ENDOREVERSIBLE AND CONVENTIONAL ANALYSIS OF A RANKINE POWER CYCLE

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Abstract. Endoreversible engines are irreversible engines where all irreversibilities are restricted to the coupling of the engine with the external heat source. Finite energy sources vary their temperature and affect, consequently, the cycle efficiency. Endoreversible analysis can be suitable to study the power cycle of solar plants that convert solar radiation into heat of a thermal fluid and transfer the heat to a working fluid, usually water-vapor. Along this process, the thermal fluid varies its temperature from a maximum, defined by its physical properties, up to a minimum, defined by the cold source, typically cooling water. A power Rankine cycle, is analyzed using two procedures: a) Conventional analysis, by means of dedicated software and b) Endoreversible model, represented by an infinite sequence of elementary Carnot cycles. The endoreversible model is described considering large temperature variations of the heat source, with a temperature profile that represents water preheating, evaporation and vapor superheating. Maximum temperature of the elementary Carnot cycles follows the temperature profile of the heat source. The cold reservoir is at constant temperature. The mechanical work, by unit vapor mass, for preheating, evaporation and steam superheating regions can be expressed by simple differential quantities of the enthalpy transferred from the hot source to the working fluid. Comparison between both procedures is made calculating the net power and efficiency against evaporation temperature. Results obtained with the endoreversible model show good agreement with conventional analysis.

Keywords: *Rankine cycle, Endoreversible model, Sensitivity analysis*

1. INTRODUCTION

Endoreversible engines are irreversible engines where all irreversibilities are restricted to the coupling of the engine to the external world (Vos, 1992). The Curzon-Ahlborn engine in Fig.1 is an example of an endoreversible machine. It considers the case of finite rates of heat transfer to and from a Carnot engine (finite time analysis). After maximizing the power output, it is derived a simple expression for the efficiency of the power cycle, considerably different and smaller than the well known Carnot efficiency. The features of the Curzon-Ahlborn engine are: (a) Two reservoirs, one at high temperature (T_H) and one at low temperature (T_C); (b) Two irreversible components which limit the heat flows QH and QC from the hot source to the engine and from the engine to the cold source, with temperatures T_h and T_c respectively and (c) A reversible Carnot engine between the heat reservoir at temperature (T_h) and the cold reservoir at temperature (T_c).

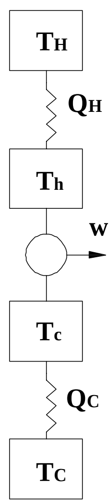

Figure 1– Curzon-Ahlborn engine.

The Rankine cycle is a thermodynamic cycle which converts heat into work. This cycle generates about 80% of all electric power used throughout the world, including solar thermal, biomass, coal and nuclear power plants. The heat is supplied externally to a closed loop, which usually applies water as the working fluid.

Ondrechen et al. (1981) discussed the production of work from a source with a finite heat capacity (finite energy analysis). They examine the conversion of heat from such a source, first by a single Carnot engine and then by a sequence of Carnot engines. The optimum values of the operating temperatures of these engines are calculated. The work production and efficiency of a sequence with an arbitrary number of engines was derived, and it is shown that the maximum available work can be extracted only when the number of cycles in the sequence becomes infinite.

ROLIM et al. (2009) developed an analytical model for a solar thermal electric generating system with linear focus parabolic collectors. In that work, the conventional Rankine cycle is treated as an endoreversible Carnot cycle, whereby the mechanical and electric power is calculated.

Solar systems convert solar radiation into heat of a thermal fluid (synthetic oil), transferred then to a working fluid, (water-vapor) (Fig.2). Along this process the thermal fluid varies its temperature from a maximum (T_H) defined by the physical properties of the thermal fluid, up to a minimum ($T_{H,\ min}$). Following this behavior, the working fluid (water-vapor) varies its temperature from a maximum (T_h) up to a minimum (T_c), defined by the cold source, typically cooling water.

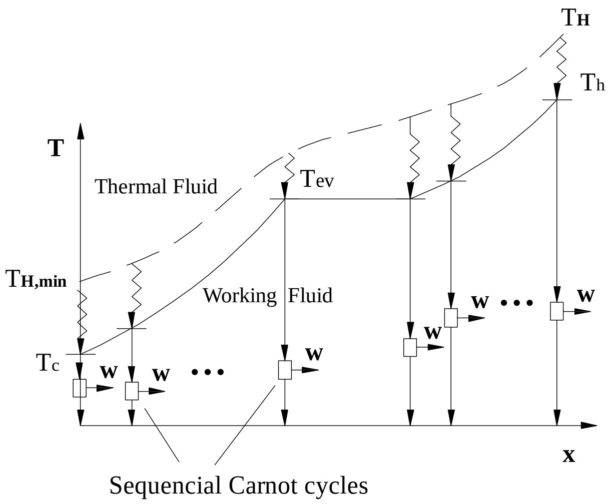

Figure 2– Temperature profile of boiler showing the sequential Carnot cycles.

We describe in this work a Rankine cycle, using the endoreversible finite energy analysis, constituted by an infinite sequence of Carnot cycles. In order to verify its validity we compare results of Net Work and Efficiency obtained with the endoreversible procedure with those calculated by a conventional analysis of a Rankine cycle. Special attention has been given to the study of the influence of the evaporation temperature on the cycle efficiency since, as shown by Rolim et al. (2009), Net Power plays the role of an optimization parameter.

2. RANKINE CYCLE

The ideal cycle for a simple engine unit of vapor is the Rankine Cycle, as showed in Fig.3. The working fluid undergoes the following series of internally reversible process (Wylen 2002):

- Process 1-2: Isentropic compression in the pump from state 1 to state 2 in the compressed region.
- Process 2-3: Heat transfer to the working fluid as it flows at constant pressure through the boiler to complete the cycle.
- Process 3-4: Isentropic expansion of the working fluid through the turbine from saturated vapor at state 3 to the condenser pressure, state 4.
- Process 4-1: Heat transfer from the working fluid as it flows at constant pressure through the condenser, coming out as saturated liquid at state 1.

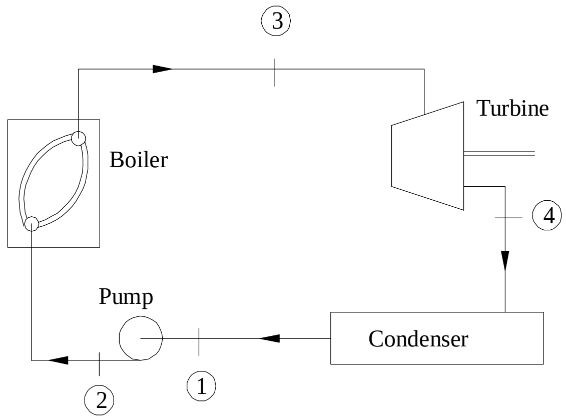

Figure 3 – Conventional Rankine Cycle

2.1 Influence of the evaporation temperature

The influence of maximum vapor pressure (or evaporating temperature T_{ev}) of the cycle is shown in Fig. 4, where two cycles are compared (1, 2, 3 and 4) and (1, 2', 3', 4').

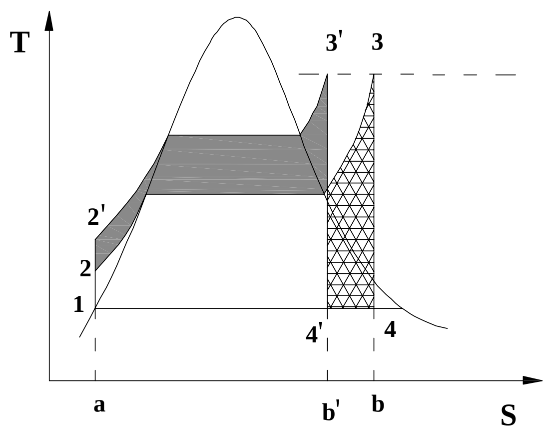

Figure 4 – Effect of maximum pressure on net work of the Rankine cycle

For each cycle the maximum temperature of the steam and the output pressure of the turbine are the same. The temperature of the states 3 and 3´ is equal to Th. The temperature of the states 1, 4 and 4´is equal to T_c.

It is verified that the cycle with higher evaporation temperature (1, 2', 3' and 4') rejects less heat than the cycle (1, 2, 3 and 4) (the difference is represented by an area equal to (4-b-b'-4'-4)).

The net work of the cycle (1, 2', 3' and 4') increases by the amount of the gray area (2-2'-3'-2) and decreases by the amount of the double-hatched area (4-4'-3-4). If those areas are approximately equal the cycle (1, 2', 3' and 4') is more efficient due to the smaller heat rejection. Thus, for the same highest and lowest temperatures (T_h) and (T_c) (corresponding to states 3 and 3´ and states 1, 4 and 4´ respectively) the choice of the evaporation temperature becomes extremely important. By means of a sensitive analysis, it will be shown that the variation of net power and efficiency of the cycle with evaporation temperature, in various cases of practical interest, reaches a maximum value.

In what follows, simulation of the Rankine cycle by conventional and endoreversible procedures are presented. For a given maximum temperature of the cycle the variation of net energy and efficiency with evaporative temperature is obtained.

2.2. Estimate of net work by conventional and endoreversible procedures

2.2.1 Conventional analysis

Neglecting the pressure loss along the pipes, the work per unit mass spent by the pump is (w_p). The work produced by the expansion of the turbine is (w_T). The heat transferred in the boiler is (q_b) and in the condenser is (q_c). They can be calculated as follows:

$$w_p = {(P_1 - P_2)}/{\rho} = h_1 - h_2 \quad (1)$$

$$w_T = h_3 - h_4 \quad (2)$$

$$q_b = h_3 - h_2 \quad (3)$$

$$q_c = h_1 - h_4 \quad (4)$$

$$w_{net} = w_T - |w_p| \quad (5)$$

where P is the pressure, ρ is the density, h is the enthalpy and w_{net} the net work. The efficiency of Rankine cycle, neglecting heat losses in the pipeline, is $\eta_{Rankine} = (w_{net})/q_b$.

Using equations (1) to (5) the simulation was implemented for different temperatures of superheated vapor (T_h) at the boiler outlet and varying the maximum pressure (or evaporation temperature - T_{ev}).

2.2.2 Endoreversible procedure

In this section we describe the endoreversible analysis to calculate the performance of a Rankine cycle. The whole cycle is composed by an infinite sequence of elementary Carnot cycles, driven by a hot source with a temperature profile shown in Fig. 1. It can be noted the existence of three regions where the water passes through, the preheating, the evaporation and the vapor superheating phase.

The efficiency of a reversible Carnot engine that works between two reservoirs with temperatures (T_h) and (T_c), is $\eta_{carnot} = 1 - T_c/T_h$ Then, for each Carnot cycle of the infinite sequence (Fig.2), the elementary mechanical work, by unit vapor mass, for the three regions (preheating, evaporation and superheating) can be expressed as:

$$dw_w = \left(1 - \frac{T_c}{T}\right) dh_w \quad (6)$$

$$dw_{ev} = \left(1 - \frac{T_c}{T_{ev}}\right) dh_{ev} \quad (7)$$

$$dw_v = \left(1 - \frac{T_c}{T}\right) dh_v \quad (8)$$

where dw_w is the elementary work produced in the preheating region, dw_{ev} in the evaporation and dw_v in the superheating region. Differential quantities of enthalpy, dh_w, dh_{ev} and dh_v, are transferred from the thermal fluid to the preheating water, evaporating water and superheating vapor. The cold reservoir is at constant temperature (T_c) The symbol T denotes the temperature of working fluid (water-vapor).

The integration of these equations yield the work during preheating (w_w), evaporation (w_{ev}) and superheating (w_v) stages of the cycle. The integration limits are T_c and T_{ev}, for w_w; T_{ev} and T_h for w_v. Along the evaporative part of the cycle the result is straightforward and equal to:

$$w_{ev} = \left(1 - \frac{T_c}{T_{ev}}\right) h_{ev} \quad (9)$$

The total work, per unit vapor mass (wt), is, then, given by:

$$w_t = w_w + w_{ev} + w_v \quad (10)$$

The integration of Eqs. (6) and (8) depends on the expressions used for enthalpy. Thermodynamic properties of water substance have been implemented using the thermodynamic property correlation of Harr, Gallagher and Kell (NBS/NRC Steam Tables ,1984). The correlations are valid up to a pressure of 815 bar. In this work, the following group of expressions is used. The superheated vapor enthalpy (h_v), vaporization enthalpy (h_{ev}) and the liquid water enthalpy (h_w) are treated as second order polynomial equations with the coefficients as function of evaporating temperature (Eqs. (11) to (13)). The equation (11), proposed for (h_v) is valid from temperature of 220 ^oC up to superheating vapor temperature of 600 ^oC.

$$h_v = a_v + b_v T + c_v T^2 \quad (11)$$

$$h_{ev} = a_v + b_v T_{ev} + c_v T_{ev}^2 - (a_w + b_w T_{ev} + c_w T_{ev}^2) \quad (12)$$

$$h_w = a_w + b_w T + c_w T^2 \quad (13)$$

where a_w, b_w, c_w, a_v, b_v and c_v are fourth-order polynomial equations of the evaporation temperature. For example, for the parameter a_w the expression is $(a_0 + a_1 T_{ev} + a_2 T_{ev}^2 + a_3 T_{ev}^3 + a_4 T_{ev}^4)$ with temperatures in Kelvin [K] and the values from a_o up to a_4 are presented in Tab. 1. The equations for obtaining the coefficients (a_w, b_w, c_w, a_v, b_v and c_v) are valid for the rage of evaporating temperature from 220 ^oC to 355 ^oC.

Table 1– Coefficients of fourth-order polynomial equations for a_w, b_w, c_w, a_v, b_v and c_v.

	a_0	a_1	a_2	a_3	a_4
a_w	7.46428551 x 10^4	-5.58680676 x 10^2	1.54711030	-1.90945392 x 10^{-3}	8.89431785 x 10^{-7}
b_w	-3.49568366 x 10^2	2.60192227	-7.18706288 x 10^{-3}	8.84344611 x 10^{-6}	-4.10568066 x 10^{-9}
c_w	4.00099163 x 10^{-1}	-2.93458559 x 10^{-3}	8.08078020 x 10^{-6}	-9.90624835x10^{-9}	4.58103012 x 10^{-12}
a_v	-2.78596543 x 10^6	2.05933741 x 10^4	-5.70709910 x 10^1	7.03806132 x 10^{-2}	-3.26259127 x 10^{-5}
b_v	6.91273722 x 10^3	-5.10236397 x 10^1	1.41318640 x 10^{-1}	-1.74142914 x 10^{-4}	8.06454407 x 10^{-8}
c_v	-4.28153038	3.16005184 x 10^{-2}	-8.74833902 x 10^{-5}	1.07742419 x 10^{-7}	-4.98597117 x 10^{-11}

3. RESULTS AND DISCUSSION

The procedures described in subsection 2.2 are used to obtain the behavior of net power and efficiency against evaporation temperature. The conventional analysis is accomplished with the EES Software. Results of the endoreversible method are obtained with the MathCad Software. The condensing pressure and temperature of superheated vapor, are kept constant while varying the evaporation temperature.

The net work and consequently the efficiency are calculated through the heat acquired by the working fluid, i.e., considering that there are no losses during heat exchange between the hot reservoir and the working fluid.

A comparison of the net work calculated by conventional and endoreversible analysis of Rankine cycle is shown in Figs. 5 to 7 for three different superheating temperatures (boiler outlet) (T_h= 400 ^oC, T_h= 500 ^oC and T_h= 600 ^oC, respectively). The net work is plotted for evaporation temperatures between 220 ^oC and 370 ^oC.

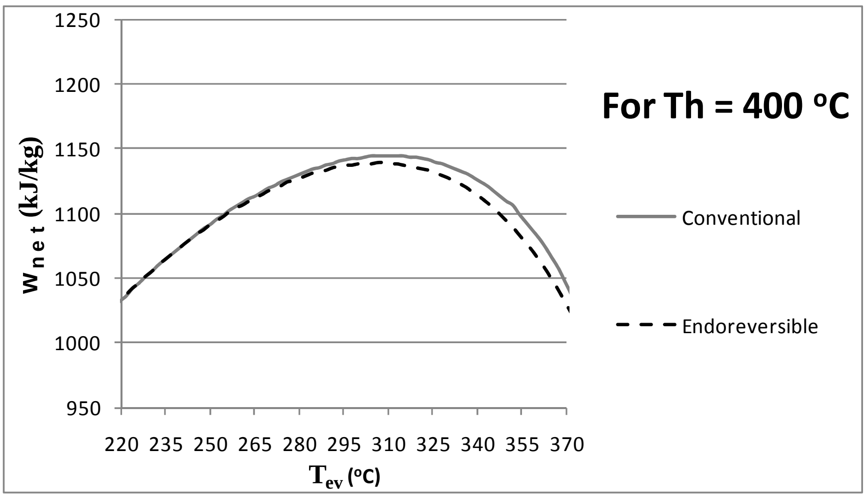

Figure 5 – Comparison between net work obtained with conventional and endoreversible procedures as a function of evaporation temperatures and maximum cycle temperature of 400 ^oC

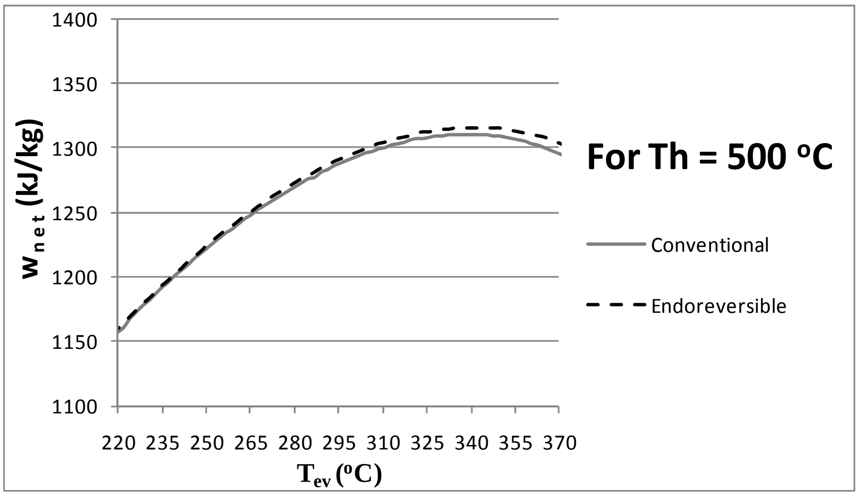

Figure 6 – Comparison between net work obtained with conventional and endoreversible procedures as a function of evaporation temperatures and maximum cycle temperature of 500 °C

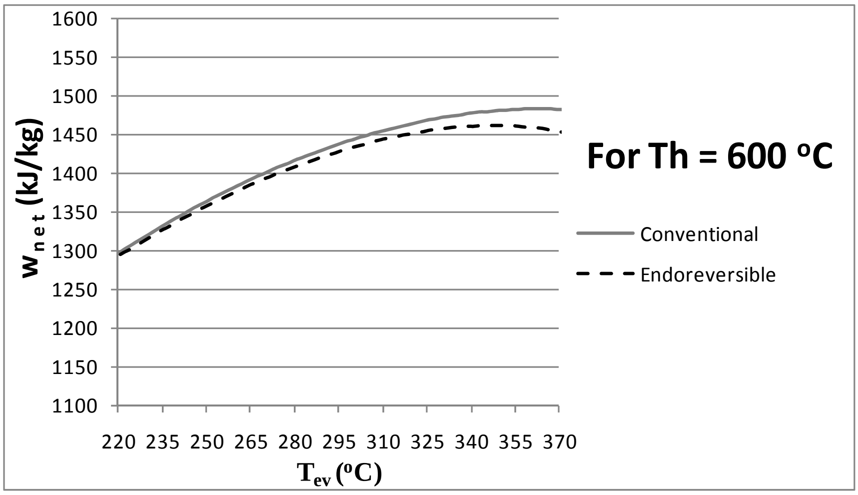

Figure 7– Comparison between net work obtained with conventional and endoreversible procedures as a function of evaporation temperatures and maximum cycle temperature of 600 °C

It can be observed that for the superheating temperatures of 400 and 500 °C the net work produced by the Rankine cycle presents maximum values for evaporation temperatures around 310 °C and 340 °C respectively. As the superheating temperature increases (i.e. the case of 600 °C), the point of maximum net work tends to reach the critical point of water that is 374.15 °C.

The curves obtained using the endoreversible model are quite satisfactory. When compared to the conventional procedure. The maximum local deviation observed is of 2.1%, for superheating temperature of 600 °C. The root mean square deviation (RMSD) calculated between the curves produced with the two methods are 0.8%, 0.3% and 1.0%, for superheating temperatures of 400 °C, 500 °C and 600 °C, respectively.

The cycle efficiency is given by the ratio of net work (w_{net}) and the enthalpy variation between inlet and outlet temperature of boiler (see Fig 3):

$$\eta = {w_{net}}/{(h_3 - h_2)} \tag{14}$$

Results obtained for the efficiency of the cycle, calculated with the two methods, are shown in Figs. 8 to 10. It is verified that only the cycle with superheating temperature of 400 °C reaches a maximum efficiency, at the evaporation temperature of 365 °C.

The differences between the curves of efficiencies calculated with both methods are lower than that observed for the net work. The maximum local deviation between the efficiency curves is 0.6% for the superheating temperature of 500°C. The (RMSD) calculated between the curves of efficiencies are 0.1%, 0.3% and 0.2%, for superheating temperatures of 400 °C, 500 °C and 600 °C, respectively.

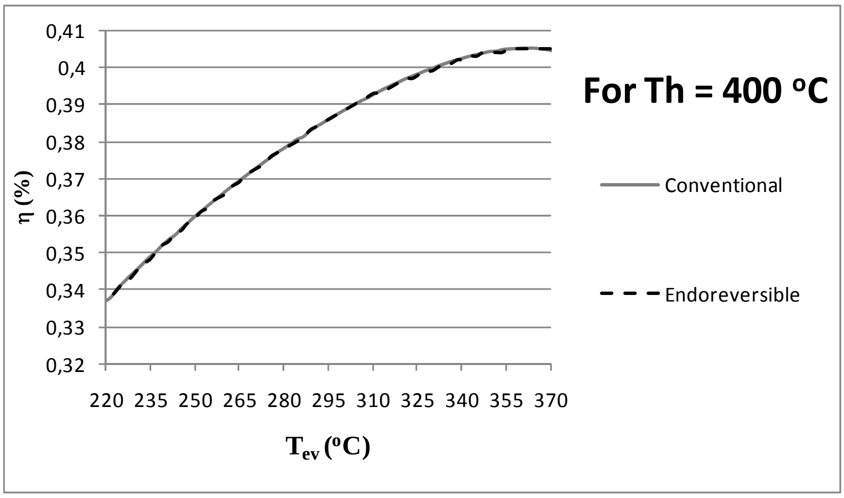

Figure 8 – Efficiency of Rankine cycle using conventional and endoreversible procedures for maximum temperature of 400 °C.

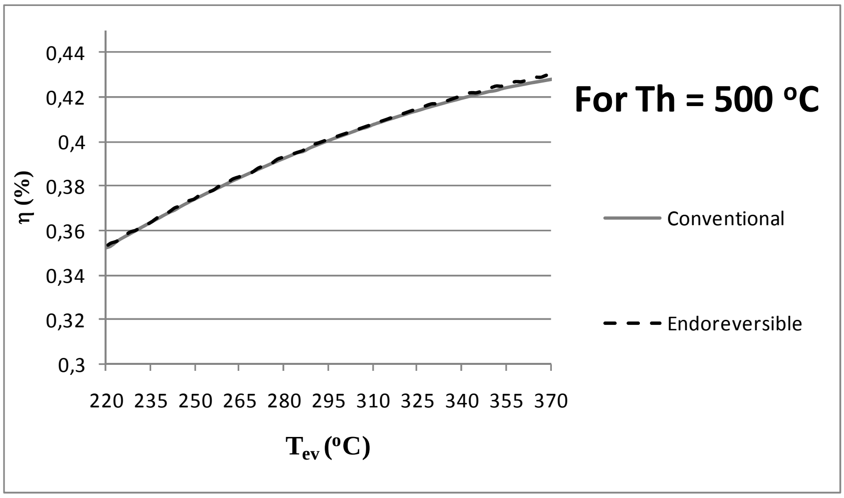

Figure 9 – Efficiency of Rankine cycle using conventional and endoreversible procedures for maximum temperature of 500 °C.

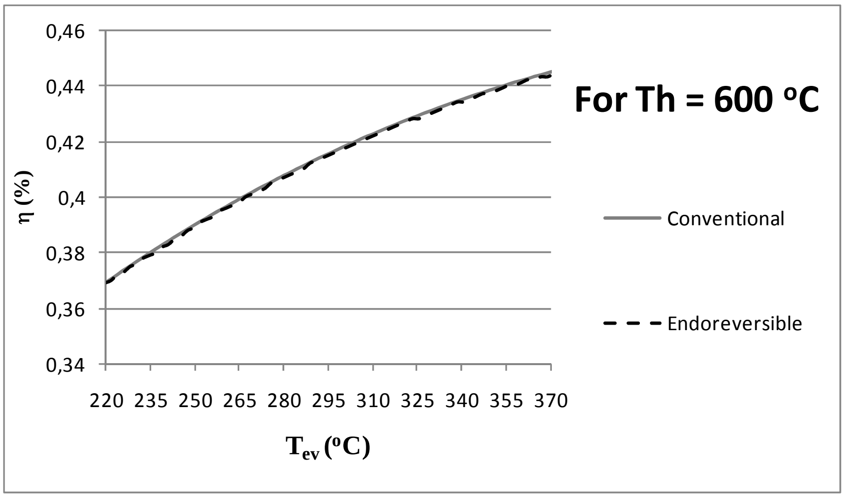

Figure 10 – Efficiency of Rankine cycle using conventional and endoreversible procedures for maximum temperature of 600 °C.

Under the conditions established in this study (Section 2.1) it is verified that the temperature that defines the maximum net work does not agree with the temperature for maximum efficiency. Considering the interest for choosing the best operating evaporation temperature, an election has to be made between the best condition either for net work or efficiency. Maximum net work is related with the shortest return time for the plant building investment and maximum efficiency with the shortest time to recover primary energy expenses. If a large difference exists between them, the best choice might be an intermediate temperature. The presented simulations suggest, for superheating temperature of 400 ºC, the election of an evaporating temperature in the interval between 310 º C and 365 ºC; for vapor at 500 ºC T_{ev} should be between 340 ºC and 374 ºC and for superheating temperature of 600 ºC T_{ev} should be somewhat near the critical temperature of water.

Finally it is interesting to compare those efficiencies (Figs. 8 to 10) with the Carnot efficiencies calculated with the superheated vapor temperature and condensation temperature (saturation pressure of 8 kPa). For the superheating temperatures of 400 ^oC, 500 ^oC and 600 ^oC, Carnot efficiencies are equal to 53.0%, 59.0% and 64.0% while the maximum efficiencies that can actually be obtained with the Rankine cycle are 40.5% (Fig. 8), 43.0% (Fig. 9) and 44.5% (Fig 10), respectively.

4. CONCLUSIONS

An analytic solution describing an endoreversible model to simulate a Rankine cycle has been derived. The model is given by a set of three equations, enabling to calculate the net power and efficiency of the cycle. In order to verify its validity, results of the endoreversible model are compared with those generated using conventional analysis of the Rankine cycle. Net work and efficiency calculated with endoreversible and conventional procedures, as a function of the evaporation temperature, show very good agreement (RMSD less than 1.0%).

Using 400 ^oC and 500 ^oC superheating temperatures it is shown that the net work reaches a clear maximum at 310 ^oC and 340 ^oC. As the superheating temperature increases to 600 °C (Fig. 7) this maximum occurs at the critical point of water (374.15 ^oC). For the efficiency, only in the cycle with superheating temperature of 400 ^oC a maximum is observed at 365 ^oC.

Under the conditions established in this study (Section 2.1) it is verified that the evaporation temperature which corresponds to maximum net work does not agree with the temperature for maximum efficiency. If a significant difference exists between them, the best choice for the most rapid return of the investment in plant building and in the primary source expenses might be an intermediate temperature.

A comparison between calculated efficiencies for the Rankine cycle with the Carnot efficiencies shows that for superheating temperatures of 400 ^oC, 500 ^oC and 600 ^oC, Carnot efficiencies are equal to 53.0%, 59.0% and 64.0% while the maximum efficiencies that can actually be obtained with the Rankine cycle are 40.5%, 43.0% and 44.5%, respectively.

Systems using Rankine cycle to convert thermal into electric energy can be simulated with good accuracy with the endoreversible model. Its simplicity enables to simulate systems subject to large variations of the primary energy source, like solar plants, in a more simple way than that required by conventional procedures.

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