DETECTION OF TRANSIENT VIBRATION COMPONENTS IN MECHANICAL SYSTEMS USING TIME-FREQUENCY DISTRIBUTIONS

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Abstract. *Transient vibration component of mechanical systems, such as, amplitude of frequency modulation are not easily detected by using the conventional Fourier analysis. Therefore, it is necessary to use appropriate analysis techniques in order to extract the signal components in both time and frequency domains. Hence, the objective of this work is to apply time-frequency energy distributions, namely, the Wigner Distribution and the Choi-Williams Distribution in the processing of the mechanical systems response in the time domain. These time-frequency analysis techniques will be applied in the study of the vibration of the mechanical systems in two different situations. In the first case the objective is to extract amplitude modulation components of vibration signals generated during the metal machining, or turning process. In the second situation, the time-frequency distributions were applied in the analysis of the frequency modulation components caused by the acceleration of a rotating machine. The results proved that the Wigner Distribution and the Choi-Williams Distribution have some advantages when compared to the conventional spectral analysis in the extraction of the transient vibration components.*

Keywords*: mechanical systems, signal analysis, time-frequency distributions, transient vibration.*

1. INTRODUCTION

The conventional spectral analysis is a powerful technique to be used in the study of the mechanical systems vibration (Cohen, 1995). By using the Fourier analysis, it is possible to extract the signal frequency components, such as, natural frequencies of structures or vibration frequencies caused by the operation of rotating machines (Braun, 1986). In most cases, the Fourier Transform (FT) is used in the analysis of mechanical systems by operating in stationary steady. On the other hand, the transient vibration signals with frequency components that change with the time are not easily detected by using the traditional spectral analysis. In the practice, rotating machines by operating in non-constant rotation and non-linear mechanical systems produce transient vibration components that should also be identified using the appropriate analysis techniques. Furthermore, faults in mechanical systems, as for example, cracks, wear, malfunctions of machine elements, etc., also cause non-stationary vibration signals (Braun, 1986).

Indeed, the main drawback of the FT is that it assumes signal frequencies do not change with the time (Cohen, 1995). In this context, several methodologies of transient vibration analysis of mechanical systems have been proposed in the literature (Gregoris and Yu, 1994, Cohen, 1995). A simple way of study the transient vibration components is to employ the Short Time Fourier Transform (STFT), that is, by multiplying the signal by a window and to apply the FT in "small pieces" of the response in the time domain (Cohen, 1995). Although the STFT is a time-frequency technique largely used in the analysis of transient signals, it is impossible to select a window function with short time duration and the narrowband simultaneously, according to the uncertainly principle (Guimarães and Silva, 2009). In this sense, the Continuous Wavelet Transform (CWT) has several advantages when compared to the STFT (Gregoris and Yu, 1994). For example, the analyst of signals may select the type of more appropriate wavelet depending on the application and the signals which will be studied. Moreover, as the CWT employs a multi-resolution analysis with wavelets of different sizes, the user does not need select the band size of frequency of the window to be used in the signal processing. Although the abovementioned techniques are extremely useful in the transient signals analysis, the resolution of the time-frequency map is not sufficiently small to extract all transient components of the vibration signal (Cohen, 1995, Guimarães, 2000).

The quadratic time-frequency energy distributions usually have better resolution when compared the linear time-frequency maps, such as the STFT and CWT (Cohen, 1995). Hence, the objective of this work is to use the Wigner Distribution (WD) and the Choi-Williams Distribution (CWD) in the detection of the amplitude and frequency modulation components in the vibration signal from mechanical systems. In a previous work, Guimarães and Silva (2009) have used only the STFT in the analysis of the transient vibration components from rotor after to turn-off the rotating machine. Therefore, it is expected that all frequency modulation components caused by the rotor vibration are detected in the WD and the CWD. In the other case, Guimarães et all. (2011) have used the Power Cepstrum for the extraction of vibration amplitude modulation components generated during the cutting process in the shafts turning. In this work, the WD and the CWD will be used to extract the frequency components caused by the vibration from machine tool, as well as, the amplitude modulation components generated by the metal cutting process. Moreover, the main advantages and drawbacks of the WD and the CWD will be compared in both cases.

2. TIME-FREQUENCY DISTRIBUTIONS

2.1. Wigner Distribution

The Wigner Distribution (WD) is a generalization of the Spectral Density Function (PS) (Bendat and Piersol, 1986). If the PS is applied to the mechanical system response, it measures the frequency energy density of the vibration signal. Therefore, by using the PS, the signal analyst may verify what vibration components have more energy in the frequency domain. By definition, the PS is the Fourier Transform (FT) of the Autocorrelation Function of the signal, x(t). The Autocorrelation Function, R(τ), is given by (Bendat and Piersol, 1986):

$$R(\tau) = E[x(t)x(t+\tau)] \tag{1}$$

where the symbol τ represents the delay time and the symbol E[] is the expected value of the product between x(t) and x(t+τ). The PS is defined by:

$$PS(f) = \frac{1}{2\pi}\int_{0}^{\infty} R(\tau)e^{-j2\pi ft}\,dt \tag{2}$$

where j is the pure imaginary and f represents frequency in Hz. The WD is an energy density function model in both time and frequency domains. Hence, by applying the WD to the vibration signal measured of a mechanical system, it is possible to identify when and how the frequencies components are changing with respect to the time. In the WD, the Autocorrelation Function is calculated for each time instant τ, by considering the signal samples in the past and the future. Instead of equation (1), the Autocorrelation Function in the WD is defined by (Cohen, 1995):

$$R(t,\tau) = \int_{0}^{\infty} x\left(t+\frac{\tau}{2}\right)x^{*}\left(t-\frac{\tau}{2}\right)d\tau \tag{3}$$

where the symbol * denotes the conjugate complex of the analytic signal x(t). The Wigner Distribution, WD(t,f) is the FT of the Autocorrelation Function, R(t,τ),

$$WD(t,f) = \frac{1}{2\pi}\int_{0}^{\infty} R(t,\tau)e^{-j2\pi ft}\,dt \tag{4}$$

Since the WD is a quadratic time-frequency representation, it has several advantages when compared to the STFT and CWT (Cohen, 1995). For example, in the WD, the time and frequency resolution is much smaller than the resolution provided by the linear time-frequency representations (Cohen, 1989). Hence, the time-frequency distribution provided by the WD yields a detailed map of the transient components from signal. On the other hand, the main drawback of the WD is the presence of interference cross-terms in the time-frequency plane (Cohen, 1995). For example, if the vibration signal is the sum of two components, $x(t) = x_1(t) + x_2(t)$, the WD of x(t) is:

$$WD[x(t)] = WD[x_1(t)] + WD[x_2(t)] + 2\,\mathrm{Re}\{WD[x_1(t)x_2(t)]\} \tag{5}$$

where $2\mathrm{Re}\{WD[x_1(t)x_2(t)]\}$represents the cross-term between the two signal components, $x_1(t)$ and $x_2(t)$. In fact, for each pair of components, there is always a cross term in time-frequency distribution (Cohen, 1995). These cross-terms are considered as "undesirable artificial components" caused only by the quadratic nature of the WD. Hence, for a multi-component vibration signal, the interference terms may difficult the interpretation of the signal components.

A way of smoothing the cross-terms is to define the WD with a window in the time domain called Pseudo Wigner Distribution (PWD). However, the windowing applied in the WD destroys some of desirable properties, such as, the instantaneous frequency of the WD. Furthermore, if two or more components are placed into the window, the cross-terms caused by these components will not be suppressed. In these cases, others time-frequency distributions should be used for the attenuation of the interference terms (Cohen, 1995).

2.2. Choi-Williams Distribution

The Choi-Williams Distribution (CWD) is a quadratic time-frequency representation able to attenuate the cross-terms for multi-component signals (Choi and Williams, 1989). For the formulation of the CWD, it is necessary to discuss the concept of the Ambiguity Function (Cohen, 1989). Let the Autocorrelation Function, R(t,τ) has given by the equation (3). The Ambiguity Function, A(θ,τ), is the Inverse Fourier Transform of R(t,τ):

$$A(\theta,\tau)=\int_{0}^{\infty} x\left(t+\frac{\tau}{2}\right)x^{*}\left(t-\frac{\tau}{2}\right)e^{j\omega t}\,dt \tag{6}$$

where ω is the frequency in rad/s and θ is the frequency in the ambiguity domain. The ambiguity domain is a different manner of interpreting the signal components in the time-frequency plane. In this domain, the cross-terms will always be localized in the regions far from the origin of the ambiguity domain. On the other hand, the signals components or auto-terms are always placed on the axes of this domain. Using this idea, Choi and Williams (1989) have proposed a function in the ambiguity domain as follows:

$$\phi(\theta,\tau)=\exp\left(\frac{-\theta^{2}\tau^{2}}{\sigma}\right) \tag{10}$$

where σ represents a parameter of suppressing of interference terms in the CWD. In the signal analysis context, the function ø(θ,τ), called kernel, can be defined by any mathematical equation that the analyst desires. Cohen (1995) yields a list of several models of kernels which are used in the definition of well-known time-frequency representations, as the STFT, WD, CWT and CWD. According to this concept, it is possible to filter the cross-terms between the several signal components, by using the Characteristic Function Formulation, M(θ,τ) (Cohen, 1995):

$$M(\theta,\tau)=\phi(\theta,\tau)A(\theta,\tau) \tag{11}$$

and any time-frequency representation can be derived from the two-dimensional Fourier Transform of M(θ,τ):

$$C(t,\omega)=\frac{1}{4\pi^{2}}\int_{0}^{\infty}\int_{0}^{\infty} M(\theta,\tau)e^{-j\theta t-j\tau\omega}\,d\theta d\omega \tag{12}$$

which yields the Cohen's general class, C(t,ω), a manner of generalizing all linear and quadratic time-frequency representations. By inserting the Choi-Williams kernel has defined by the equation (10) in the equations (11) and (12), one has the Choi-Williams Distribution (1989):

$$CWD(t,\omega)=\frac{1}{4\pi^{3/2}}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{\sqrt{\tau^{2}/\sigma}}\,x\left(t+\frac{\tau}{2}\right)x^{*}\left(t-\frac{\tau}{2}\right)\exp\left[-j\tau\omega-\sigma(t-\tau)^{2}/\tau^{2}\right]dt\,d\tau \tag{13}$$

which will be applied to the vibration response of some mechanical systems have considered in this work. Hence, if the signal has several components, the cross-terms will be suppressed and some of desired properties of WD will be preserved. A serious disadvantage of the CWD is the high computational cost has required in the calculation of the time-frequency matrix in the discretized form of the equation (13). If the vibration signal does not have several frequency components, it may be more appropriate to use the WD instead of the CWD.

3. CHARACTERISTICS OF THE VIBRATION SIGNALS FROM MECHANICAL SYSTEMS

The time-frequency distributions have above discussed will be applied to the experimental vibration signals measured in complex mechanical systems. The objective is to study the ability of these signal analysis tools in the extraction of the transient vibration components generated by the operation of these systems. Furthermore, it will also be studied the influence of the experimental noise and the interference terms in the analysis of the WD and CWD have applied to the vibration signals.

3.1. Vibration of a rotating machine

The response signal measured in a rotating machine by operating in stationary steady has several harmonics caused by the rotor vibration modes (Guimarães, 2009). In this situation, these frequency components may be easily detected using the Power Spectrum of the vibration signal. During the start-up and the stopping of the rotating machine, the acceleration and the passing of the rotation frequency by the critical speed increases the rotor vibration amplitude (Braun, 1986). These effects would be difficult to be detected using the traditional Fourier analysis. Hence, it will be used the WD and the CWD for detecting of frequency modulation component caused by the acceleration of the rotor, as well as, the increasing of the vibration amplitude due to the rotor critical speed and the natural frequencies of the rotor foundation.

3.2. Vibration generated in the turning process

The vibration generated during the metal machining, such as, the turning process is highly nonlinear with several frequencies components (Sick, 2002). Usually, the vibration signals measured during the machining process depends on the cutting parameters, as for example, cutting speed, piece material, etc. Moreover, there are several noise sources that may contaminate the vibration response. Guimarães et al (2011) have demonstrated that the turning process produces vibration amplitude modulation components caused by the contact between the cutting tool and the metal piece. The amplitude of these components and the repetition pattern of modulation are associated with the cutting parameters and the piece cutting speed. The WD and CWD will also be employed in the detection of the frequency components and the vibration amplitude modulation caused by the turning process.

4. MEASUREMENT AND PROCESSING OF THE VIBRATION SIGNALS

4.1. Acquisition of the vibration signals

The rotating machine has considered in this work is a rotor with two disks driven by an electrical motor (Guimarães and Silva, 2009). In this case, the vibration signals were measured using an accelerometer placed on the electrical motor case. The table 1 shows the parameters used in the acquisition of the vibration data. As the accelerometer was mounted on the motor case, the vibration signal may be contaminated by the noise, as the motor case vibration, the rotor foundation vibration, etc.

Table 1. Parameters values used in the acquisition of vibration signals from rotor.

Sampling Frequency	Number of Points	Acquisition Time	Sensitivity of Accelerometer
1250Hz	1024	0.8s	$1.05 mV/m/s^2$

For the analysis of the turning process, the vibration signals were measured by using an accelerometer has attached to the toolholder of the machine tool (Guimarães et al., 2011). The signal measured in the turning process also is extremely noisy since, there are several signal vibration and noise sources that usually contaminate the response in the time domain (Sick, 2002). The table 2 illustrates the parameters values used for the measuring of the vibration response from turning process.

Table 2. Parameters values used in the acquisition of vibration signals from turning process.

Sampling Frequency	Number of Points	Acquisition Time	Sensitivity of Accelerometer
2000Hz	1024	0.6s	$1.05 mV/m/s^2$

In both cases, it was used an accelerometer from 4214 model Bruel & Kjaer manufacturer for the acquisition of the vibration data. This model of accelerometer has an integrated signal pre-amplifier for increasing the output signal gain. Thus, it is not necessary to use a signal conditioning unit to increase the gain of output data. Therefore, the signals were directly connected to a data acquisition board from National Instruments manufacturer. Subsequently, the software Labview® was used to save the data file in a format txt. In the acquisition, the data were automatically filtered by the board in order to avoid the presence of "aliasing frequencies" in the vibration signal.

4.2. Signal processing

After the acquisition of the vibration data, they were processed in the software Matlab® version 2009, using the WD and CWD algorithms. In the bilinear time-frequency distributions, the sampling frequency used in the acquisition of the vibration data should be four times larger than the maxima frequency of the signal (Cohen, 1995). This is because of definition of the auto-correlation function in the discretized form, $R(t,\tau)$, used in the WD and CWD. For the processing of the time-frequency tools, it was used the functions wig2 (Wigner Distribution) and the wig2c (Choi-Williams Distribution) available in the Higher Order Spectral Analysis toolbox from Matlab®. The discretized vibration signals in the time domain were processed without any previous treatment, such as, filtering or windowing. After the processing of the WD and CWD, the contours of the time-frequency maps were generated and studied.

5. ANALYSIS OF THE RESULTS

5.1. Analysis of the rotating machine vibration

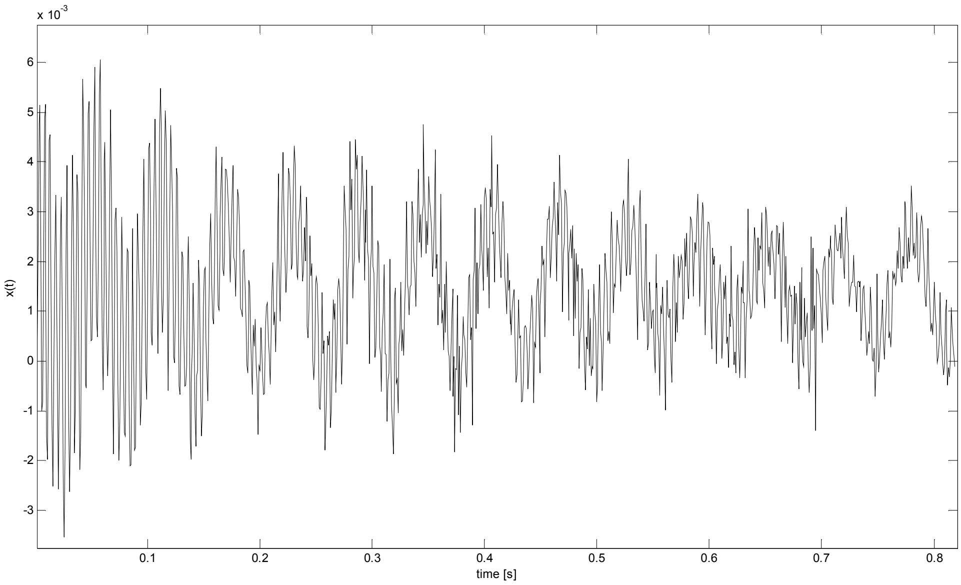

Figure 1. Vibration signal in the time domain measured after stopping the rotating machine.

The figure 1 shows the vibration signal in the time domain measured after stopping the rotating machine. It is interesting to note that the vibration amplitude decays with respect to the time due to the diminishing from rotating frequency and unbalanced forces. The figures 2 and 3 illustrate the WD and the CWD of the rotor vibration signal. I can be seen in these figures the presence of two frequencies components, 17 Hz and 205 Hz, approximately. The frequency of 17 Hz represents one of the natural frequencies from the rotor foundation. This frequency was identified using the experimental modal analysis has applied to the rotating machine (Guimarães and Silva, 2009).

By the analysis of the time-frequency maps, it can be observed that the frequency component of 17 Hz is constant with the time. Although this component is one of the resonances from rotor foundation, it was caused by the excitation forces after to turn-off the rotating machine. The frequency of 205 Hz has also observed in the figures is decreasing with respect to the time. This component is displayed by a slightly declined line in the WD and the CWD and the angular coefficient from this line represents the angular acceleration from rotor during the stopping. Furthermore, both time-frequency maps show that the vibration signal amplitude in the time-frequency domain is maximum in about 0.05s and 205 Hz. Thus, it is believed that this component is one of the harmonics from rotor critical frequency.

The time-frequency maps provided by the WD and the CWD have some advantages when compared to the STFT (Guimarães and Silva, 2009). For example, by using the CWT and STFT, it would not be possible to extract all transient vibration components, as can be seen in the time-frequency plane displayed by the WD and the CWD. In the WD showed in the fig. 2, the time-frequency map shows the behaviour with the time of the components of 17 Hz and 205 Hz and several interference terms. These cross-terms might difficult the analysis of the signal components. After filtering the interference terms in the CWD, the rotor vibration components are more easily identified in the time-frequency plane. One disadvantage of the CWD is that time-frequency distribution does not display the instantaneous frequency from signal (Lee et al., 2001). In the WD, the variation with respect to the time of the frequency component of 205 is clearly observed. On the other hand, in the CWD this component is partially destroyed.

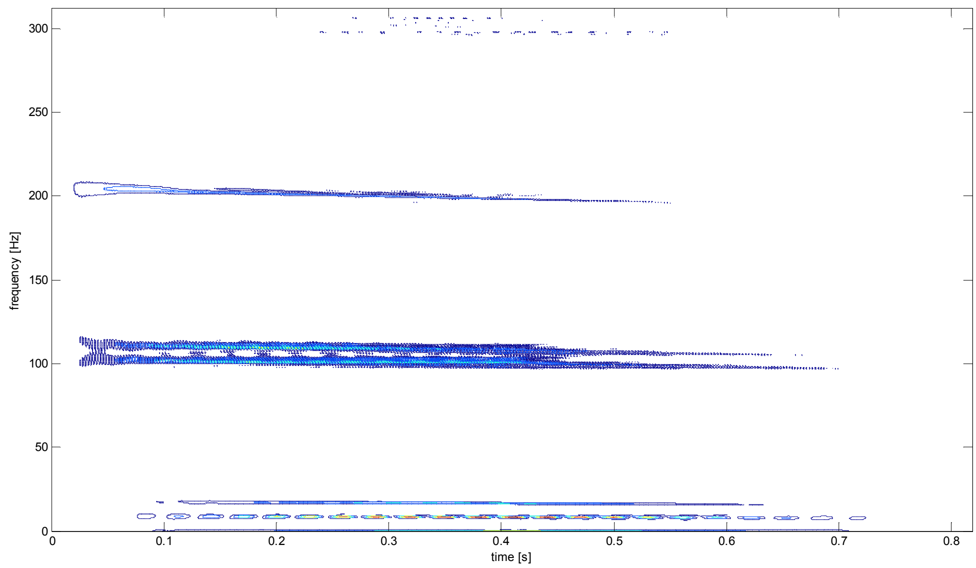

Figure 2. Wigner Distribution of the rotating machine vibration.

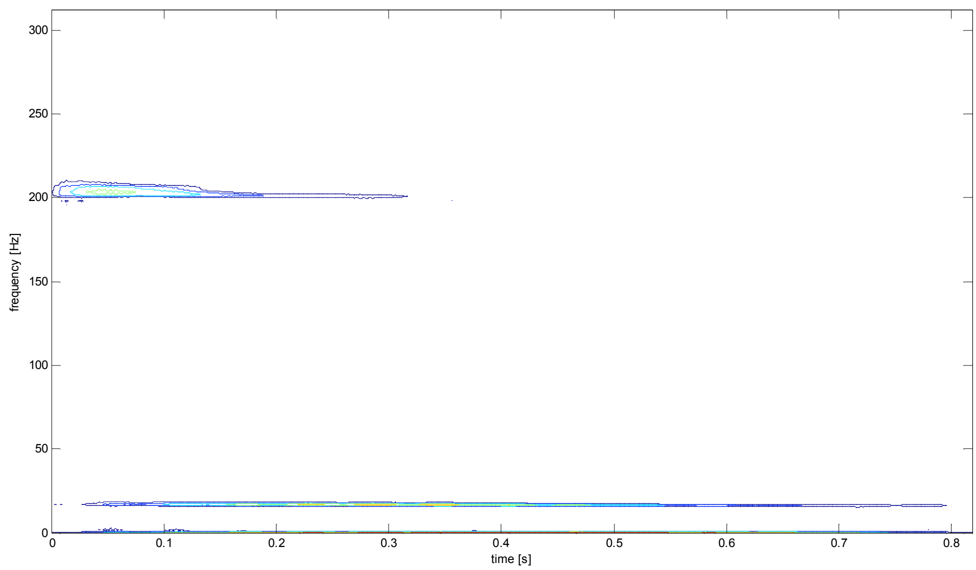

Figure 3. Choi-Williams Distribution of the rotating machine vibration.

5.2. Analysis of the turning process vibration

The figure 4 shows the vibration signal measured the turning process for a rotating of the piece of 630 rpm (10.5Hz). This signal in the time domain display a random pattern, since the vibration measured during the machining process has several components caused by the contact between the piece and the tool, by the movements of the mechanisms from tool machine, etc. Because of this, it not possible to identify any frequency component produced by the turning of the piece using the WD. Although the WD of the vibration signal is not shown, the interference terms mask the transient pattern in the time-frequency plane. Moreover, the noise measured during the process also difficult the detection of the components.

After filtering the cross-terms in the CWD, the components from vibration signal are easily detected in the fig. 5. For example, the time-frequency plane of the CWD displays some "vertical lines" caused by the short time events in the measured vibration signal. It is interesting to note that some of these transient components have periodicity pattern with a repetition period equals to the 0.09 s. This period of 0.09 s is associated with the piece rotation in the turning process, 1/10.5 Hz. It means that the machining process causes a vibration with amplitude modulation due to the cutting of the piece. This source of amplitude modulation could be identified by using others signal analysis techniques, as for example, the Power Cepstrum (Guimarães et al., 2011). In the time-frequency representations, it can be seen as the transient vibration caused by the modulation amplitude, as the frequency components produced by the turning process. It is observed in the fig. 6, two frequency components, 120 and 360 Hz probably caused by noise due to the sensor instrumentation. The both frequency components were provided by the other noise sources from tool machine.

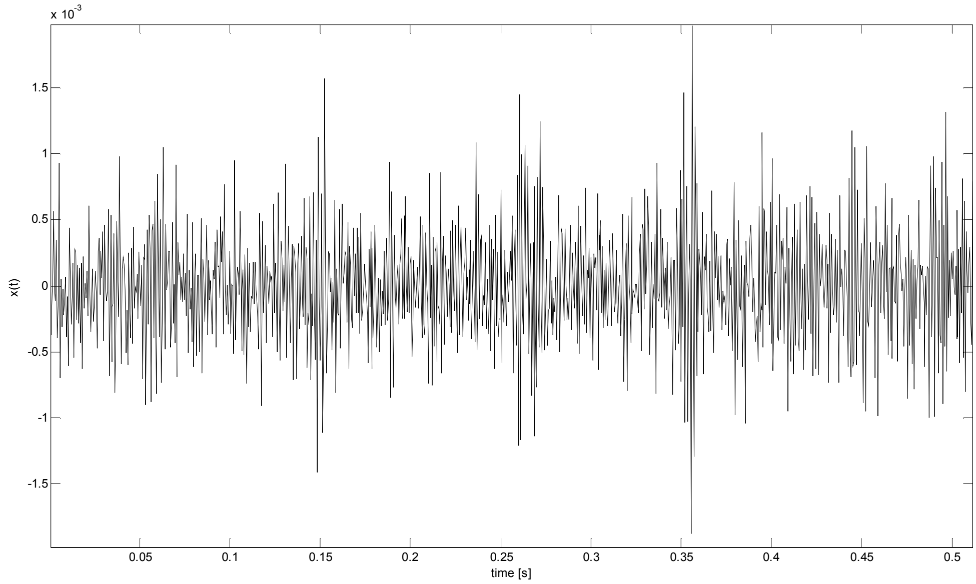

Figure 4. Vibration signal in the time domain measured during the turning process.

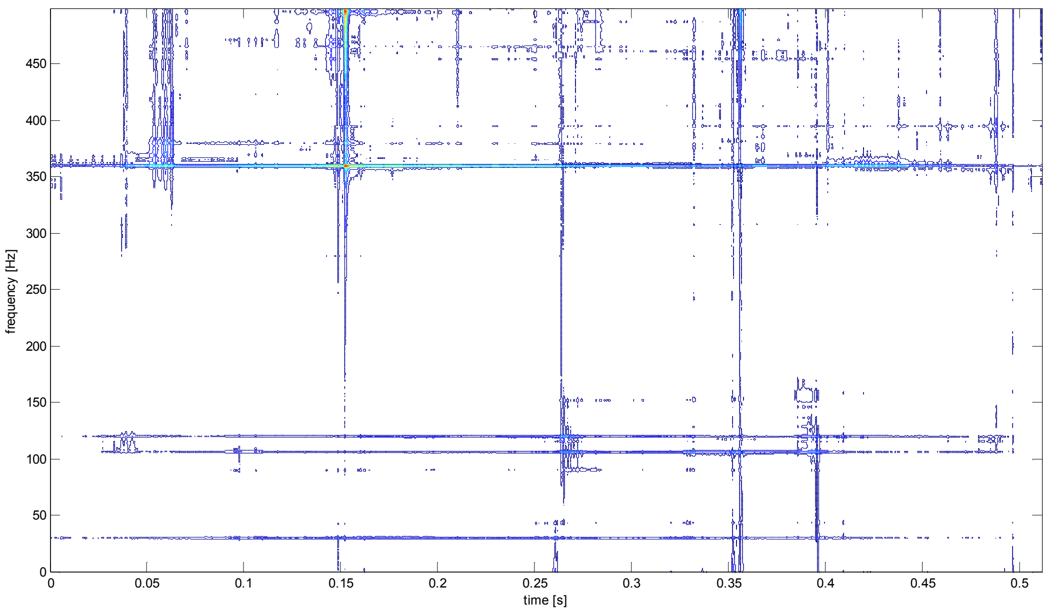

Figure 5. Choi-Williams Distribution of the vibration signal measured during the turning process with σ=0.05.

6. CONCLUSIONS

In this work, it was applied the Wigner Distribution (WD) and the Choi-Williams Distribution (CWD) for the identification of the frequency and amplitude modulation components of the mechanical systems response. In the first study case, the two abovementioned techniques were used in the frequency modulation vibration analysis due to the stopping of a rotating machine. In this situation, because of the instantaneous frequency property, the time-frequency map provided by the WD displayed the correct behaviour of the transient vibration caused by the acceleration from rotating machine. From the time-frequency maps, it was possible to identify some of natural frequencies of rotating machine foundation and the critical frequency of the rotor. The CWD was able to attenuate the cross-terms between the frequency components of the rotor vibration. Nevertheless, the CWD did not work well in the identification of the vibration signal feature like “chirp” due to rotor stopping.

In the second case, the WD and the CWD were used in the detection of amplitude modulation vibration components measured during the turning process of a piece. For this situation, it was not possible to detect any frequency component in the time-frequency plane provided by the WD because of the multi-component nature from vibration signal. In fact, the cross-terms in the WD from turning process vibration masked the vibration components. Otherwise, these interference terms were suppressed in the time-frequency map provided by the CWD. Hence, the CWD illustrated the frequency components generated by the tool machine and the amplitude modulation components due to the contact between the piece and the tool in the metal cutting process.

7. ACKNOWLEDGEMENTS

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