MATHEMATICAL AND COMPUTATIONAL MODELING FOR RADIATION OF A COMPACT TUBULAR MICROALGAE PHOTOBIOREACTOR

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Abstract. *The mathematical and computational modeling of a photobioreactor for the determination of the temperature transient behavior in compact tubular microalgae photobioreactors is presented. The model combines theoretical concepts of thermodynamics with classical theoretical and empirical correlations of Fluid Mechanics and Heat Transfer. The physical domain is discretized with the Volume Element Model (VEM) through which the physical system (reactor pipes) is divided in lumped volumes, such that only one time dependent ordinary differential equation, ODE, results for temperature, based on the first law of thermodynamics. The energetic interactions between the volumes are established through heat transfer empirical correlations for convection, conduction and radiation. Temperature is one of the most important parameters to be controlled in microalgae growth. Microalgae that are cultivated outside their growth temperature range may have a low growth rate or die. For this reason a system numerical simulation based on the operating conditions and environmental factors is desirable, in order to predict the transient algae growth temperature distribution along the reactor pipes. The resulting ODE's set was solved using a fourth order Runge-Kutta method with time step of 10 seconds.*

Keywords*: Radiation, Photobioreactor, Temperature, Simulation.*

1. INTRODUCTION

Numerous investigations have been made using microalgae. Due to its wide biodiversity, it has become the source for various applications such as: increasing the nutritional value of food and feed, capture the CO_2, sewage treatment and biofuel (Lourenço, 2006). The production of biofuels through the oil removed from the cells of microalgae is a field of study on the rise, and according to Xu et al. (2006) biodiesel can be a competitive alternative to conventional diesel. Given this broad field of study, the Center for Research and Development of Self-Sustainable Energy (NPDEAS) located at the Federal University of Parana is building a pilot plant with a compact tubular photobioreactor (Fig 1), an operations laboratory, and a biotechnology laboratory for cultivating microalgae, removing the oil and converting it into biodiesel.

In order to achieve a satisfactory growth rate of microalgae in compact photobioreactors tubes, it is necessary to know in advance the effects that can cause variations in solar radiation, temperature, nutrients, cellular agitation, and pH in the photobioreactor during the cultivation of microalgae. Nutrients, pH, and cellular agitation of the culture medium can be controlled easily using standard tools and techniques. However, the solar radiation in the photobioreactor depends on several uncontrollable factors such as incident radiation, air speed, cloud cover, latitude, time, day and year in which the culture is being grown. If these effects are known in advanced, the control parameters and structure of the photobioreactor can be optimized in order to improve conditions for the growth of microalgae.

Few models exist in the literature for numerical simulation and mathematical modeling of tubular photobioreactors for the cultivation of microalgae. The models that do exist typically focus on the mathematical modeling of the growth rate of microalgae. In more specific cases, software packages such as CFD (Fluent) and Matlab are used to perform numerical approximations of the trajectory of microalgae in a tubular photobioreactor or simulate the 3D behavior of light particles into tubes of a photobioreactor (PAPACEK et al., 2003 and Perner and Nocht-Posten, 2007).

This work is a complement of the work presented by Ribeiro et al. (2010), where it was proposed a mathematical model and computational procedure to calculate and visualize (plot) the temperature profiles inside the tubes of a compact tubular photobioreactor. Here it was added to the previous model the radiation effects on the rector's tube walls. The VEM (Vargas et al., 2001) is used in the discretization of the computational domain. The analysis of radiation variations on the tube walls is of paramount importance for the cultivation of microalgae. The temperature in the fluid (microalgae + nutrients + H2O + O2) depends mainly on the incident solar radiation on the walls of the photobioreactor tubes and if it not is in the range suitable for the cultivation can result in inhibition of cell growth or even cell death of microalgae.

Figure 1 – Photobioreactor at NPDEAS

2. MATHEMATICAL AND COMPUTATIONAL MODEL

The mathematical and computational model for the tubular photobioreactors proposed in this paper accounts for the prediction of the incident radiation on the tube walls. In the current model, the radiation in the photobioreactor directly affects the temperature and consequently the growth of microalgal cells. Fundamental theories combined with the Volume Element Model, which simplifies the partial differential equations into ordinary differential equations, provides a mathematical model which is handled numerically with substantially low computational effort.

The model is used to calculate the fluid temperature inside the transparent tubes of the photobioreactor. This calculation depends on air temperature, velocity of fluid flow, direct sun radiation, diffuse radiation, air velocity and geometry of the photobioreactor (Fig. 2).

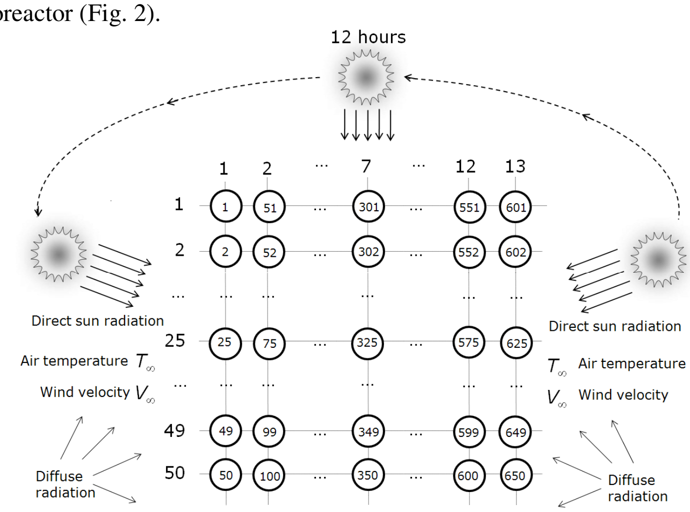

Figure 2 – Photobioreactor geometry and ambient conditions.

2.1. Photobioreactor

A large number of possible geometries for the transparent tubes of a photobioreactor can be found in the literature. This article studies the geometry presented by Vargas et al. (2007), for a tubular photo bioreactor with 3,250 meters of total tube length distributed in a compact geometry with 13 columns and 50 rows of tubes (Fig. 2). Each tube is 5 meters long (Fig. 3) with a radius of 0.03 meters.

A schematic representation of the photobioreactor is shown in Fig. 3. The main components are: a) transparent tubes where the microalgae will growth, b) tank capable of accumulating a large capacity of the culture medium from the tubes (in some cases the reservoir also serves as a de-gasser), c) pump for circulating the culture medium of microalgae in the tubes and reservoir and d) valve to control the flow.

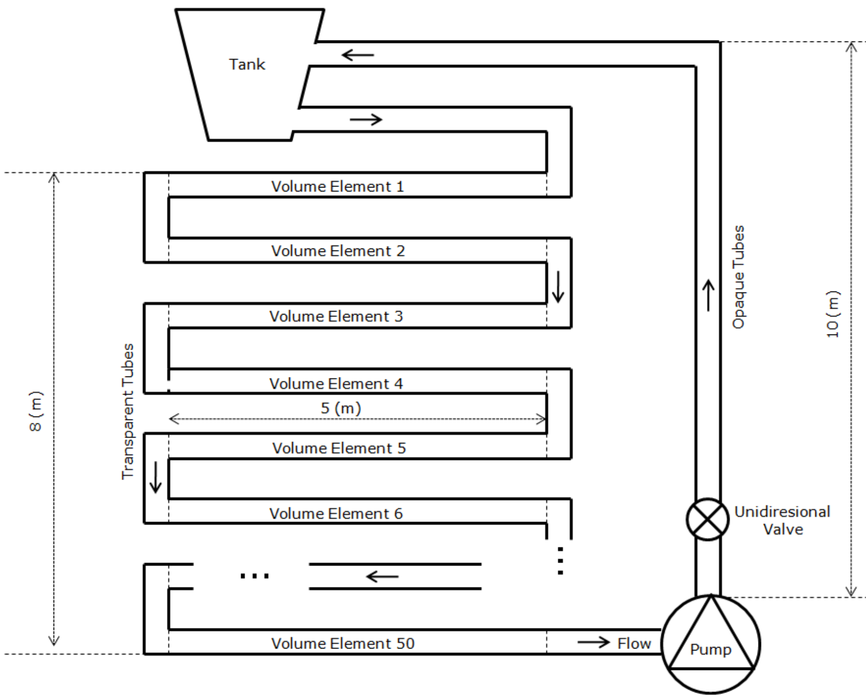

Figure 3 – Simplified schematic of the photobioreactor

2.2. Mathematical modeling

The primary goal for combining accurate numerical methods with low computational time and user-friendly interfaces is for use in the development of simplified mathematical and computational tools for improving the science and technology of equipment and processes.

This work presents the development and testing of a new system of compact photobioreactors. The system is modeled using the Volume Element Method, which subdivides the domain into Volume Element (*VE*) or volume centered cells as shown in Fig. 4. This method produces a single differential equation for each cell by applying the principles of energy and species conservation. The energetic interactions between cells are established through empirical correlations of heat transfer by convection, conduction, and radiation.

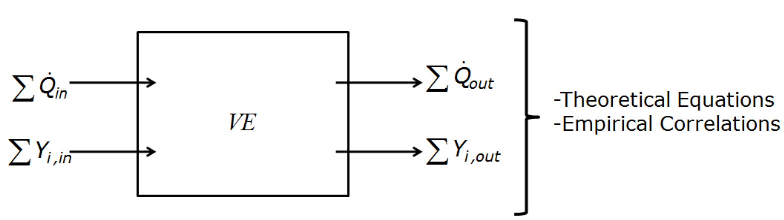

Figure 4 – Volume centered cells.

In the current formulation, algae growth is assumed to occur only in the transparent pipes. The mathematical domain for the transparent tubes is divided in two Volume Elements types (Fig. 5): a) VE_w for the walls of transparent tubes, and b) VE_f for the fluid that flows inside the transparent tubes.

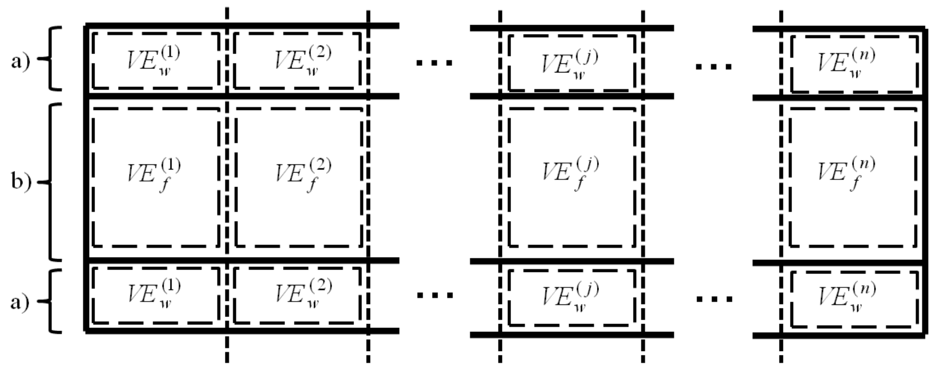

Figure 5 – VE used with the transparent pipes model

Since temperature plays a key role in microalgal growth its mathematical modeling is of paramount importance for the design of photobioreactor. The proposed mathematical model used to determine the temperature variations between the VE_w depends on the global solar radiation and the short-wave solar radiation reaching the earth surface. This short wave radiation consists of two components: incident solar radiation (Rad_{inc}) – fraction of global radiation that did not interact with the atmosphere – and diffuse radiation (Rad_{dif}) – fraction of global radiation that interacted with constituents of the atmosphere and is re-radiated in all directions. The maximum proportion of diffuse radiation occurs during the sunset and sunrise and also in overcast days.

The incident solar radiation and diffuse radiation are calculated by:

$$Rad_{inc} = G_i - G_i(0.9neb + 0.1) \tag{1}$$

$$Rad_{dif} = r_d G_i \tag{2}$$

where *neb* is the cloudiness, r_d is the percentage of diffuse radiation and G_i (Kreith, 1969) is the global solar radiation per unit area (W/m^2).

The *0.1* value represents the percentage of radiation that is absorbed and scattered through the layer of 144,833.89 meters thick air, water vapor, carbon dioxide, and dust that surrounds the earth and *0.9 neb* represent the percentage of radiation lost by the haziness of the sky.

The decrease of solar radiation by the atmosphere depends on the length of the path which, in turn, depends on the position of the sun. The radiant energy incident on a surface on earth G_n, due to sunlight, can be estimated by

$$G_n = G_0 \tau_a^{m_a} \tag{3}$$

where τ_a is the transmission coefficient for unit air mass and is m_a relative air mass, defined as the ratio of the length of current path for the shortest possible path.

The value of τ_a varies with the condition of the sky, ranging from 0.81 to a clear day, to 0.62 on a cloudy day. The value of m_a depends on the sun's position, given by the *zenith* z distance, i. e. the angle between the zenith and the direction of the sun (z).

If the receiving surface is not normal to the direction of the sun, the radiation incident per unit area is reduced by G_i cosine of θ, the angle between the direction of the sun and the surface normal, or

$$G_i = G_n \cos(\theta) \tag{4}$$

When the receiving surface is horizontal, then $\cos(\theta) = \cos(z)$. When the sun is seen from Earth the zenith angle varies with latitude φ of the location, time of day in hours angle h_z and declination sun of δ_s. Then the cosine of z is shown in Eq (5).

$$\cos z = sen(\varphi)sen(\delta_s) + \cos(\varphi)\cos(\delta_s)\cos(h_z) \tag{5}$$

Combining equations Eqs. (4) and (5), we obtain the radiant energy ratio received by the horizontal surface. The global solar radiation is given

$$G_i = G_n\left(sen(\varphi)sen(\delta_s) + \cos(\varphi)\cos(\delta_s)\cos(h_z)\right) \tag{6}$$

According to Fig. 5, the mathematical model that describes the energy balance in the VE_w and VE_f is presented below:

a) VE for the walls of the transparent tubes:

The proposed mathematical model accounts for the temperature variation between the volume elements of the wall due to the fluid flow, solar radiation and ambient interactions. For each VE_w the energy balance in the wall of the plastic tubes (Fig. 6) is calculated by:

$$\dot{Q}_{rad}^{(j)} - \dot{Q}^{(j)} - \dot{Q}_{air}^{(j)} = m_w^{(j)} C_w \frac{dT_w^{(j)}}{dt} \tag{7}$$

where C_w is the specific heat of the plastic (wall) (*J/kg.K*), m_w is the mass of the wall (*kg*), and $\dot{Q}_{rad}$ is the solar radiation reaching the walls of the tubes in the photo bioreactor (*W/m²*).

The heat transfer between the wall and air ($\dot{Q}_{air}$) is calculated by:

$$\dot{Q}_{air}^{(j)} = h_e A_e \left(T_w^{(j)} - T_\infty\right) \tag{8}$$

where h_e is the convective heat transfer coefficient between the outside wall and the air environment (*W/m²K*), A_e the area of outside wall (*m²*) and T_∞ is the temperature of air (*K*)

The amount of solar radiation received in the photobioreactor tube ($\dot{Q}_{rad}$) depends on the locality, time and day of the year and is calculated by:

$$\dot{Q}_{rad} = Rad_{inc} + Rad_{dif} \tag{9}$$

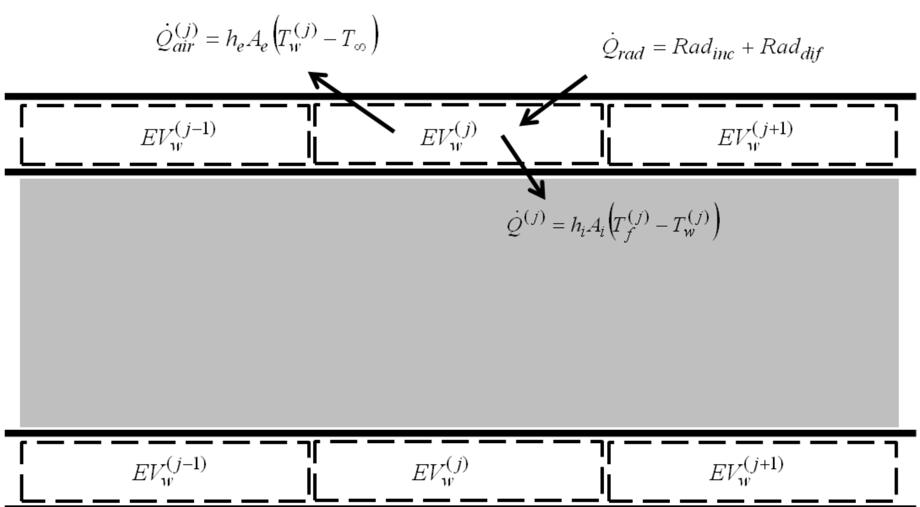

Figure 6 – VE *j* for energy equation at the wall of the transparent tubes.

b) VE for the fluid that flows inside the transparent tubes:

The First Law of Thermodynamics was used to calculate the temperature variation between the volume elements of fluid (microalgae + nutrients + H_2O + O_2) and the effects of wall temperature. For each VE_f the energy balance in the fluid by the elements (Fig. 7) is calculated by:

$$\dot{Q}^{(j)} + \dot{m}C_f T_f^{(j-1)} = m_f^{(j)} C_f \frac{dT_f^{(j)}}{dt} + \dot{m}C_f T_f^{(j)} \tag{10}$$

while the heat exchange with the fluid on the wall ($\dot{Q}$) is calculated by:

$$\dot{Q}^{(j)} = h_i A_i \left(T_f^{(j)} - T_w^{(j)} \right) \tag{11}$$

where $\dot{m}$ is mass flow (kg/s), m_f is the mass of the fluid (kg), C_f is the specific heat of the fluid (*J/kgK*), h_i is convective heat transfer coefficient between the fluid and the inner wall of the tube (W/m^2K), A_i the area of the inner wall of the tube (m^2), T_w the temperature of the wall (*K*) and T_f the temperature of the fluid (*K*).

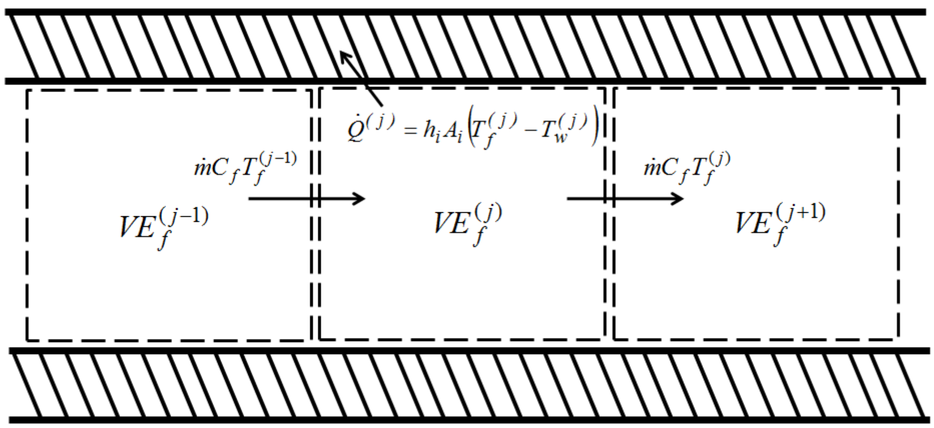

Figure 7 – VE *j* for the equation of conservation of energy in the fluid.

3. NUNERICAL METHODS

Simulations were made in the time interval from 12 hours to 13 hours, with four different time steps (0.1 sec, 1 sec, 5 sec and 10 sec) in order to analyze whether the results diverge (Fig. 8). As the error between the four time steps in this third decimal place and that in this problem is only important the value numeric integer. So to solve the set of transient ordinary differential equations formed by Eqs (7) and (10) for all simulation time is solved numerically with a fourth order Runge-Kutta method with time step of 10 seconds (Kincaid and Cheney, 1991).

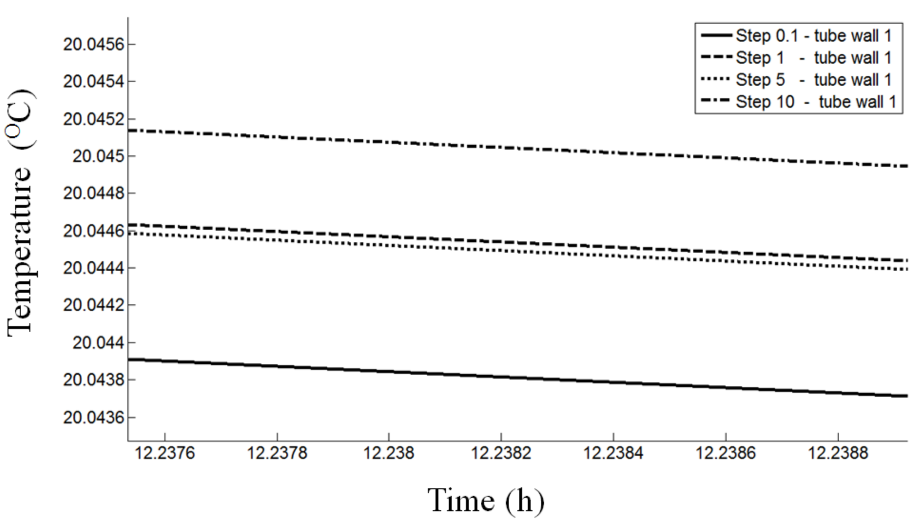

Figure 8 – Four different time steps (0.1 sec, 1 sec, 5 sec and 10 sec).

The initial and operating conditions of the problem are:

Table 1. Initial conditions.

$D_0 = 1$ (*day*)	$T_{H2O,0} = 296$ (*K*)
$t_0 = 6$ (*hour*)	$C_0 = 0{,}2$ (kg/m^3)
$D_f = 10$ (*day*)	$neb = 0$ %
$t_f = 6$ (*hour*)	

Table 2. Operating conditions.

$V_r = 2$ (m^3)	$v = 0.01$ (*m/s*)
$r_i = 0.029$ (*m*)	$L_{T.transp} = 3250$ (*m*)
$r_e = 0.030$ (*m*)	$C_f = 4181$ (*J/(kg.K)*)
$D_H = 4\ r_i$ (*m*)	$C_w = 980$ (*J/(kg.K)*)
$D_V = 4\ r_i$ (*m*)	$h_e = 50$ ($W/(m^2K)$)
$N_{col} = 13$	$h_i = 200$ ($W/(m^2K)$)
$N_{lin} = 50$	$\Phi = -25{,}45$

where the not yet defined variables are D_0 is initial of simulation day (*days*), D_f is final of simulation day (*day*), t_0 is initial time of simulation (*hours*), t_f is final time of simulation (*hour*), $T_{H2O,0}$ is water temperature initial (*K*), C_0 is microalgae concentration initial (kg/m^2), V_r is volume of reservoir (m^3), v is wind speed (*m/s*), r_i is inner radius of the tubes (*m*), r_e is outer radius of the tubes (*m*), D_H is distance between two in horizontal tubes (*m*), D_V is distance between two in vertical tubes (*m*), N_{col} is number of columns of tubes, N_{lin} is number of lines of tubes, Φ is latitude of the photobioreactor, $L_{T.\mathrm{transp}}$ is length of transparent tubes (*m*).

4. RESULTS

The numerical simulation of the photobioreactor began at 6 am and lasted 216 hours (9 days). This final result shows the local radiation reaching the photobioreactor as a function of time. Figure 9 shows the result of 216 hours of simulation of the radiation at the site where the photobioreactor was constructed.

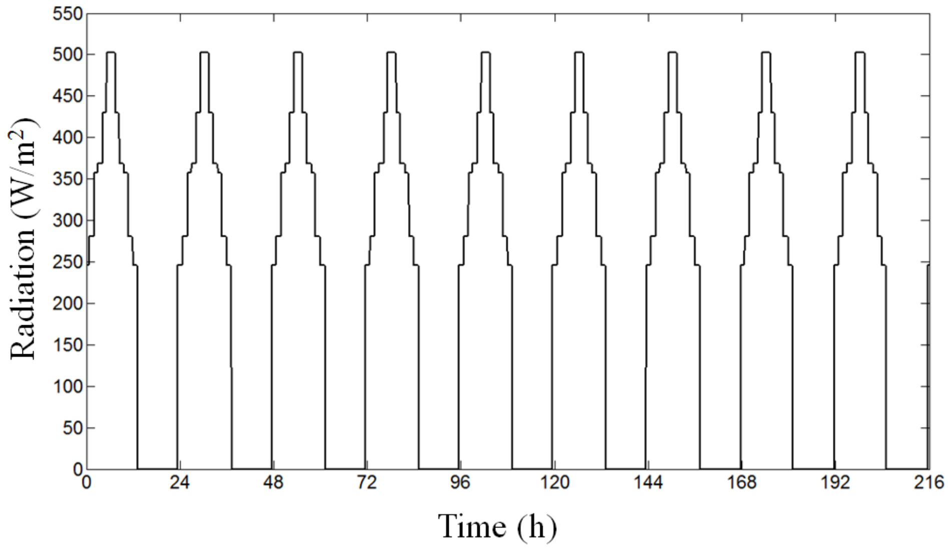

Figure 9 – Local radiation reaching the photobioreactor.

The radiation incident on each tube of the photobioreactor was calculated using computer graphics techniques. The resulting radiation field is shown in Fig. 10.

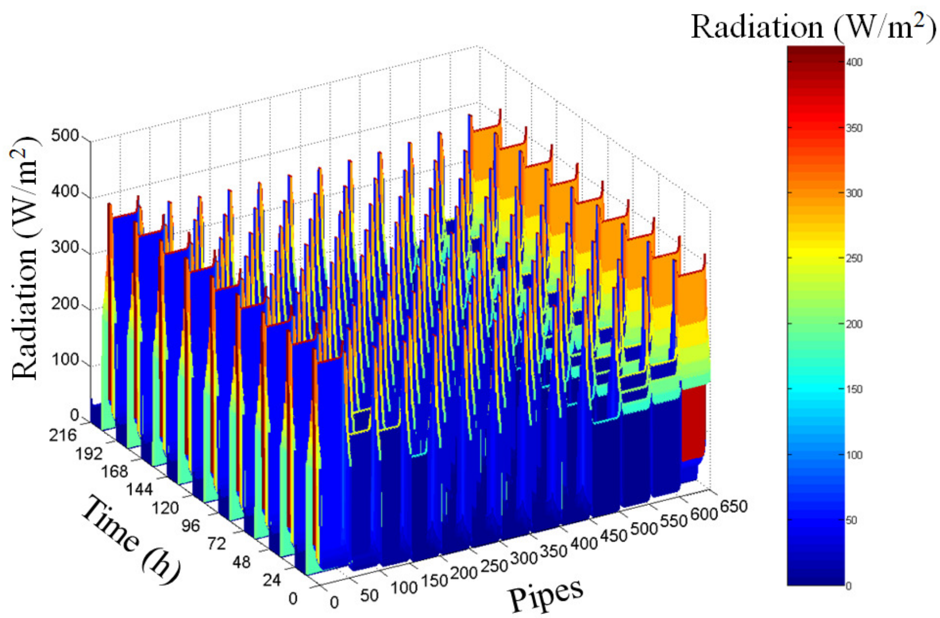

Figure 10 – Incident radiation in the tubes of the photobioreactor during the simulation time.

The same result can be shown in a different way in Fig. 11. In this figure, the y-axis represents the 216 hours of simulation, the x-axis the tubes of the photobioreactor.

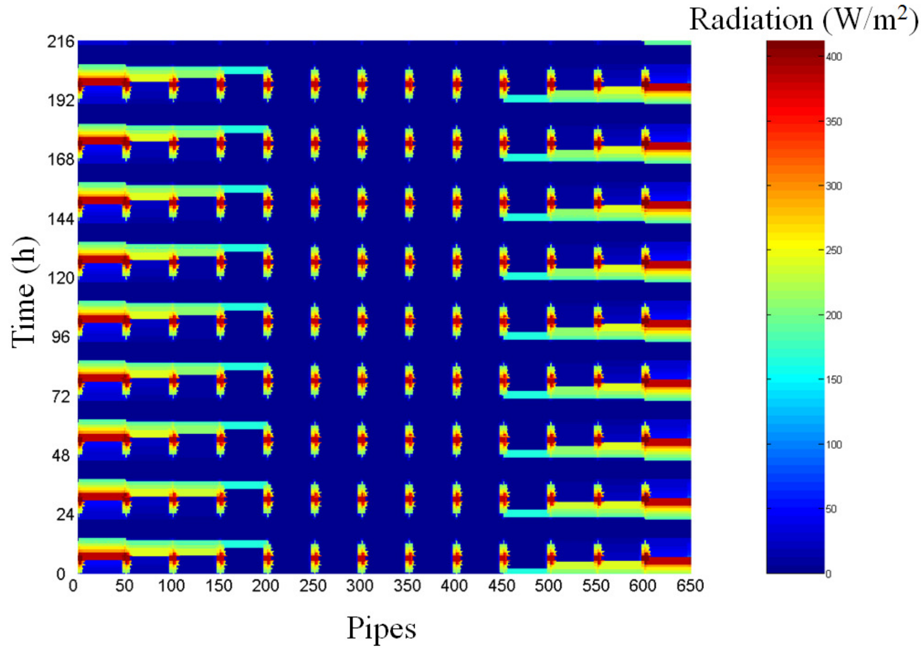

Figure 11 – Visualization of boundary lines of the incident radiation in the tubes of photobioreactor.

With the radiation field results for the 650 tubes of photobioreactor it is possible to calculate the temperature in the tubes of the reactor. Figure. 12 shows the wall and fluid temperatures of tube-1.

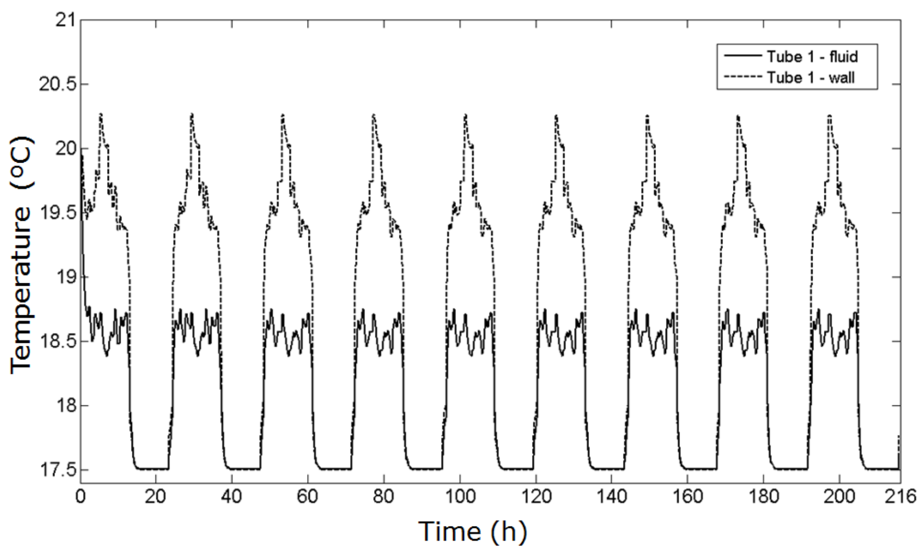

Figure 12 – Wall radiation for Tube-1 (Ribeiro et al. (2010)).

Goldman and Ryther (1976) have studded the growth of microalgae in relation to temperature of their environment and concluded that the best temperature range for growth of microalgae is between 12 °C and 28 °C. The temperature is also responsible for the overcome (dominating) of some species of microalgae in relation to other. The computer simulations performed shoed that all calculated fluid temperatures are within the range of growth of microalgae.

5. CONCLUSIONS

In this work it was analyzed the solar radiation dependence with the tube position and day time (position of the sun) in an algae photobioreactor. The main goal was to calculate the temperature of the fluid and verified if it remains within the best growth temperature range. This temperature is difficult to control, but a very important parameter for the microalgae growth. The prediction of the temperature profiles along the tubes length and its position with respected to the light source (sun light in the current work) is of great importance for the design of this type of bioreactors. The Volume Element Method has been used to formulate the problem and the resulting ordinary differential system of equations was solved with a fourth order Runge-Kutta method. Low computational effort with the inclusion of the main physical phenomena is the major advantage of the proposed method.

Presented results showed that the current model and numerical tool can predict the incident radiation, temperature variations inside the reactor, thus the resulting application is expected to be a useful tool for simulation and design of microalgae phobioreactors.

6. ACKNOWLEDGEMENTS

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