DEVELOPMENT OF A COMPUTATIONAL TOOL USING SOFTWARE EXCEL FOR THE TEACHING ASSISTANCE IN METHOD OF FINITE DIFFERENCES FOR PLOTTING THE DISTRIBUTION OF TEMPERATURES

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Abstract. The finite differences method is a numerical method widely used in solving engineering problems. This paper presents a computational tool for teaching implemented in MICROSOFT EXCEL™, on account of its availability, versatility, easy handling and especially its fast response, with the aim of plotting the temperature distribution in two-dimensional square plate in steady state.

Keywords: finite differences method, computational tool, temperature distribution

1. INTRODUCTION

The development of high-speed computers allowed numerical methods were increasingly used in practical engineering problems where the analytical methods were not so effective.

According to Silva Neto (2007), the classical analytical methods allow calculating the exact response of the structure variables at every point. These solutions, however, are only known for some cases, fleeing found practical applications in day-to-day. It is necessary to use approximate procedures that can be applied in general character, regardless of how the structure and boundary conditions, provided that within the acceptable accuracy of the engineering problem. This alternative path to conventional analytical procedures is the central idea of the Finite Differences Method (FDM). Its efficiency and development are due to two factors: applications of matrix methods in mechanics and the use of computers.

According to Hoffman (2001), the FDM is a numerical procedure that solves partial differential equations (PDE) by discretizing the continuous physical domain into a finite discrete mesh, approaching each partial derivative in PDE by algebraic finite differences approximations. By substituting these approximations in the PDE, it will be obtained an algebraic equation of finite differences for the dependent variable.

The main objective was develop a didactic tool for better understanding of the FDM through plotting temperature distribution for steady two-dimensional square plates, for this it was used the software MICROSOFT EXCEL™, commonly used by students engineering due to their availability, versatility, easy handling.

2. THEORECTICAL FOUNDATIONS

Often the engineers are faced with problems that the mathematical model has no exact solution or it's extremely complex so its analytic solution becomes impractical, like problems of heat transfer (nonlinear). If it is possible to accept an approximate result, numerical methods may be adequate tools for its solution.

Among a range of numerical methods, the most commonly used in engineering are: the FDM, finite element method and finite volume method. The FDM can be used to solve boundary value or initial value problems, involving partial or ordinary differential equations. Thus, this method can be used to solve the equations of concentrated or distributed models. The technique consists in replacing each derivative or differential equations by FDM approximation or finite increments of variables, as shown in the equation below.

$$\frac{dT}{dx} \cong \frac{\Delta T}{\Delta x} \tag{1}$$

The purpose of the FDM is transform a problem compounded by differential equations in other one formed by algebraic equations. The first step is called the domain discretization of the independent variable. The discretization consists in dividing the domain into a specific number of subdomains. For a semi-infinite domain, there are infinite subdomains. When the domain is finite, the number "i" is also a subdomain. In any case, it must be stipulated the number of points that define the subdomains, which in the case of a finite domain, are equal to "i +1" in number.

The second step is to generate approximations for the derivatives of the dependent variables that appear at the differentials in discrete points. Finally, apply the ordinary differential equations to discrete points, replacing the approximations obtained.

Thus, for this study, which involves a problem of temperature distribution on a thin square plate with a discrete mesh in steady state, Fig. 1, we have a two-dimensional problem described by the differential equation of heat conduction or simply the Laplace Equation as Eq. (2).
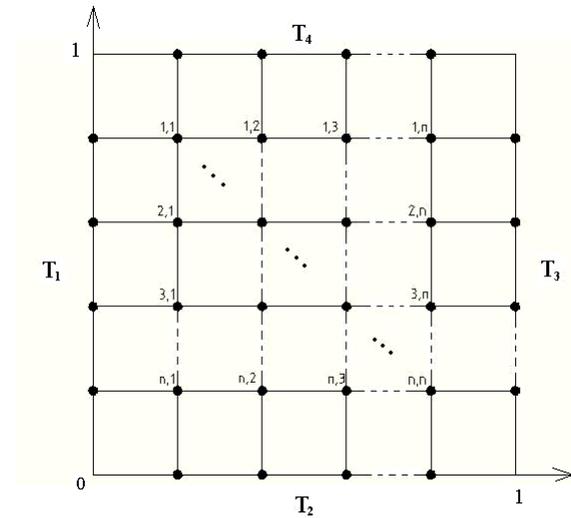
Figure 1. Thin square plate with generic discrete mesh

$$\nabla^2 T(x, y) = \frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} = 0 \tag{2}$$

Where x and y are spatial coordinates and T is the temperature.
The boundary conditions of the first kind are given by:

$$T(0, y) = T_1 \ , \ T(x,0) = T_2 \ , \ T(1, y) = T_3 \ , \ T(x,1) = T_4 \tag{3}$$

Note that T_1, T_2, T_3 and T_4 are known and, therefore, will be the input data.
If Eq. (2) is expanded in Taylor series and assuming that since the cubic term from all other elements of the series will be negligible, we obtain:

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0 \tag{4}$$

Where the index i +1 is related to the forward element, i-1 with the backward, j +1 with the above and j-1 with below. For simplicity, we can admit $\Delta x = \Delta y$. This is easily done when the plate are discretized by a square and uniform mesh. Thus, Eq. (4) is solved for $T_{i,j}$.
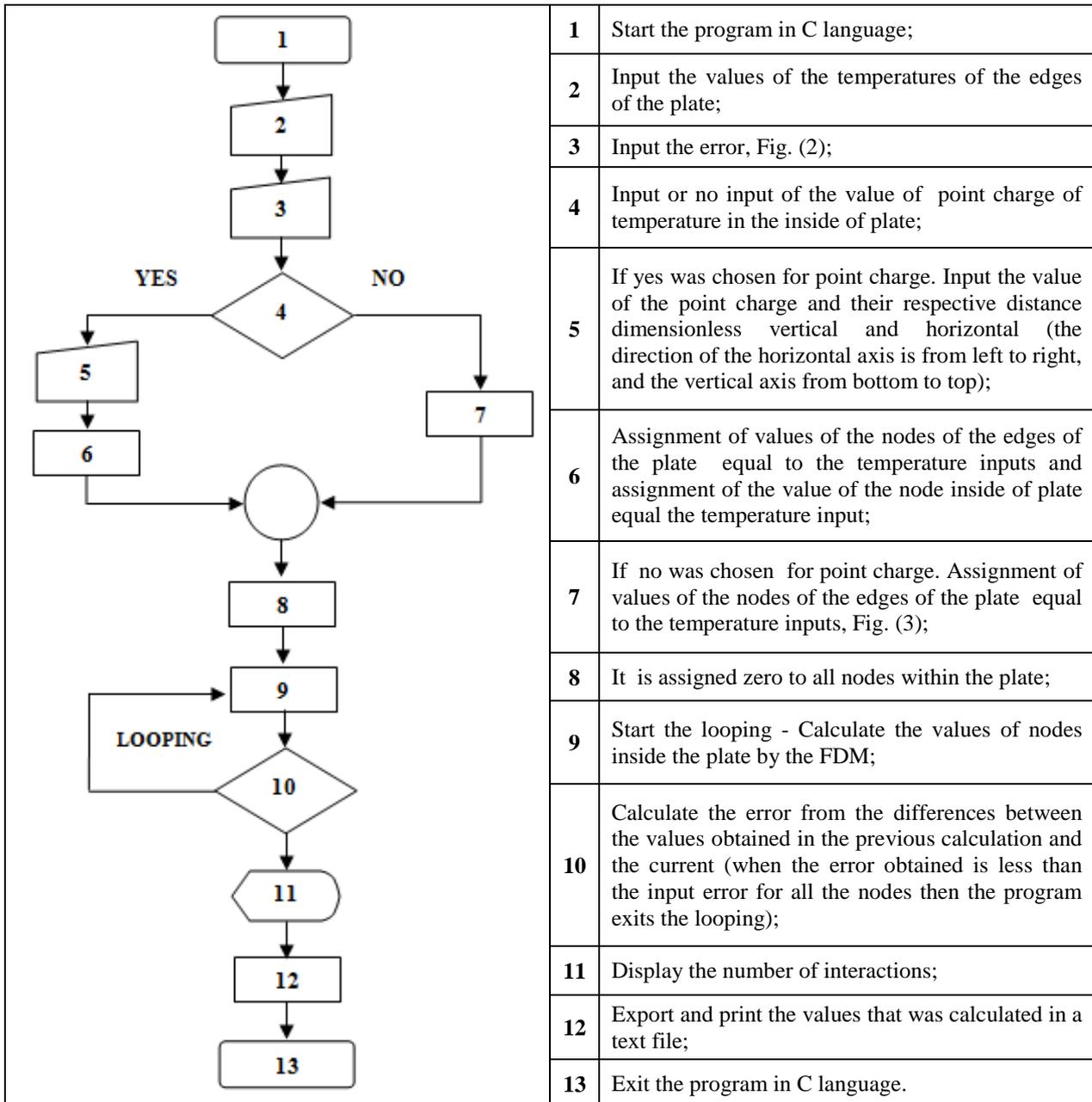
$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4} \tag{5}$$

The above equation provides the temperature at position i, j, depending on the arithmetic average of the temperatures forward, backward, above and below. When you have an equation of this type, where the new value is calculated in terms of unknown values, it is called the method implicit. Then, it can be obtained a linear system, where Eq. (5) is the fundamental equation that might be expanded for each point inside the plate, easily programmable and was used to develop the computer code discussed in this paper.

3. PROCEDURE

Just below is a block diagram representing the operation of the algorithm developed in C language. The algorithm using the equations of the FDM and works with a fixed mesh of 40.401 elements . After calculation shows the values of 40,000 nodes.

Table 1. Block diagram of the algorithm in C Language.

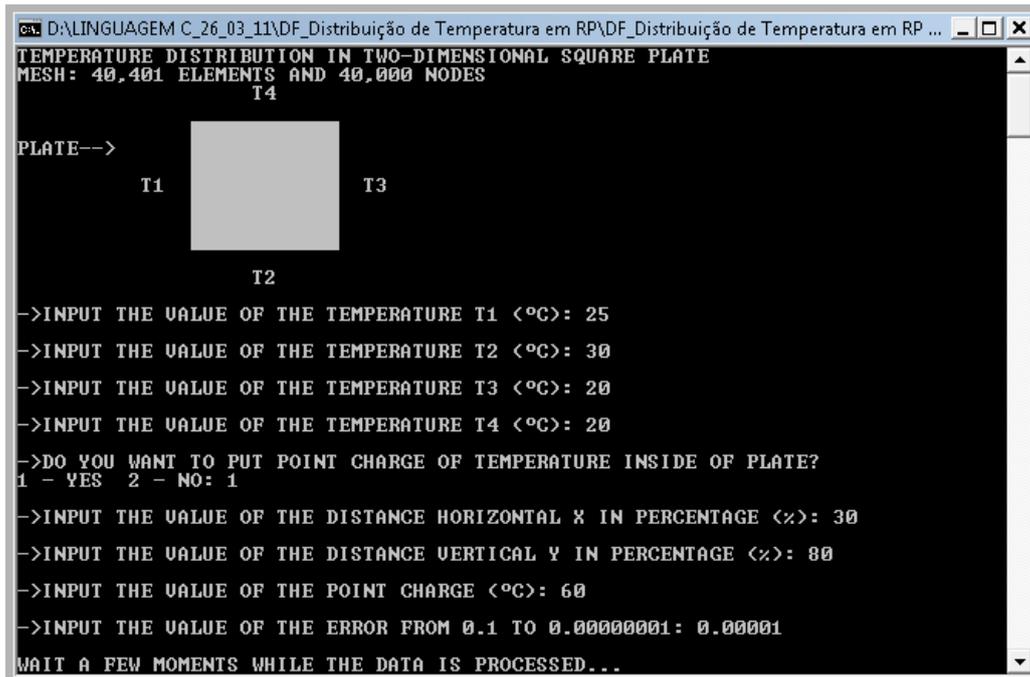|	
1	Start the program in C language;
2	Input the values of the temperatures of the edges of the plate;
3	Input the error, Fig. (2);
4	Input or no input of the value of point charge of temperature in the inside of plate;
5	If yes was chosen for point charge. Input the value of the point charge and their respective distance dimensionless vertical and horizontal (the direction of the horizontal axis is from left to right, and the vertical axis from bottom to top);
6	Assignment of values of the nodes of the edges of the plate equal to the temperature inputs and assignment of the value of the node inside of plate equal the temperature input;
7	If no was chosen for point charge. Assignment of values of the nodes of the edges of the plate equal to the temperature inputs, Fig. (3);
8	It is assigned zero to all nodes within the plate;
9	Start the looping - Calculate the values of nodes inside the plate by the FDM;
10	Calculate the error from the differences between the values obtained in the previous calculation and the current (when the error obtained is less than the input error for all the nodes then the program exits the looping);
11	Display the number of interactions;
12	Export and print the values that was calculated in a text file;
13	Exit the program in C language.

Figure 2. Input the values in the program
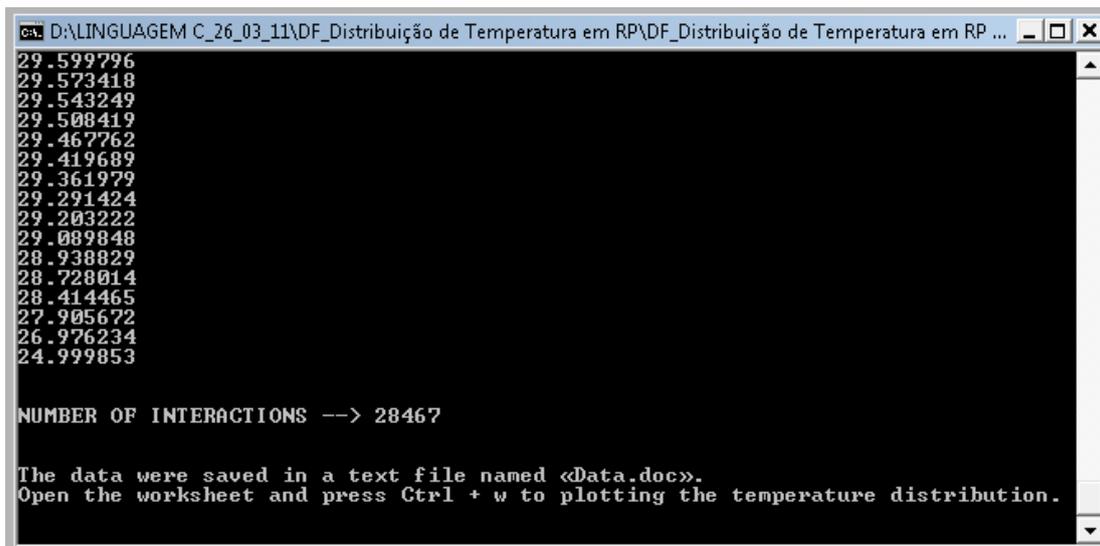

Figure 3. Output of the data and number of interactions

After the closure of the program in C language. The user must open the *MICROSOFT* EXCEL™ worksheet and activate the macro of the worksheet with the command Ctrl + w. Then, the data is imported from text file and is plotted temperature distribution with color scales.
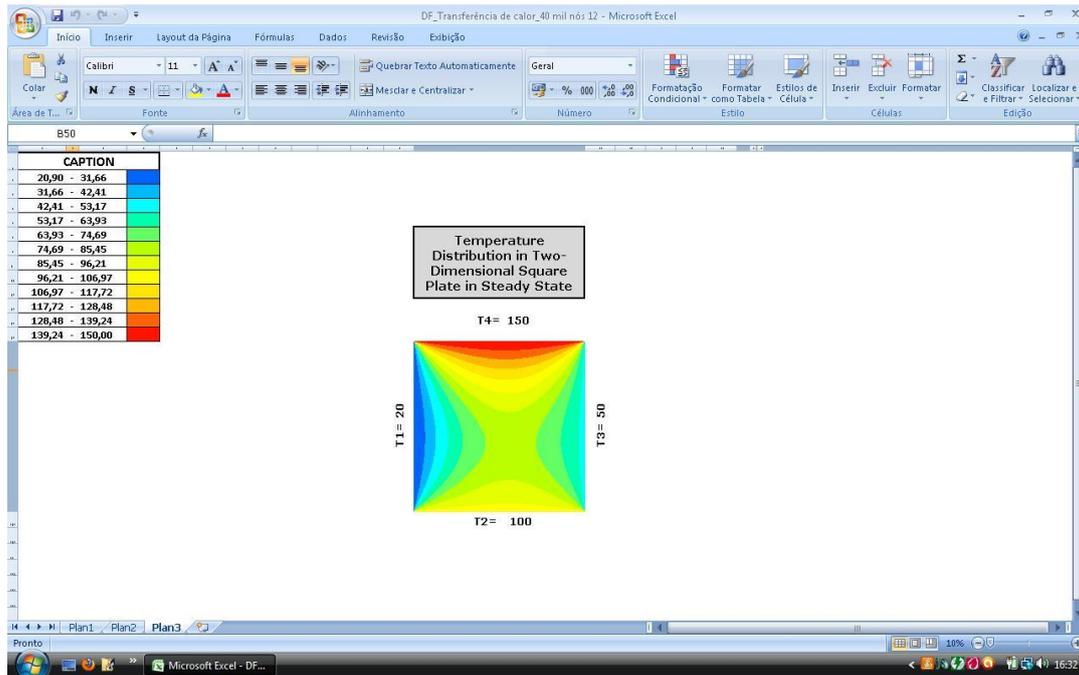
Figure 4. The *MICROSOFT* EXCEL™ worksheet and plotting of temperature distribution with color scales

4. VALIDATION

The algorithm made for this study operates with a fixed mesh of 40000 elements. To validate this algorithm it worked with a mesh of 9 elements and 4 nodes. The results was compared with the analytical solution.
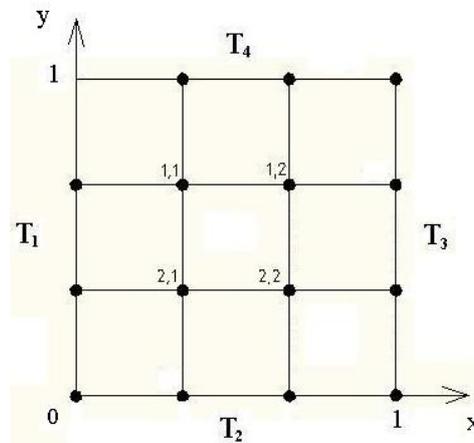
Figure 5. Thin square plate with mesh of 9 elements

This problem has the following system of linear equations:

$$T_{1,1} = \frac{20 + 25 + T_{1,2} + T_{2,1}}{4} \tag{6}$$

$$T_{1,2} = \frac{20 + 20 + T_{1,1} + T_{2,2}}{4} \tag{7}$$

$$T_{2,1} = \frac{25 + 30 + T_{1,1} + T_{2,2}}{4} \tag{8}$$

$$T_{2,2} = \frac{30 + 20 + T_{1,2} + T_{2,1}}{4} \tag{9}$$

which after mathematical manipulations and organizing the system, the system is a matrix of $A \times T = b$, where

$$A := \begin{pmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix} \quad T := \begin{pmatrix} T_{1,1} \\ T_{1,2} \\ T_{2,1} \\ T_{2,2} \end{pmatrix} \quad b := \begin{pmatrix} 45 \\ 40 \\ 55 \\ 50 \end{pmatrix}$$

(10)

the vector T is the solution of above system and is obtained of the following matrix product

$$T = A^{-1} \times b$$

(11)

calculating this product it can get the solution below

$$T = \begin{pmatrix} 23.125 \\ 21.875 \\ 25.625 \\ 24.375 \end{pmatrix}$$

(12)

Just below the program running on a mesh of 9 elements and showing the values of the nodes calculated by the FDM.
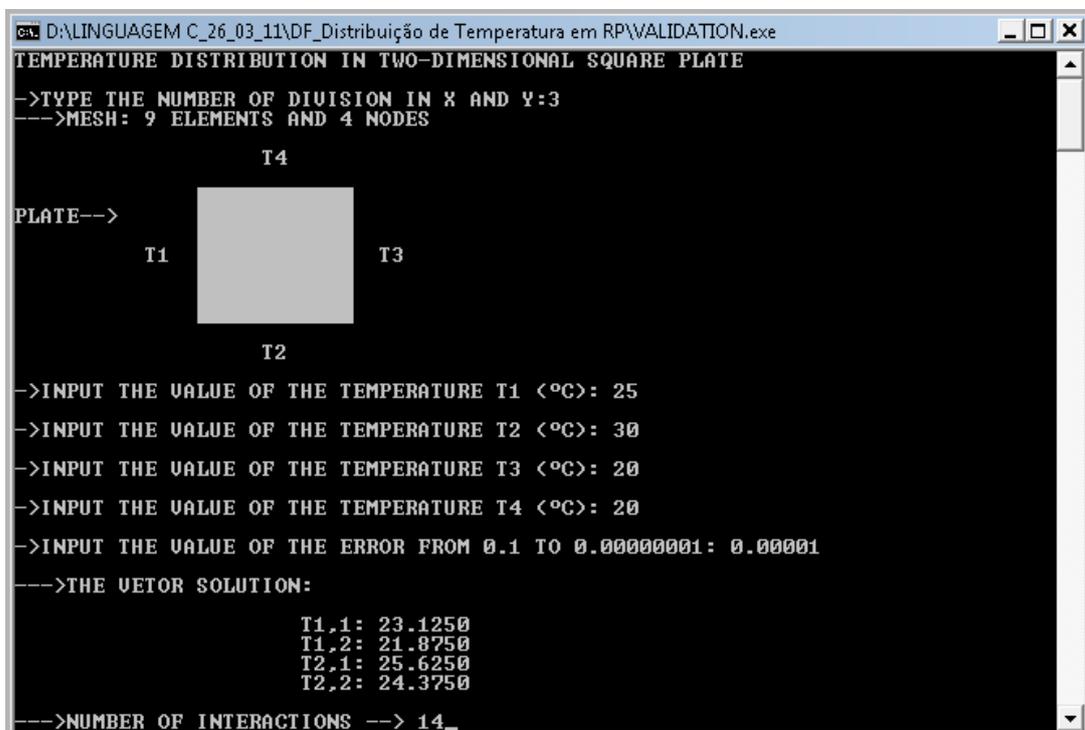
Figure 6. Validation of the algorithm

We can see in the picture above that the results of the numerical solution obtained by C-language program with an error of 10^{-5} are the same results with the analytical solution. Thus the algorithm is valid.

5. RESULTS

The results of the various input data were arranged in the photo below, with and without point charge inside the board.
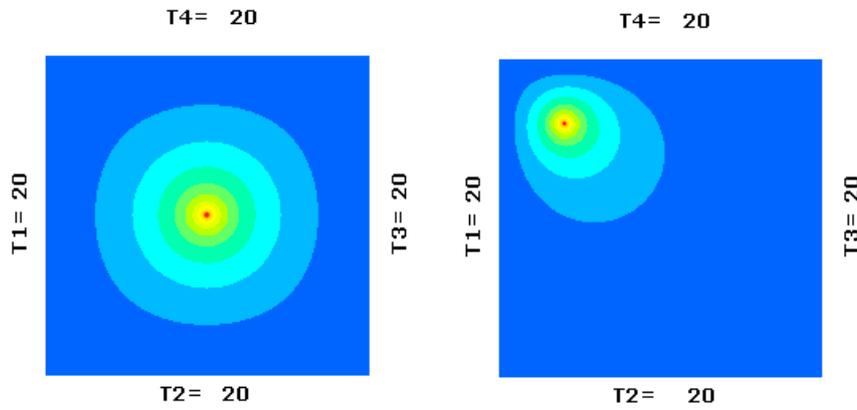
Figure 7. Left - Point charge at the center of plate of 100. Right - Plotting with point charge of 100
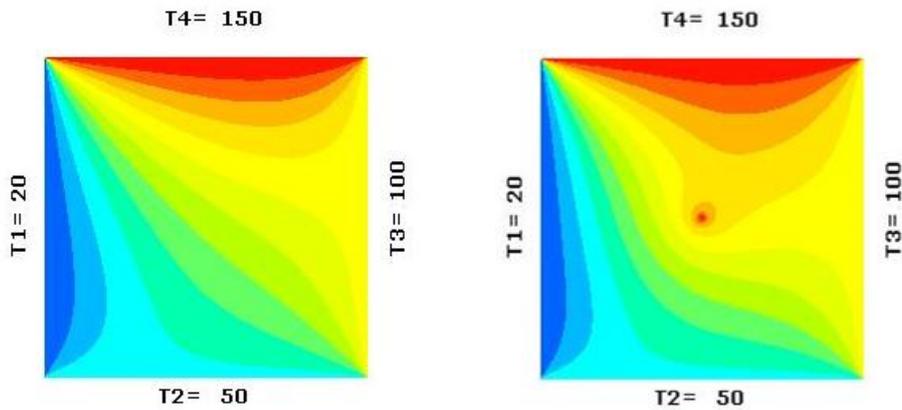
Figure 8. Left - plotting without point charge. Right - plotting with point charge of 150
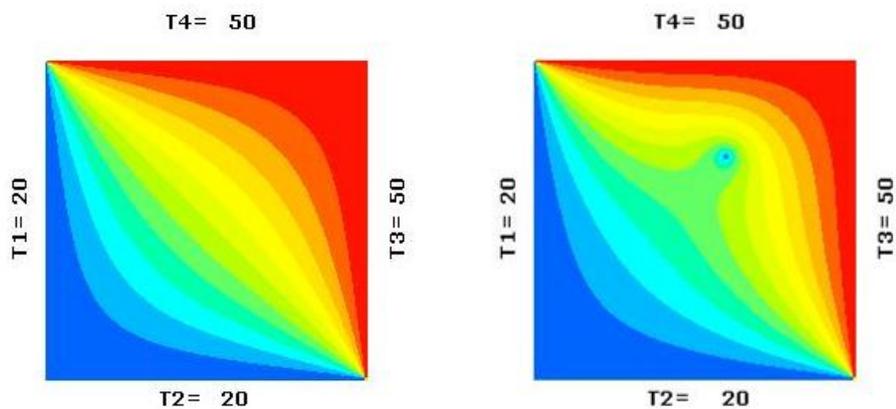
Figure 9. Left - plotting without point charge. Right - plotting with point charge of 20
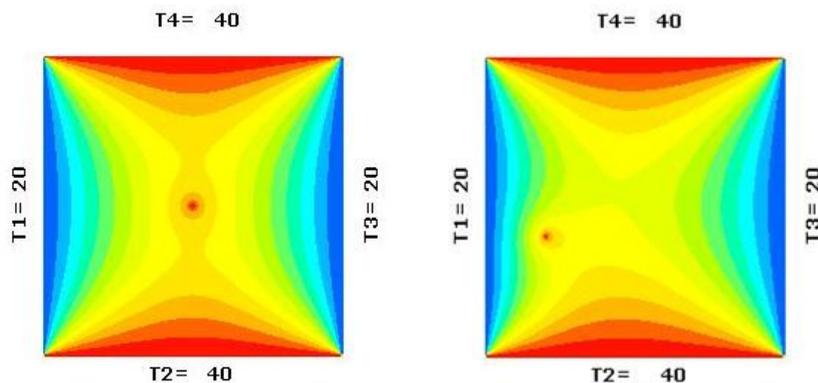
Figure 10. Left - plotting with point charge of 40 at the center of plate. Right - plotting with point charge of 40
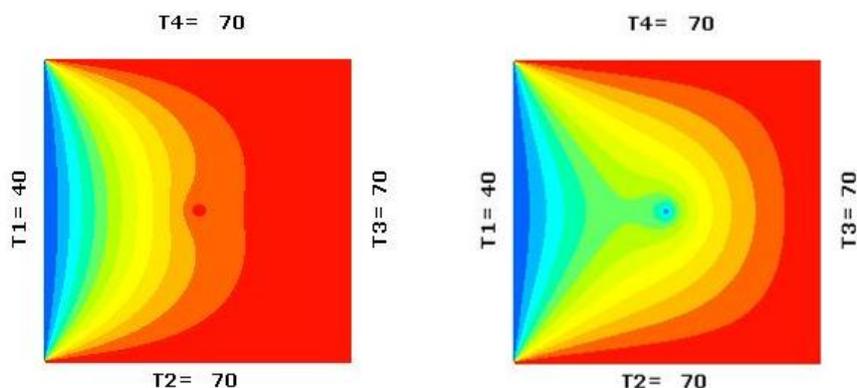

Figure 11. Left - plotting with point charge of 70. Right - plotting with point charge of 40

6. CONCLUSION

This study shows that it is possible to use the software MICROSOFT EXCEL™ to plotting data of temperature distribution of efficiently way. Plots were shown in photographs from various input data, with equal edges temperatures and different edges temperatures and with or without point charge inside the plate. This tool also offers the possibility of placing a point charge of temperature anywhere on the square plate, which offers many applications for analysis of temperature distribution.

Finally the computational tool obtained in this study could help anyone who wanted to learn the FDM, and also shows another option for using the software MICROSOFT EXCEL™, software extremely versatile used by almost all engineering students.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

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9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.