# Analysis of the Aerodynamic Parameters Change Influence on the Controller Design of Assisted Light Airplanes 

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Abstract. The objective of the present work is to analyze some aspects related to assisting techniques in order to aid the control design of light aircraft. Specifically, it is analyzed the influence of the atmospheric characteristics and initial aircraft conditions on the optimum gains of the controller, i. e., the choice of the gains that better aid the control in simple tasks, such as climb and descent missions. Furthermore, it is obtained what type of aerodynamic parameters, e. g., derivative of lift in relation to the angle of attack, influence the most on the controller design. This analysis is done in one case study.

Keywords: control, assistance, aircraft, gain, optimization.

## 1. INTRODUCTION

General aviation is a branch of technology that is lacking studies. Many improvements were made over the past years for larger aircraft, mainly executive or commercial ones. But the light aircraft evolved in a much lower rate on important aspects, such as: safety, performance, efficiency, accessibility and others.

Acknowledging this issue, the Center for Aeronautical Studies (CEA) of the Universidade Federal de Minas Gerais (UFMG) in Brazil, an institution that studies, researches, designs and builds light aircraft since 1963, is studying pilot assisting techniques for light aircraft. The main goal is to develop control technologies, analogous to Fly-by-wire, that turns the task of piloting requiring no continuous control compensations by the pilot. That type of control technique allows for a lower risk, due to the diminished pilot workload, which is notable when he is doing many tasks such as piloting, navigating and communicating at the same time.

According to AOPA(2005) a total of 19.7 fatalities per million hours flown happened for general aviation aircraft in 2004. According to $\operatorname{NTSB}(2000)$, using data from 1991 to 2000 , a total of 6.5 fatalities per million hours flown happened for scheduled Air carriers, which means, FAA Part 121 certified airlines.

Another problem in general aviation aircraft is that it is difficult to operate the airplane with the best efficiency (minimum fuel consumption) when there is a high pilot workload. With the aid of a computer, this task is done automatically, just like on airliners.

Knowing this gap between general aircraft and airliners, CEA started a research in pilot assisting techniques for light aircraft, funded by Brazilian agency CNPQ. The objective of the research is to bring some technology from the commercial airplanes to light aircraft, in an effort to lower this gap and popularize general aviation. The research encompass studying and developing a fly-by-wire control with a dynamical model of a specific airplane and the further installation of hardware and testing the system in the real aircraft, priming for safety, simplicity and cost.

The aircraft used in the model and control development is an ACS-100 Sora, which can be seen in Fig. 1 below.
The present work deals, specifically, with the control design. From the work of Fielding(2000), it is known that the aircraft nonlinear model is highly dependent on factors such as Air density, aircraft center of gravity position and angle of attack. Hence, the controller design must face this change in the plant in order to be effective (OGATA, 2009).

It was used the computational longitudinal model of the aircraft and the controller from the work of SILVA(2009). A gain scheduling (Fielding, 2000) for the PID controller was proposed and the results analyzed (OGATA, 2009). For this gain scheduling, the factors that were analyzed were: i) the initial true airspeed (TAS) (Mccormick, 1994), ii) the variation of the initial air density, determined by the pressure altitude of the airplane (set on 1000 m ) and iii) the C.G. position in percentage of the Mean Aerodynamic Chord (Etkin, 1995).

Another issue that was studied is the influence of some aerodynamical longitudinal parameters of the aircraft in the optimum gains, in order to inspect if the system is robust for applying in a prototype. In another words, if the calculated parameters diverge from the real ones, it is desired to know the optimum gain variation of the PID controller.


Figure 1. ACS-100 Sora aircraft with testing hardware.

## 2. METHODOLOGY

Using the longitudinal model of the ACS-100 Sora aircraft as shown in Silva(2009), a controller was adapted. On his study, Silva(2009) tested 13 strategies for assisted longitudinal control with 34 volunteers in a simulator. After undergoing the test in a specified trajectory, the volunteer was asked to give a grade for the aircraft response according to the Cooper-Harper scale (Cooper and Harper, 1986). Using this data and the integrated error in the trajectory of each volunteer, Silva(2009) concluded that the best strategy for the controller was the strategy 12 . On strategy 12, the stick longitudinal movement changes the setpoint of the velocity angle, which is the angle of the velocity vector in relation to the horizontal reference. The throttle control changes the setpoint of the airspeed.

When the pilot changes the setpoint with the controls, two PID (Proportional-Integral-Derivative) controllers actuate both the elevator (PID1) and the engine throttle input (PID2) simultaneously and independently trying to attend the setpoint of velocity angle and airspeed. The cross dependence of the model, which is the dependence of the velocity angle output to the engine throttle input and the dependence of the airspeed output to the elevator control, could be neglected by observing the system's response. Hence, the controllers could be analyzed independently.

The choice of the optimum gains of the controller is made using optimization techniques. The objective function (Chapra and Canale, 2005) takes into account the step response of the control assisted airplane model. A step of $1^{\circ}$ is done on the setpoint and the aircraft dynamic response of the velocity angle is analyzed. For the other PID, a step of 1 $\mathrm{m} / \mathrm{s}$ is done on the setpoint and a similar analysis is done. Factors such as overshoot, settling time, rise time and peak time (Ogata, 2009) are normalized from known good values in practice and summed, forming the multidisciplinary objective function. The 6 variables to optimize are the proportional, derivative and integral gains of the two PID's.

Before the step response analysis is done, an algorithm trims the aircraft on the desired initial conditions.
For each initial value of TAS, $\Delta \rho$ (variation of the initial air density) and $x_{C G}$ (C.G. position in percentage of the Mean Aerodynamic Chord), the optimization is done and the values of the optimum 6 gains are determined.

On the second part of the work, the previously determined values of $C L_{\alpha}$ (derivative of lift in relation to the angle of attack, used between $0^{\circ}$ and $5^{\circ}$ ) of the wing and tail and the $d \varepsilon / d \alpha$ (derivative of the downwash angle in relation to the angle of attack) are changed and the new values of controller gains are determined. The objective of the second part is to analyze the influence of the parameter calculation error on the controller, providing a means of inspecting the robustness of the system.

## 3. THE MODEL

The ACS-100 Sora aircraft model is divided into three main blocks: Aircraft, Propulsion and Dynamics. The aircraft state: x (horizontal position), y (vertical position), $\theta$ (pitch angle), $\mathrm{u}_{\mathrm{X}}$ (horizontal position), $\mathrm{u}_{\mathrm{Y}}$ (vertical position) and $\dot{\theta}$ (pitch angular velocity) is determined for each time step, given an initial condition.

The Aircraft model calculates the aerodynamic forces and moments given the aircraft state, specifically, $\mathrm{u}_{\mathrm{X}}, \mathrm{u}_{\mathrm{Y}}, \theta$ and $\dot{\theta}$. From $\mathrm{u}_{\mathrm{X}}$ and $\mathrm{u}_{\mathrm{Y}}$ one can determine $\mathbf{V}_{\mathbf{r}}$ the relative air velocity vector and $\gamma$, the velocity angle of the aircraft, which will be used by the controller model. From $\theta$ and $\gamma$, one might determine $\alpha$, the angle of attack of the aircraft. Given the initial air density, the airspeed, and $\alpha$, it is possible to obtain the lift, moment and drag of the wing-body combinations, by some nonlinear lookup tables, given the wing planform, aircraft dimensions and airfoil by semiempirical methods. For the tail lift determination, two extra factors must be taken into account, namely: the downwash effect (Etkin, 1995) and the longitudinal angular velocity. The downwash of the wing changes the actual angle of attack of the tail, this real angle of attack must be determined in order to apply the tail lift and drag coefficient lookup tables. Because of the tail distance from the C.G., when there is a longitudinal angular velocity, a vertical relative airspeed is observed on the tail, hence, the relative velocity vector at the tail will be a little deviated from the aircraft's. For each step, knowing the previous information, this model calculates the aerodynamic forces and moments and delivers the information to the Dynamics model.

The Propulsion model calculates the thrust force and propeller-engine RPM with the engine throttle, airspeed and Atmosphere conditions as inputs. The engine BHP (brake horse power) for each condition is determined by interpolation from the engine chart data (115 hp Lycoming). The propeller $C_{p}$ (Power coefficient) and $C_{T}$ (thrust coefficient) in function of the propeller pitch and advance ratio (Mccormick, 1994) are determined from the manufacturer empirical data. Knowing the moment of inertia of the propeller and the power coefficient, it is possible to obtain the propeller-engine rpm in each step of the simulation, integrating the angular momentum equation with a fifth order fixed step runge-kutta algorithm (Chapra and Canale, 2005).

The Dynamics model solves the state space 3 degree of freedom longitudinal equations of motion of the aircraft (Newton's second law) (Etkin, 1995). Given the aircraft mass, longitudinal moment of inertia, and the forces and moments calculated in the Aircraft model, the system of differential equations is solved using the same runge-kutta algorithm.

## 4. THE CONTROLLER

After balancing the aircraft on the initial conditions the entire model is run, beginning with the controller model. That model generates the control laws, which are: Elevator deflection and engine throttle, necessary to the aircraft respond to the step on the setpoint of $\gamma$ and airspeed. The error signal, which is the difference with the setpoint and the aircraft actual response, is passed to both PID controllers, that calculates next step control law for the aircraft, trying to minimize next step's error. The control law is saturated, because there are maxima and minima to the elevator deflection ( $+30^{\circ}$ and $-30^{\circ}$ ) and engine throttle ( $0 \%$ and $100 \%$ ). The initial gains of the controllers were determined using Ziegler-Nichols method (OGATA, 2009). These gains were used as initial guesses for the optimization algorithm

## 5. THE OPTIMIZATION ALGORITHM

After the aircraft is trimmed on the initial conditions, an optimization algorithm changes the values of the 6 gains, in order to find the optima of the objective function.

The numerical algorithm used is called pattern search (Audet and Dennis, 2003). This is a type of direct search algorithm, independent of the function gradient. In brief, it tries some values of the variables (in this case, the 6 gains) in a domain radius and in the center of this radius, if the value of the function on the center is lower than on the borders, then the domain radius is diminished and the algorithm repeats. If the value of the function on the center is bigger than anyone of the border, the domain is expanded and the point of the border that has the lower value is picked as the next center point of the algorithm. This process repeats until optimization criteria are reached. The optimization criteria are $1 \%$ variation of the tried variable values or fifty iterations. Those criteria were observed in practice and showed themselves efficient ones.

## 6. THE GAIN SCHEDULING PROCESS

In order to obtain a gain scheculing diagram, it was done the whole previous process for each initial condition. It was chosen 3 values of TAS, $\Delta \rho$ and $x_{C G}$ yielding 27 initial conditions. The values of TAS were chosen respecting the airplane stol speed and maximum speed for each $\Delta \rho$. The values of $\Delta \rho$ were chosen based on one side on the operational ceiling of the aircraft, which is 14000 ft , on the other side on a lower temperature for Mean Sea Level ISA
atmosphere. The values of $\Delta \rho$ were $-0.3,0$ and $0.2\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ and the values of $x_{C G}$ were $0.1,0.22$ and 0.35 . The values of the initial TAS for each $\Delta \rho$ are shown on Tab. 1 below.

Table 1. Values of initial TAS function of $\Delta \rho$

|  | $\Delta \boldsymbol{\rho}\left(\mathbf{k g} / \mathbf{m}^{\mathbf{3}}\right)$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{- 0 . 3}$ | $\mathbf{0}$ | $\mathbf{0 . 2}$ |
|  | 49 | 43 | 39 |
| TAS | 49 | 59 | 53 |
|  | 67 | 75 | 67 |
|  | 75 |  |  |

## 7. THE PARAMETER VARIATION PROCESS

The optima gains for an average condition $\left(\Delta \rho=0 \mathrm{~kg} / \mathrm{m}^{3}, x_{C G}=0.25\right.$ and TAS $\left.=60 \mathrm{~m} / \mathrm{s}\right)$ are determined. After that, some aerodynamic parameters have their values changed and a new set of optima gains are determined. This process is repeated for each combination of parameter change.

The parameters to be considered are: $\left(C L_{\alpha}\right)_{\text {Wing }},\left(C L_{\alpha}\right)_{\text {Tail }}$ and $d \varepsilon / d \alpha$. Changes of $0 \%,+10 \%$ and $-10 \%$ for each are applied, yielding 27 different scenarios. For each of the 27 different scenarios, the mean of the absolute value of the percentual variation on the 6 gains are computed.

## 8. RESULTS

Figure 2, below, shows the step response of the controlled airplane before and after the optimization for one initial condition.

The results are shown in Fig. 3 to 9 . Figures 3 to 8 show each of the 6 gains variation with the three initial conditions of the gain scheduling. Figure 9 shows the mean percentage gain variation with each of the three parameters. The gains on the y coordinate are of the type kij, knowing that " i " is the number of the PID and " j " is the number corresponding to each type of controller: 1 for proportional, 2 for integral and 3 for derivative.


Figure 2. Step response before (left) and after (right) the gain optimization.


Figure 3. Proportional gain of PID1.


Figure 4. Integral gain of PID1.


Figure 5. Derivative gain of PID1.


Figure 6. Proportional gain of PID2.


Figure 7. Integral gain of PID2.


Figure 8. Derivative gain of PID2.


Figure 9. Mean percentage gain variation.
It was possible to observe variations of the magnitude of 500 times on the modulus of the gain value (integral controller of PID2). This shows the importance of performing the gain scheduling. For some values of initial conditions, the initial controller response suffered high oscillations, or even divergence, hence, reassuring gain scheduling as a safety requisite for control assisted aircraft.

From Fig. 3, it is possible to see that, for lower values of TAS, the proportional controller increases his value significantly. That is because of the higher angles of attack, when the derivative of the $\mathrm{C}_{\mathrm{L}} \mathrm{x} \alpha$ curve is lower, hence, it is needed more elevator deflection to keep the velocity angle.

From Fig. 9, it is concluded that for a variation of $-10 \%$ on $\left(C L_{\alpha}\right)_{\text {Wing }}, 10 \%$ on $\left(C L_{\alpha}\right)_{\text {Tail }}$ and $-10 \%$ on $d \varepsilon / d \alpha$ yields a mean variation of $80 \%$ on the optimum gains of the controller. This happens because the $\left(C L_{\alpha}\right)_{\text {Tail }}$ has a stabilizing effect and the other parameters have an unstabilizing effect, according to the following equation taken from Etkin(1995):

$$
\begin{equation*}
K n=-\frac{\partial C_{M}}{\partial C_{L}}=x_{C A}-x_{C G}+\frac{\bar{V}\left(C L_{\alpha}\right)_{\text {Tail }}}{\left(C L_{\alpha}\right)_{\text {Wing }}}\left(1-\frac{d \varepsilon}{d \alpha}\right) . \tag{1}
\end{equation*}
$$

Knowing that $x_{C A}$ is the Aerodynamic Center in relation to the mean aerodynamic chord and $\bar{V}$ is the tail volume ratio, an attribute of the dimensions of the aircraft (Etkin, 1995). Kn represents the tendency of the aircraft in staying at the same pitch condition. The bigger Kn, the more control is needed for the step, what would explain the previous observation.

In order to obtain the most influential parameter for the gain determination, it is considered the value of the gain variation function of each parameter, given that the others are maintained zero. Furthermore, $\left(C L_{\alpha}\right)_{\text {wing }}-10 \%$ variation causes a $20 \%$ change on the mean gains, $\left(C L_{\alpha}\right)_{\text {Tail }} 10 \%$ variation causes a $30 \%$ change on the mean gains, $d \varepsilon / d \alpha-10 \%$ variation causes a $15 \%$ change on the mean gains.

Hence, it is reasonable to assure that the tail parameter is the most important for the control design, followed by the wing parameter and then the downwash parameter.

## 9. CONCLUSION

It was analyzed the PID control design of the fly-by-wire scheme of the ACS-100 Sora aircraft. Diagrams of proportional, integral and derivative gains were proposed varying the air density, initial true airspeed and center of gravity position. An important observation was that a huge variation (on the order of 500 times) of some gain values can occur, clarifying the importance of such design practice.

Furthermore, it was determined which aerodynamic parameters of the aircraft influence the most on the controller design. The tail derivative of lift in relation to its angle of attack was the most important, followed by the wing
parameter and then the downwash parameter. This information has the power to give aircraft designers a clue about how to spend efforts for parameter estimation and calculation.

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