AN APPROACH FOR THE GLOBAL SIZING AND GEOMETRY TRUSS OPTIMIZATION

André Jacomel Torii, ajtorii@hotmail.com
Programa de Pós-Graduação em Métodos Numéricos em Engenharia (PPGMNE), Universidade Federal do Paraná, Curitiba, Brazil.

Rafael Holdorf Lopez
Laboratoire de Mécanique de Rouen, Institut National des Sciences Appliquées (INSA) de Rouen, France.

Marco Antônio Luersen
Laboratório de Mecânica Estrutural (LaMEs), Centro de Inovação Tecnológica da UTFPR (CITEC), Universidade Tecnológica Federal do Paraná (UTFPR), Curitiba, Brazil.

Abstract. *This manuscript describes an approach for the global optimization of truss sizing and geometry that is based on a probabilistic restart procedure coupled with a local gradient-based search method. The resulting algorithm is able to guarantee local optimality and to asymptotically converge to the global optimum. As a result a set of local optima (eventually containing the global one) is obtained. The optimization problem searches for a truss structure of minimum volume, subject to stress constraints. The design variables are the bars cross-section areas and some nodal coordinates. To evaluate the proposed technique, three numerical examples are presented and the results are discussed.*

Keywords: *global optimization; geometry optimization; size optimization; stress constraints; truss optimization.*

1. INTRODUCTION

The truss sizing optimization problem (where the member's cross-sections areas are taken as design variable) can be stated, in many cases, as a Linear Programming (LP) problem (Hemp, 1973; Pedersen, 1970; Pedersen, 1993). One of the interesting characteristics of LP problems is that, when they are feasible, they always have a single global optimum or a convex set of local optima that are all global optima (Nocedal and Wright, 1999). That is, in the case of sizing optimization problems that are stated as LP problems, every optimum solution found is indeed a global one.

The truss geometry optimization problem (where the nodal coordinates are taken as design variables) is, instead, non linear by its nature, and therefore it cannot be stated as a LP problem. Consequently, one cannot know in advance if the problem being studied has a single global optimum or several local optima. In fact, many authors already pointed out that in many relevant cases several local optima may exist, which are not the global optimum (Achtziger, 1997; Achtziger and Stolpe, 2007; Rozvany, 1997; Torii et al., 2011).

In this context, there is an increasing effort on applying global optimization procedures to the problem of simultaneous optimization of truss geometry and sizing (Achtziger and Stolpe, 2007; Dominguez et al., 2006; Luh and Lin, 2008; Rahami et al., 2009; Schutte and Groenwold, 2003; Zheng et al., 2003; Torii et al., 2011). Most metaheuristics (e.g. Genetic Algorithms and Simulated Annealing) have the ability to search for global optima, and consequently, these techniques have been applied extensively to the problem studied here. However, these techniques do not guarantee local optimality (Arora, 2004) and require, in general, a large amount of calculations (Arora, 2004). Consequently, the solutions given by metaheuristics frequently present some kind of residual, such as bars that clearly do not compose an optimum solution.

Gradient based algorithms (e.g. Sequential Quadratic Programming and Interior Point Methods) were also successfully applied to the problem being addressed (Achtziger, 1997; Achtziger, 2006; Achtziger, 2007; Kocvara and Zowe, 1996; Torii and Biondini, 2009, Torii et al., 2011). These techniques are efficient for local searches and local optimality can be guarantee (Arora, 2004; Nocedal and Wright, 1999). That is, the solutions given by this approach do not present residuals and, in most cases, require fewer calculations than most metaheuristics. However, such techniques are not global optimization algorithms, and consequently may give poor results for problems that present several local optima.

This paper presents a local-global strategy for the simultaneous geometry and sizing optimization of truss structures. The optimization problem searches for a structure of minimum volume, subject to stress constraints. Local search is performed by gradient based techniques and thus local optimality is guarantee. In order to find the global minimum, the local search is made global by a probabilistic restart procedure presented by Luersen and Le Riche (2004). In this approach, a spatial probability of starting a local search is built based on past searches. As a result, when the optimization algorithm ends a list with several local optima (eventually the global optimum) is obtained. It should be pointed out that the restart procedure is not purely a random one, but is based on information obtained from previous results. The optimization problem solved here is described in details in the forthcoming paper by Torii et al. (2011), where probabilistic aspects are also taken into account.

2. FORMULATION OF THE OPTIMIZATION PROBLEM

The optimization problem is posed as the minimization of the volume of the structure subject to stress constraints by taking the nodal coordinates and cross section areas as design variables (Torii et al., 2011):

Find: **x** and **A**

that gives

$$\min V(\mathbf{x},\mathbf{A}) = \mathbf{A}^T\mathbf{L}(\mathbf{x}), \quad (1)$$

subject to

$$g_j \quad = +\sigma_j - \sigma_t \le 0 \quad (j = 1,2,...,m), \quad (2)$$

$$g_{j+m} = -\sigma_j + \sigma_c \le 0 \quad (j = 1,2,...,m), \quad (3)$$

where V is the volume of the structure, **x** is the vector of nodal coordinates, **A** is the vector of member areas, **L** is the vector of member lengths, g_j are stress constraints, σ_j is the stress on member j, σ_t is the yielding stress in tension, σ_c is the yielding stress in compression and m is the number of members subjected to stress constraints. In this paper, buckling constraints are not considered.

For convenience, the design variables **A** and **x** can be grouped into a single design vector **X**, and the constraints from Eq. (2) and Eq. (3) can be grouped into a single vector of constraints **g**. In this way, the previous problem is rewritten as follows:

Find: **X**

that gives

$$\min V(\mathbf{X}) = \mathbf{A}^T\mathbf{L}, \quad (4)$$

subject to

$$\mathbf{g} \le \mathbf{0}, \quad (5)$$

where **g** is a vector with $2m$ components since there are two constraints defined for each bar of the structure. When the structure is subject to more than one loading condition than we have one vector constraint as given by Eq. (5) for each loading condition. This problem is discussed in details by Torii et al. (2011).

Bounds on design variables are defined as shown in Fig. 1, by prescribing how far each node can be moved from its original position. Sensitivity analysis can then be carried out using some finite difference scheme or the adjoint method (Torii et al., 2011).

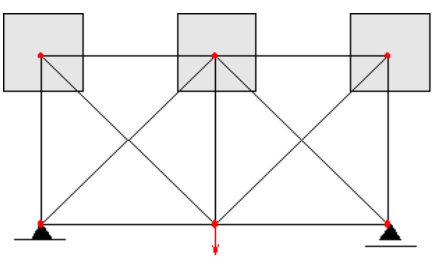


Figure 1: Bounds on nodal coordinates defined locally for each node.

3. GLOBAL OPTIMIZATION BY MENAS OF A LOCAL-RESTART STRATEGY

As already mentioned in the introduction of this paper, the problem of geometry and sizing optimization of truss structures may present local minima that are not global minima. Under this condition, deterministic optimization algorithms such as gradient methods, Newton methods or sequential simplex methods, may not converge to the global minimum of the problem. Then, the use of a global optimization algorithm is required.

In this framework, stochastic or hybrid stochastic/deterministic methods are often used. Well known examples of the former are: pure random search, genetic algorithm, and simulated annealing. Among these methods, the simplest approach is furnished by the pure random search, where a trial point is randomly generated at each iteration. It is accepted or rejected according to its performance: accepted if better than the current design, rejected otherwise. This simple procedure leads to a very high computational cost and several classes of global optimization algorithms have been developed in order to increase the efficiency of the search. One of them are the hybrid stochastic/deterministic methods where a local optimizer, such as the deterministic methods cited above, is combined with a global optimizer. For instance, when working with regular continuous objective functions, local optimizers can be turned into asymptotically global ones by restarting the search from a random initial point (Solis and Wets, 1981).

Here we use a local-global optimization strategy where the restart procedure uses an adaptive probability density function constructed using the memory of past local searches. The local search is performed using Sequential Quadratic Programming (SQP) (Nocedal and Wright, 1999). The search is then turned into an asymptotically global one applying the probabilistic restart procedure proposed by Luersen and Le Riche (2004).

Consider that the probability of having sampled a point $\mathbf{X}$ is described by a Gaussian-Parzen-window approach (Duda et al., 2001):

$$P(\mathbf{X}) = \frac{1}{N}\sum_{i=1}^{N} p_i(\mathbf{X}), \tag{6}$$

where N is the number of points $\mathbf{S}_{(i)}$ already sampled. Such points come from the memory kept from the previous local searches, being, in the present version of the algorithm, all the starting points and local optima already found. $p_i(\mathbf{X})$ is the normal multidimensional probability density function given by:

$$p_i(\mathbf{X}) = \frac{1}{(2\Pi)^{\frac{n}{2}} \det(\mathbf{\Sigma})^{\frac{1}{2}}} \times \exp\left(-\frac{1}{2}\left(\mathbf{X}-\mathbf{S}_{(i)}\right)^{\mathrm{T}} \mathbf{\Sigma}^{-1}\left(\mathbf{X}-\mathbf{S}_{(i)}\right)\right), \tag{7}$$

where n is the problem dimension (number of variables) and $\mathbf{\Sigma}$ is the covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_n^2 \end{bmatrix}, \tag{8}$$

and variances are estimated by the relation:

$$\sigma_n^2 = \beta\left(X_j^{\max} - X_j^{\min}\right)^2, \tag{9}$$

where β is a positive parameter that controls the length of the Gaussians, and $X_j^{\max}$ and $X_j^{\min}$ are the bounds of the jth design variable. To keep the method simple, such variances are kept constant during the optimization. At the first local search, the initial point $\mathbf{X_0}$ is given by the user. Then, after every local minimum is found, M points are randomly sampled ($\mathbf{X}_1$, $\mathbf{X}_2$, … , $\mathbf{X}_M$) and the one that gives the minimum value for Eq.(6) is selected as the initial point to restart the next local search. The stopping criterion of the global optimization is the maximum number of local searches, $n_{\max}$, defined a priori by the user.

4. NUMERICAL RESULTS

In this section, three numerical examples are solved in order to demonstrate the main aspects of the local-global approach presented. The Young modulus, the maximum allowed stress in tension and compression for all the examples are E = 200GPa, σ_t = 250 MPa and σ_c = 250 MPa, respectively. Besides, self weight of the structures is not considered. Also, for all examples shown here, the lower bound for the cross-section areas is equal to 0.1mm^2 and, in the pos-processing visualization, when a bar cross-section is smaller than 0.3mm^2, the correspondent member is not shown. Finally, the parameters used in the optimization algorithm are shown in Table 1.

Table 1. Parameters used for the optimization algorithm.

Parameter	Value
M	1000
n_{max}	20
β	0.001
Tolerance on constraints, objective function and design variables used by local search algorithm	10^{-9}

4.1. Example 1

Figure 2 presents a ground structure that is subjected to the loading F = 10kN. The ground structure has the dimensions L_x=2,000mm and L_y=1,000mm. The initial cross-section areas for the first local search are taken as 250mm^2.

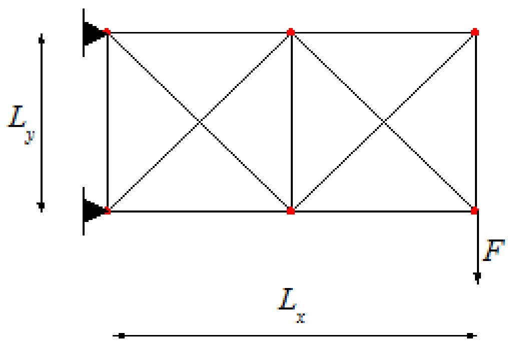
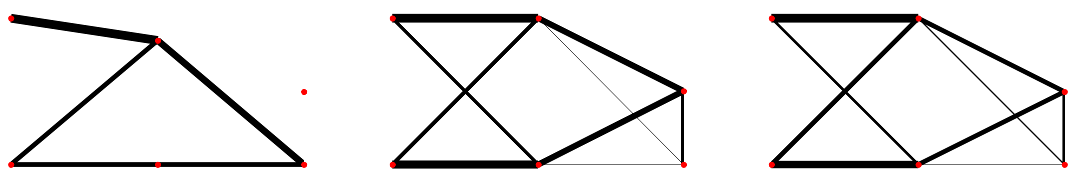


Figure 2: Ground structure of the Example 1.

For the optimization problem, the nodes of the upper chord (except the node of the support) are allowed to be moved up and down to positions as far as 500mm, in both directions, from its original positions. The results are presented in Fig. 3, showing that at least three local minima exist for this problem. It seems that the structure from Fig. 3(a) is the global optimum of this problem, since no better solution was found by the algorithm. Besides, note that the structures from Fig. 3(b) and Fig. 3(c) are slightly different, but have the same volume. It is important to note that when the restart procedure was not used, the optimum structure found by the optimization algorithm was that presented in Fig. 3(c). Besides, other local optima were also found for this example that are not presented here. However, they are similar to the ones presented in Fig. 3(b) and Fig. 3(c).



V = 3.1395E+5 mm^3
A_{max} = 80.8572 mm^2
(a)

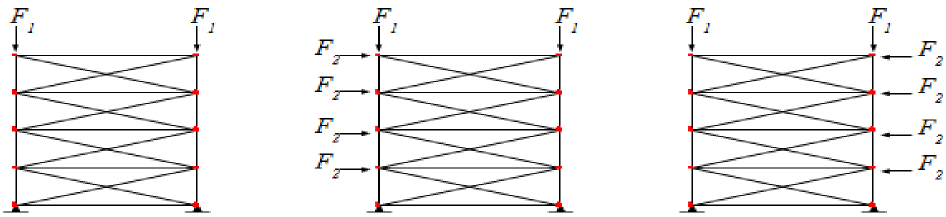
V = 3.2020E+5 mm^3
A_{max} = 60.9112 mm^2
(b)

V = 3.2020E+5 mm^3
A_{max} = 64.2791 mm^2
(c)

Figure 3: Local optima found and their correspondent volumes V and maximum cross-section area A_{max} when allowing the nodes to be moved by 500mm from its original positions (Example 1).

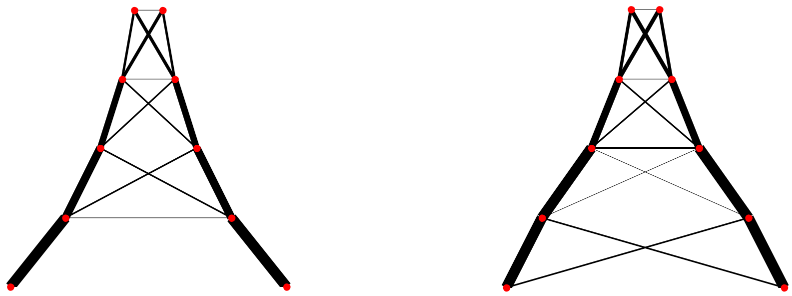
4.2. Example 2

Figure 4 presents a ground structure that is subjected to three loading conditions. The ground structure has a total height of 4,000mm and a total width of 4,000mm. All nodes (except the nodes of the supports) are allowed to be moved left and right by the optimization algorithm, to positions as far as 1,800mm from its original position. The vertical and horizontal force values are F_1 = 10kN and F_2 = 5kN, respectively. Finally, symmetry of the structure about a middle vertical axis is enforced for the nodal coordinates. Figure 5 shows the minima found for this problem. As can be seen, it appears to have two local minima. We also note that, for the two solutions found, the cross section areas are symmetric about the middle vertical axis, despite this condition was not enforced.



Loading condition 1 Loading condition 2 Loading condition 3

Figure 4: Ground structure of the Example 2, subjected to three loading conditions.



V = 9.8592E+5 mm3
Amax = 115.1772 mm2
(a)

V = 9.9894E+5 mm3
Amax = 96.2070 mm2
(b)

Figure 5: Local optima for Example 2 and their correspondent volumes V and maximum cross-section area Amax.

4.3. Example 3

The last example is that presented in Fig. 6. The ground structure has a total height of 1,000mm and a total width of 8,000mm. All nodes of the upper chord are allowed to be moved up and down by the optimization algorithm, to positions as far as 10,000mm from its original position. The applied force is F = 10kN. The initial values of the cross-section areas for the first local search are taken as 250mm^2. Symmetry of the structure according to the vertical axis is enforced in this example.

The results are presented in Fig. 7, and it can be seen that this problem has several local optima. It seems that the structure from Fig. 7(a) and Fig. 7(b) are the two global optima for this case. These two solutions are essentially the same, since the allowable stresses in tension and compression are the same. The same is true for the solutions from Fig. 7(c) and Fig. 7(d). It is important to note that the optimum solution found when no restart procedure was employed was that of Fig. 7(g), that is actually, among the solutions found, the worst one. This puts in evidence the importance of using global optimization strategies.

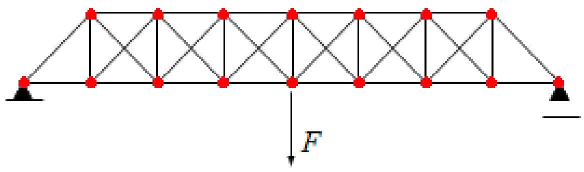
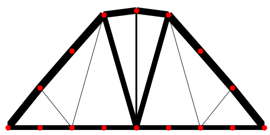
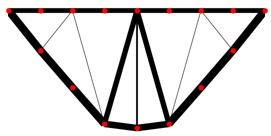


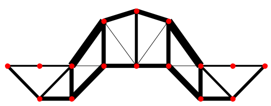
Figure 6: Ground structure of the Example 3.



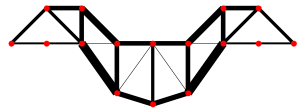
V = 0.5772 E+6 mm^3
A_{max} = 25.6859 mm^2
(a)



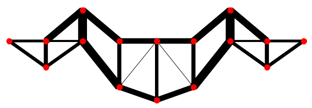
V = 0.5772 E+6 mm^3
A_{max} = 25.6859 mm^2
(b)



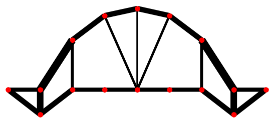
V = 1.1939 E+6 mm^3
A_{max} = 74.0320 mm^2
(c)



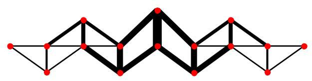
V = 1.1939 E+6 mm^3
A_{max} = 74.0320 mm^2
(d)



V = 1.4026 E+6 mm^3
A_{max} = 79.9947 mm^2
(e)



V = 0.8777 E+6 mm^3
A_{max} = 47.6602 mm^2
(f)



V = 1.9992E+6 mm^3
A_{max} = 160.0268 mm^2
(g)

Figure 7: Local optima for Example 3 and their correspondent volumes V and maximum cross-section area A_{max}.

5. CONCLUDING REMARKS

This paper addressed the development of a global optimization technique for truss structures which. The optimization strategy is based on a probabilistic restart procedure coupled with a local search algorithm. The resulting algorithm is able to guarantee local optimality and to asymptotically converge to the global optimum, as the number of restarts increases. Besides, the restart procedure is based on information from the previous iterations, and is not a purely random one, which may reduce computer time in the global optimization process. The main advantage of the procedure proposed here is that the local search can be made by efficient gradient based algorithms, thus ensuring that the solutions found by the algorithm are residual free.

At the end of the optimization procedure, the approach proposed here presents a set of local optima that are, in general, different from one another. In some cases the designer may find some local optima more appealing than others (for some reasons apart from optimality) and thus choose some local optima instead of the global one.

The examples presented demonstrated that even for simple cases local minima, that are not global minima, may exist. Some of these local minima may even appear to be global optima at first glance. Besides, some problems may present several local optima, and in many cases the optimum solution found by a single local search is not the global one. This highlights the importance of considering global optimization procedures for the problem being addressed, since this can lead to much improved solutions.

6. REFERENCES

Achtziger., W, 1997, "Topology optimization of discrete structures: in introduction in view of computational and nonsmooth aspects". In: *Rozvany GIN (ed) Topology optimization in structural mechanics*, Springer-Verlag, Wien.

Achtziger., W., 2006, "Simultaneous optimization of truss topology and geometry, revisited". In: *Bendsøe M, Olhoff N, Sigmund O (eds) IUTAM Symposium on topological design optimization of structures, machines and materials: status and perspectives*, Springer, Berlin Heidelberg New York, pp 413–423.

Achtziger., W., 2007, "On simultaneous optimization of truss geometry and topology". *Structural and Multidisciplinary Optimization*, 33:285–305.

Achtziger, W, Stolpe, M,. 2007, "Truss topology optimization with discrete design variables - Guaranteed global optimality and benchmark examples". *Structural and Multidisciplinary Optimization*, 34:1–20.

Arora, J.S., 2004, "*Introduction to optimum design"*, Elsevier, San Diego.

Dominguez, A., Stiharu, I., Sedaghati, R., 2006, "Practical design optimization of truss structures using the genetic algorithms". *Research in Engineering Design,* 17:73–84.

Duda, O.R., Hart, P. E., Stork, D.G., 2001, "*Pattern classification"*, John Wiley and Sons, New York.

Hemp, W.S., 1973, "Optimum structures". Clarendon, Oxford.

Kocvara, M., Zowe, J., 1996, "How mathematics can help in design of mechanical structures". In: *Griffiths DF and Watson GA Watson (eds) Numerical Analysis. Longman*, Harlow, pp 76-93.

Luersen, M.A., Le Riche, R., 2004, "Globalized Nelder-Mead method for engineering optimization", *Computers & Structures*, 82:2251-2260.

Luh, G.C., Lin, C.Y., 2008, "Optimal design of truss structures using ant algorithm". *Structural and Multidisciplinary Optimization*, 36:365–379.

Nocedal, J., Wright, S.J., 1999, "*Numerical optimization"*, Springer-Verlag, Berlin.

Pedersen, P., 1970, "On the minimum mass layout of trusses". *Symposium on structural optimization*, AGARD Conference Proceedings, 36:189-192.

Pedersen, P., 1993, "Topology optimization of three-dimensional trusses". In: *Bendsøe MP, Mota Soares CA (eds) Topology design of structures*, Kluwer, Dordrecht, pp 19–30.

Rahami, H., Kaveh, A., Gholipour, Y., 2009, "Sizing, geometry and topology of trusses via force method and genetic algorithm", *Engineering Structures*, 30:2360-2369.

Rozvany, G.I.N., 1997, "Aims, scope, basic concepts and methods of topology optimization". In:*Rozvany GIN (ed) Topology optimization in structural mechanics*, Springer-Verlag, Wien.

Schutte, J.F., Groenwold, A.A., 2003, "Sizing design of truss structures using particle swarms." *Structural and Multidisciplinary Optimization*, 25:261–269.

Solis, F.J., Wets, R.J.B., 1981, "Minimization by Random Search Techniques". *Mathematics of Operations Research*, 6:19-30.

Torii, A.J., Biondini, F., 2009, "A simple geometry optimization method for statically indeterminate trusses". *8th World Congress on Structural and Multidisciplinary Optimization*, Lisbon.

Torii, A.J., Lopez, R.H., Biondini, F., 2011, "An approach to reliability-based shape and topology optimization of truss structures", Engineering Optimization (accepted for publication).

Zheng, Q.Z., Querin, O.M., Barton, D.C., 2003, "Geometry and sizing optimisation of discrete structure using the genetic programming method". *Structural and Multidisciplinary Optimization*, 25:261–269.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.