where $\overline{\mathbf{Z}}_k^{(1)} = \mathbf{C}(\mathbf{A}+\mathbf{G}\mathbf{C})^{(k-1)}(\mathbf{B}+\mathbf{G}\mathbf{D})$ and $\overline{\mathbf{Z}}_k^{(2)} = \mathbf{C}(\mathbf{A}+\mathbf{G}\mathbf{C})^{(k-1)}\mathbf{G}$. Now $\mathbf{Z}_1 = \mathbf{C}\mathbf{B} = \mathbf{C}(\mathbf{B}+\mathbf{G}\mathbf{D}) - (\mathbf{C}\mathbf{G})\mathbf{D}$, so that the Markov parameter $\mathbf{Z}_1$ is given by

$$\mathbf{Z}_1 = \overline{\mathbf{Z}}_1^{(1)} - \overline{\mathbf{Z}}_1^{(2)}\mathbf{D} \tag{9}$$

To obtain the Markov parameter $\mathbf{Z}_2$, first consider $\overline{\mathbf{Z}}_2^{(1)} = \mathbf{C}(\mathbf{A}+\mathbf{G}\mathbf{C})(\mathbf{B}+\mathbf{G}\mathbf{D})$. This can be expanded and rearranged to give

$$\mathbf{Z}_2 = \overline{\mathbf{Z}}_2^{(1)} - \overline{\mathbf{Z}}_1^{(2)} \mathbf{Z}_1 - \overline{\mathbf{Z}}_2^{(2)} \mathbf{D} \tag{10}$$

In a similar way the Markov parameter $\mathbf{Z}_3$ is found to be

$$\mathbf{Z}_3 = \overline{\mathbf{Z}}_3^{(1)} - \overline{\mathbf{Z}}_1^{(2)} \mathbf{Z}_2 - \overline{\mathbf{Z}}_2^{(2)} \mathbf{Z}_1 \mathbf{D} - \overline{\mathbf{Z}}_3^{(2)} \mathbf{D} \tag{11}$$

By induction, the general relationship between the actual system Markov parameters and the observer parameter is

$$\begin{aligned}
\mathbf{Z}_0 &= \mathbf{D} \\
\mathbf{Z}_k &= \overline{\mathbf{Z}}_k^{(1)} - \sum_{i=1}^{k} \overline{\mathbf{Z}}_i^{(2)} \mathbf{Z}_{k-i}; \quad \text{for } k = 1, ..., p \\
\mathbf{Z}_k &= -\sum_{i=1}^{p} \overline{\mathbf{Z}}_i^{(2)} \mathbf{Z}_{k-i}; \quad \text{for } k = p+1, ..., \infty
\end{aligned} \tag{12a,b}$$

where p is the number of Markov´s parameters of the observer. This must be chosen such that $mp > n$, where m is the number of outputs and n is the order of the system, (Alves, 2005).

The observer Markov parameters can be used as the basis for computing the system Markov parameters. Indeed, the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{G}$ are embedded in the observer Markov parameter sequence.

Knowledge of the system Markov parameters allows a state-space realization as discussed in section 2.2.

2.5. Observer Gain Markov Parameters

The observer gain $\mathbf{G}$ can be identified by the following procedure. First, let

$$\mathbf{Y}_k^o = \mathbf{C}\mathbf{A}^{k-1}\mathbf{G}; \quad k=1,2,3,... \tag{13}$$

In terms of the observer Markov parameters $\mathbf{Y}_1^o$ is given by

$$\mathbf{Y}_1^o = \mathbf{C}\mathbf{G} = \overline{\mathbf{Z}}_1^{(2)} \tag{14}$$

Then, $\mathbf{Y}_2^o$ is obtained by considering that

$$\begin{aligned}
\overline{\mathbf{Z}}_2^{(2)} &= \mathbf{C}\overline{\mathbf{A}}\mathbf{G} = (\mathbf{C}\mathbf{A}\mathbf{G} + \mathbf{C}\mathbf{G}\mathbf{C}\mathbf{G}) \\
&= \mathbf{Y}_2^o + \overline{\mathbf{Z}}_1^{(2)}\mathbf{Y}_1^o
\end{aligned} \tag{15}$$

which yields

$$\mathbf{Y}_2^o = \overline{\mathbf{Z}}_2^{(2)}\overline{\mathbf{Z}}_1^{(2)}\mathbf{Y}_1^o \tag{16}$$

Similarly,

$$\begin{aligned}
\overline{\mathbf{Z}}_3^{(2)} &= \mathbf{C}\overline{\mathbf{A}}^2\mathbf{G} = (\mathbf{C}\mathbf{A}^2\mathbf{G} + \mathbf{C}\mathbf{G}\mathbf{A}\mathbf{G} + \mathbf{C}\overline{\mathbf{A}}\mathbf{G}\mathbf{C}\mathbf{G}) \\
&= \mathbf{Y}_3^o + \overline{\mathbf{Z}}_1^{(2)}\mathbf{Y}_2^o + \overline{\mathbf{Z}}_2^{(2)}\mathbf{Y}_1^o \\
\mathbf{Y}_3^o &= \overline{\mathbf{Z}}_3^{(2)} - \overline{\mathbf{Z}}_1^2\mathbf{Y}_2^o - \overline{\mathbf{Z}}_2^{(2)}\mathbf{Y}_1^o
\end{aligned} \tag{17}$$

The general relationship is given by

$$\begin{aligned}
\mathbf{Y}_1^o &= \mathbf{C}\mathbf{G} = \overline{\mathbf{Z}}_1^{(2)} \\
\mathbf{Y}_k^o &= \overline{\mathbf{Z}}_k^2 - \sum_{i=1}^{k-1} \overline{\mathbf{Z}}_i^{(2)} \mathbf{Y}_{k-1}^o; \quad \text{for } k=2, ..., p \\
\mathbf{Y}_k^o &= -\sum_{i=1}^{P} \overline{\mathbf{Z}}_i^{(2)} \mathbf{Y}_{k-1}^o; \quad \text{for } k=p+1, ..., \infty
\end{aligned} \tag{18}$$

After obtaining $\mathbf{Y}_k^o = \mathbf{C}\mathbf{A}^{p-2}\mathbf{G}$, $k = 1,2,3,... k$, the observer gain $\mathbf{G}$ can be computed by

$$\mathbf{G} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{Y}^o \tag{19}$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^k \end{bmatrix}, \quad \mathbf{Y}^o = \begin{bmatrix} \mathbf{Y}_1^o \\ \mathbf{Y}_2^o \\ \mathbf{Y}_3^o \\ \vdots \\ \mathbf{Y}_{k+1}^o \end{bmatrix} = \begin{bmatrix} \mathbf{CG} \\ \mathbf{CAG} \\ \mathbf{CA}^2\mathbf{G} \\ \vdots \\ \mathbf{CA}^k\mathbf{G} \end{bmatrix} \tag{20}$$

2.6. Experimental Identification of the Model

The techniques described in the previous subsections were applied to identify the state-space matrices ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$) of the aluminium cantilever beam with dimensions $35.4 \times 42.0 \times 2.0$ mm, shown in figure 1. The system was excited by white noise, over a frequency range of 0 to 500 Hz, using a Lead-Zirconate-Titanate (PZT) actuator with dimensions: $42 \times 23 \times 0.2$ mm. The vibration was measured using a Polyvinilidene-Fluoride (PVDF) patch of dimensions $30.0 \times 10.0 \times 0.2$ mm positioned at the root of the cantilever.
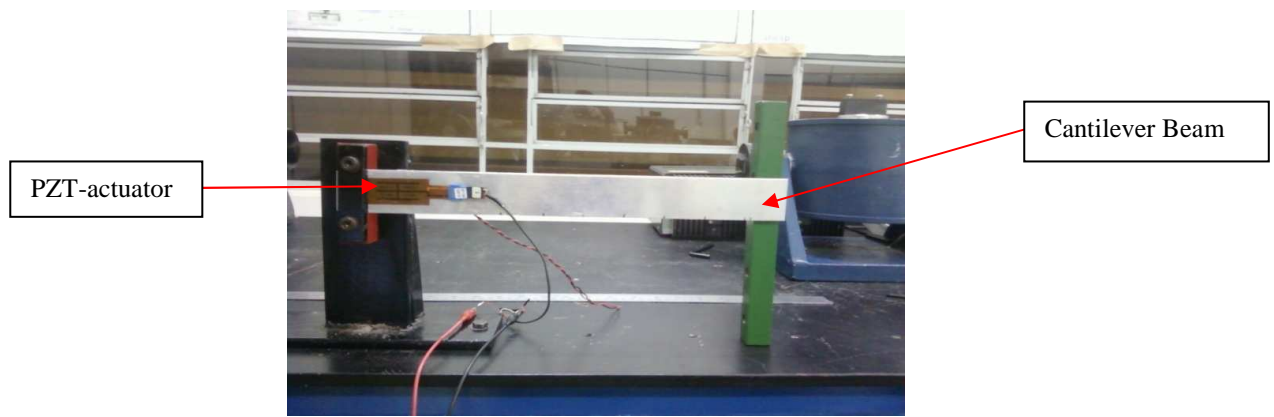

Figure 1. Cantilever beam with PZT and PVDF coupled. (The PVDF element is attached to the other side of the cantilever beam and is co-located with the actuator)

Using the method presented in section 2, the Markov parameters of the observer and the system were calculated, and consequently a state-space model of the cantilever beam was determined. As it was intended to control only the first two modes of the system, it was necessary to reduce the state-space model to a fourth order model. The Hankel norm model reduction technique (Gawronski, 1998), was used. This method allows the mapping of past inputs and future outputs through states of the system and quantifies the individual contribution of each state. The states that contribute least are discarded. The identified system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ of the reduced order model are given by

$$\mathbf{A} = \begin{bmatrix} -4.7608 & -487.3184 & -2.5197 & -7.9764 \\ 496.2083 & -4.2912 & -1.2954 & 4.8282 \\ 0 & 0 & -1.0591 & 79.9404 \\ 0 & 0 & -81.1069 & -2.2728 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.2160 \\ -0.3655 \\ -0.0346 \\ 0.2747 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} -0.4143 & -0.0730 & -0.2213 & -0.1695 \end{bmatrix}, \quad \mathbf{D} = 0$$

The measurement of the frequency response of the system (in terms of the voltage applied to the PZT actuator and the voltage measured from the PVDF patch) together with the reconstructed frequency response from the model are shown in figure 2.

From figure 2, it can be seen that the frequency response of the identified model is a reasonable match to the frequency response of the actual system for the first two modes.
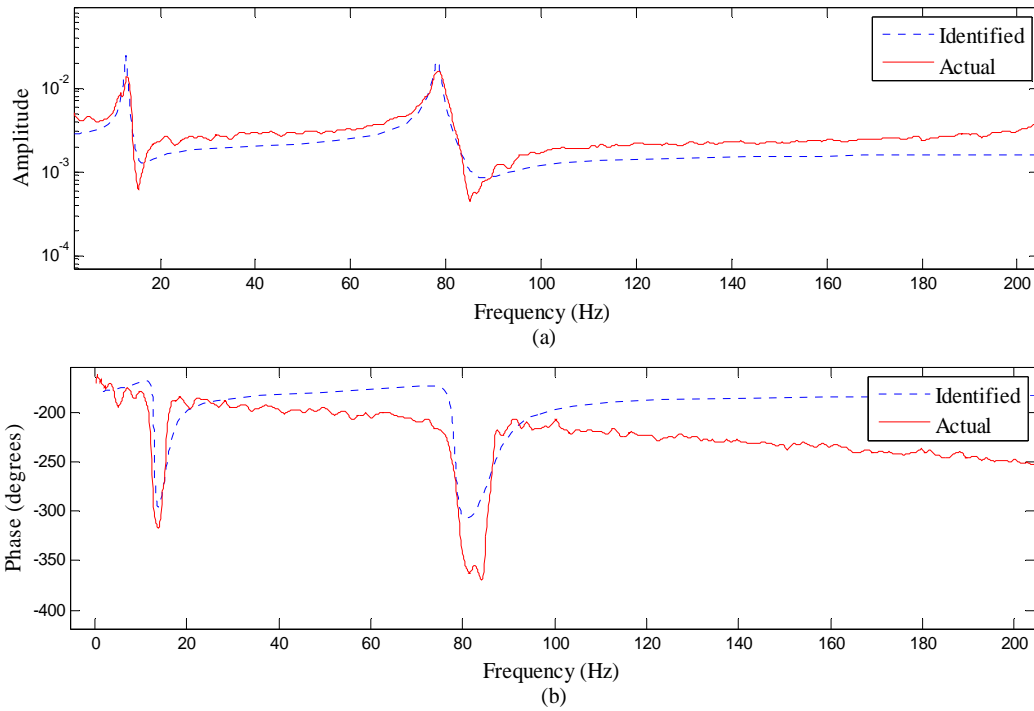
Figure 2. The measured (actual) and the reconstructed frequency response of the system (identified) from the state space model (a) magnitude, (b) phase.

3. CONTROL DESIGN AND IMPLEMENTATION

In this section, the controller design and implementation using the results of the previous section are discussed. An LQR controller was chosen as the structure and the controller are both linear. The cantilever beam shown in Fig. 1 beam was used in which the PZT patch was used as the control actuator and the PVDF patch as the sensor. The controller was implemented using dSpace and a PC. This board was connected to the control system into the computer, which returned a response to the amplifier connected to the PZT. This is shown in figure 3.

3.1. Controller Design

Consider the discrete-time state-space system description given in equation (1a,b). In this case the input vector is given by

$$\mathbf{u}(k) = -\mathbf{Kx}(k) \tag{21}$$

and $\mathbf{K}$ is the matrix of gains which are to be obtained by minimization a performance index. The quadratic performance index J with summation limits 0 to ∞ (the infinite-horizon case) which is to be minimized, is given by (Anderson *et al.,* 1989)

$$J = \sum_{k=0}^{\infty}(\mathbf{x}^{T}(k)\mathbf{Qx}(k) + \mathbf{u}^{T}(k)\mathbf{Ru}(k)) \tag{22}$$

where the matrices $\mathbf{Q} \geq 0$ and $\mathbf{R} > 0$ determine the relative importance of the state $\mathbf{x}$ and the control effort $\mathbf{u}$ respectively. They also determine the relative importance of the error and the control effort. In the practical case considered here, the matrix of gains $\mathbf{K}$ that was determined was for a finite-horizon case .

The unknown elements of the matrix $\mathbf{K}$ are determined so as to minimize the performance index, so $\mathbf{u}(k) = -\mathbf{Kx}(k)$ is optimal for any initial state $\mathbf{x}(0)$. To determine the elements of matrix $\mathbf{K}$ which minimizes the performance index, it is necessary to solve the Riccati matrix equation given by (Anderson *et al.,* 1989)

$$\mathbf{A}^{T}(\mathbf{P}-\mathbf{PB}(\mathbf{R}+\mathbf{B}^{T}\mathbf{PB})^{-1}\mathbf{B}^{T}\mathbf{P})\mathbf{A} + \mathbf{Q} - \mathbf{P} = 0 \tag{23}$$

where matrices **A** and **B** are the state and the input matrices, respectively (Skogestad *et al.,* 2007). The matrix **P** is the unique positive definite solution to Eq. (23). After determining the matrix **P**, the matrix of gains **K**, can be found from

$$\mathbf{K} = (\mathbf{R} + \mathbf{B}^T\mathbf{PA})^{-1}\mathbf{B}^T\mathbf{PA} \qquad (24)$$

For LQR control of the cantilever the matrix **Q** was chosen to be the identity matrix of order 4 and **R** was chosen to be 1. The matrix **K** was then determined to be:

$$\mathbf{K} = [-1039 \quad -604 \quad 9 \quad -20]$$

3.2. Experimental Implementation

The closed-loop block diagram of the cantilever beam under control is shown in figure 3.
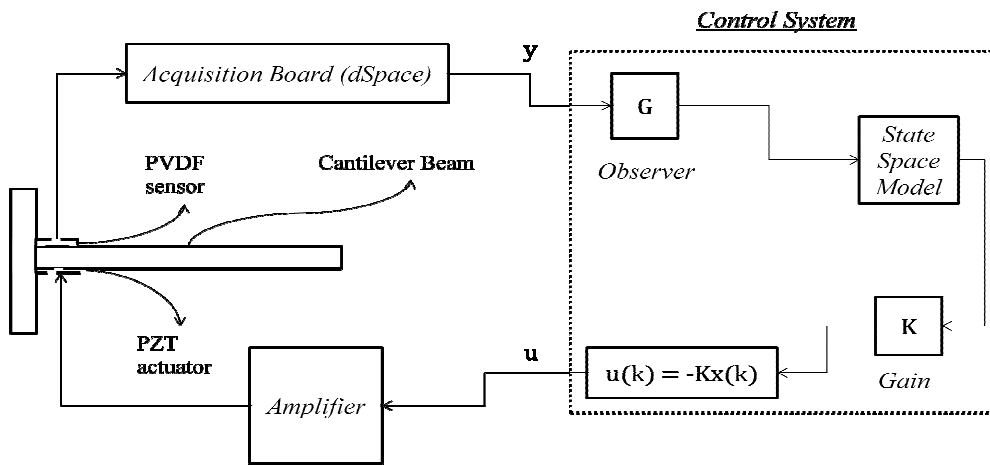

Figure 3. The closed-loop feedback system.

The beam was disturbed by displacing the tip by one centimeter and then letting it freely vibrate. Initially the controller was turned off and the results of this test can be seen in figure 4a. It can be seen that the damping in the system is quite light as it takes a long time for the vibration to decay away. Close examination of the time response showed that the beam was vibrating primarily in its fundamental mode, and the damping ratio for this mode was estimated to be 0.0682 using the method of logarithmic decrement.

The experiment was then repeated but this time with the controller turned on. The results are shown in figure 4b. It can be seen that the vibration decays away much more quickly demonstrating the effectiveness of the controller. The damping ratio in this case was found to be 0.2176. It is thus clear that the main effect of the control was to add more damping to the system.
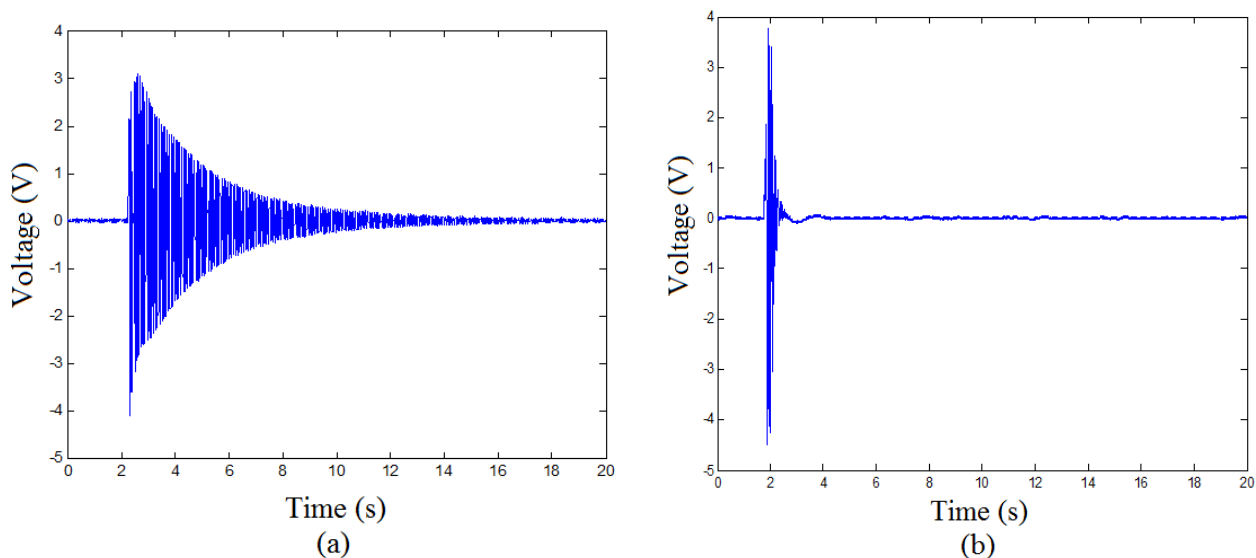
Figure 4. Vibration of the cantilever beam measured using the PVDF sensor, (a) without and (b) with control.

4. CONCLUSIONS

This paper has described an experimental study into the control of a cantilever beam. LQR control was implemented, which required a state-space model of the system. This was achieved using the ERA/OKID system identification technique with the beam being driven with white noise. The subsequent model was reduced to a fourth order model as only the first two modes of the beam were targeted for control. A displacement step input was applied to the tip of the beam and the controller, which sensed vibration using a PVDF patch at the root of the beam, and applied a control force through a collocated PZT actuator, significantly reduced the time for the transient vibration to decay away. The damping ratio of the first mode was increased from 0.0682 to 0.2176 demonstrating the efficacy of the control strategy.

5. ACKNOWLEDGEMENTS

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