

## STATE SPACE RECONSTRUCTION APPLIED TO MULTIPARAMETER CHAOS CONTROL METHODS

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**Abstract.** *Chaos is a kind of nonlinear system response that has a dense set of unstable periodic orbits (UPOs) embedded in a chaotic attractor. The idea of the chaos control is to explore the UPO stabilization obtaining dynamical systems that may quickly react to some new situation, changing conditions and their response. The OGY (Ott-Grebogi-Yorke) method achieves system stabilization by using small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as a Poincaré section. This paper investigates the state space reconstruction applied to a multiparameter (MP) method based on OGY approach in order to control chaotic behavior using different control parameters. As an application of the proposed multiparameter general formulation it is presented an uncoupled approach where the control parameters do not influence the system dynamics when they are not active. This method is applied to control chaos in a nonlinear pendulum. Results show that the proposed procedure is a good alternative for chaos control since it provides a more effective UPO stabilization than the classical single-parameter OGY approach.*

**Keywords:** *Chaos control, nonlinear dynamics, pendulum, state space reconstruction*

### 1. INTRODUCTION

Chaos is a kind of nonlinear system response that has a dense set of unstable periodic orbits (UPOs) embedded in a chaotic attractor. The idea of the chaos control is to explore the UPO stabilization obtaining dynamical systems that may quickly react to some new situation, changing conditions and their response. Chaos control may be understood as the use of tiny perturbations for the stabilization of UPOs embedded in a chaotic attractor. Chaos control methods may be classified as discrete or continuous techniques. The first chaos control method was proposed by Ott *et al.* (1990), nowadays known as the OGY method as a tribute of their authors (Ott-Grebogi-Yorke). This is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section (Grebogi & Lai, 1997; Shinbrot *et al.*, 1993). On the other hand, continuous methods are exemplified by the so called delayed feedback control, proposed by Pyragas (1992), which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and a delayed state of the system. There are many improvements of the OGY method that aim to overcome some of its original limitations, as for example: control of high periodic and high unstable UPO (Otani & Jones, 1997; Ritz *et al.*, 1997 and Hübinger *et al.*, 1994), control using time delay coordinates (Dressler & Nitsche, 1992; So & Ott, 1995; Korte *et al.*, 1995; and Pereira-Pinto *et al.*, 2004), control using different control parameters (de Paula & Savi, 2007; Otani & Jones, 1997; Barreto & Grebogi, 1995).

This contribution considers the application of the uncoupled approach of semi-continuous multiparameter (SC-MP) chaos control method, method built upon the OGY method (De Paula & Savi, 2007), using state space reconstruction. As an application of the general formulation a two-parameter control of a nonlinear pendulum is carried out. Is it considered that only the scalar time series of pendulum position is available and system dynamics is reconstructed by using delay coordinates method. Results show that the procedure is a good alternative for chaos control since it provides an effective UPO stabilization.

### 2. MULTIPARAMETER CHAOS CONTROL METHOD

A chaos control method may be understood as a two stage technique. The first step is known as learning stage where the unstable periodic orbits are identified and some system characteristics are evaluated. After that, there is the control stage where the desirable UPOs are stabilized.

The OGY approach is described considering a discrete system of the form of a map  $\xi^{n+1} = F(\xi^n, p)$ , where  $p \in \mathfrak{R}$  is an accessible parameter for control. This is equivalent to a parameter dependent map associated with a general surface, usually a Poincaré section. The control idea is to monitor the system dynamics until the neighborhood of a

desirable point is reached. After that, a proper small change in the parameter  $p$  causes the next state  $\xi^{i+1}$  to fall into the stable direction of the desirable point. In order to find the proper variation in the control parameter,  $\delta p$ , it is considered a linearized version of the dynamical system near this control point. The linearization has a homeomorphism with the nonlinear problem that is assured by the Hartman-Grobman theorem (Savi, 2006). The semi-continuous control method introduces as many intermediate control stations as it is necessary to achieve stabilization of a desirable UPO. In order to use  $N$  control stations per forcing period  $T$ , one introduces  $N$  equally spaced successive Poincaré sections  $\Sigma_n$  ( $n=1, \dots, N$ ).

The semi-continuous multiparameter (SC-MP) chaos control method considers  $N_p$  different control parameters,  $p_i$  ( $i=1, \dots, N_p$ ). By considering a specific control station, only one of those control parameters actuates. Under this assumption, the map  $F$ , that establishes the relation of the system behavior between the control stations  $\Sigma_n$  and  $\Sigma_{n+1}$ , depends on all control parameters. Although only one parameter actuates in each section, it is assumed the influence of all control parameters based on their positions in station  $\Sigma_n$ . On this basis,

$$\xi^{n+1} = F(\xi^n, P^n) \quad (1)$$

where  $P^n$  is a vector with all control parameters. By using a first order Taylor expansion, one obtains the linear behavior of the map  $F$  in the neighborhood of the control point  $\xi_C^n$  and around the control parameter reference position,  $P_0$ , is defined by.

$$\delta \xi^{n+1} = D_{\xi^n} F(\xi^n, P^n) \Big|_{\xi^n = \xi_C^n, P^n = P_0} \delta \xi^n + D_{P^n} F(\xi^n, P^n) \Big|_{\xi^n = \xi_C^n, P^n = P_0} \delta P^n \quad (2)$$

This equation may be rewritten as follows

$$\delta \xi^{n+1} = J^n \delta \xi^n + W^n \delta P^n \quad (3)$$

where  $\delta \xi^{n+1} = \xi^{n+1} - \xi_C^{n+1}$ ,  $\delta \xi^n = \xi^n - \xi_C^n$ ,  $\delta P^n = P^n - P_0$  is the control actuation,  $J^n = D_{\xi^n} F(\xi^n, P^n) \Big|_{\xi^n = \xi_C^n, P^n = P_0}$  is the Jacobian matrix and  $W^n = D_{P^n} F(\xi^n, P^n) \Big|_{\xi^n = \xi_C^n, P^n = P_0}$  is the sensitivity

matrix which each column is related to a control parameter. In order to evaluate the influence of all parameters actuation, it is assumed that the system response for all parameters actuation is given by a linear combination of the system responses when each parameter actuates isolated and the others are fixed at their reference value. Therefore,

$$\delta P^n = B^n \delta p^n \quad (4)$$

where  $B^n$  is defined as a  $[N_p \times N_p]$  diagonal matrix formed by the weighting parameters, *i.e.*,  $diag(B^n)_i = \beta_i^n$ . This can be understood considering that each parameter influence is related to a vector with components  $q_i = W_i^n \delta p_i^n = W_i^n (p_i^n - p_{0i})$ , and the general actuation is given by:

$$q = \beta_1 q_1 + \beta_2 q_2 + \dots + \beta_{N_p} q_{N_p} \quad (5)$$

and  $\beta_i$  weights each parameter influence in the system response. Notice that  $q$  may be written as follows:

$$q = \beta_1^n W_1^n \delta p_1^n + \beta_2^n W_2^n \delta p_2^n + \dots + \beta_{N_p}^n W_{N_p}^n \delta p_{N_p}^n = W^n B^n \delta p^n \quad (6)$$

Moreover, by assuming that only one parameter actuates in each control station it is possible to define active parameters, represented by subscript  $a$ ,  $\delta P_a^n = B_a^n \delta p_a^n$  (actuates in station  $\Sigma_n$ ), and passive parameters, represented by

subscript  $p$ ,  $\delta P_p^n = B_p^n \delta p_p^n$  (does not actuate in station  $\Sigma_n$ ). At this point, it is assumed a weighting matrix for active parameter,  $B_a^n$ , and other for passive parameters,  $B_p^n$ . Therefore,

$$\delta \xi^{n+1} = J^n \delta \xi^n + W^n \delta P_a^n + W^n \delta P_p^n \quad (7)$$

Now, it is necessary to align the vector  $\delta \xi^{n+1}$  with the stable direction  $v_s^{n+1}$ :

$$\delta \xi^{n+1} = \alpha v_s^{n+1} \quad (8)$$

where  $\alpha \in \Re$  needs to be satisfied as follows:

$$J^n \delta \xi^n + W^n \delta P_a^n + W^n \delta P_p^n = \alpha v_s^{n+1} \quad (9)$$

Therefore, once the unknown variables are  $\alpha$  and the non-vanishing term of the vector  $\delta P_a^n$ , one obtains the following system:

$$\begin{bmatrix} \delta P_a^n \\ \alpha \end{bmatrix} = -[W^n - v_s^{n+1}]^{-1} [J^n \quad W^n] \begin{bmatrix} \delta \xi^n \\ \delta P_p^n \end{bmatrix} \quad (10)$$

The solution of this system furnishes the necessary values for the system stabilization:  $\alpha$  and  $\delta p_{ai}^n$ , where  $\delta p_{ai}^n$  is related to the non-vanishing element of the vector  $\delta P_a^n$ . Notice that the actuation is given by:  $\delta p_{ai}^n = \delta P_{ai}^n / \beta_{ai}^n$ .

A particular case of this control procedure has uncoupled control parameters meaning that each parameter returns to the reference value when it becomes passive. Moreover, since there is only one active parameter in each control station, the system response to parameter actuation is the same as when it actuates alone. Under this assumption, passive influence vanishes and active vector is weighted by 1, which is represented by:

$$B_p^n = 0 \text{ and } B_a^n = I \quad (11)$$

where  $I$  is the identity matrix.

Therefore, the map  $F$ , that establishes the relation of the system behavior between the control stations  $\Sigma_n$  and  $\Sigma_{n+1}$ , is just a function of the active parameters,  $\xi^{n+1} = F(\xi^n, P_a^n)$ , and the linear behavior of the map  $F$  in the neighborhood of the control point  $\xi_C^n$  and around the control parameter reference positions,  $P_0$ , is now defined by:

$$\delta \xi^{n+1} = J^n \delta \xi^n + W^n \delta P_a^n \quad (12)$$

where the sensitivity matrix  $W^n$  is the same of the previous case. Moreover, since  $B_a^n = I$ , it follows that  $\delta P_a^n = \delta p_a^n$ , thus the value of  $\delta P_a^n$  corresponds to the real perturbation necessary to stabilize the system. In order to align the vector  $\delta \xi^{n+1}$  with the stable direction, the following system is obtained:

$$\begin{bmatrix} \delta P_a^n \\ \alpha \end{bmatrix} = -[W^n - v_s^{n+1}]^{-1} J^n \delta \xi^n \quad (13)$$

## 2.1. State Space Reconstruction

A time series of a dynamical system can be understood as a time evolution of an observable variable of the system. It can be a state variable or a representation of that. An essential point related to the time series analysis is that it contains all information related to system dynamics. Therefore, the dynamics can be reconstructed by a scalar time series. There are different alternatives to perform the state space reconstruction. The method of delay coordinates is an

alternative employed in this paper. Basically, this method may be used to construct a vector time series that is equivalent to the original dynamics from a topological point of view. The state space reconstruction needs to form a coordinate system to capture the structure of orbits in state space, which could be done using lagged variables. Then, it is possible to use a collection of time delays to create a vector in a  $D_e$ -dimensional space. The application of this approach is associated with the determination of delay parameters, time delay,  $\tau$ , and embedding dimension,  $D_e$ . The average mutual information method is an alternative to determine time delay (Fraser & Swinney, 1986) while the false nearest neighbors method is used to estimate embedding dimension (Rhodes & Morari, 1997).

In terms of control purposes, it should be highlighted that the state space reconstruction by delay coordinates method causes the map  $F$  to be dependent on all control parameters perturbations performed in the time interval  $t^n - \tau \leq t \leq t^n$ , where  $\tau$  is the time delay (Dressler & Nitsche, 1992). Thus, it is necessary to consider perturbations until  $\delta \mathcal{P}^{n-r}$ , where  $r$  is the biggest value so that  $\delta \mathcal{P}^{n-r}$  is inside the considered interval ( $t^n - \tau \leq t \leq t^n$ ). Therefore, the use of state reconstructed by delay coordinates method implies that:

$$\xi^{n+1} = F(\xi^n, P^n, P^{n-1}, \dots, P^{n-r}) \quad (14)$$

By considering the same steps employed from equation (1) to equation (3), it is obtained:

$$\delta \xi^{n+1} = J^n \delta \xi^n + W_0^n \delta P^n + W_1^n \delta P^{n-1} + \dots + W_r^n \delta P^{n-r} \quad (15)$$

where  $J^n = D_{\xi^n} F(\xi^n, \delta P^n, \delta P^{n-1}, \dots, \delta P^{n-r})$  and  $w_i^n = D_{\mathcal{P}^{n-i}} F(\xi^n, \delta P^n, \delta P^{n-1}, \dots, \delta P^{n-r})$ . By considering active and passive control parameters:

$$\delta \xi^{n+1} = J^n \delta \xi^n + W_{a0}^n \delta P_a^n + W_{p0}^n \delta P_p^n + W_{a1}^n \delta P_a^{n-1} + W_{p1}^n \delta P_p^{n-1} + \dots + W_{ar}^n \delta P_a^{n-r} + W_{pr}^n \delta P_p^{n-r} \quad (16)$$

In order to obtain system stabilization, the same procedure presented at section 2 must be considered and the vector  $\delta \xi^{n+1}$  has to be aligned with the stable direction  $v_s^{n+1}$ .

### 3. SIMULATION RESULTS

As an application to the proposed chaos control procedure, a system with high instability characteristic is of concern. A nonlinear pendulum actuated by two different control parameters is considered. The motivation of the proposed pendulum is an experimental set up discussed in De Paula *et al.* (2006) that proposed a mathematical model to describe the pendulum dynamical behavior. Basically, the pendulum consists of an aluminum disc with a lumped mass. An electric motor harmonically excites the pendulum via a string-spring device, which provides torsional stiffness to the system.

The mathematical model for the pendulum dynamics describes the time evolution of the angular position,  $\phi$ , assuming that  $\varpi$  is the forcing frequency,  $I$  is the total inertia of rotating parts,  $k$  is the spring stiffness,  $\zeta$  represents the viscous damping coefficient and  $\mu$  the dry friction coefficient,  $m$  is the lumped mass,  $a$  defines the position of the guide of the string with respect to the motor,  $b$  is the length of the excitation arm of the motor,  $D$  is the diameter of the metallic disc and  $d$  is the diameter of the driving pulley. The equation of motion is given by (De Paula *et al.*, 2006):

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{kd^2}{2I} & \frac{\zeta}{I} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ \frac{kd}{2I} (\Delta f(t) - \Delta l_1) - \frac{mgD \sin(x_1)}{2I} - \frac{2\mu}{\pi I} \arctan(qx_2) \end{bmatrix} \quad (14)$$

where  $\Delta f(t) = \sqrt{a^2 + b^2 + \Delta l_2^2 - 2ab \cos(\varpi t) - 2b\Delta l_2 \sin(\varpi t)} - (a - b)$  and  $\Delta l_1$  and  $\Delta l_2$  correspond to actuations.

Numerical simulations of the pendulum dynamics are in close agreement with experimental data by assuming parameters used in De Paula *et al.* (2006):  $a = 1.6 \times 10^{-1}$  m;  $b = 6.0 \times 10^{-2}$  m;  $d = 4.8 \times 10^{-2}$  m;  $D = 9.5 \times 10^{-2}$  m;  $m = 1.47 \times 10^{-2}$  kg;  $I = 1.738 \times 10^{-4}$  kg m<sup>2</sup>;  $k = 2.47$  N/m;  $\zeta = 2.386 \times 10^{-5}$  kg m<sup>2</sup> s<sup>-1</sup>;  $\mu = 1.272 \times 10^{-4}$  Nm;  $\varpi = 5.61$  rad/s.

Position and velocity time series are obtained from numerical integration of the mathematical model with  $\varpi = 5.61$  rad/s, a frequency related to chaotic behavior. UPOs embedded in chaotic attractor are identified by using the close return method (Auerbach *et al.*, 1987). This identification consists in the first step of the learning stage being common to all control methods.

It is assumed that a scalar time series of angular position is acquired with sampling time  $2\pi/(120\varpi)$ , where  $\varpi$  is the forcing frequency. For  $\varpi=5.61\text{rad/s}$ , the sampling time is  $\tau_s \approx 9.3 \times 10^{-3}\text{s}$ . In order to reconstruct the dynamics of the system from time series, the method of delay coordinates is employed. The average mutual information method is employed to determine time delay (Fraser & Swinney, 1986) while the false nearest neighbors method is used to estimate embedding dimension (Rhodes & Morari, 1997). Thus,  $\kappa$  is determined by the minimum value of  $I(\kappa)$  curve, shown in Figure 1(a), and  $D_e$  is determined by Figure 1(b) when the false nearest neighbors percentage is approximately zero. Therefore, it is obtained that  $\kappa \approx 32$  and  $D_e = 3$ . Figure 2 shows the reconstructed state space and Poincaré section related to chaotic behavior, employing these immersion parameters. Under this assumptions, note that the time delay is  $\tau = \kappa\tau_s = 32\tau_s$  while the embedding dimension is  $D_e=3$ .

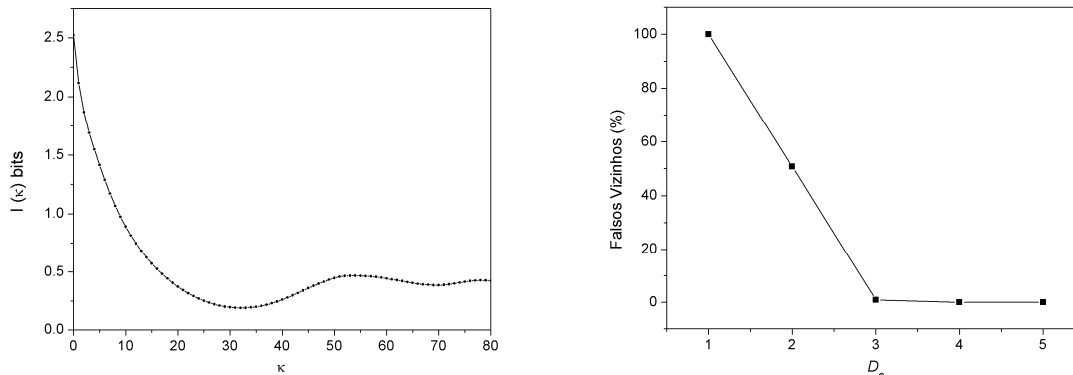


Figure 1: Delay parameters determination. Left: time delay,  $\kappa$ ; and Right: embedding dimension,  $D_e$ .

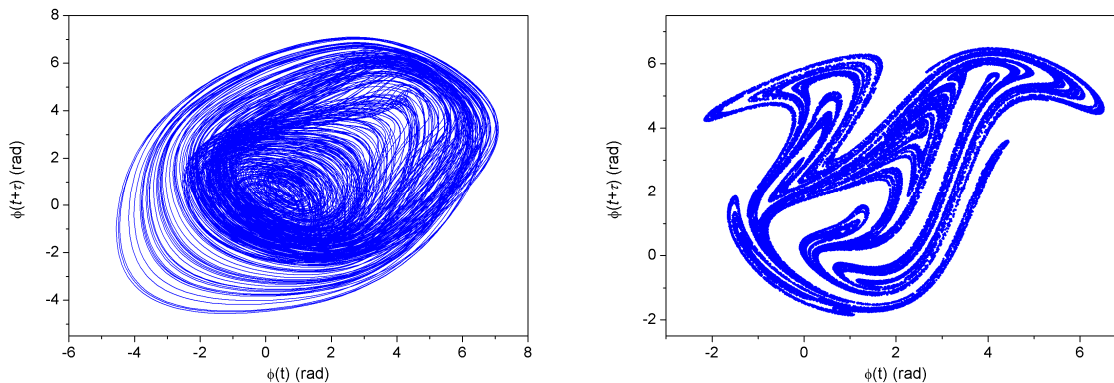


Figure 2: Reconstructed dynamics. Left: Phase space; and Right: Poincaré section.

At this point, the capability of the uncoupled approach of the SC-MP to stabilize UPOs using delay coordinates reconstruction is of concern. With this aim, it is considered 4 control sections ( $S_1, S_2, S_3$  and  $S_4$ ) uniformly distributed in one forcing period. Moreover, once the signal is sampled 120 times per forcing cycle, the time interval between two consecutive control sections,  $\tau_\Sigma$ , correspond to 30 samples,  $\tau_\Sigma = 30\tau_s$ . On the other hand, the time delay is  $\tau = 32\tau_s$ . Thus, to include all control parameters influence in the interval  $t^n - 32\tau_s \leq t \leq t^n$  it is necessary to consider the perturbations  $\delta P^n$ ,  $\delta P^{n-1}$  and  $\delta P^{n-2}$ . This implies that only sensitivity matrixes  $W_0^n$ ,  $W_1^n$  and  $W_2^n$  should be determined during the learning stage. If  $\tau$  is smaller than  $\tau_\Sigma$ , only the influence of  $\delta P^n$  and  $\delta P^{n-1}$  would be enough.

A control rule is defined for the stabilization of four different UPO in the following sequence: a period-7 orbit during the first 500 periods, a period-5 from period 500 to 1000, a period-1 from 1000 to 1500 and, finally a period-6, from period 1500 to 2000. Maximum perturbation of  $|\Delta I_{1\text{max}}|=5\text{mm}$  and  $|\Delta I_{2\text{max}}|=15\text{mm}$  are assumed with reference position being  $\Delta I_{10}=\Delta I_{20}=0\text{mm}$ . Figure 3 presents the UPOs of the control rule at the considered control sections  $S_1, S_2, S_3$  and  $S_4$ .

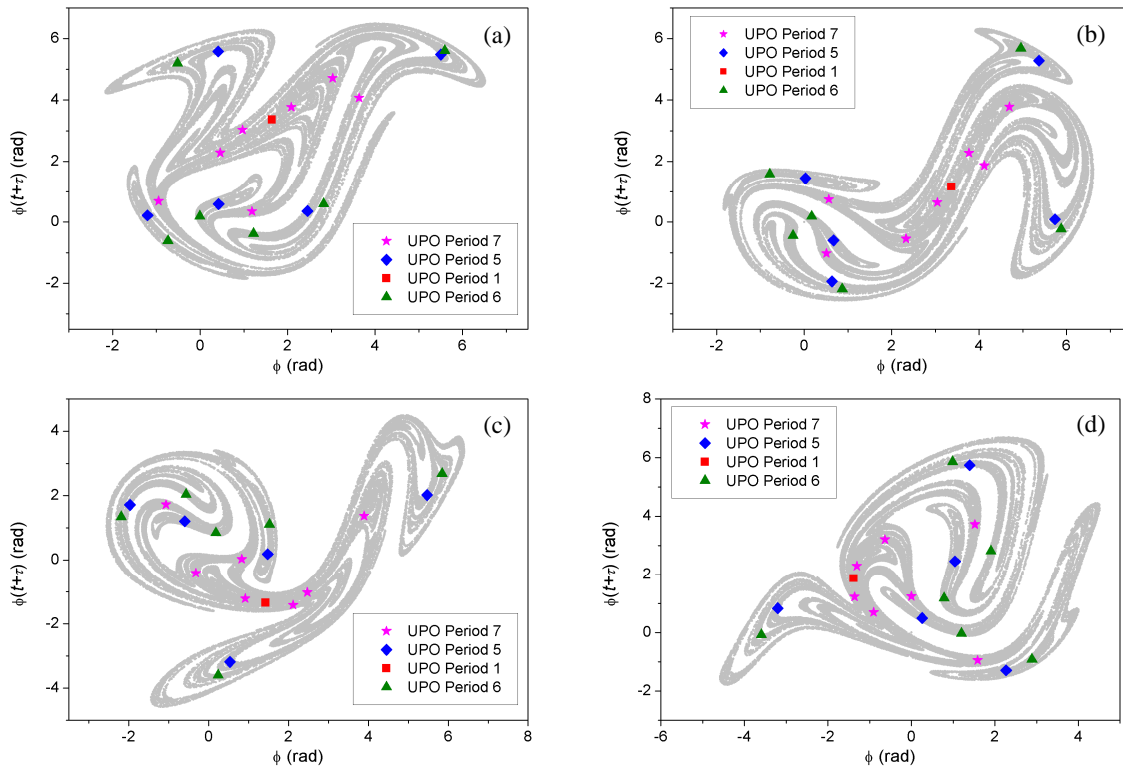


Figure 3: UPOs to be stabilized by the control rule at: (a)  $S_1$ ; (b)  $S_2$ ; (c)  $S_3$ ; and (d)  $S_4$ .

After determining the fixed points of the UPOs at control sections, the local dynamics expressed by the Jacobian matrix and the sensitivity matrix of each fixed points at each control station are determined using the least-square fit method (Auerbach *et al.*, 1987; Otani & Jones, 1997). Sensitivity matrixes  $W_0^n$ ,  $W_1^n$  and  $W_2^n$  are estimated by the procedure described in Dressler & Nitsche (1992). After that, the SVD technique is employed for determining the stable and unstable directions near the next fixed point. After the learning stage, the control stage starts and parameters perturbations are calculated by using Equation (16).

Figure 5(a) and Figure 6(a) show system time evolution at control stations  $S_1$  and  $S_2$ , respectively, while Figure 5(b) and Figure 6(b) show the actuators behavior in the same control stations. These results show that this control approach is effective to stabilize all orbits of the control rule

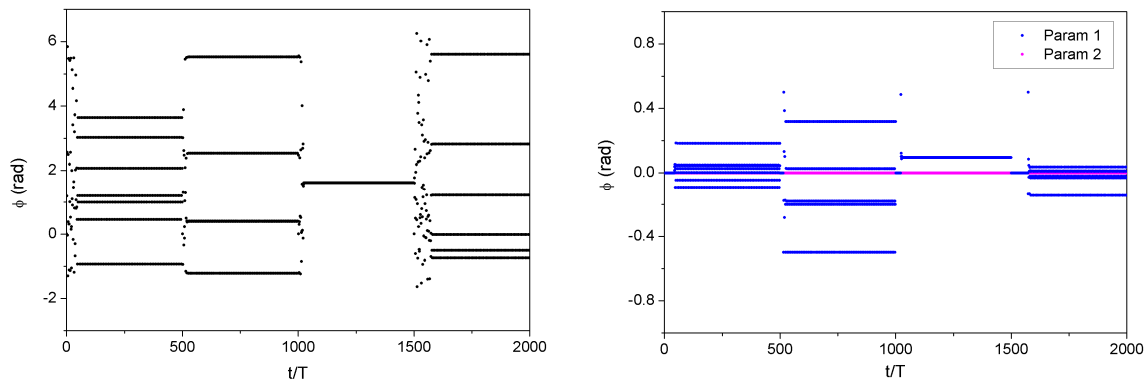


Figure 5: System stabilization at  $S_1$ . Left: Position; and Right: Control signal.

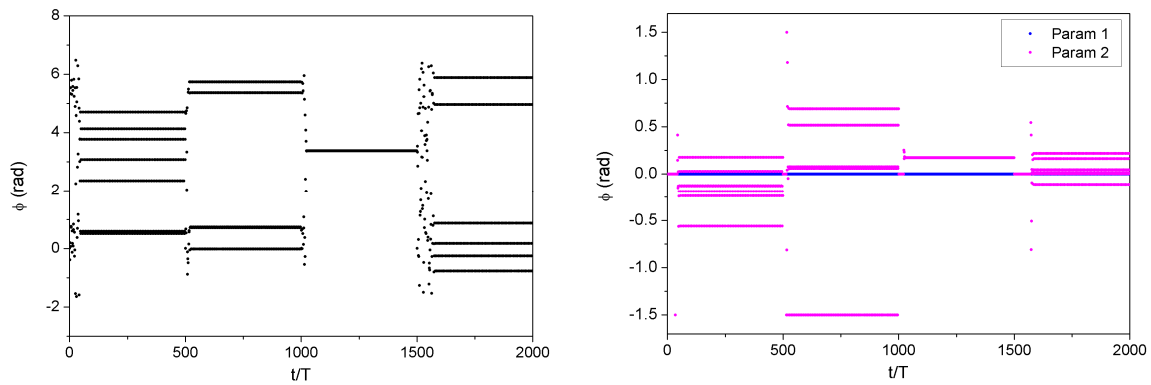


Figure 6: System stabilization at  $S_2$ . Left: Position; and Right: Control signal.

Details of the stabilized UPOs of periodicity 7, 5, 1 and 6 are presented at Figures 7-10, respectively, showing the orbit phase space, temporal evolution of pendulum position and control perturbations. It can be observed that the controller is able to stabilize all UPOs of the control rule. Moreover, after a transient time the perturbation values become periodic.

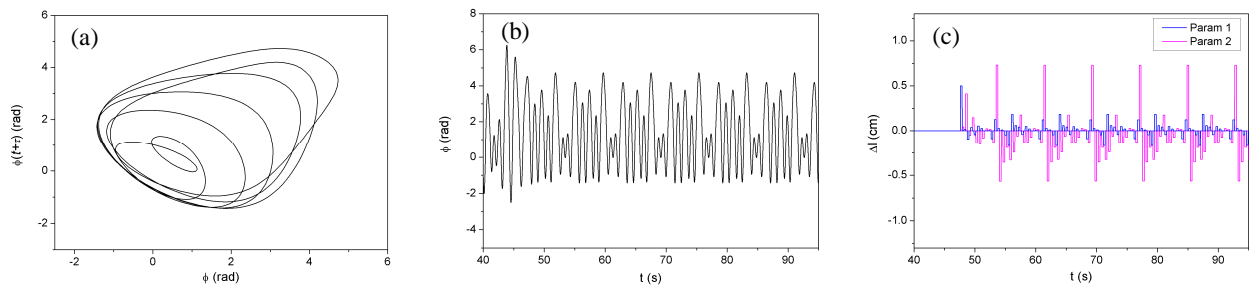


Figure 7: Period-7 UPO stabilization details: (a) Phase space; (b) Position; (c) Perturbations.

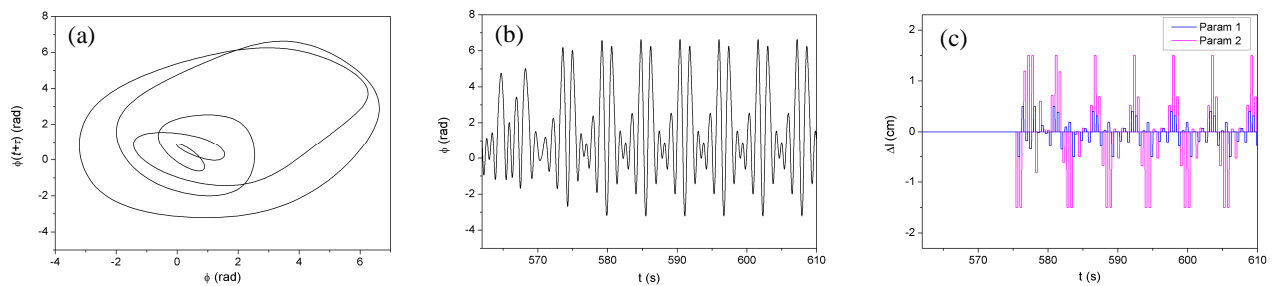


Figure 8: Period-5 UPO stabilization details: (a) Phase space; (b) Position; (c) Perturbations.

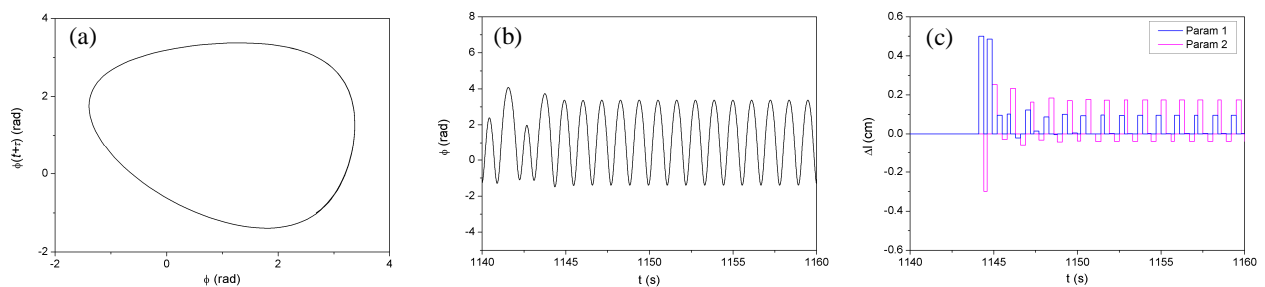


Figure 9: Period-1 UPO stabilization details: (a) Phase space; (b) Position; (c) Perturbations.

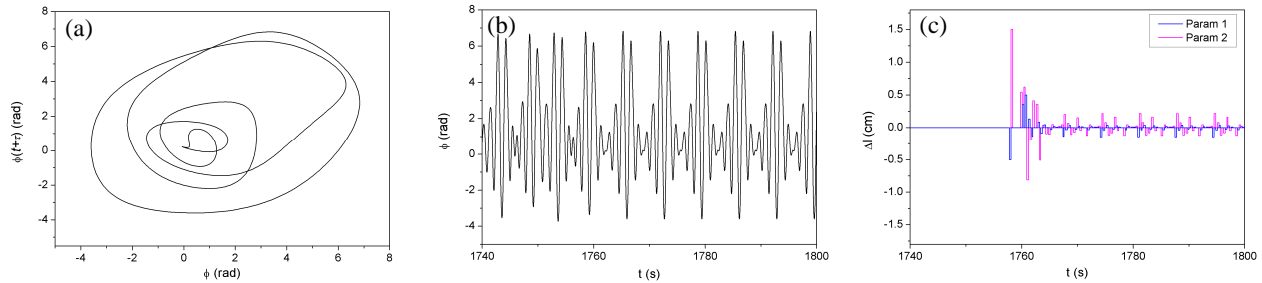


Figure 10: Period-6 UPO stabilization details: (a) Phase space; (b) Position; (c) Perturbations.

The obtained results show that it is possible to achieve the stabilization of UPOs from scalar time series by employing the uncoupled approach of the MP-SC using delay coordinates to reconstruct system dynamics.

#### 4. CONCLUSIONS

This contribution presents the application of the uncoupled approach of semi-continuous multiparameter method to a nonlinear pendulum. Two different control parameters are of concern and only time series of the pendulum position is available. Under these conditions, it is necessary to perform the state space reconstruction. The method of delay coordinates is employed to reconstruct system dynamics. The stabilization of some identified UPOs is successfully achieved showing the possibility of using such approach to control chaotic behavior in mechanical systems using state space reconstruction.

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