NONLINEAR ANALYSIS OF SAGGED CABLES WITH MOVING MASSES

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Abstract. *The paper deals with the nonlinear geometrical analyses of sagged cables subjected to moving masses traveling with constant velocities. The main scope of the paper is to present a new numerical approach for the coupled system mass-cable. There are several possible engineering applications, such as cable cars, maintenance systems of transmission lines and transport buckets of commodities. The structural systems will be numerically modeled with the aid of a numerical formulation, based on the finite elements method that uses the geometrical nonlinear positional concept. The moving mass attached to the cable problem is an extreme case of masses acting on flexible surfaces. Some recent papers deals with the linear analysis of the moving mass attached to the cable. Thus, it becomes necessary the development of more accuracy numerical methods, capable to supply a better reliability to the projects. It will be analyzed sagged cables with parabolic geometry. It will be analyzed the mechanical behavior for different horizontal constants velocities of the moving mass, up to the limit of the speed of longitudinal propagation of waves in the cable. There will be plotted graphs for the moving mass trajectories of the cable, the middle span vibrations and the normal forces in some finite elements. The preliminary results point to the hardening of the coupled mass-cable system, as the mass velocity increases, and for variations of the normal forces in the finites elements. Moreover, above certain velocities the equilibrium state of the coupled system mass-cable presents qualitative changes.*

Keywords*: nonlinear analysis, moving mass, cables*

1. INTRODUCTION

With the aid of numerical nonlinear formulations, specially based on the Finite Elements Method, it is possible to model complex mechanical problems, such as the structural systems subjected to moving masses. There are no exact solutions available for the sagged cables subject to moving masses, not even for the linear analysis. Due to the complexity of the real physical problem, i.e. the aerodynamic iteration and the friction between the mass and the cable, the numerical formulations become essential for the analysis. The first common aspect involving the numerical formulations is related with the critical velocity of the traveling mass, limited by the wave propagation speed of the cable. During the time-marching process the geometrical configuration of the structural system changes. Usually, as the velocity of the traveling mass approaches to the critical velocity the numerical analysis loses its accuracy.

Some recent papers regarding numerical analysis of such systems stand out. Al-Qassab et al. (2003) had presented a general formulation that has been derived using the Hamilton's principle. The cable configuration was not restricted to small sags and the moving mass particle was assumed to travel along the cable with general motion, i.e. the formulation is fully nonlinear The solution was obtained using the Galerkin procedure with two methods of representation, i.e Fourier and Wavelet. Bajer and Dyniewicz (2008) present a linear space–time approach to analyze a straight cable under a moving mass. The authors present both a semi-analytical solution and a finite element formulation developed by the Galerkin method. Wang and Rega (2010) followed the formulation presented Al-Qassab et al. (2003), extending it for the transient response of the suspended cable subjected to a sequence of masses moving with constant velocity. The authors pointed that the inertia forces of the moving mass enhance the maximum midspan displacement, and that the relevant effects play a more important role in the transient response when the mass of the moving mass increases. However, the increase of mass velocity may decrease the maximum displacement in a large range of mass ratio values (Wang and Rega, 2010). The three previous papers considered inertial, Coriolis and centrifugal terms in the formulations. Nonetheless, another important paper (Wu, 2005) proves that the influence of the Coriolis force is minor, when compared with the total structural numerical response, for beam elements. In this sense, the proposed formulation presented in this paper intends to verify the possibility of to consider integrally the inertial moving mass term, without the separation in the three acceleration terms.

Figure 1 presents a horizontal sagged cable subjected to a moving mass traveling with horizontal constant velocity. Where the variables presented in Fig. 1 are related with the mass density of the cable (ρ), the cross-sectional area of the cable (A), the horizontal length of the cable (*L*), the tensional force applied in each cable element, the constant horizontal velocity of the traveling mass ($\dot{x}_m$), the concentrated mass (*m*), the normal forces (*N*) and the vertical force related with the moving mass (*P*).

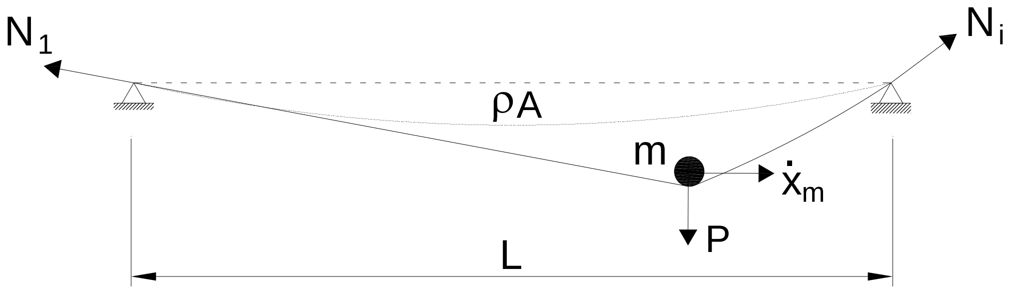

Figure 1. Sagged cable subjected to a moving mass

2. NONLINEAR DYNAMIC ANALYSIS FOR MOVING MASS

The proposed formulation is based on the minimum potential energy theorem. The total potential energy (Π) of the system, presented in Eq. (1), can be written in terms of the strain energy (*U*), the kinetic energy (*K*), the energy related with the moving mass inertia (Q_m), the dissipative damping term (*Q*) and the potential energy of the forces due to the motion of the moving mass (*F*).

$$\Pi = U + K + Q_m + Q - F \tag{1}$$

The strain energy (*U*) can be written for the reference volume *V* as:

$$U = \int_V \int_{\varepsilon_{\ln}} \sigma_{\ln} d\varepsilon_{\ln} dV = \int_V u dV \tag{2}$$

In Eq. (2), the strain measure used here is logarithmic (ε_{ln}) and its associated stress conjugate is related with the Cauchy's stress tensor ($\boldsymbol{\sigma}$) by the stretching ratio (λ) and the Young's modulus (*E*). The geometrical nonlinear formulation follows the notation presented in Greco and Ferreira (2009).

$$\sigma_{\ln} = \sigma\lambda = E(\lambda - 1)\lambda \tag{3}$$

The kinetic energy term is given by Eq. (4).

$$K = \frac{1}{2}\int_V \rho \dot{x}_i \dot{x}_i dV \tag{4}$$

According to Greco and Coda (2006), the damping term is only proportional to mass and it must be interpreted as a term that measures the amount of energy dissipated in the mechanical system that restores the stationary character of the functional. The dissipative term related with damping is written in its differential form as follows:

$$\frac{\partial Q(x,t)}{\partial X_i} = \int_V \frac{\partial}{\partial X_i} q(x,t) dV = \int_V \lambda_m \rho \dot{x}_i dV \tag{5}$$

where *q(x,t)* is the specific dissipative functional, λ_m is a constant related with the damping, proportional to the mass of the elastic body and X_i is the nodal point position associated with a Cartesian system of coordinates fixed in space, as presented in Greco et al. (2006). The kinematics of the used nonlinear model is exact. The proposed nonlinear dynamic formulation treats space and time as independent variables. Nonetheless, it is possible to analyze higher mass velocities. The three inertial effects (obtained from the Renaudot formula) are associated with the space-time approach, i.e. the separation between transverse acceleration, the Coriolis acceleration and the centrifugal acceleration and this differentiation was not performed directly in the proposed formulation. The energy term related with the moving mass inertia and its potential energy were obtained from Bajer and Dyniewicz (2008).

$$\frac{\partial Q_m(x,t)}{\partial X_i} = \delta(x_i - \dot{x}_m t) m \ddot{x}_i \tag{6}$$

$$F = \delta_{ik}\left(x_k - \dot{x}_m t\right)P \tag{7}$$

Where $\dot{x}_m$ the constant velocity of the moving mass and δ_{ik} is the Kronecker delta. According to Graff (1991), a velocity equal to that of the falling mass is imparted to the structure at $x_k = \dot{x}_m t$. Here, the effect of the inertial force was considered concentrated node-by-node. This approach has presented better results than the solution given by the distributed force among the finite elements for high traveling mass velocities.

The total potential energy of the moving mass and its inertial force applied in the system results in:

$$\Pi = \int_V u dV + \frac{1}{2}\int_V \rho \dot{x}_i \dot{x}_i dV + Q + Q_m - F \tag{8}$$

The nonlinear equilibrium equation is obtained through the application of the minimum potential energy theorem.

$$\begin{aligned} \frac{\partial \Pi}{\partial X_S} &= \int_V \frac{\partial u(\xi, X_i)}{\partial X_S} dV + \int_V \rho \dot{x}_i(\xi, X_i)\frac{\partial \dot{x}_i(\xi, X_i)}{\partial X_S} dV + \int_V \lambda_m \rho \dot{x}_S(\xi, X_i) dV \\ &+ \delta\left(x - \dot{x}_m t\right)m - \delta\left(x - \dot{x}_m t\right)P = 0 \end{aligned} \tag{9}$$

where ξ is a dimensionless parameter (varying from 0 to 1) used to map the finite element strain.

Considering the minimization of total functional energy, it is possible rewrite a semi-discrete time dynamic equilibrium equation, for the actual instant of time (*S+1*).

$$\frac{\partial \Pi}{\partial \mathbf{X}} = \left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} + \mathbf{M}\ddot{\mathbf{X}}_{S+1} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\ddot{\mathbf{X}}_{S+1} + C\dot{\mathbf{X}}_{S+1} = \delta\left(\mathbf{X} - \dot{x}_m t\right)P = 0 \tag{10}$$

2.1. Central differences algorithm

The central differences algorithm is an explicit time integration method, very fast with high numerical algorithmic damping (not controllable) that is directly dependent on the time step. It is suitable for dynamical systems with high degree of freedom or complex structural analysis involving severe nonlinear behavior. Another important application of the central differences algorithm is regarding the initialization of some implicit time integration methods, such as α-HHT method, Houlbout's method and Park's method. The algorithm is based on Taylor series expansion for $\mathbf{X}_{S+1}$ and $\mathbf{X}_{S-1}$ vectors for time $S\Delta t$ and neglecting terms above second order. It is possible to approximate velocity and acceleration for a time step *S* as follows:

$$\dot{\mathbf{X}}_S = \frac{1}{2\Delta t}(\mathbf{X}_{S+1} - \mathbf{X}_{S-1}) \tag{11}$$

$$\ddot{\mathbf{X}}_S = \frac{1}{\Delta t^2}(\mathbf{X}_{S+1} - 2\mathbf{X}_S + \mathbf{X}_{S-1}) \tag{12}$$

The solution for time *S+1* is obtained by replacing the approximations presented in Eqs. (11) and (12) in the equilibrium equation. Written for the time step *S*, the results in the explicit form of march in time is given by:

$$\left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} + \mathbf{M}\ddot{\mathbf{X}}_S + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\ddot{\mathbf{X}}_S + C\dot{\mathbf{X}}_S = \delta\left(\mathbf{X} - \dot{x}_m t\right)P \tag{13}$$

$$\left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} + \frac{\left[\mathbf{M} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\right]}{\Delta t^2}(\mathbf{X}_{S+1} - 2\mathbf{X}_S + \mathbf{X}_{S-1}) + \frac{\mathbf{C}}{2\Delta t}(\mathbf{X}_{S+1} - \mathbf{X}_{S-1}) = \delta\left(\mathbf{X} - \dot{x}_m t\right)P \tag{14}$$

Rearranging terms, the following nonlinear equilibrium equation can be obtained:

$$\left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} + \left\{\frac{\left[\mathbf{M} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\right]}{\Delta t^2} + \frac{\mathbf{C}}{2\Delta t}\right\}\mathbf{X}_{S+1} + \mathbf{R}_S + \mathbf{R}_{S-1} = \delta\left(\mathbf{X} - \dot{x}_m t\right)P \tag{15}$$

Where vectors $\mathbf{R}_S$ and $\mathbf{R}_{S-1}$ represent dynamic contribution of the variables from the past.

$$\mathbf{R}_s = \frac{-2}{\Delta t^2}\left[\mathbf{M} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\right]\mathbf{X}_s \tag{16}$$

$$\mathbf{R}_{s-1} = \left[\frac{\left[\mathbf{M} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\right]}{\Delta t^2} - \frac{\mathbf{C}}{2\Delta t}\right]\mathbf{X}_{s-1} \tag{17}$$

In relation to nodal positions, the second derivative of the energy function gives the Hessian matrix for the current time interval is given by Eq. (18).

$$\left.\frac{\partial^2 \Pi}{\partial \mathbf{X}^2}\right|_{S+1} = \nabla \mathbf{g}(\mathbf{X_0}) = \left.\frac{\partial^2 \mathbf{U_T}}{\partial \mathbf{X}^2}\right|_{S+1} + \frac{\left[\mathbf{M} + \delta\left(\mathbf{X} - \dot{x}_m t\right)m\right]}{\Delta t^2} + \frac{\mathbf{C}}{2\Delta t} \tag{18}$$

The Newton-Raphson method applied to Eq. (15) corrects the nodal positions during the iterations.

$$\mathbf{g(X)} = 0 = \mathbf{g(X_0)} + \nabla \mathbf{g(X_0)}\Delta\mathbf{X} \tag{19}$$

The proposed set of equations, based on Greco et al. (2006), was developed for the finite element space truss and it was initially proposed to solve static problems. It applies the principle of minimum potential energy to derive equations of equilibrium, which according to Toklu (2004) it is a successful technique for analysis of trusses experiencing large deformations before and after loss of stability. The cables finite elements were modeled by truss elements and considering the initial strain of the cables, as presented in Greco and Ferreira (2009).

During the iterative process, the position must be corrected according to values from Eq. (19) and thus velocity and acceleration, as follows:

$$\mathbf{X}_{S+1} = \mathbf{X}_{S+1} + \Delta\mathbf{X} \tag{20}$$

At this point, it must be emphasized that $\mathbf{X}_{S+1} = \mathbf{X}_S$ is considered at the beginning of the time interval.

$$\dot{\mathbf{X}}_{S+1} = \frac{\mathbf{X}_{S+1} - \mathbf{X}_S}{\Delta t} \tag{21}$$

$$\ddot{\mathbf{X}}_{S+1} = \frac{\dot{\mathbf{X}}_{S+1} - \dot{\mathbf{X}}_S}{\Delta t} \tag{22}$$

It should be observed that computation of $\mathbf{X}_{S+1}$ involves $\mathbf{X}_S$ and $\mathbf{X}_{S-1}$. Thus, to compute the solution to time *S-1*, a special start procedure must be used. Since $\mathbf{X}_0$ and $\dot{\mathbf{X}}_0$ are provided, $\ddot{\mathbf{X}}_0$ can be computed using Eq. (10) at time $t = 0$.

$$\mathbf{X}_{-1} = \mathbf{X}_0 - \Delta t\dot{\mathbf{X}}_0 + \frac{\Delta t^2}{2}\ddot{\mathbf{X}}_0 \tag{23}$$

$$\ddot{\mathbf{X}}_0 = -\mathbf{M}^{-1}\left(\mathbf{C}\dot{\mathbf{X}}_0 + \left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_0\right) \tag{24}$$

According to Argyris and Mlejnek (1991), the implicit algorithms present higher accuracy and stability than the explicit central differences method, especially for larger time period analyses. One classical time integration scheme used to introduce numerical damping in nonlinear formulations is the Wilson-θ algorithm. But, the main problem regarding the Wilson-θ algorithm is related with its numerical damping. The method removes both high and low frequency spurious oscillations. In order to stabilize the response, only high frequencies numerical damping is desirable.

To perform the damping on the high frequency spurious oscillations only, one can use the α-method of Hilber, Hughes and Taylor (α-HHT).

2.2. α-HHT algorithm

The α-HHT method is a direct time integration scheme based on the introducing of a numerical damping parameter factor (α) into the equilibrium equation, Hilber et al. (1977), which takes the modified form as follows:

$$\left.\frac{\partial \Pi}{\partial \mathbf{X}}\right|_{S+1} = \mathbf{M}\ddot{\mathbf{X}}_{S+1} + \delta(\mathbf{X} - \dot{x}_m t) m \ddot{\mathbf{X}}_{S+1} + (1+\alpha)\mathbf{C}\dot{\mathbf{X}}_{S+1} + (1+\alpha)\left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} = (1+\alpha)\delta(\mathbf{X} - \dot{x}_m t)P + \alpha \mathbf{C}\dot{X}_S + \alpha \frac{\partial \mathbf{U}}{\partial \mathbf{X}} \quad (25)$$

The Newmark equations are maintained and replaced in the modified equilibrium equation:

$$\mathbf{X}_{S+1} = \mathbf{X}_S + \Delta t \dot{\mathbf{X}}_S + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \ddot{\mathbf{X}}_S + \beta \ddot{\mathbf{X}}_{S+1} \right] \quad (26)$$

$$\dot{\mathbf{X}}_{S+1} = \dot{\mathbf{X}}_S + \Delta t (1-\gamma) \ddot{\mathbf{X}}_S + \gamma \Delta t \ddot{\mathbf{X}}_{S+1} \quad (27)$$

Where α, γ and β are numerical control parameters. The method will possess stability and order properties for the values:

$$-\frac{1}{3} \le \alpha \le 0 \,,\; \gamma = \frac{(1-2\alpha)}{2} \text{ and } \beta = \frac{(1-\alpha)^2}{4} \quad (28)$$

It is important to note that the lower the value of α, the greater the numerical damping induced in the solution. At the limit, $\alpha=0$ leads to the trapezoidal rule (Newmark algorithm). Isolating acceleration of the current time interval in Eq. (26), one has:

$$\ddot{\mathbf{X}}_{S+1} = \frac{\mathbf{X}_{S+1}}{\beta \Delta t^2} - \frac{\mathbf{X}_S}{\beta \Delta t^2} - \frac{\dot{\mathbf{X}}_S}{\beta \Delta t} - \left(\frac{1}{2\beta} - 1 \right) \ddot{\mathbf{X}}_S \quad (29)$$

As in the previous procedure, the terms $\mathbf{Q}_S$ and $\mathbf{R}_S$ represent dynamic contributions of variables of the past and are given by the expressions:

$$\mathbf{Q}_S = \frac{\mathbf{X}_S}{\beta \Delta t^2} + \frac{\dot{\mathbf{X}}_S}{\beta \Delta t} + \left(\frac{1}{2\beta} - 1 \right) \ddot{\mathbf{X}}_S \quad (30)$$

$$\mathbf{R}_S = \dot{\mathbf{X}}_S + \Delta t (1-\gamma) \ddot{\mathbf{X}}_S \quad (31)$$

Substituting approximation for position and velocity, Eq. (26) and (27), in Eq. (26), one has:

$$\begin{aligned} \left.\frac{\partial \Pi}{\partial \mathbf{X}}\right|_{S+1} &= (1+\alpha)\left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_{S+1} - (1+\alpha)\delta(\mathbf{X} - \dot{x}_m t)P + \frac{[\mathbf{M} + \delta(\mathbf{X} - \dot{x}_m t)m]}{\beta \Delta t^2}\mathbf{X}_{S+1} - [\mathbf{M} + \delta(\mathbf{X} - \dot{x}_m t)m]\mathbf{Q}_S + (1+\alpha)\mathbf{C}\mathbf{R}_S \\ &+ (1+\alpha)\mathbf{C}\frac{\gamma}{\beta \Delta t}\mathbf{X}_{S+1} - (1+\alpha)\gamma \Delta t \mathbf{C}\mathbf{Q}_S - \alpha \mathbf{C}\dot{\mathbf{X}}_S - \alpha \left.\frac{\partial \mathbf{U}}{\partial \mathbf{X}}\right|_S = 0 \end{aligned} \quad (32)$$

The derivative of Eq. (32) in relation to nodal positions of the current time instant gives the Hessian matrix.

$$\left.\frac{\partial^2 \Pi}{\partial X^2}\right|_{S+1} = \nabla \mathbf{g}(\mathbf{X_0}) = (1+\alpha)\left.\frac{\partial^2 \mathbf{U}}{\partial \mathbf{X}^2}\right|_{S+1} + \frac{[\mathbf{M} + \delta(\mathbf{X} - \dot{x}_m t)m]}{\beta \Delta t^2} + (1+\alpha)\frac{\gamma}{\beta \Delta t}\mathbf{C} \quad (33)$$

The corrections are given as follows:

$$\mathbf{X}_{S+1} = \mathbf{X}_{S+1} + \Delta\mathbf{X} \tag{34}$$

$$\dot{\mathbf{X}}_{S+1} = \mathbf{R}_S + \gamma\Delta t\left(\frac{\mathbf{X}_{S+1}}{\beta\Delta t^2} - \mathbf{Q}_S\right) \tag{35}$$

$$\ddot{\mathbf{X}}_{S+1} = \frac{\mathbf{X}_{S+1}}{\beta\Delta t^2} - \mathbf{Q}_S \tag{36}$$

Where $\mathbf{X}_0$ and $\dot{\mathbf{X}}_0$ are given for the initial time interval; $\ddot{\mathbf{X}}_0$ can be evaluated from the equilibrium equation.

$$\ddot{\mathbf{X}}_0 = \mathbf{M}^{-1}\left[(1+\alpha)\left(-C\dot{\mathbf{X}}_0 - \left.\frac{\partial\mathbf{U}}{\partial X}\right|_0\right) + \alpha\left(C\dot{\mathbf{X}}_{-1} + \left.\frac{\partial\mathbf{U}}{\partial\mathbf{X}}\right|_{-1}\right)\right] \tag{37}$$

The necessary variables of the step of time previous to the initial step (*S-1*) can be calculated using the central differences algorithm, as follows:

$$\mathbf{X}_{-1} = \mathbf{X}_0 - \Delta t\dot{\mathbf{X}}_0 + \frac{\Delta t^2}{2}\ddot{\mathbf{X}}_0 \tag{38}$$

$$\dot{\mathbf{X}}_{-1} = \frac{\mathbf{X}_0 - \mathbf{X}_{-1}}{\Delta t} \tag{39}$$

$$\ddot{\mathbf{X}}_{-1} = \frac{\dot{\mathbf{X}}_0 - \dot{\mathbf{X}}_{-1}}{\Delta t} \tag{40}$$

The main problem of the direct time integration schemes for moving mass formulations is the time integration steps used in the analysis. To vary the velocity of the moving mass is necessary to vary the time step used in the analysis (larger the time step, lower the velocity). It is well-know that for small time steps the numerical formulations present better results, but for this kind of analysis small time steps leads to faster velocities, that can cause dynamical instabilities in the system.

The velocity of the moving mass can be evaluated by dividing the horizontal finite element length by the time step used.

$$\dot{x}_m = \frac{L_{FE}}{\Delta t} \tag{41}$$

To vary the velocity of the moving mass is necessary to vary the time step used in the analysis (larger the time step, lower the velocity). It is well-know that for small time steps the numerical formulations present better results, but for this kind of analysis small time steps leave to faster (sometimes instable) velocities.

3. NUMERICAL RESULTS

The geometry of the cables analyzed in this paper is parabolic. According to Irvine (1981), up to a sag-to-span relation of 1/8 the parabolic geometry is reasonably valid. For larger ratios, the catenary solution should be taken, considering the self weight of the cable distributed among the finite elements (Luongo, 2010).

For the numerical analysis, a small algorithmic numerical damping of *α=-0.0001* was adopted to stabilize the dynamical response. Following Bajer and Dyniewicz (2008) analysis, the length, the cross-section area, the horizontal span, the density, the Young's modulus and the initial normal force prescribed in the cable were considered with non-dimensional unitary dimensions. The initial normal forces acing on cables are prescribed by initial unitary strains applied in the finite elements, as presented in Greco and Ferreira (2009). For the inclined sagged cable, a vertical unevenness of 0.1 between the supports was considered. Also, the minimum initial vertical position co-ordinate occurs at -0.08 (0.4 horizontally from the left support).

The unitary cable density was used to obtain a critical velocity equal to 1.0. Depending on the mass matrix distribution, consistent or lumped, the response may change. The moving mass is also unitary. The cable celerity is defined as:

$$c = \sqrt{\frac{N}{m}} \tag{42}$$

For the non-dimensional space, the celerity coincides with the longitudinal wave propagation speed defined as:

$$v = \sqrt{\frac{E}{\rho}} \tag{43}$$

3.1. Horizontal sagged cable

The trajectories of the mass attached to the sagged cable with aligned supports, for different traveling speeds ($\dot{x}_m = v_m$), are presented in Fig. 2.

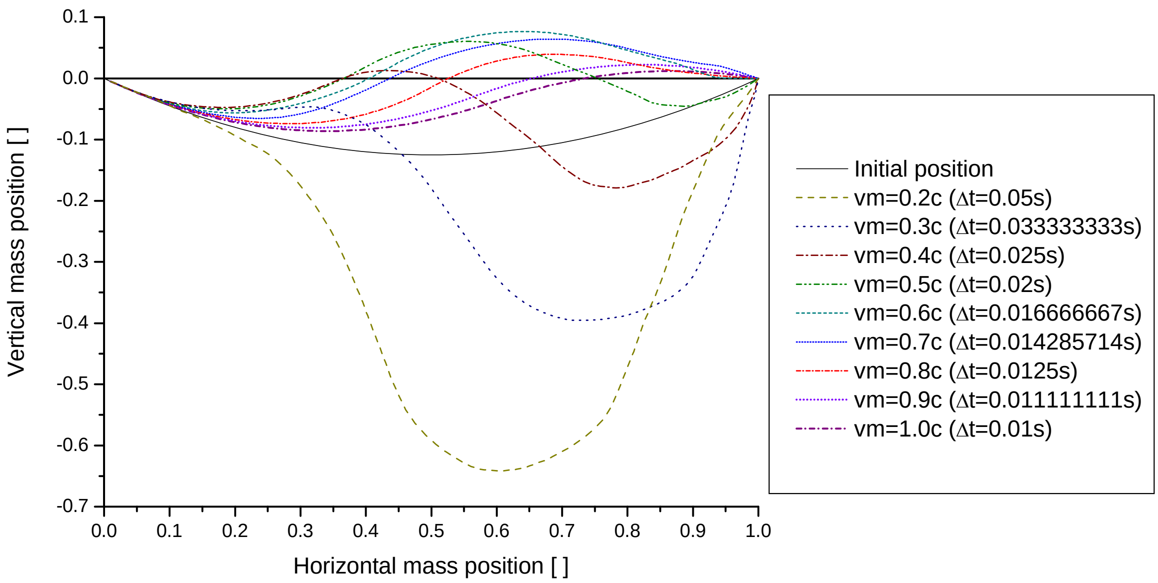

Figure 2. Moving mass trajectory along the span of the cable obtained for the horizontal sagged cable

The numerical results, presented in Fig. 2, indicate smaller vertical displacement amplitudes as the mass velocity increase. Another interesting aspect is related with the slopes of the response curves, i.e. after the mass velocity equal 0.4c all the displacements are smaller than the initial cable position.

3.2. Inclined sagged cable

The trajectories of the mass attached to the sagged cable with supports at different levels, for different traveling speeds ($\dot{x}_m = v_m$), are presented in Fig. 3. The height difference among the two supports is of 0.1.

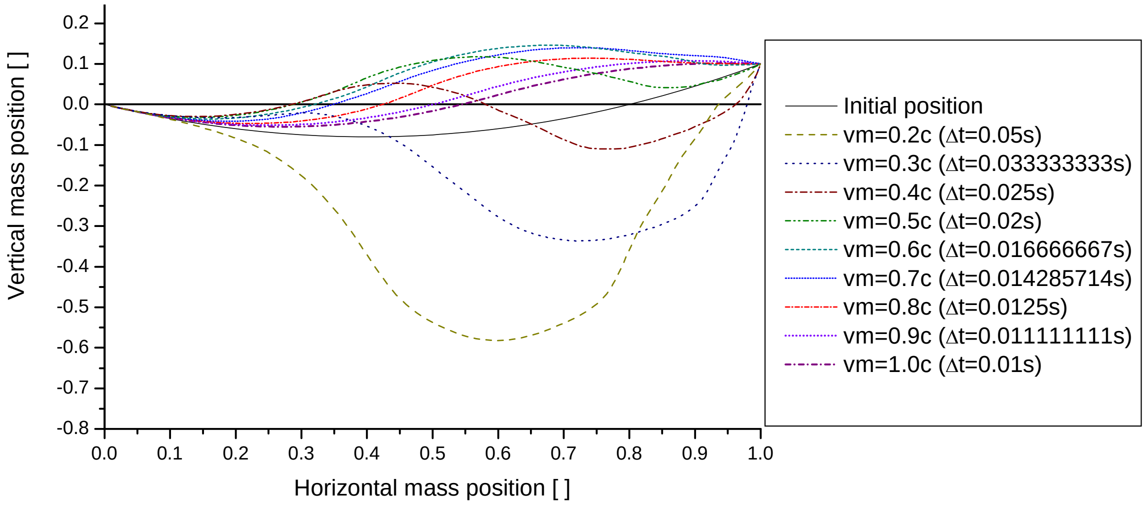

Figure 3. Moving mass trajectory along the span of the cable obtained for the horizontal sagged cable

Again, the numerical results, presented in Fig. 3, indicate smaller vertical displacement amplitudes as the mass velocity increase. Moreover, again, after the mass velocity equal 0.4c all the displacements are smaller than the initial cable position.

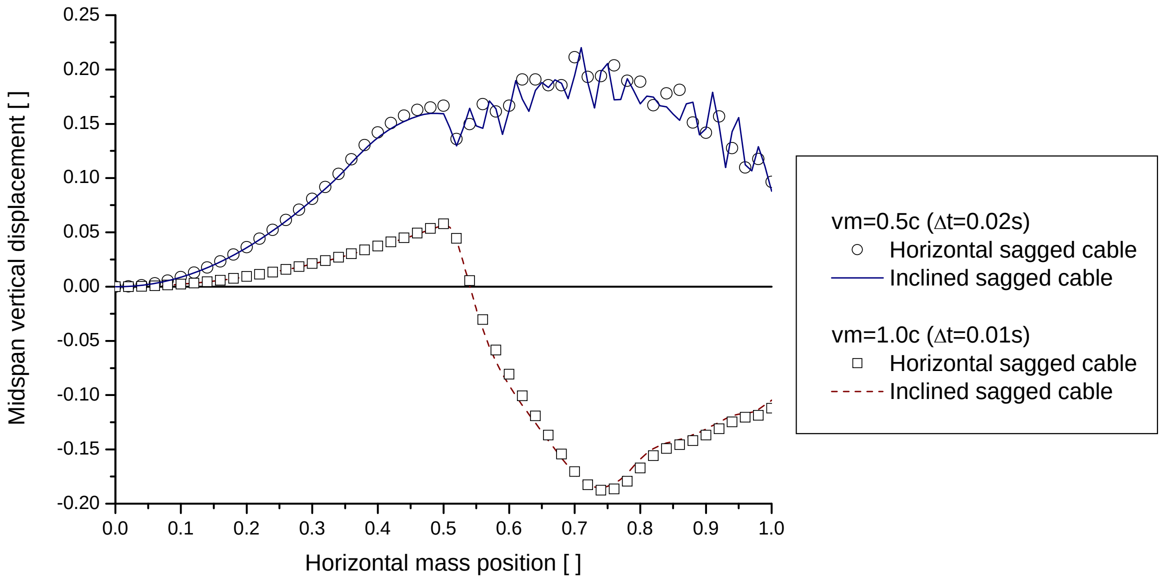

Figure 4. Midspan vertical displacement obtained for the horizontal and the inclined sagged cables

Figure 4 presents the midspan vertical displacement for both horizontal and inclined support conditions. Two traveling mass velocities were considered in the analysis. For the same velocity, the responses of the horizontal and the inclined cables were close, with some deviations. It is possible to note that for the lower speed (v_m=0.5c) some oscillations occur after the mass pass through the midspan. The responses for the higher speed (v_m=1.0c) were more smooth, for the same situation.

Figure 5 presents the last Finite Element normal force for both horizontal and inclined support conditions. Two traveling mass velocities were considered in the analysis. Again, for the same velocity, the responses of the horizontal and the inclined cables were close, with some little deviations. For the lower speed (v_m=0.5c), similar oscillations occur in the response after the mass pass through the midspan. For all the analyzed traveling mass speeds, the cables were still in traction.

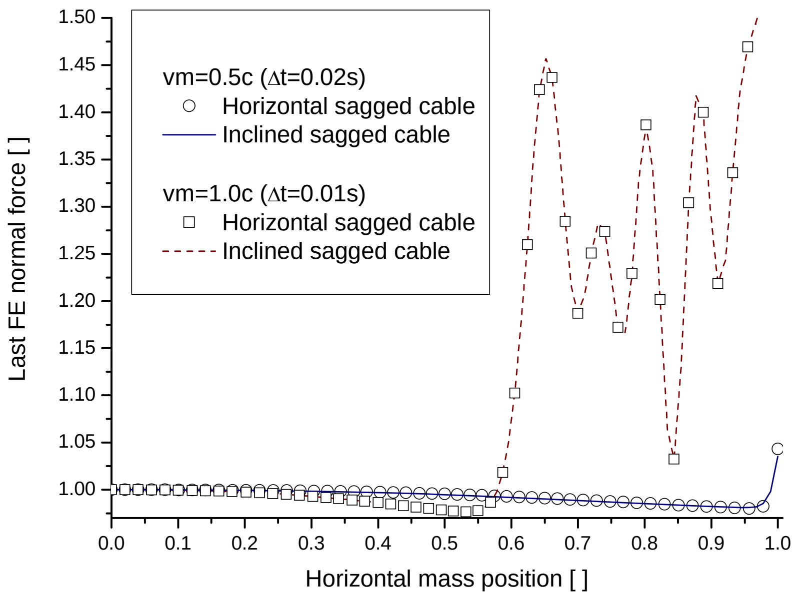

Figure 5. Last Finite Element normal force obtained for the horizontal and the inclined sagged cables

4. CONCLUSIONS

The implemented numerical formulation presented in this paper presents good results for analyses of sagged cables subjected to moving masses traveling with constant velocities. It can be stated based on the results presented in the literature, specially thinking on three important references, i.e. Wang and Rega (2010), Bajer and Dyniewicz (2008) and Al-Qassab et al. (2003).

Specifically, observing the results presented in Figs. 2 to 5, it can be stated that for smaller velocities the geometric nonlinear behavior is preponderant in relation with the inertial effects. The high speed traveling mass problem is more stable for the proposed technique; that is one of the most important issue related with the problem.

The direct comparison with the numerical results presented in Bajer and Dyniewicz (2008) is not feasible. But, for small deflections the nonlinear response converges to the linear response. Bajer and Dyniewicz (2008) justify the use of the space-time approach due numerical difficulties found in formulations based on the separation between time and space variables. The proposed nonlinear dynamic formulation treats space and time as independent variables. Nonetheless, it is possible to analyze higher mass velocities. The numerical divergence for the string problem traveled by high speed masses, pointed in the paper of Bajer and Dyniewicz (2008), was not observed using the proposed formulation. No numerical response divergence was observed in the results obtained from the proposed formulation. In the case of a beam the divergence rate is lower than that of a string, due to the type of the differential equation (Bajer and Dyniewicz, 2008). Regarding the mechanical behavior of the beam subjected to moving masses, new possibilities for further researchers are opened.

An interesting aspect is related with the normal forces, i.e. they change in the nonlinear program. This aspect can be explored on further studies. Another interesting analysis can be performed after the mass course through the cable, i.e the free vibration problem and its influence in the mechanical behavior of the cable.

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6. REFERENCES

Al-Qassab M., Nair S. and O'Leary J., 2003, "Dynamics of an elastic cable carrying a moving mass particle", Nonlinear Dynamics, Vol. 33, pp. 11-32.

Argyris, J. and Mlejnek, H.P.,1991, "Dynamics of structures: Texts on computational mechanics", Vol. 5, North-Holland, Amsterdam, Holand.

Bajer, C.I. and Dyniewicz, B., 2008, "Space-time approach to numerical analysis of a string with a moving mass", International Journal for Numerical Methods in Engineering, Vol. 76, pp. 1528-1543.

Graff, K.F., 2001, "Wave motions in elastic solids", Dover Publications, Toronto, Canada.

Greco, M. and Ferreira, I.P., 2009, "The logarithmic strain measure applied to the nonlinear positional formulation for space truss analysis", Finite Element in Analysis and Design, Vol. 45, pp. 632-639.

Greco, M. and Coda, H.B., 2006, "Positional FEM formulation for flexible multi-body dynamic analysis", Journal of Sound and Vibration, Vol. 290, pp. 1141-1174.

Greco, M., Gesualdo, F.A.R., Venturini, W.S. and Coda, H.B., 2006, "Nonlinear positional formulation for space truss analysis", Finite Elements in Analysis and Design, Vol. 42, pp. 1079-1086.

Hilber, H.M., Hughes, T.J.R. and Taylor, R., 1977, "Improved numerical dissipation for time integration algorithms in structural dynamics", Earthquake Engineering and Structural Dynamics, Vol. 5, pp. 283-292.

Irvine, H.M., 1981, "Cable structures", MIT Press, Cambridge, USA.

Luongo, A.,2010, Private Communication, University of L'Aquila, Italy.

Toklu, Y.C., 2004, "Nonlinear analysis of trusses through energy minimization", Computers & Structures, Vol. 82, pp. 1581-1589.

Wang, L. and Rega, G., 2010, "Modelling and transient planar dynamics of suspended cables with moving mass". International Journal of Solids and Structures, Vol. 47, pp. 2733–2744.

Wu, JJ., 2005, "Dynamic analysis of an inclined beam due to moving loads". Journal of Sound and Vibration, Vol. 288, pp. 107–131.

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