DESIGN OF ELECTROTHERMOMECHANICAL (ETM) MEMS USING TOPOLOGY OPTIMIZATION METHOD CONSIDERING THE THERMAL TRANSIENT RESPONSE

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Abstract. *Electrothermomechanical (ETM) Microsystems (MEMS) are systems in micrometric scale which operate based on thermoelastic effect deformation induced by heating the structure by means of an electrical current. As a fast, damped and null (at steady state) transient response is desirable with the aim of improving ETM efficiency, it is necessary to minimize the response time of the thermal effect which is a slower phenomena among different physics involved in the ETM structure. This can be achieved by changing the ETM structural topology. Thus, in this work, the Topology Optimization Method (TOM) is applied for ETM MEMS design, taking into account transient thermal response in order to reduce their response time and to maximize their output displacement. The TOM combines optimization techniques with the Finite Element Method (FEM) to distribute material in a fixed design domain in order to extremize a cost function subjected to some inherent constraints of the problem. The modeling of ETM MEMS is obtained by solving the governing equations using the linear FEM based on four-node isoparametric elements implemented through MATLAB. Non-dependent material properties with temperature are considered in the finite element models. The electrical problem is solved by considering a steady current static analysis and in the transient state thermal problem; the model temperature distribution is a time variable function. In the elastic domain, the mass and the damping effects are neglected, thus resulting in a quasi-static problem. In the Topology Optimization formulation, a material model is based on the Solid Isotropic Microstructure with Penalization (SIMP) model combined with a sensitivity filter as a solution control to reduce mesh dependence and checkerboard problems intrinsic to the TOM. Sequential Linear Programming (SLP) is used for solving the non-linear optimization problem. Two-dimensional results are presented to illustrate the method.*

Keywords*: Topology optimization, Electrothermomechanical microsystems, Transient thermal response, Finite element modeling.*

1. INTRODUCTION

Microelectromechanical systems (MEMS) are mechanical systems, typically sensors and actuators, integrated with electronic circuits (Sigmund, 2001) whose major dimensions range from hundreds of microns to a few millimeters. Several actuation principles are used in MEMS, such as electrostatic, piezoelectric, shape memory alloy-based and electrothermomechanical (ETM) effects. ETM MEMS are microsystems that operate based on the thermoelastic effect induced by the Joule heating of the structure. They are usually fabricated using micromachining and etching processes, and optimal design techniques have been proposed (Sigmund, 2001; Sigmund, 2001a; Moulton and Ananthasuresh, 2001). Potential applications include nanotube manipulation in transmission electron microscopes (Sardan *et al*., 2008), medical instruments, micropumps, micromotors, snap-fit mechanisms (Li *et al.*, 2004), etc.

Among the three physical effects that govern the movement of ETM microsystems, i.e., the electrical, thermal and mechanical effects, the thermal effect is the slowest. This fact has been recognized in the literature for structures with major dimensions in the order of magnitude of millimeters (Rubio *et al.*, 2009) or even bigger structures (Li *et al.*, 2004), and thus proper time-transient analysis has been introduced to study and design such thermally actuated devices (Li *et al.*, 2004).

In the work by Li *et al.*, (2004), the output displacement of a thermomechanical actuator has been maximized, while taking into account time-transient effects. However, since a fast response is desirable in order to improve ETM efficiency, it is also important to minimize the response time related to the thermal effect of a thermally excited actuator. This can be achieved by changing the ETM structural topology.

Thus, the Topology Optimization Method (TOM) is applied here to reduce the response time and maximize the output displacement of an ETM microactuator, taking into account time-transient thermal response. The TOM combines optimization techniques with the FEM to distribute material in a fixed design domain, in order to maximize a cost function subjected to some constraints, representing physical restraints (i.e., TOM solves an optimization problem). Non-dependent material properties with temperature are employed in the finite element models. In the Topology Optimization formulation, the Solid Isotropic Material with Penalization (SIMP) model is used.

Sequential Linear Programming (SLP) solves the non-linear optimization problem. Furthermore, a smoothing filter is implemented as solution control, to reduce mesh dependence and checkerboard problems intrinsic to the TOM. Two-dimensional results are presented to showcase the proposed method.

This paper is organized as follows. In Section 2, the finite element formulation applied to the ETM structure is introduced. In Section 3, the TOM is discussed, the optimization problem is introduced and the computation of sensitivities, an important step of the optimization algorithm, is described. In Section 4, implementation details are provided. Preliminary results are presented in Section 5, and concluding remarks are offered in Section 6.

2. FINITE ELEMENT FORMULATION

In the modeling, it is assumed that strains are small under a plane-stress condition, the structures are two-dimensional (given their reduced thickness), the temperature distribution is a time variable function and non-dependent material properties with temperature are considered. Thus, the system is weakly coupled. In the elastic domain, the mass and the damping effects are neglected, therefore resulting in a quasi-static problem. The two-dimensional domain is discretized by means of four-node isoparametric finite elements. The FEM formulations for the electrical, thermal and elastic domains, in that order, are (Li *et al.*, 2004; Ananthasuresh, 2003; Yin and Ananthasuresh, 2002; Mankame and Ananthasuresh, 2001; Sigmund, 2001):

$$\boldsymbol{K_o}(\boldsymbol{\rho})\boldsymbol{V}(\boldsymbol{\rho})=\boldsymbol{F_o} \tag{1}$$

$$\boldsymbol{C_1}(\boldsymbol{\rho})\dot{\boldsymbol{T}}(\boldsymbol{\rho},t)+\boldsymbol{K_1}(\boldsymbol{\rho})\boldsymbol{T}(\boldsymbol{\rho},t)=\boldsymbol{F_1}(\boldsymbol{V}(\boldsymbol{\rho}),\boldsymbol{\rho}) \tag{2}$$

$$\boldsymbol{K_2}(\boldsymbol{\rho})\boldsymbol{U}(\boldsymbol{\rho},t)=\boldsymbol{F_2}(\boldsymbol{T}(\boldsymbol{\rho},t),\boldsymbol{\rho},t) \tag{3}$$

where, index 0, 1 and 2 refer to electrical, thermal and elastic problems, respectively. The total number of elements in the discretization is N, while the total number of nodes is n. $\boldsymbol{K_o}(\boldsymbol{\rho})$, $\boldsymbol{K_1}(\boldsymbol{\rho})$ and $\boldsymbol{C_1}(\boldsymbol{\rho})$ (n x n) are global electrical, thermal conductivity and heat capacity matrices, respectively, and $\boldsymbol{K_2}(\boldsymbol{\rho})$ ($2n$ x $2n$) is the global stiffness matrix. $\boldsymbol{V}$, $\boldsymbol{T}$ and $\dot{\boldsymbol{T}}$ (n x 1) are the voltage, temperature and time derivative of temperature output vectors, respectively; and $\boldsymbol{U}$ ($2n$ x 1) is the displacements output vector. $\boldsymbol{F_o}$, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are the electrical, thermal and structural load vectors, respectively. $\boldsymbol{\rho}$ is the design variables vector as defined by SIMP material model (see Section 3).

The global matrices are defined as:

$$\boldsymbol{K_o}(\boldsymbol{\rho})=\mathop{A}_{e=1}^{N}\boldsymbol{k_{0e}}(\rho_e)=\mathop{A}_{e=1}^{N}\left[\sigma_o(\rho_e)\int_{\forall_e}B_{o\,ip}^{T}B_{o_{pj}}d\forall_e\right]_e \tag{4}$$

$$\boldsymbol{C_1}(\boldsymbol{\rho})=\mathop{A}_{e=1}^{N}\boldsymbol{c_{1e}}(\rho_e)=\mathop{A}_{e=1}^{N}\left[\alpha_1(\rho_e)\int_{\forall_e}N_{o_i}N_{o_j}d\forall_e\right]_e \tag{5}$$

$$\boldsymbol{K_1}(\boldsymbol{\rho})=\mathop{A}_{e=1}^{N}\boldsymbol{k_{1e}}(\rho_e)=\mathop{A}_{e=1}^{N}\left[\left[\sigma_1(\rho_e)\int_{\forall_e}B_{o\,ip}^{T}B_{o_{pj}}d\forall_e\right]+\left[h_1(\rho_e)\int_{S_e}N_{o_i}N_{o_j}dS_e\right]+\left[\sum_{m=1}^{v}(1-\rho_m)h_1(\rho_e)\oint_{L_e}N_iN_jt_e dL_e\right]\right]_e \tag{6}$$

$$\boldsymbol{K_2}(\boldsymbol{\rho})=\mathop{A}_{e=1}^{N}\boldsymbol{k_{2e}}(\rho_e)=\mathop{A}_{e=1}^{N}\left[\int_{\forall_e}B_{2\,rp}^{T}D_{2pi}(\rho_e)B_{2is}d\forall_e\right]_e \tag{7}$$

where A is a FEM assembly operator (Hughes, 1987), index e refers to variables and properties of the finite element. As electrical and thermal domains are scalar field problems, the size elementary matrices $\boldsymbol{k_{0e}}$, $\boldsymbol{c_{1e}}$ and $\boldsymbol{k_{1e}}$ are m x m, where m is the finite element degrees of freedom (DOF). On the other hand, the displacement field in the structural problem is a vector, so the elementary matrix $\boldsymbol{k_{2e}}$ is 2m x 2m. ρ_e is the element pseudo-density function (see Section 3). $\sigma_o(\rho_e)$, $\alpha_1(\rho_e)$, $\sigma_1(\rho_e)$, $h_1(\rho_e)$ and $\boldsymbol{D_2}(\rho_e)$ are the electrical conductivity, thermal capacity, thermal conductivity, convection coefficient and stress-strain constitutive tensor of the element, respectively. $\boldsymbol{N_o}$ is a shape functions vector, $\boldsymbol{B_o}$ and $\boldsymbol{B_2}$ are voltage or temperature gradient and strain-displacement matrices, in that order. $\forall_e$, S_e and t_eL_e are the element volume, surface area and contour area being t_e element thickness. The third term on the right hand side, in Eq. (6), corresponds to convection from the side surfaces. The summation is over v neighboring elements and ρ_m is the design variable of the

m^{th} neighboring element. If $\rho_m = 1$ there is no side convection (Yin and Ananthasuresh, 2002). Considering the discretized domain as mentioned above, the global electric, thermal and elastic load vectors can be expressed as:

$$\boldsymbol{F_o} = \mathop{A}_{e=1}^{N} \boldsymbol{f_{oe}} = \mathop{A}_{e=1}^{N} \left\{ \oint_{L_e} N_{o_i}(-J^p) t_e dL_e + I_i^p \right\}_e \tag{8}$$

$$\boldsymbol{F_1}(\boldsymbol{V}(\boldsymbol{\rho}),\boldsymbol{\rho}) = \mathop{A}_{e=1}^{N} \boldsymbol{f_{1e}}\left(\boldsymbol{V_e}(\rho_e),\rho_e\right) = \mathop{A}_{e=1}^{N} \left\{ \begin{array}{c} h_1(\rho_e) T^p \int_{S_e} N_{o_i} dS_e + \sum_{m=1}^{v} (1-\rho_m) h_1(\rho_e) T^p \oint_{L_e} N_{o_i} t_e dL_e \\ + \sigma_o(\rho_e) \int_{\forall_e} N_{o_i} V_k B_{o_{kp}}^T B_{o_{pj}} V_j d\forall_e + Q_i^p \end{array} \right\}_e \tag{9}$$

$$\boldsymbol{F_2}(\boldsymbol{T}(\boldsymbol{\rho},t),\boldsymbol{\rho},t) = \mathop{A}_{e=1}^{N} \boldsymbol{f_{2e}}\left(\boldsymbol{T_e}(\rho_e,t),\rho_e,t\right) = \mathop{A}_{e=1}^{N} \left\{ \int_{\forall_e} B_{2_{rp}}^T D_{2_{pi}}(\rho_e) \alpha_{2_i} \left(N_{o_j} T_j - T^p \right) d\forall_e + \oint_{L_e} N_{2_{ri}}^T q_i^p t_e dL_e + P_r^p \right\}_e \tag{10}$$

where, J^p is prescribed current density, I^p is prescribed current nodal, T^p is prescribed temperature, Q^p is prescribed heat flux, $\boldsymbol{\alpha_2}$ is thermal expansion vector, $\boldsymbol{N_2}$ is shape functions matrix, $\boldsymbol{q^p}$ is prescribed traction force vector and $\boldsymbol{P^p}$ is prescribed nodal force.

The FEM electrical problem is solved by considering a steady current static analysis. For solving thermal transient and quasi-static structural problem, Hilber-Hughes-Taylor (HHT) α-method (Cornwell and Malkus, 1992) is implemented in order to avoid fictitious fluctuations in the time response.

3. FORMULATION OF TOPOLOGY OPTIMIZATION

The TOM is a powerful structural optimization technique which determines a constrained material distribution in a given design domain in order to fulfill predefined optimization objectives. Constraints are related to the amount of material to be used, maximum allowed stresses, etc. Thus, TOM solves an optimization problem. It combines the FEM with an optimization algorithm and allows for holes or empty regions in the structure. Pseudo-densities at each point of the domain are usually the design or optimization variables. In other words, in order to provide the best structure, design variables are governed by the function (Bendsøe and Sigmund, 2003):

$$\chi(\Omega) \begin{cases} 1, & \text{solid material} \\ 0, & \text{void} \end{cases} \tag{11}$$

This is essentially an ill-posed optimization problem, with multiple local minima. TOM regularizes the optimization problem, in the sense that it introduces relaxation (i.e., χ may assume intermediate values). It is achieved by setting an appropriate continuous material model. Then, some sort of penalization scheme, together with constraints is introduced to favor discrete solutions.

In this work, the SIMP (Solid Isotropic Material with Penalization) material model is employed. By applying the FEM, the following equations are finally obtained:

$$\begin{aligned} \sigma_{eo} &= \sigma_o(\rho_e) = (\rho_e)^{p_{\sigma_o}} \sigma_{o0} \\ \alpha_{e1} &= \alpha_1(\rho_e) = (\rho_e)^{p_{\alpha_1}} \alpha_{10} \\ \sigma_{e1} &= \sigma_1(\rho_e) = (\rho_e)^{p_{\sigma_1}} \sigma_{10} \\ h_{e1} &= h_1(\rho_e) = (\rho_e)^{p_{h_1}} h_{10} \\ D_{e2} &= D_2(\rho_e) = (\rho_e)^{p_{D_2}} D_{20} \end{aligned} \tag{12}$$

where $\rho_e \in (0, 1]$ is the relaxed pseudo-density function, and σ_{o0}, α_{10}, σ_{10}, h_{10} and D_{20} are the base material properties defined in the e^{th} finite element. Notice that ρ_e is higher than 0, which avoids singularities. The parameters $p_{\sigma o}$, $p_{\alpha 1}$, $p_{\sigma 1}$, p_{h1}, p_{D2} are the corresponding penalization coefficients (Bendsøe and Sigmund, 2003). In this work, the coefficients are arbitrarily chosen, following a heuristic criterion. For more elaborate criteria, Kim *et al.* (2010) is suggested.

For further detail on the TOM, Bendsøe and Sigmund (2003) is recommended. In the next subsections, a formulation for the optimization problem is proposed.

3.1. Design Problem Formulation

In the ETM MEMS design, kinematic and structural requirements must be considered. The kinematic requirement consists of maximizing the output displacement of a point along a certain direction for a given voltage excitation. The structural requirement is the stiffness maximization, since the object manipulated exerts a reaction force that affects the entire structure. Therefore, the optimization problem is to distribute a given amount of material in the design domain, considering the transient thermal response, in order to obtain a maximum output displacement at a certain point in a structure with stiffness *K* (see Fig. 1). This requirement is implemented using a spring of stiffness *K* which is a design control (Sigmund, 2001). Taking into account thermal transient-steady, transient time (t_f) is improved with the aim of maximize the displacement from the excitation instant until response has stabilized.

The optimization problem is defined as:

$$max_{\rho}\, F_{ETM}=\int_0^{t_f} u_{out}(\rho,t,T(\rho,t))dt \quad \textit{(Ouput displacement integral time)} \tag{13}$$

$$\text{subject to} \begin{cases} \sum_{e=1}^{N} \rho_e \forall_e \le V^* & \textit{(Material volume constraint)} \\ 0 \le \rho_{min} \le \rho \le 1 & \textit{(Design variables box constraint)} \\ \boldsymbol{K_o V=F_o} & \textit{(Electric static equilibrium)} \\ \boldsymbol{C_1 \dot{T}+K_1 T=F_1} & \textit{(Thermal transient equilibrium)} \\ \boldsymbol{K_2 U=F_2} & \textit{(Structural quasi-static equilibrium)} \end{cases} \tag{14}$$

In the previous optimization problem, volume restriction limits the amount of material in the ETM MEMS and V^* is the constraint on material volume. The first term in volume restriction is the value of the material volume obtained after optimization and depends on the pseudo-density values (ρ_e) of each point in the optimized domain. Time t_f is the interval transient state and its value must be greater than the minimum time to produce a temperature difference for the system to begin to deform and smaller than maximum time to produce plastic deformation (Li *et al.,* 2004).

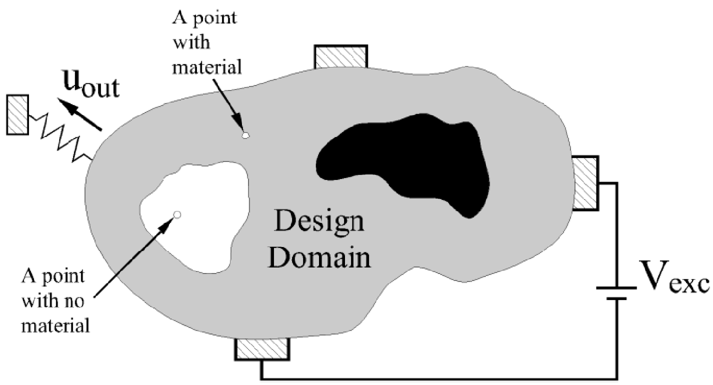


Figure 1 – Design problem in ETM MEMS.

Volume constraint is used to produce structures formed by thin bars and narrow members, reducing manufacturing time. Furthermore, it contributes to obtaining almost discrete structures, i.e., with pseudo-density values close to 0 or 1 in the case of static problems. This is because the volume constraint makes intermediate density areas "uneconomical" (Sigmund 2001). As will be seen, the restriction has the same effect on dynamic problems. This could be attributed to the fact that the dynamic problem can be considered as a weighted sum of static problems.

3.2. Sensitivity Analysis

The sensitivity analysis provides the gradients for the objective function and constraints in relation to design variables to be used solving the optimization problem (see Section 4). With the purpose of obtaining the sensitivities, direct and adjoint methods are applied (Haftka and Gürdal, 1992). In this work, the adjoint method is implemented.

Expressing the output displacement sensitivity in this way:

$$\frac{\boldsymbol{dF_{ETM}}}{\boldsymbol{d\rho}}=\int_0^{t_f}\left(\frac{\boldsymbol{du_{out}}}{\boldsymbol{d\rho}}\right)dt=\int_0^{t_f}\left(\boldsymbol{L_2^T}\frac{\boldsymbol{dU}}{\boldsymbol{d\rho}}\right)dt \tag{15}$$

where, $\boldsymbol{L_2^T}$ is a vector consisting of zeros except for the position corresponding to the degree of freedom (DOF) of the output direction in the structural problem, which value is one. Based on the discrete form of FEM equations and using the adjoint method, the sensitivity to design variables can be expressed as (Chen and Tong, 2004):

$$\frac{d\boldsymbol{F_{ETM}}}{d\boldsymbol{\rho}} = \int_0^{t_f} \left(\boldsymbol{L_2^T} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{\rho}} + \boldsymbol{L_2^T} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{T}} \frac{d\boldsymbol{T}}{d\boldsymbol{\rho}} \right) dt = \int_0^{t_f} \left(\boldsymbol{L_2^T} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{\rho}} + \boldsymbol{\Lambda_t^T R} \right) dt \tag{16}$$

being $\boldsymbol{\Lambda_t^T} = \boldsymbol{L_2^T} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{T}}$ an adjoint vector that satisfies:

$$-\boldsymbol{C_1}\dot{\boldsymbol{\Lambda}}_{\mathbf{t}} + \boldsymbol{K_1}\boldsymbol{\Lambda}_{\mathbf{t}} = \left(\boldsymbol{L_2^T} \frac{\partial \boldsymbol{U}}{\partial \boldsymbol{T}} \right)^{\mathrm{T}} \tag{17}$$

with the condition at the final time t_f:

$$\Lambda_t(t_f) = 0 \tag{18}$$

Equation (17) is calculated for each step of the integration using the HHT α-method (Cornwell and Malkus, 1992) converting the final value problem into an initial value problem for a variable substitution $\tau = t_f - t$ (Dahl *et al.* 2008). $\boldsymbol{R}$ is calculated by:

$$\boldsymbol{R} = \frac{d\boldsymbol{F_1}}{d\rho} - \frac{d\boldsymbol{C_1}}{d\rho}\dot{\boldsymbol{T}} - \frac{d\boldsymbol{K_1}}{d\rho}\boldsymbol{T} \tag{19}$$

Finally, the optimization problem sensitivity, taking into account that $\boldsymbol{F_1}$ depends on the voltage vector $\boldsymbol{V}$ and also the design variable vector $\boldsymbol{\rho}$:

$$\frac{d\boldsymbol{F_{ETM}}}{d\rho} = \int_0^{t_f} \left(\boldsymbol{\Lambda_2^T} \left(\frac{\partial \boldsymbol{F_2}}{\partial \rho} - \frac{d\boldsymbol{K_2}}{d\rho}\boldsymbol{U} \right) + \boldsymbol{\Lambda_1^T} \left(\frac{\partial \boldsymbol{F_1}}{\partial \rho} - \frac{d\boldsymbol{C_1}}{d\rho}\dot{\mathbf{T}} - \frac{d\boldsymbol{K_1}}{d\rho}\boldsymbol{T} \right) + \boldsymbol{\Lambda_o^T} \left(\frac{d\boldsymbol{F_o}}{d\rho} - \frac{d\boldsymbol{K_o}}{d\rho}\boldsymbol{V} \right) \right) dt \tag{20}$$

where:

$$\boldsymbol{\Lambda_2^T} = \boldsymbol{L_2^T K_2^{-1}} \qquad \boldsymbol{\Lambda_1^T} = \boldsymbol{\Lambda_t^T} \qquad \boldsymbol{\Lambda_o^T} = \boldsymbol{\Lambda_t^T} \frac{\partial \boldsymbol{F_1}}{\partial V} \boldsymbol{K_o^{-1}} \tag{21}$$

being $\boldsymbol{\Lambda_0}$, $\boldsymbol{\Lambda_1}$ (size n) and $\boldsymbol{\Lambda_2}$ (size 2n) adjoint vectors.

4. NUMERICAL IMPLEMENTATION

The algorithm used for the optimization process is shown in Fig. 2. Primarily, the initial domain is discretized by finite elements and designs variables (ρ_e) are defined with a uniform values guess and the FEM problem is solved.

A Sequential Linear Programming (SLP) algorithm (Haftka and Gürdal, 1992) iteratively solves the non-linear optimization problem. The objective function – Eq. (13) – at each SLP iteration is linearized, around the current design point ρ. This linearization requires the objective function sensitivities previously calculated – see Eq. (20) and (21). A Linear Programming (LP) algorithm solves the linearized problem; therefore, a new approximation is obtained. In addition, box constraints or moving limits for each design variable are applied in the linearized problem to assure that calculated solution of the original non-linear problem is a good approximation. The range of values within the moving limits is reduced if the corresponding design variable oscillates or stagnates, and it is increased otherwise. The range may be 5 to 15% of the original values. In order to avoid numerical problems or singularities, a lower bound ρ_{min} is specified as 10^{-3}. As a consequence, numerically regions with $\rho_e = \rho_{min}$ can be considered void regions.

After linear optimization, a new set of design variables ρ is obtained and updated in the design domain. The SLP iterative process is continued until a convergence criterion is achieved. For the purpose of avoiding TOM-related problems, such as checkerboard patterns and mesh dependency (Bendsøe and Sigmund, 2003) a filter has been implemented. The filter (Andreassen *et al.* 2011) modifies the sensitivities according to the amount of neighboring elements considered by a specific radius. The final result dependence on the finite element mesh refinement is minimized. The final topologies are obtained using the continuation method (Bendsøe and Sigmund, 2003) because of ETM MEMS design problem is highly non-convex. The continuation method minimizes the problem of the multiple local maximum, allowing TOM to find a solution close to the global maximum. In this method, the penalty coefficients vary increasing 0.1 by iteration until a maximum value is reached.

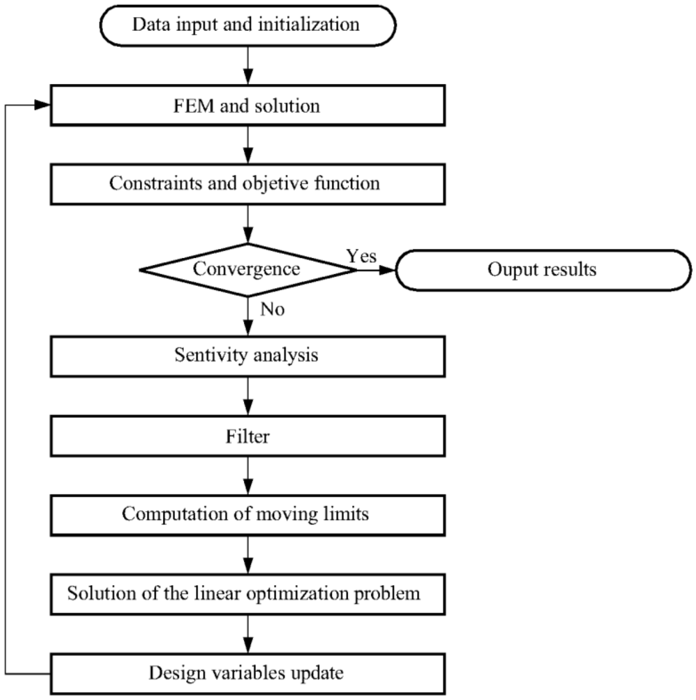


Figure 2 – Flowchart of the optimization algorithm.

5. RESULTS

In order to illustrate the TOM ability in the ETM MEMS design, an example is presented. For the example, nickel is assumed as material. Material properties and other useful data for topology optimization are shown in Tab. 1.

Table 1. Data used in the numerical example.

Description	**Value**
Electrical Conductivity (S/m)(1)	6.4×10^{6}
Thermal Conductivity (W/m K)(1)	90.7
Specific Heat (J/Kg K)(2)	456
Mass Density (Kg/m^3)(2)	8890
Young's Modulus (Pa)(1)	2×10^{11}
Poisson's Modulus(1)	0.31
Thermal Expansion Coefficient (1/K)(1)	15×10^{-6}
Excitation Voltage (V)	0.2
Environment temperature (K)(3)	300
Convection Coefficient (W/m^2 K)(4)	18.7×10^{3}
Transient Time (s)	0.1
Thickness (m)	15×10^{-5}
Volume Domain (m^3)	2.4×10^{-9}
Volume Constraint (%)	30
Design Variables Initial Guess	0.3
Stiffness Spring K (N/ m)	1×10^{5}

(1): Sigmund (2001a); (2): ASM International Handbook Committee (1990); (3): Rubio (2005); (4): Jonsmann *et al.* (1999).

The design domain and boundary conditions for a microactuator are shown in Fig. 3. The dimensions are in millimeters and environmental temperature is assumed at the mechanical supports. The design domain is discretized by 1600 finite elements. The optimization is performed in transient-state (dynamic optimization) and the result is compared with steady-state optimization (static optimization), (Rubio *et al.*, 2009; Rubio, 2005). The static formulation has been implemented by authors just for comparative purposes. The radius of the implemented filter (Andreassen *et al.*, 2011) encompasses two neighboring elements. The non-intuitive final topologies and their interpretations are shown in Fig. 4 and Fig. 5, respectively. In Fig. 6, convergence curves for objective function are shown.

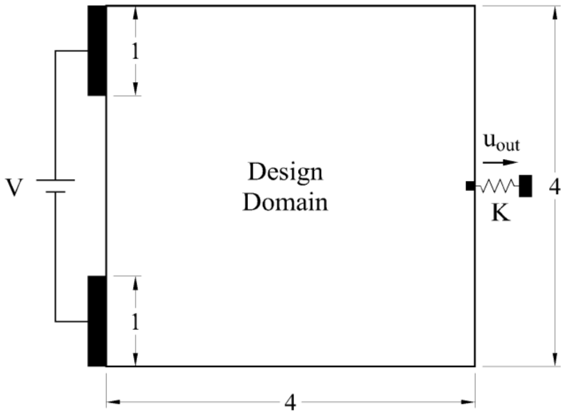
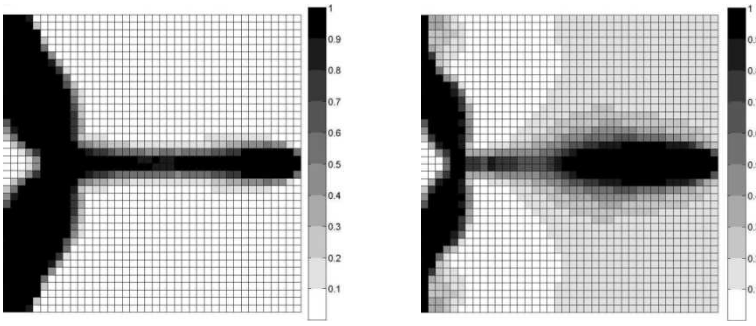


Figure 3 – Design domain and boundary conditions for the example.

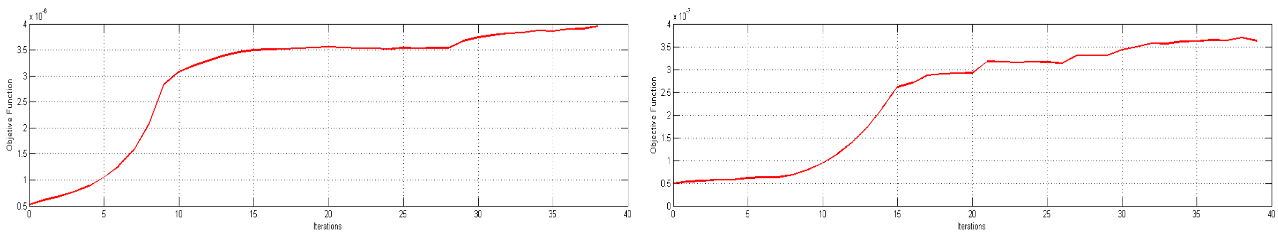


(a) (b)

Figure 4 – Final topologies: (a) Static optimization; (b) Dynamic optimization.

(a) (b)

Figure 5 – Interpreted topologies: (a) Static optimization; (b) Dynamic optimization.

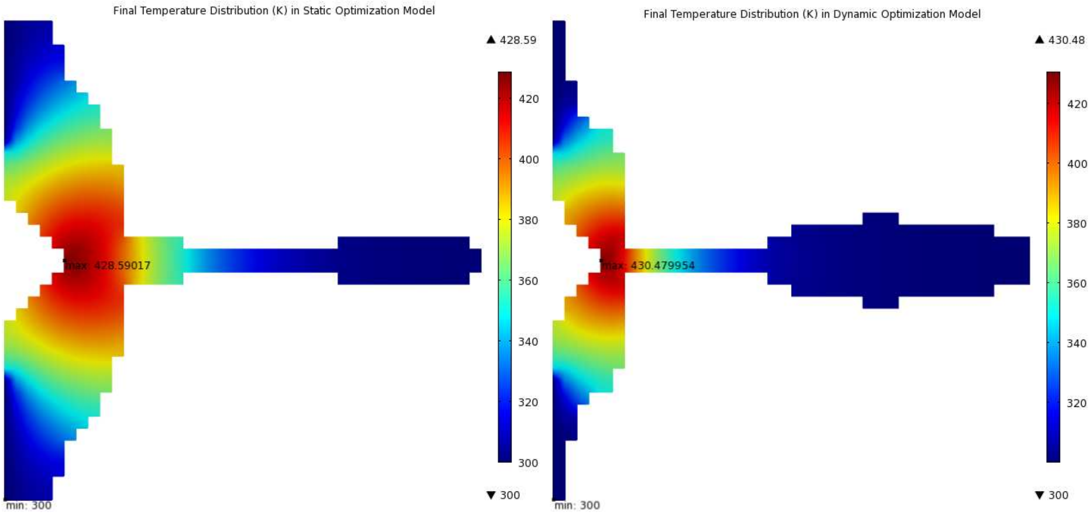


(a) (b)

Figure 6 – Convergence curves for the objective function: (a) Static optimization; (b) Dynamic optimization.

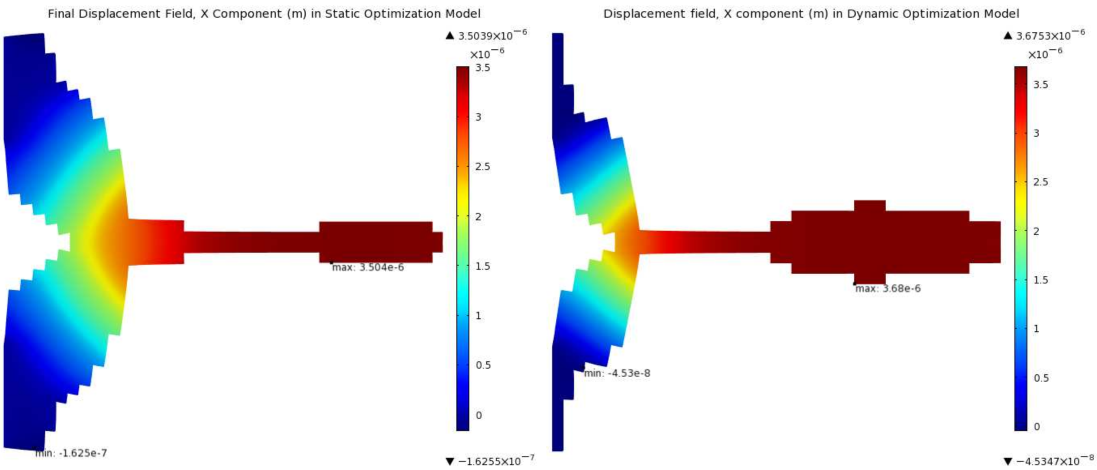
In addition, deformed topologies simulated by finite element software COMSOL Multiphysics with the temperature distribution are plotted in Fig. 7 and Fig. 8, respectively. Notice that regions close to the anchors (on the left) are active regions, in the sense that they have higher temperatures and, consequently, deformations. On the other hand, regions far from the anchors are passive. Similar behaviors were detected by Rubio *et al.* (2009).

Figure 9 shows the temporal behavior of output displacement for static and dynamically optimized microactuators. To compare their responses, they are simulated in transient analysis under the same parameters (see Tab. 1).



(a) (b)

Figure 7 – Final temperature distributions: (a) Static optimization; (b) Dynamic optimization.



(a) (b)

Figure 8 – Final displacement distributions in the X component: (a) Static optimization; (b) Dynamic optimization.

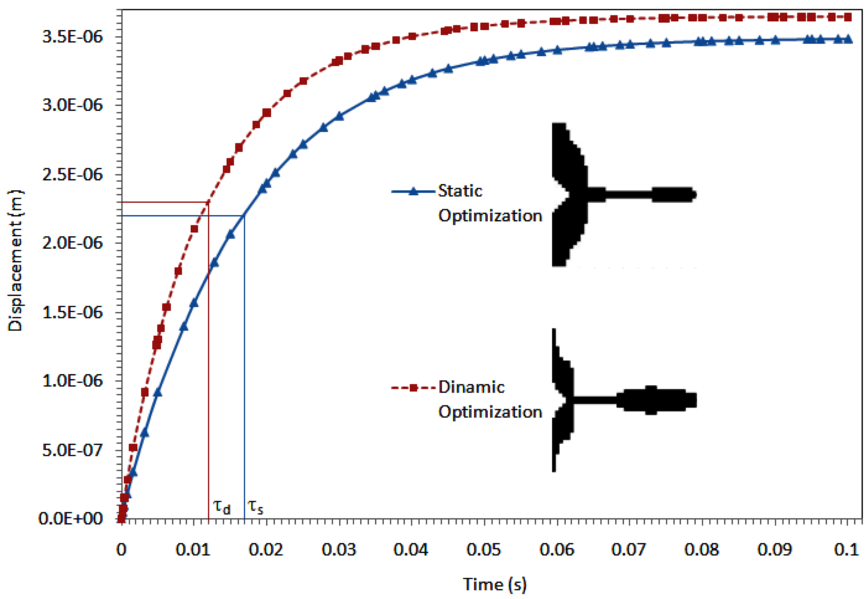


Figure 9 – Output displacement vs. time for static and dynamic optimization.

To characterize the microactuators, their time constants (τ) were calculated (see Fig. 9). τ is the time that system takes to reach 63.2% of its final displacement (response at settling time), representing the system speed. Settling time is defined as required time for the system response to reach and to remain in a range around the final value within ± 5% (Ogata, 1998). Settling time is taken as the transient time (t_f). The final optimization results are presented in Tab. 2.

Table 2. Optimization results for the numerical example.

Description	Static Optimization	Dynamic Optimization
Final Volume (m^3)	5.70×10^{-10}	4.29×10^{-10}
Final Volume Constraint Value	23.75%	17.87%
Time Constant τ (s)	1.69×10^{-2}	1.20×10^{-2}
Output Displacement (m)	3.48×10^{-6}	3.64×10^{-6}

Table 2 shows that dynamic optimization provides better results than static optimization in both the output displacement and time response. Output displacement increased by 4.6% while time constant decreased by 29.0%.

6. CONCLUSIONS

In this work, the TOM is applied to increase the response speed and to maximize the output displacement of an ETM microactuator, taking into account time-transient thermal response. The SIMP model is used and an algorithm based on SLP solves the optimization problem. Furthermore, a smoothing filter is implemented as a solution control, reducing checkerboard problems, as can be seen in Section 5. Two-dimensional results show the ability of the proposed method.

It was shown that TOM successfully improved the dynamic characteristics of the studied microsystem, while fulfilling the imposed constraints. A reduction of 29.0% in the time constant was obtained, in addition to an increase in the output displacement of 4.6%. The reduction in the time constant could be attributed to the fact that dynamic optimization uses every transient state to maximize the response gradually.

In future works, our intention is to test other thermal transient problem solvers in order to reduce the runtime of the optimization algorithm for a constant accuracy. In addition, the TOM algorithm will be applied to design new microactuators considering distinct boundary conditions, materials and domain aspect ratios.

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