STUDY OF A BISTABLE FLOW ON TWO PARALLEL CIRCULAR CYLINDERS WITH A MIXTURE MODEL APPROACH

Alexandre Vagtinski de Paula, vagtinski@mecanica.ufrgs.br
Sergio Viçosa Möller, svmoller@ufrgs.br
Programa de Pós-Graduação em Eng. Mecânica – PROMEC
Universidade Federal do Rio Grande do Sul – UFRGS
Rua Sarmento Leite, 425, Fone/Fax: +55 (51) 3308-3228
90050-170 Porto Alegre, RS, Brasil

Abstract. *Bistable flow is found in turbulent cross-flow impinging on two side-by-side circular cylinders, where the flow who emanates through the gap between them presents a flip-flopping wake characterized by a biased flow switching at irregular intervals. The study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries, like banks of tubes or rods, found in the nuclear and process industries, which are the most common geometry used in heat exchangers. To better characterize this behavior, which can represent an additional source of dynamic instabilities, this work presents a study where a finite mixture model is applied to analyze the probabilistic distribution functions (PDF) of the phenomenon. Velocity time series, obtained in an aerodynamic channel using a double hot wire anemometer probe, are used as input data to obtain the PDF of the axial and transverse velocity components. The experiments were performed with a Reynolds number of 4.36×10^4 and a pitch-to-diameter ratio of p/d=1.26, where "p" is the pitch or the distance between the centers of the cylinders and "d" the diameter. The bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the time series as observed data. The observed data is classified according to a family of PDF, and the number of clusters is considered fixed and equal to two. An expectation-maximization (EM) algorithm is applied to estimate the maximum log-likelihood function, to know what the probability distribution function is more likely to have produced the observed data. Results show that in the changes between the flow modes, the increase in the axial velocity component is accompanied by the increase of the transverse component. The velocity components PDF present the predominance of two major concentrations, but with different shapes. A value that represents the mean frequency of the changes between the flow modes, called pseudo-frequency, analyzed after decomposition of the signal through a discrete wavelet transform could be defined. An analysis of length growth of the PDF to their level of reconstruction by discrete wavelet transform (DWT) of velocity components shows a gradual loss of information on small scales. Comparisons between the probabilities of occurrences of the velocity signals together with the DWT show a similar behavior, but there is no evident correlation between the changes with time. The bivariate scenario provides a joint analysis of the time series, and shows the relationship between the PDF in the measurement plane.*

Keywords*: Bistable Flow, Hot Wires, Finite Mixture Model.*

1. INTRODUCTION

Flows around circular cylinders nearly disposed are found in several engineering applications, like shell and tube heat exchangers, pipelines, electric power transmission lines and systems for the collection and raising of petroleum deposits. The turbulent flow impinging on circular cylinders placed side-by-size presents a floppy and random phenomenon that changes the flow mode. This is the so called bistable flow, which is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, which generates two dominant Strouhal numbers, each one associated with one of the two wakes formed: the wide wake is associated with a lower Strouhal number and the narrow wake with a higher one. The study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries, as in the case of banks of tubes or rods, which are found in the nuclear and process industries, being the most common geometry used in heat exchangers.

New studies are justified since flow induced vibration and structure-fluid interactions are very dependent of the arrangement or configuration of the cylinders and since bistability can be an additional excitation mechanism on the tubes. Thus, new techniques can be applied to better understand and classify this phenomenon.

Finite mixture models are statistical tools applied in many knowledge areas to perform the PDF-estimation of an incomplete data problem. Recently the mixture of skew distributions has been found to be effective in the treatment of heterogeneous data with high asymmetry across subclasses. Through this approach, bistable phenomenon can be considered as an incomplete data problem, and to know what the probability distribution function is more likely to have produced the observed data, maximum likelihood estimation (MLE) can be applied in experimental time series, used as observed data.

Wavelet analysis is a useful tool for the study of transient turbulent signals (Indrusiak *et al.*, 2005; Olinto *et al.*, 2009), in special the switching phenomenon in two side-by-side cylinders (Alam *et al.*, 2003), and its use combined with flow visualization techniques helps in the comprehension of the phenomena studied in laboratory conditions.

2. THE BISTABLE EFFECT

According to Sumner *et al.* (1999), the cross steady flow trough circular cylinder with same diameter (d) placed side-by-side can present a wake with different modes depending on distances between its centers, called pitch (p). Different flow behaviors can be found for different pitch-to-diameter ratios p/d. For intermediate pitch ratios (1.2 < p/d < 2.0), the flow is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, as shown schematically in Fig. 1a and Fig. 1b. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the wide wake is associated with a lower frequency and the narrow wake with a higher one. The switching of the gap flow, which is biased toward the cylinder, from one side to other at irregular time intervals, is therefore known as a flip-flopping regime or bistable flow regime (Bearman and Wadcock, 1973). Figure 1 presents a link between the wakes patterns (Figs. 1a and 1b) and a velocity measurement technique, performed by the hot wire anemometry technique (Fig. 1c). The velocity signals are measured downstream the cylinders, along the tangent to their external generatrixes, where one switching mode can be observed (modes 1 and 2). Previous studies show that this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature. The transition between the asymmetric states is completely random and it is not associated with a natural frequency (Kim and Durbin, 1988). Through a dimensional analysis was observed that the mean time between the transitions is on order 10^3 times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. The authors conclude that there is no correlation between the bistable feature and the vortex shedding, due to the fact that Strouhal numbers are relatively independent from the Reynolds numbers. Wavelet analysis is a useful tool for the study of transient turbulent signals (Indrusiak *et al.*, 2005; Olinto *et al.*, 2009), and in special the switching phenomenon in two side-by-side cylinders (Alam *et al.*, 2003).

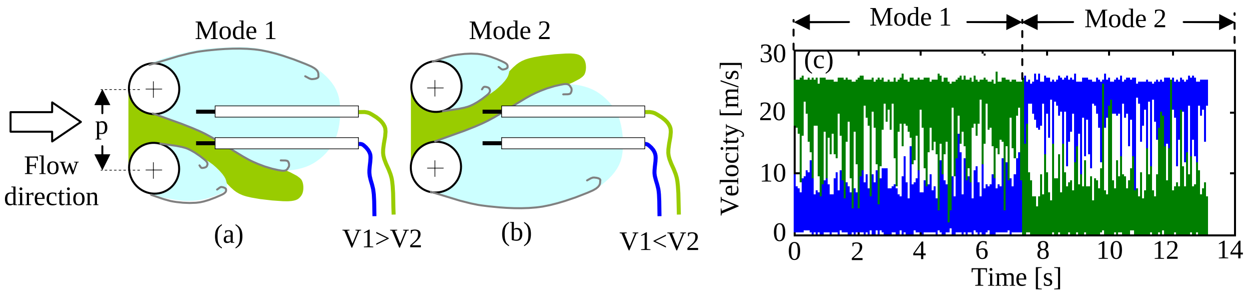

Figure 1. Bistability scheme for (a) mode 1 and (b) mode 2, and their respective characteristic hot wire anemometry signals (c).

3. OBJECTIVES

The objective of the present work is to apply the techniques cited above in experimental time series of bistable flow of the simplified geometry of two circular cylinders placed side-by-size to better comprehend the switching of the gap flow and to classify the data according to a representative PDF in a mixture model approach.

4. METHODOLOGY

A finite mixture model is applied to classify the observed data according to a family of probability density functions (PDF), where the input data are the time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel. In this context, the bistable phenomenon is considered as an incomplete data problem and PDF-estimation is performed using the observed data (time series), where the number of clusters was considered equal to two. An expectation-maximization (EM) algorithm (Dempster *et al.*, 1977) together with a Monte Carlo (MC) method is applied to estimate the maximum log-likelihood function (Wei and Tanner, 1990) according to a skew Student's PDF (Azzalini and Capitanio, 2003), to know what the probability distribution function is more likely to have produced the observed data. According to Lin (2010), finite mixture models have become more frequently used to provide a natural framework for unobserved heterogeneity in a population. Lin *et al.* (2007) comment that a finite mixture model using the Student's t distribution has been recognized as a robust extension of normal mixtures. The joint time-frequency domain analysis was made trough wavelet transform, which is applied to time varying signals, where the stationary hypothesis cannot be maintained. A discrete wavelet transform (DWT) is used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level. A continuous wavelet transform (CWT) is used to analyze the energy content of a signal through the so called spectrogram. Daubechies "db20" functions were used as bases of wavelet transforms.

4.1. Aerodynamic channel

The velocity of the flow and its fluctuations, as well as the angle of deviation of the flow, are measured by means of the DANTEC *StreamLine* constant hot-wire anemometry system, with aid of a double hot wire probe (type DANTEC 55P71 Special), with straight/slant wires (the straight wire is placed perpendicularly to the flow, and the slant forms a 45° with the axial plane). The measurements were performed aligning the probes along the tangent to the external generatrixes of a cylinder (Fig. 2b). The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1% in the test section. Upstream the test section, placed in one of the side walls, there is a Pitot tube, which measures the reference velocity of the non-perturbed flow. The data acquisition is performed by a 16-bit board (NATIONAL INSTRUMENTS 9215-A) with USB interface, which converts the analogical signal to digital series. The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), and are rigidly attached to the top wall of test section. Their extremities are covered to avoid acoustic resonance phenomenon. The probe support is positioned with a 3D table placed 200 mm downstream the end of the channel (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 4.36×10^4, computed with the tube diameters and the gap velocity. The pitch-to-diameter ratio is p/d=1.26.

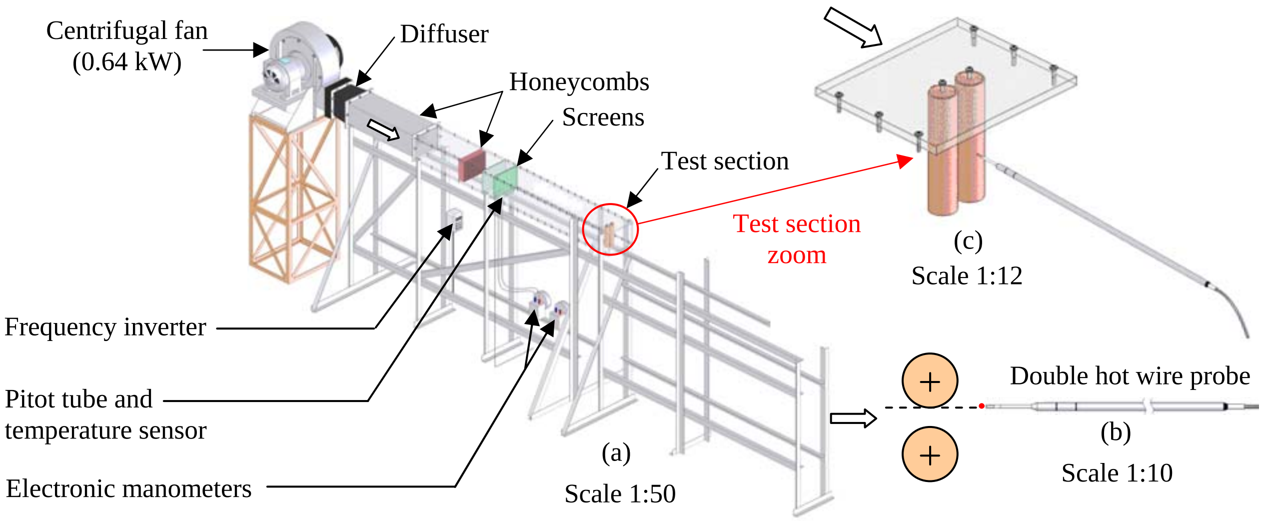

Figure 2. Schematic view of (a) the aerodynamic channel, (b) test section and (c) probe position.

4.2. Finite mixture model simulations

The experimental time series were used as input data to the finite mixture model simulations. A skew Student's t PDF was used in the estimation the maximum log-likelihood function, which is view an inference problem. A *Q* function is defined as the expected complete data log-likelihood function. An expectation-maximization (EM) algorithm is an iterative method for the computation of the maximizer of the posterior density, which has two steps: the E-step, which is the current estimative of the *Q* function; and the M-step, which is the maximizer of the *Q* function. Due to difficulties in evaluating the E-step, caused by the complexity of the target distribution, which does not admit a close-form solution to the *Q* function, the iterations may be executed by a MC process, performing independent draws of the missing values from the conditional distribution and then approximating the *Q* function. The *Q* function is maximized in the M-step over the parameters vector and a new estimative is calculated. As MC error is introduced at the E-step, the monotonicity property is lost. To assess the convergence of the algorithm is assessed through the increasing of the MC sample size with the number of iterations, and its stability was monitored with a tolerance of 10^{-3}.

5. RESULTS

Figure 3 show the time series of axial velocity, transversal velocity and the angle of deviation of the flow. Figure 3a shows several changes between two distinct velocity levels from the axial component, concerning to 3.0 m/s (wide near-wake - mode 1) and 18.6 m/s (narrow near-wake - mode 2). These changes are accompanied by the transversal component (Fig. 3b), and as the flow changes direction, from the wide near-wake to the narrow near-wake mode, the angle of incidence tends to have smaller fluctuations (Fig. 3c).

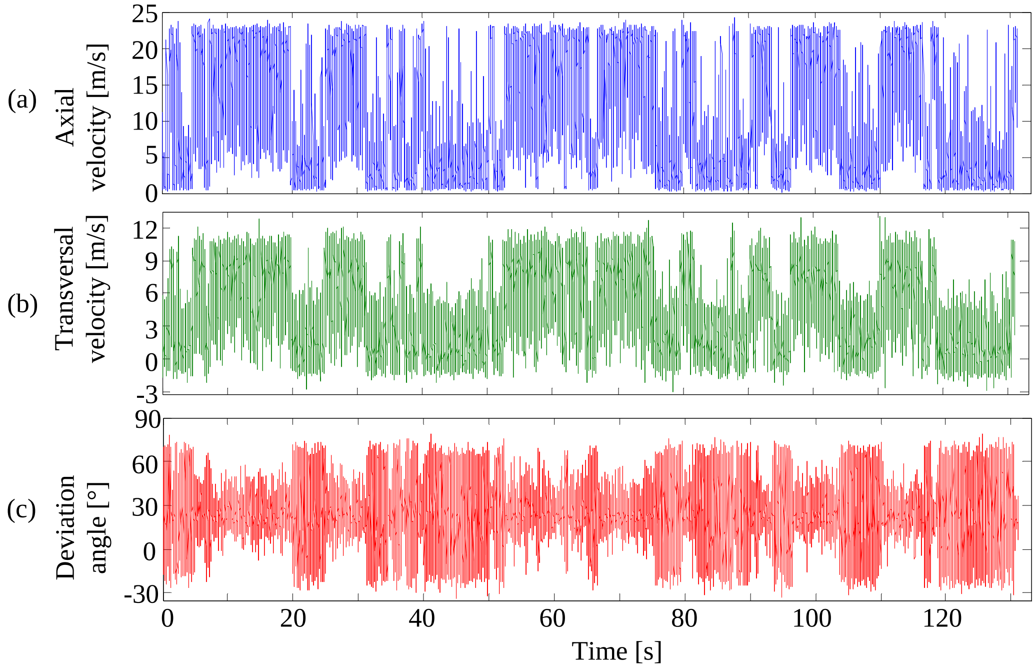

Figure 3. (a) Axial velocity signal, (b) transversal velocity signal and (c) deviation angle of the flow.

A discrete reconstruction of the velocity component via wavelet transform (DWT) for several bandwidth frequencies is presented in Fig. 4. It is possible to observe that the reconstruction level (n) which best represent the phenomenon is that of n = 9 (Fig. 4a), where the flow changes above 0.976 Hz are filtered. Figures 4b to 4d present the results for the reconstruction levels n = 10 to n = 12, respectively, where the lower frequencies do not identify all the flow changes. More details about the bandwidth frequencies of each reconstruction are presented in Tab. 1a.

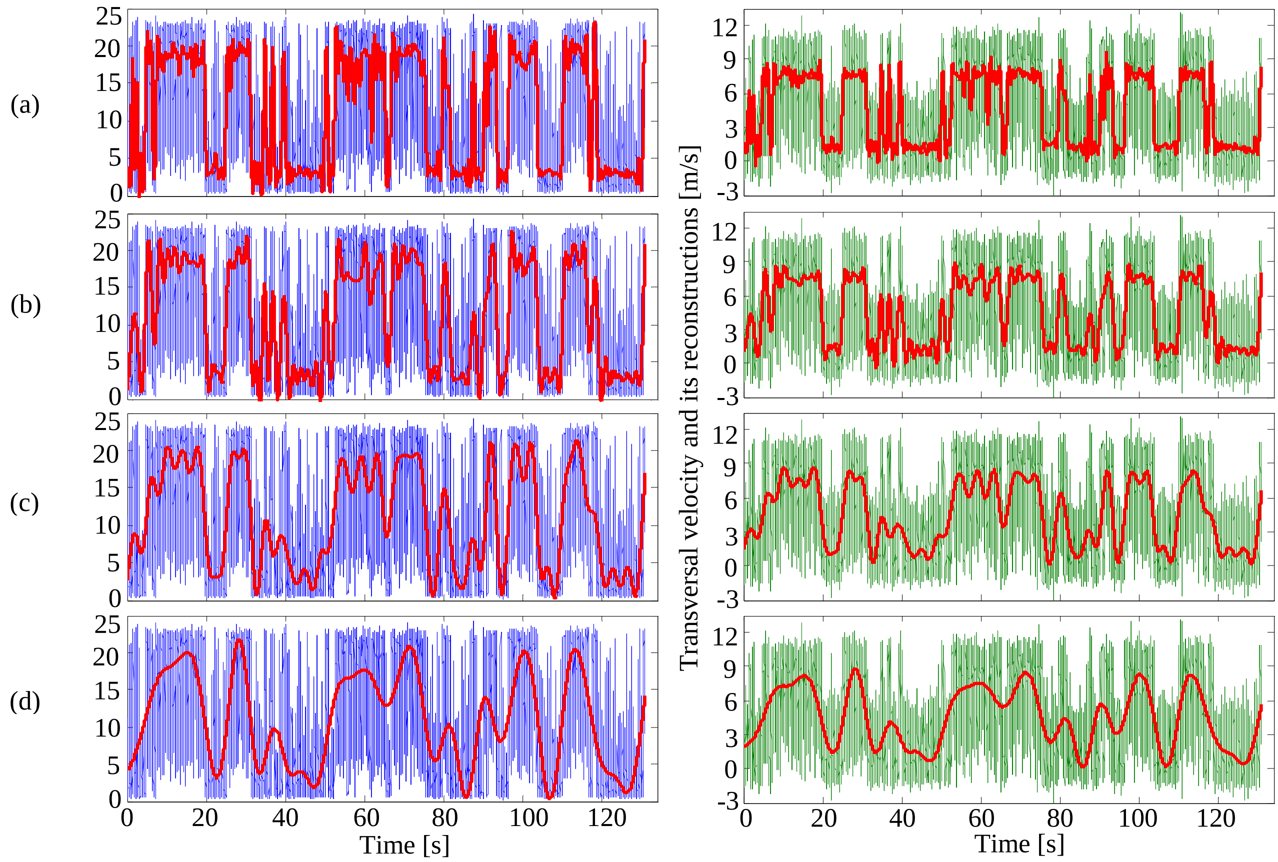

Figure 4. Velocity components and its reconstructions via DWT (red lines).
Levels of reconstruction: (a) n = 9, (b) n = 10, (c) n = 11 and (d) n = 12.

A complete discrete wavelet decomposition of the time series with level 9 is presented in Fig. 5. It is possible to observe that the mean feature of the velocity signals are the several changes between the two distinct velocity levels (Figs. 5a and 5b). The deviation angle is better expressed by its fluctuations, since the low frequencies until the higher one, and the reconstructed signal presents no special characteristic, if not the average angle of deviation of flow (Fig. 5c).

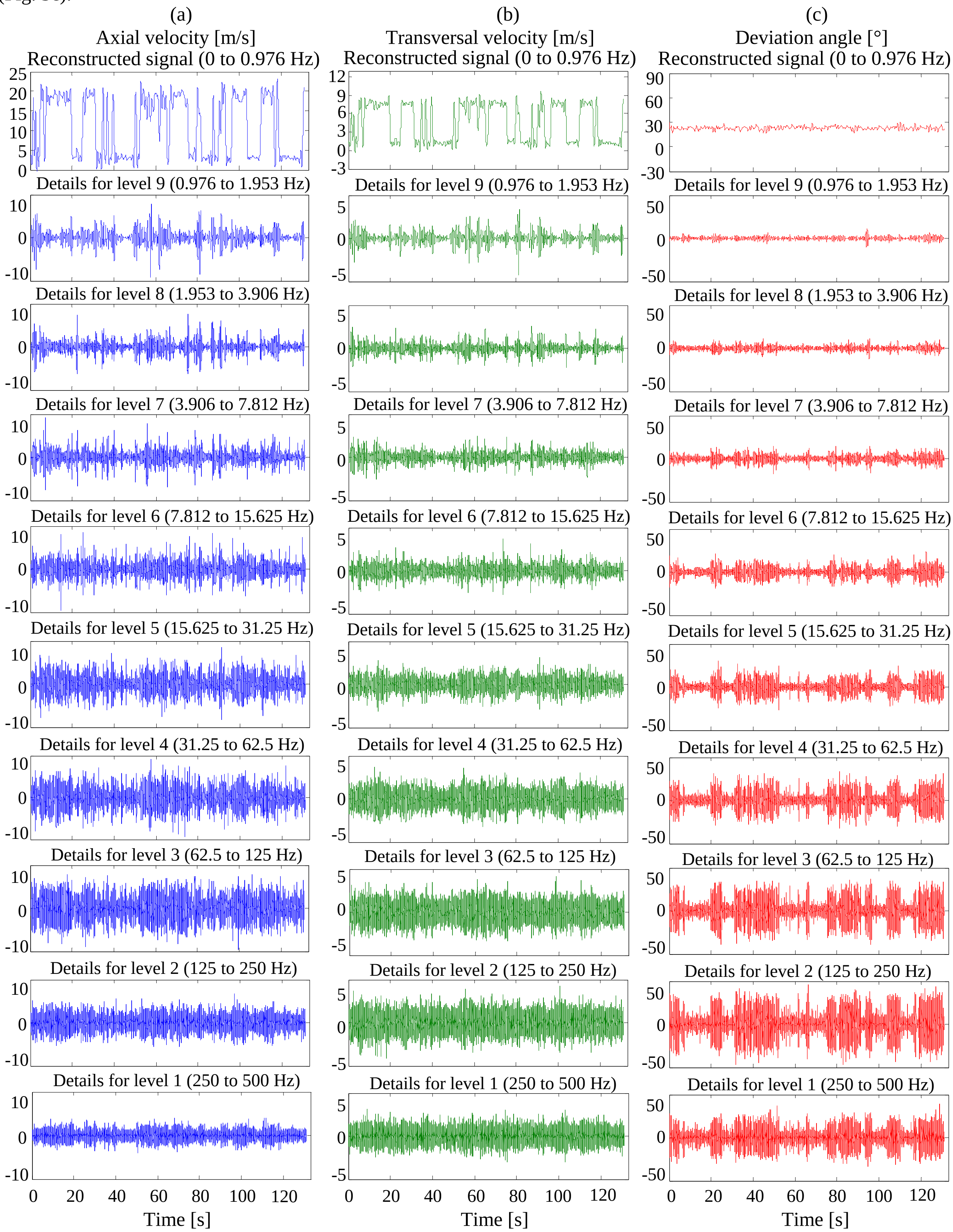

Figure 5. Reconstruction of the time series with DWT, type Db20, level 9.
(a) Axial velocity signal, (b) transversal velocity signal and (c) deviation angle of the flow.

As literature refers that bistability phenomenon has no natural period between the changes of mean velocity levels, it becomes interesting to establish some parameter to express such variation in a generally way. A possible methodology to analyze this behavior is through a pseudo-frequency concept, which refers a probable mean frequency of the changes. An automated algorithm was written to calculate the pseudo-frequency from the reconstructed velocity time series, with the same reconstruction levels of Fig. 4, and the results are presented in Tab. 1a. The best result is for n = 9, which means that above 0.976 Hz the changes of direction of the flow will not be considered as "stable", and will not be accounted to the pseudo-frequency calculation of the bistable phenomenon. For this case, the calculated pseudo-frequency value was approximately 0.15 Hz, what means that during the 131 seconds of acquisition occurred about 39 changes of the flow mode, in other words, 19.5 cycles or periods were completed, with a mean time of 6.72 seconds.

The velocity series are divided in several blocks according to Tab. 1b, with length N, overlap of 25% (ov), and its PDF are analyzed in a color scale plot, for better identification of concentrations, together with the reconstructed signals via DWT (Fig. 6). It is observed that they vary continuously between the two flow modes. The PDF of the velocity signals present similar behaviors, and the results are also best represented by the reconstruction of level n = 9. In a few words, Fig. 6 presents an analysis of length growth of the PDF to their level of reconstruction by DWT of velocity components, where results show a gradual loss of information on small scales.

Table 1. (a) Values for the pseudo-frequency determination. (b) Analysis of several blocks of PDF.

(a)					
Reconstruction level (n)	Bandwidth [Hz]	Pseudo-frequency [Hz]	Mean time of the flow changes [s]	Number of the flow changes	Number of completed cycles
9	0 to 0.976	**0.149**	6.722	39	19.5
10	0 to 0.488	0.126	7.944	33	16.5
11	0 to 0.244	0.065	15.420	17	8.5
12	0 to 0.122	0.050	20.165	13	6.5

(b)		
Length (N)	Overlap size (ov)	Numbers of PDF
1024	256	509
2048	512	253
4096	1024	125
8192	2048	61

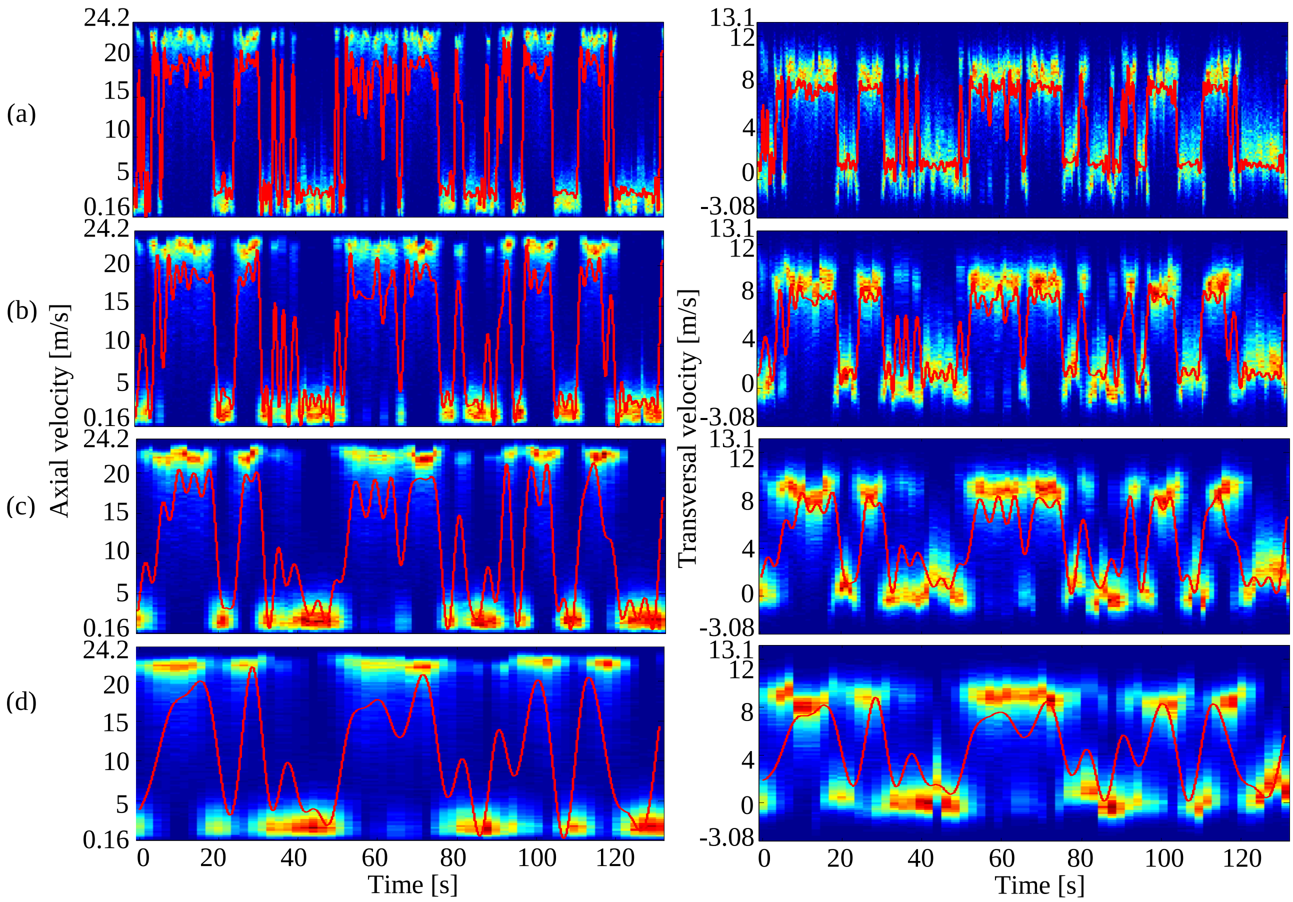

Figure 6. PDF of the velocity series (color scale) together with DWT reconstructions (red lines):
(a) N = 1024, ov = 256, n = 9; (b) N = 2048, ov = 512, n = 10;
(c) N = 4096, ov = 1024, n = 11; (d) N = 8192, ov = 2048, n = 12.

The time variation of the PDF of the velocity components for mode 1 and its respective reconstructions via DWT with n = 9 are presented in Fig. 7, where a very similar behavior is observed. However, there is no evident correlation between the changes with time.

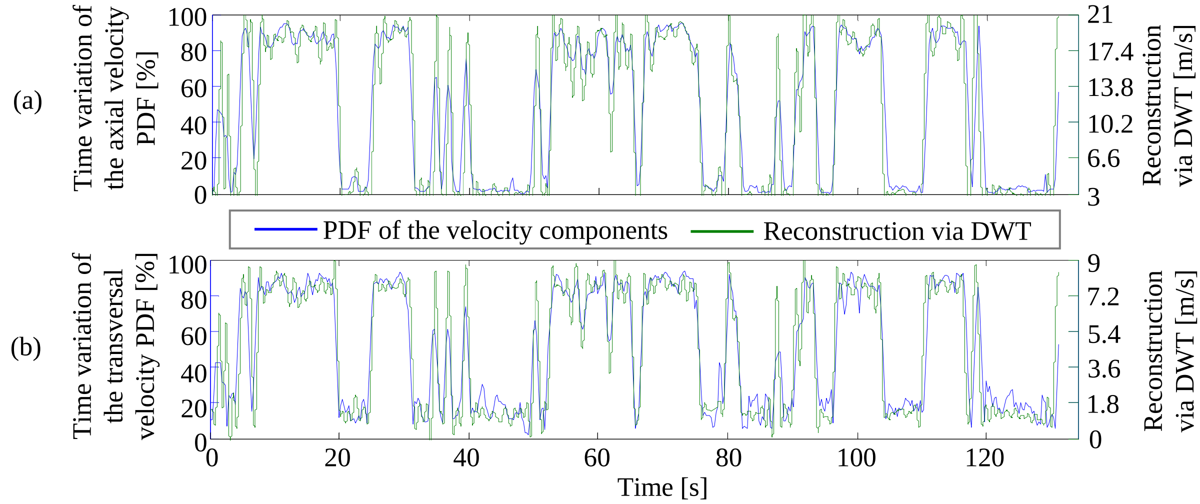

Figure 7. Time variation of the PDF of the velocity components for mode 1 and its respective reconstructions via DWT with n = 9: (a) axial velocity, (b) transversal velocity.

The PDF of whole velocity components present the predominance of two major states of energy, with different shapes (Fig. 8a and Fig. 8b). The PDF of the angle of deviation of the flow presents a concentration around a single value (Fig. 8c). The results with the finite mixture model simulation to the MLE estimation via EM algorithm is a parameter vector, with all the numerical values needed to express what skew Student's t PDF is more likely to have produced the observed data. For a visual comparison the number of intervals in each PDF variable in simulations was of 100. Figure 8 shows the results of the simulations, where is observed a goodness of fit. However, the fit for axial velocity signal presents small differences with the experimental PDF, which may be caused by the inference of same degrees of freedom for both flow modes. A search for individual degrees of freedom for each mode and also increasing the numbers of clusters could enable better fit results. The numerical results with the finite mixture model simulation are presented in Tab. 2, where there is a remarkable difference between the skewness values between the velocity signals.

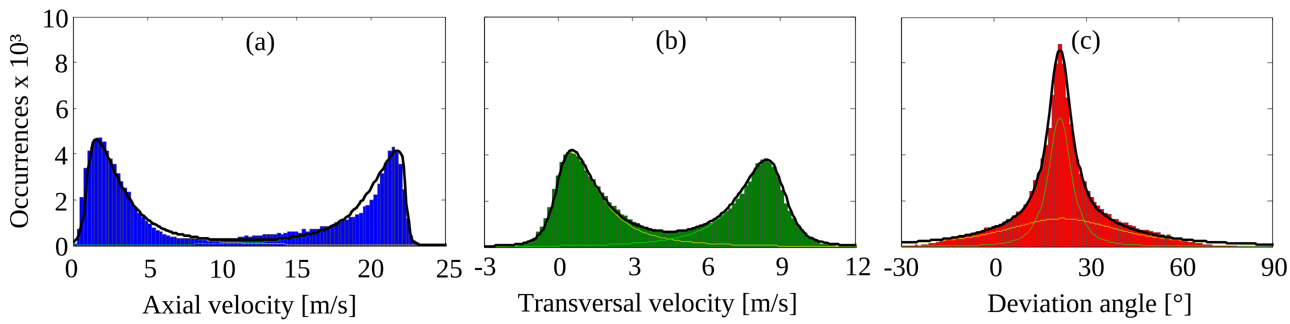

Figure 8. PDF of the time series (vertical bars) and MLE simulations via EM algorithm (solid lines): (a) axial velocity, (b) transversal velocity and (c) deviation angle of the flow.

Table 2. Numerical results of the MLE simulations of Fig. 8 via EM algorithm.

	Mean		Standard deviation		Skewness		Max. probability	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
Axial velocity [m/s]	3.02	18.67	2.18	3.94	3.33	-1.48	0.49	0.51
Transv. velocity [m/s]	1.31	7.89	1.26	1.81	0.92	-1.29	0.52	0.48
Deviation angle [°]	22.14	22.64	23.31	7.44	0.09	0.52	0.32	0.68

Flow visualizations performed in water channel for p/d=1.26 and p/d=1.6 with a Reynolds number of 7.5×10^3 are shown in Fig. 9. The tubes have 60 mm of diameter, where colored ink is injected in three different plans, according to the feature to be studied. More details of the experimental apparatus are presented in De Paula et al., 2009. From the top plan view (Fig. 9a) the formation of a large wake behind one of the tubes (blue ink) and a narrow wake behind the other tube (red ink) is observed which refers to mode 1. After the switching of the gap flow there is the formation of the mode 2 (Fig. 9b). Figure 9c show the result of the three simultaneous ink injection plans, from a top plan view, where the flow behavior is predominantly two-dimensional. The same conclusions can be drawn from frontal view visualization, with a slight elevation (Fig. 9d) and from a side view (Fig. 9e). This feature enables a bivariate analysis of the PDF in the measurement plan.

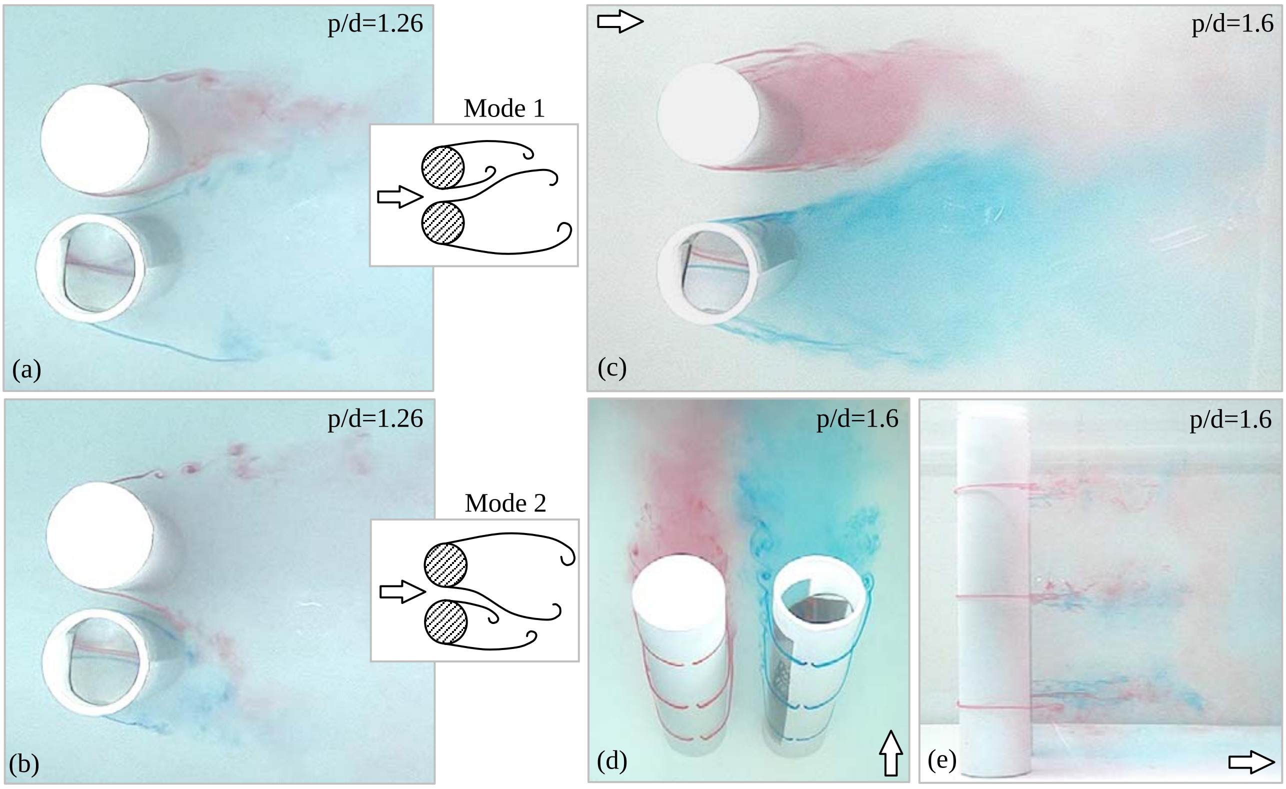

Figure 9. Flow visualizations in water channel of two side-by-side circular cylinders with p/d=1.26 and p/d=1.6. Top plan view: (a), (b) and (c). (d) Frontal view, with a slight elevation. (e) Side view. Re = 7.5×10^3.

Figure 10a presents a plot of the ordered pair of data, with both axial and transversal velocity components, where the points are dispersed over a large area. The bivariate velocity PDF is shown in Fig. 10b, and presents two prominences. The higher velocity mode seems to be present in larger quantity in this case.

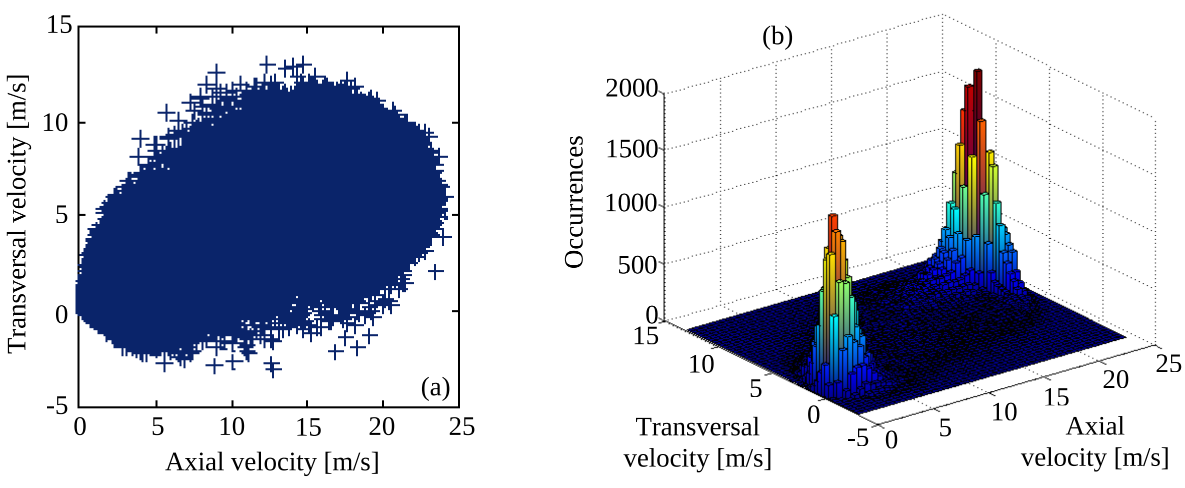

Figure 10. (a) Ordered pair of data points of the flow velocity components. (b) Bivariate velocity PDF.

Figure 11 presents the reconstructions performed with DWT for levels 5 to 10 performed for joint axial and transversal velocity components, and show the path or temporal trajectory of the flow in the measurement plan. As higher is the reconstruction level, the less is the data correlation in the plan. This feature can be correlated to the space phase concept, where a particular state of a dynamical system can be represented as a point in this phase space. As the time varies, a path is formed in the phase space.

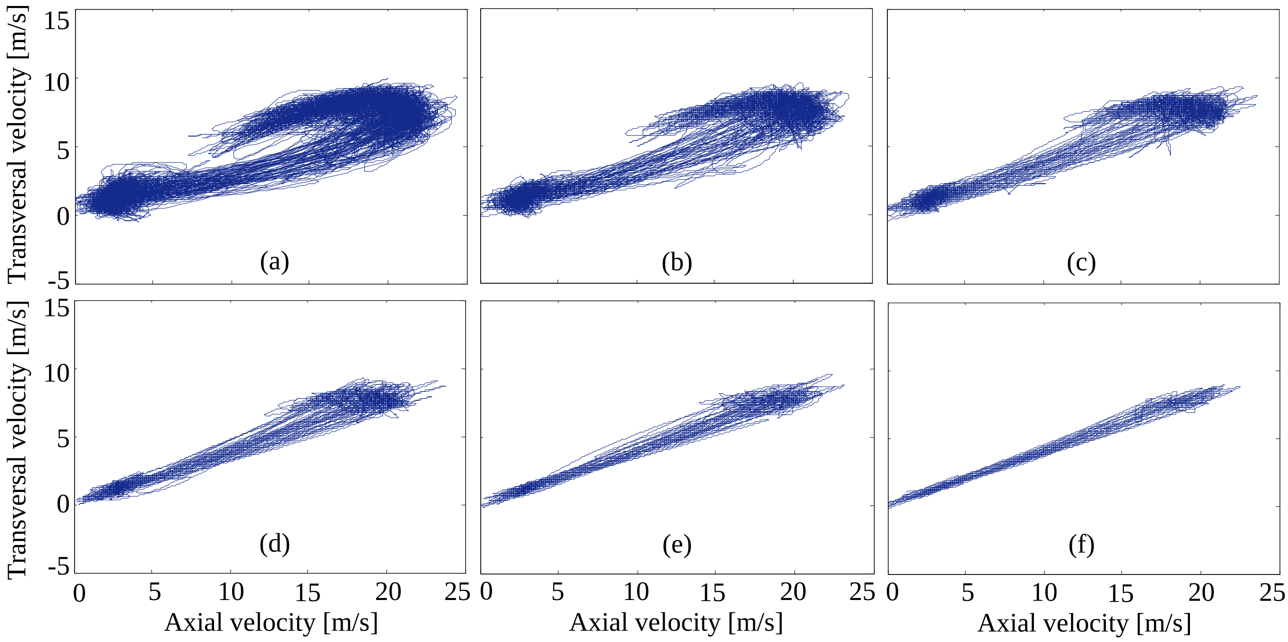

Figure 11. DWT for some reconstruction levels of the bivariate case.
Reconstruction levels: (a) n = 5, (b) n = 6, (c) n = 7, (d) n = 8, (e) n = 9, (f) n = 10.

A double-well energy model (DWEM) is used to succinctly describe the bistable behavior of the system (velocity signals). Two wells of energy, related to the wide and narrow flow modes are separated by a barrier. The behavior of the time series is well expressed by this model, which has two minima (M1 and M2) and a point of maximum, or transition state (TS), as shown in Figs. 12a 12b and 12c. The model is based on the probability distribution function of the problem, normalized by its maximum value. For the univariate case the results are presented in Figs. 12d and 12e, for the axial and transversal velocity components, respectively. For the bivariate case the results are presented in Fig. 12f, where an energy transition level (in orange color) is formed, which means that when the flow leaves one of the stable wells, it reaches the transition level and can move to the other well or back to the same again. The higher velocity mode presents a high nonsymmetrical shape (in cross-PDF terms), what suggests the presence of more than two flow modes, as stated by Alam at al., 2003, who found that there was an intermediate flow of short duration in which the gap flow was oriented parallel to the free-stream flow.

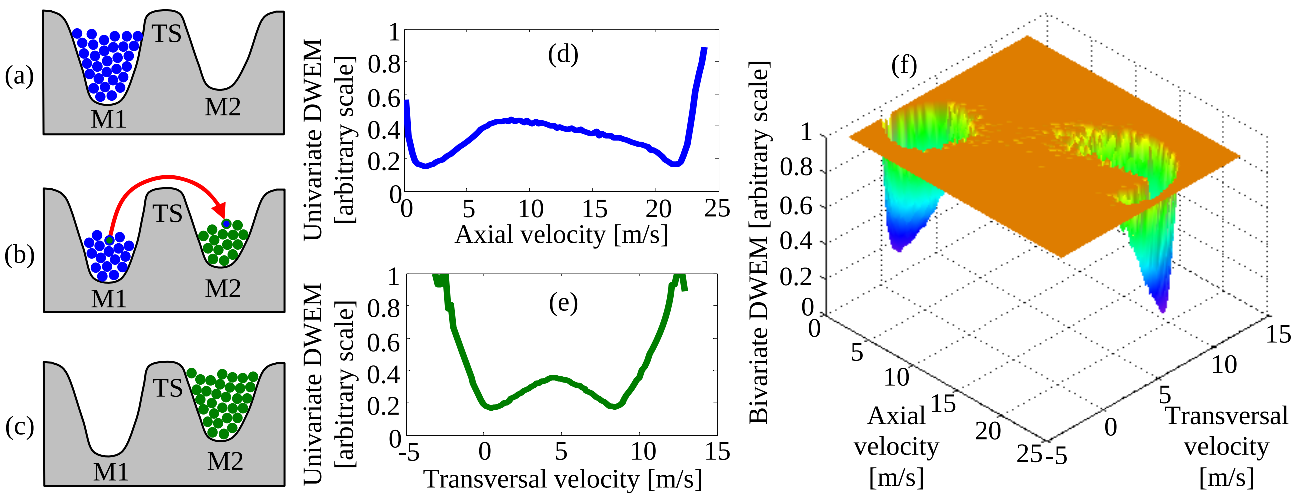

Figure 12. (a - c) Schematic transition between the double-well energy models. (d – e)) Univariate double-well energy model for the axial and transversal velocity components, respectively. (f) Bivariate double-well energy model.

6. CONCLUSIONS

This work presents a study about the bistable phenomenon which occurs in turbulent flows impinging on two side-by-side circular cylinders. With the purpose of knowing what the probability distribution function is more likely to have produced the observed data, a finite mixture model tool is applied, where an expectation-maximization algorithm performs the maximum likelihood estimation according to a skew Student's t PDF with aid of a Monte Carlo method. Results with the hot wire anemometry technique show that in the changes between the flow modes the increase in the axial velocity component is accompanied by the increased of the transverse component, and their PDF present the predominance of two major states of energy, with different shapes. The angle of deviation of the flow tends to have smaller fluctuations when the flow direction changes from the wide near-wake to the narrow near-wake, and their PDF presents a concentration around a single value. A value that represents the probable mean frequency of the changes between the flow modes, called pseudo-frequency, analyzed after decomposition of the signal through a discrete wavelet transform could be defined. Comparisons between the probabilities of occurrences of the velocity signals together with the discrete wavelet transform show a similar behavior, but there is no evident correlation between the changes with time. This analysis could be possible through the use of wavelet transforms, which are valuable tools for the analysis of transient turbulent signals. Results with the finite mixture model are useful to determine the numerical values of the shape of the both PDF modes and the results show a goodness of fit. The numerical results of the simulations present a remarkable difference between the skewness values between the velocity signals. Flow visualizations show that the flow behavior is predominantly two-dimensional. An energy model was adopted to describe the bistable behavior of the velocity components, where the higher velocity mode, which seems to be present in larger quantity than the lower one, has a high nonsymmetrical shape, what suggests the presence of more than two flow modes. New simulations are in progress to search for individual degrees of freedom for each flow mode and increasing the numbers of clusters for the univariate case, and to perform the bivariate fit of the phenomenon.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Alam, M.M., Moriya, M. and Sakamoto, H., 2003, "Aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon", Journal of Fluids and Structures, Vol. 18, pp. 325–346.

Azzalini, A. and Capitanio, A., 2003, "Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution", Journal of the Royal Statistical Society, Series B, Vol. 65, Part 2, pp. 367–389.

Bearman, P.W. and Wadcock, A.J., 1973, "The interaction between a pair of circular cylinders normal to a stream", Journal of Fluid Mechanics, Vol. 61, Part 3, pp. 499-511.

De Paula, A.V., Endres, L.A.M. and Möller, S.V., 2009, "Some features of the turbulent flow in tube banks of triangular arrangement", Proceedings of the 20th International Conference on Structural Mechanics in Reactor Technology, Espoo, Finland.

Dempster, A.P., Laird, N.M. and Rubin, D.B., 1977, "Maximum likelihood from incomplete data via the EM algorithm", Journal of the Royal Statistical Society, Series B, Vol. 39, No 1, pp. 1–38.

Indrusiak, M.L.S., Goulart, J.V., Olinto, C.R. and Möller, S.V., 2005, "Wavelet time–frequency analysis of accelerating and decelerating flows in a tube bank", Nuclear Engineering and Design , Vol. 235, pp. 1875–1887.

Kim, H.J. and Durbin, P.A., 1988, "Investigation of the flow between a pair of circular cylinders in the flopping regime", Journal of Fluid Mechanics, Vol. 196, pp. 431-448.

Lin, T.I., 2010, "Robust mixture modeling using multivariate skew t distributions", Statistics and Computing, Vol. 20, pp. 343-356.

Lin, T.I., Lee, J.C. and Hsieh, W.J., 2007, "Robust mixture modeling using the skew t distribution", Statistics and Computing, Vol. 17, pp. 81-92.

Olinto, C.R., Indrusiak, M.L.S., Endres, L.A.M. and Möller, S.V., 2009, "Experimental study of the characteristics of the flow in the first rows of tube banks", Nuclear Engineering and Design, Vol. 239, pp. 2022–2034.

Sumner, D., Wong, S.S.T., Price, S.J. and Païdoussis, M.P., 1999, "Fluid behavior of side-by-side circular cylinders in steady cross-flow", Journal of Fluids and Structures, Vol. 13, pp. 309-338.

Wei, G.C.G. and Tanner, M. A., 1990, "A Monte Carlo implementation of the EM algorithm and the Poor Man's Data Augmentation Algorithms", Journal of the Statistical Association, Vol. 85, No. 411, pp. 699-704.

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