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STUDY OF AN FILTER APPLIED THE SOLUTION OF THE CONVECTION FORCED - VIA INTEGRAL TRANSFORM

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Abstract. *Initially it is made a general formulation of the convection forced transient in a completely developed flow and in thermal development, with the use of a periodic temporal function in the entrance condition and a boundary layer that presents convection in the walls. The Generalized Integral Transform Technique is used to find a complete solution of equation of the energy problem, considering the term of axial diffusion. Next, it is solved the problem through a filter, and it is done an analysis of the complete solution of the original problem, supplying thus, a solution of low cost that presents satisfactory results when compared to results that already exist in literature. The computational code developed through the MATHEMATICA system, allows refined analysis of the results.*

Keywords: *convection forced transient, Integral transform , equation of the energy*

1. INTRODUCTION

In engineering, heating and cooling fluids that flow inside conduits are among the most important processes of heat transfer. It may be noted that the solutions found in this study may be useful in the design of any machine or components which are present on the forced convection phenomena inside, especially in so called regions of entry thermal and hydrodynamic fluid. . Therefore, electronic components, condensers, evaporators, heat exchangers, among others are examples of the possible usefulness of this work. More than that, the growing need for miniaturization and optimization of these, has become a global necessity. Moreover, the motivation of this study, far from being a purely academic exercise, is due to an immediate practical necessity which is used in the fields of nuclear energy, transport processes involving chemical reactions, spatial and automatic control.

Another important factor in this study is the growing need for exact solutions to engineering problems, increasingly complex, in a short interval of time. For this reason, the theoretical approach has been gaining ground on trial and the traditional analytic methods. This occurs because the first trial is usually delayed, beyond the fact of being very expensive because for each experiment have been new spending such as: use, modify the prototype, among others, where all of this is due to a possible change in the problem under review, the second because traditional analytic methods have certain limitations which lowers the difficulty by mathematical simplifications, which sometimes make the models too far from real cases having utility in terms of academic or didactic, but rarely objective practical application, and finally, and most important, because with the development of digital computers with processing speeds getting bigger, has advanced significantly in the simulation of problems in fluid mechanics and heat transfer, thus enabling a lower cost and minimizing time to work .

The advancement of numerical simulation techniques has allowed the opening of new directions in research on convection, given that the amount of movement of energy in fluids is modeled by complicated equations, usually nonlinear, which had only numerical solutions. As an example, one can cite the equations of boundary layer, which is a group of convective-diffusive equations, derived from simplifications made in the Navier-Stokes equations, when the terms are neglected longitudinal diffusion and the pressure gradient across the main flow . These simplifications do not allow a full reproduction of reality for high Reynolds numbers and points relatively far from the duct entrance.

The major drawback of numerical methods is due to the fact that a natural loss of physical sensitivity of the proposed problem, besides providing a high computational cost, considering that to achieve a good accuracy is need of a mesh with a high number of points , thus impeding certain solutions.

It has been used to Generalized Integral Transform Technique, which is a hybrid method that has analytical – numerical solution. The basic idea is to transform a system of partial differential equations in a system of ordinary differential equations by the elimination of spatial dependence, with an advantage of producing a more accurate and more economical compared to numerical methods, and does not require mesh generation and to allow a control on the relative error of the results, which is established a priori and automatically controlled.

The basic steps for the implementation of the Generalized Integral Transform Technique are:

a) Choose an ideal auxiliary problem, which contains much information possible regarding the original problem.
b) From an auxiliary problem can be determined by the pair of equations Inverse Transform, which allows for connection between the auxiliary problem and the main problem.
c) The potential helper and the pair transformed - allow inverse transform the system of partial differential equation in an ordinary system equations through operators that allow the elimination of one or more spatial variables.
d) Determining the system of coupled ordinary differential equations, after the truncation of the infinite series in an order large enough for the desired precision, using routines available in numerical scientific libraries, resulting in potential processed.
e) From the transformed potential, obtained in solving the system of equations, is the original potential rescued by the inverse formula.

The periodic forced convection problem is presented in the literature with different approaches. The effects of a spatially variable velocity profile for fully developed laminar flow were accounted for by Cotta and Özisik (1986), where the GITT was employed to provide analytical solutions. In another work, Cotta *et al*. (1987) solved the slug flow problem, including walls conjugation, for circular tube and the parallel plates channel.

This work can be seen in the context of problems of forced convection transient and is considered an extension of the work done by Cotta (1994), Gondim (1997), Cavalcanti (2000), Santos (2004) and other.

2. FORMULATION OF THE PROBLEM

It is considered a hydrodynamically developed laminar flow inside a parallel plate duct, subjected to forced convection, as shown in Fig. 1. Considerations: Incompressible fluid, the physical properties constant, physical effects of viscous dissipation, negligible convection and combination with walls, and a temporal variation of the inlet temperature is the periodic type.

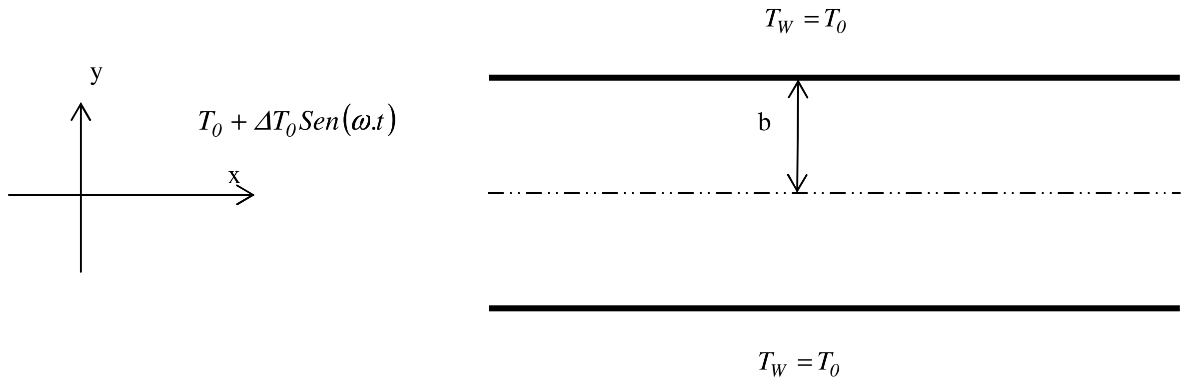

Figure 1 - Representation of the problem

The problem is mathematically defined by the energy equation:

$$\frac{\partial T^*(x^*,y^*,t^*)}{\partial t^*} + u^*(y^*)\frac{\partial T^*(x^*,y^*,t^*)}{\partial x^*} = \alpha\left(\frac{\partial^2 T^*(x^*,y^*,t^*)}{\partial x^{*2}} + \frac{\partial^2 T^*(x^*,y^*,t^*)}{\partial y^{*2}}\right)$$

$$0 < y^* < b \quad , \quad x^* > 0 \quad , t^* > 0 \tag{1}$$

With initial condition and boundary conditions given by:

$$T^*(x^*,y^*,0) = T_0 , \qquad x^* > 0, \qquad 0 \le y^* \le b \tag{2}$$

$$T^*(0,y^*,t^*) = T_0 + \Delta\theta(y^*)Sen(\omega.t^*), \qquad 0 \le y^* \le b, \qquad t^* > 0 \tag{3}$$

$$\frac{\partial T^*(x^*,0,t^*)}{\partial y^*} = 0 , \qquad x^* \ge 0 \quad , \qquad t^* > 0 \tag{4}$$

$$T^*(x^*,b,t^*) = T_0 , \qquad x^* \ge 0, \qquad t^* > 0 \tag{5}$$

$$T^*(\infty,y^*,t^*) = T_0 , \qquad 0 \le y^* \le b, \qquad t^* > 0 \tag{6}$$

3. ADIMENSIONAMENT PROBLEM

For the proposed problem, are considered the following dimensionless groups:

$$x = \frac{x^*/b}{Re.Pr} = \frac{x^*}{b.Re.Pr} \qquad y = \frac{y^*}{b}; \qquad u = \frac{u^*}{\bar{u}}; \qquad t = \frac{\alpha t^*}{b^2};$$

$$T = \frac{T^* - T_o}{\Delta T_o}; \qquad R_e = \frac{\bar{u}\, D_h}{\nu}; \qquad Pr = \frac{\nu}{\alpha}; \qquad Pe_{Dh} = Re_{Dh}.Pr = \frac{\bar{u}.D_h}{\alpha};$$

$$\Omega = \frac{\varpi b^2}{\alpha} \tag{7}$$

By using the dimensionless groups defined as applying to Eq. (1) to Eq. (6) results in:

$$\frac{\partial T(x,y,t)}{\partial t} + u(y)\frac{\partial T(x,y,t)}{\partial x} = \frac{\partial^2 T(x,y,t)}{\partial y^2} + \frac{1}{Pe^2}\frac{\partial^2 T(x,y,t)}{\partial x^2}$$

$$0 < y < 1, \qquad x > 0, \qquad t > 0 \tag{8}$$

$$T(x,y,0) = 0, \qquad x \ge 0, \qquad 0 \le y \le 1 \tag{9}$$

$$T(0,y,t) = \Delta\theta(y)Sen(\Omega.t), \qquad t > 0, \qquad 0 \le y \le 1 \tag{10}$$

$$\frac{\partial T(x,0,t)}{\partial y} = 0, \qquad t > 0 \qquad x \ge 0 \tag{11}$$

$$T(x,1,t) = 0, \quad t > 0 \qquad x \ge 0 \tag{12}$$

$$T(\infty,y,t) = 0, \ t > 0, \qquad 0 \le y \le 1 \tag{13}$$

4. COMPLETE SOLUTION FOR TRANSIENT FORCED CONVECTION THROUGH A FILTER

We used the Integral Transformation Process for solution of Eq. (3), simply operating with a filter. In order to accelerate the convergence of expansions in eigenfunctions unfolds the potential *T(x, y, t)* of the Eq. (8), as follows:

$$T(x,y,t) = \theta(x,y,t) + F(x,y) \tag{14}$$

where *F(x, y)*, solving a problem is convection, which acts as a filter to homogenize the boundary conditions of the original problem, and *Θ(x, y, t)* is the new potential to be determined.

5. FILTER CALCULATION OF F(x, y):

To determine *F(x, y)* intends to use the following issue, purely convective:

$$U(y)\frac{\partial F(x,y)}{\partial x} - \frac{\partial^2 F(x,y)}{\partial y^2} = 0 \tag{15}$$

The boundary conditions are:

$$F(0,y) = \Delta\theta(y)Sen(\Omega.t) \qquad 0 \le y \le 1 \tag{16}$$

$$\frac{\partial F(x,o)}{\partial y} = 0 \qquad x > 0 \tag{17}$$

$$F(x,1) = 0 \qquad x > 0 \tag{18}$$

$$F(\infty, y) = 0 \qquad 0 < y < 1 \tag{19}$$

6. AUXILIARY PROBLEM IN TRANSVERSE DIRECTION

To solve the Eq. (1) using the GITT is necessary as a first step to define an auxiliary eigenvalue problem, which will provide autofunção used to propose the expansion of the potential. We adopt the auxiliary problem in the *y* direction that will define the eigenfunction: $Y_n(y)$:

$$\frac{d^2 Y_n(y)}{dy^2} + \beta_n^2 Y_n(y) = 0 \tag{20}$$

$$\frac{dY_n(0)}{dy} = 0 \tag{21}$$

$$Y_n(1) = 0 \tag{22}$$

where Y_n and β_n are the eigenfunctions and eigenvalues respectively.

The solution is given by eigenfunction:

$$Y_n(y) = cos(\beta_n y) \tag{23}$$

The property of orthogonality is given by:

$$\int_0^1 \tilde{Y}_h(y)\tilde{Y}_g(y)dy = \delta_{hg} \text{ , onde } \begin{cases} \delta_{hg} = 1 \quad , & h = g \\ \delta_{hg} = 0 \quad , & h \neq g \end{cases} \tag{24}$$

6.1 Problem Transformation Filter

Applying the operator $\int_0^1 \tilde{Y}_g(y)dy$ in the filter Eq (15) we have:

$$\int_0^1 (y)_g\, U(y)\frac{\partial F(x,y)}{\partial x}dy - \int_0^1 \tilde{Y}_g(y)\frac{\partial^2 F(x,y)}{\partial y^2}dy = 0 \tag{25}$$

Applying the orthogonality property, defined by Eq. (24) yields the following relationship for the term transformed:

$$\sum_{g=1}^{\infty} \frac{d\overline{F}_h(x)}{dx} A1_{gh} - \beta^2 \overline{F}_g(x) = 0 \tag{26}$$

Where

$$A1_{gh} = \int \tilde{Y}_h(y)\tilde{Y}_g(y)U(y)dy \tag{27}$$

6.2 Solution of Equation Filter

The differential Eq. (26) has the numerical solution, so the inversion of the filter provides the solution:

$$F(x,y) = \sum_{g=1}^{\infty} \tilde{Y}_g(y)\overline{F}_h(x) \tag{28}$$

6.3 Inverse-Transform Pair

Writing the function $\Theta\ (x,\ y,\ t)$ as an expansion which is based on the eigenfunctions derived from the eigenvalue problem associated with the original problem, Eq. (20) to Eq. (24), and noting the orthogonality property we obtain the pair of formulas :

$$\overline{\theta}_n(x,t) = \int_0^1 \tilde{Y}_n(y)\theta(x,y,t)dy \qquad \text{TRANSFORMED} \tag{29}$$

$$\theta(x,y,t) = \sum_{n=1}^{nt} \tilde{Y}_n(y)\overline{\theta}_n(x,t) \qquad \text{INVERSE} \tag{30}$$

6.4 Transformation of the Problem in the direction "y"

Operating the Eq. (29) with the operator $\int_0^1 \tilde{Y}(y)dy$ and applying the Eq. (30) of the inverse in each term of Eq. (31), and using the orthogonality property Eq. (24) we have:

$$\frac{\partial\, \theta(x,y,t)}{\partial\, t} + u(y)\frac{\partial\, \theta(x,y,t)}{\partial\, x} = \frac{\partial^2\, \theta(x,y,t)}{\partial\, y^2} + \frac{1}{Pe^2}\frac{\partial^2 \theta(x,y,t)}{\partial\, x^2} + G(x,y) \tag{31}$$

Thus, the equation is transformed filtered in two-dimensional partial differential system:

$$\frac{\partial\overline{\theta}_n(x,t)}{\partial\, t} + \sum_{m=1}^{nt} A1_{nm}\frac{\partial\overline{\theta}_m(x,t)}{\partial\, x} = -\beta_n^2\overline{\theta}_n(x,t) + \frac{1}{Pe^2}\frac{\partial^2\overline{\theta}_n(x,t)}{\partial\, x^2} + \overline{G}_n(x) \tag{32}$$

where:

$$A1_{nm} = \int_0^1 \tilde{Y}_n(y)\tilde{Y}_m(y)u(y)dy \tag{33}$$

$$\overline{G_n}(x) = \int_0^1 \tilde{Y}_n(y)G(x,y)dy = \int_0^1 \tilde{Y}_n(y)\frac{1}{Pe^2}\frac{\partial^2 F(x,y)}{\partial x^2}dy \tag{34}$$

6.5 Average Temperature

The average temperature of the mixture, in its dimensionless form is given by MIKHAILOV and OZISIK (1984)

$$T_{av}\,(\xi,t) = \frac{\int_0^1 u(y)T(\xi,y,t)dy}{\int_0^1 u(y)dy} \tag{35}$$

and considering the profile of fluid velocity in fully developed flow:

$$u(y) = u_\infty = \frac{3}{2}\left(1 - y^2\right) \tag{36}$$

hence:

$$\int_0^1 u(y)dy = \frac{3}{2}\left(1 - \frac{y^2}{3}\right) = 1 \tag{37}$$

$$T_{av}(\xi,t) = \frac{3}{2}\left\{\sum_{i=1}^{nt}\tilde{X}_k(\xi)\tilde{\bar{\theta}}_i(t)\left[\int_0^1 \tilde{Y}_n(y)dy - \int_0^1 y^2\tilde{Y}_n(y)dy\right] + \sum_{g=1}^{ng}F(\xi)\left[\int_0^1 \tilde{Y}_g(y)dy - \int_0^1 y^2\tilde{Y}_g(y)dy\right]\right\} \tag{38}$$

7. RESULTS AND ANALYSIS

Was generated a MATHEMATICA code that allowed the symbolic manipulation, numerical and analytical, of the mathematical model, simplifying and optimizing calculations made on the problem. The results were compared with results existing in literature.

At first, in order to validate the present work, we determined the convergence for the filter as shown in Tab. 1. Subsequently we obtain the behavior of temperature for various locations along the channel and compared with results obtained by Cheroto (1995), and later by Santos (2002) as shown in Tab. 2.

Table 1. Convergence of the average temperature along the centerline to the Problem Filter: Bi = 10^5
Ω = 0.06491 and $\Delta\theta$ (y) = 1-y^2, Re = 452 and Pr = 0.7.

X	2 terms	4 terms	6 terms	8 terms
0	0.993828	0.999074	0.999714	0.999878
0.1	0.858043	0.856261	0.856218	0.856212
0.2	0.714331	0.713325	0.713292	0.713289
0.3	0.591992	0.591261	0.591234	0.59123
0.4	0.490322	0.489733	0.489711	0.489708
0.5	0.406079	0.405601	0.40558	0.405577
0.6	0.336309	0.335915	0.335897	0.335895
0.7	0.278525	0.2782	0.278186	0.278184
0.8	0.230669	0.230402	0.230389	0.230388
0.9	0.191037	0.190816	0.190805	0.190805
1	0.158213	0.158031	0.158023	0.158023

Table 2. Comparison of results using the same conditions used by Cheroto (1995) with Pe= 316.4,
Ω = 0.06491 and Bi = 10^5

X/D_h	**CHEROTO**	**PRESENT WORK**
1	0.8642	0.8642
3	0.7223	0.7224
6	0.5677	0.5677
9	0.4480	0.4480
12	0.3537	0.3537
15	0.2792	0.2793
18	0.2205	0.2205
21	0.1740	0.1741
24	0.1374	0.1374
27	0.1085	0.1085
30	0.0856	0.0857
33	0.0676	0.0676
36	0.0534	0.0534
39	0.0421	0.0422

The present work obtains the behavior of the average temperature exactly equal to that achieved by Cheroto (1995), the graph in Fig. 2 shows the behavior of the average temperature along the channel.

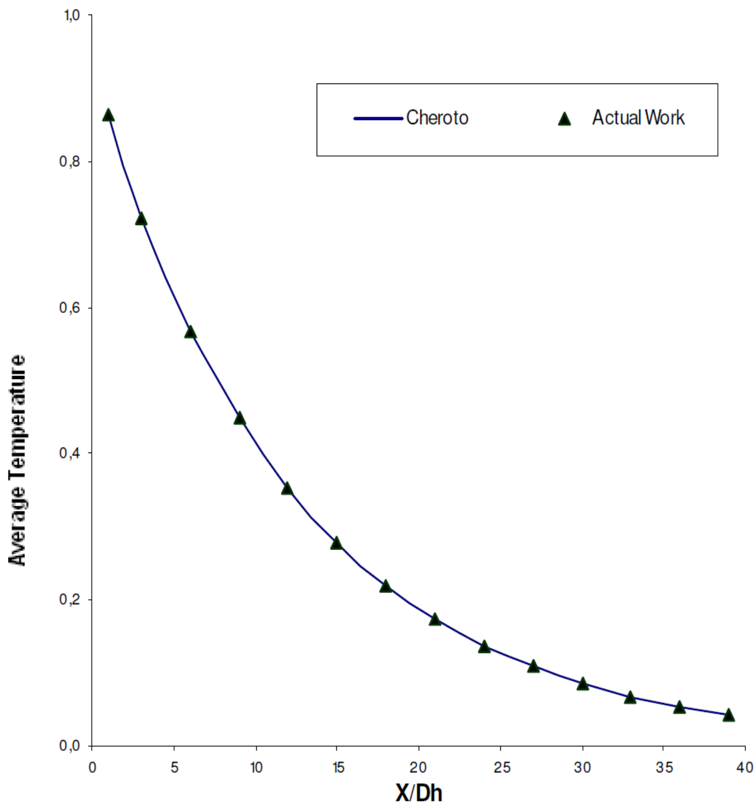

Figure 2. Behavior of the average temperature along the channel with $\Omega = 0.06491$, Pe = 316.4 and Bi = 10^5.

Table 3 and Tab.4 compares these results using the same conditions used by Santos (2002), as: Bi = 10^5, Ω = 0.06491 and $\Delta\theta$ (y) = $1-y^2$, Re = 452 and Pr = 0.7.

Table 3. Comparison temperature along the centerline between the present work and Santos (2002). Bi = 10^5, $\Omega = 0.06491$ and $\Delta\theta$ (y) = $1-y^2$, Re = 452 and Pr = 0.7.

X	2 terms	4 terms	6 terms	8 terms	12 terms	Santos (2002)
0	0.993828	0.999074	0.999714	0.999878	0.999878	0.999814
0.1	0.858043	0.856261	0.856218	0.856212	0.856213	0.856219
0.2	0.714332	0.713325	0.713292	0.713289	0.713291	0.713293
0.3	0.591992	0.591236	0.591234	0.591233	0.591234	0.591239
0.4	0.49032	0.489733	0.489711	0.489711	0.489712	0.489719
0.5	0.40608	0.405601	0.40558	0.405581	0.405578	0.40559
0.6	0.33631	0.335915	0.335897	0.335899	0.335901	0.335909
0.7	0.278526	0.27828	0.278186	0.278188	0.27819	0.278199
0.8	0.23067	0.230402	0.230389	0.230393	0.230395	0.230404
0.9	0.191038	0.190816	0.190805	0.190811	0.190813	0.19082
1.0	0.158215	0.158031	0.158023	0.158029	0.158031	0.158037

Table 4. Comparison of the average temperature along the axis between the present work and Santos (2002). Bi = 10^5, Ω= 0.06491 and $\Delta\theta$ (y) = 1-y^2 , Re = 452 and Pr = 0.7.

X	1 term	3 terms	6 terms	9 terms	12 terms	Santos (2002)
0	0.8	0.8	0.8	0.8	0.8	0.8
0.1	0.654616	0.654616	0.654617	0.654617	0.654617	0.654617
0.2	0.541396	0.541397	0.541397	0.541397	0.541398	0.5414
0.3	0.448291	0.448291	0.448292	0.448292	0.448293	0.448296
0.4	0.371259	0.37126	0.371261	0.371261	0.371262	0.371266
0.5	0.307472	0.307472	0.307474	0.307474	0.307475	0.30748
0.6	0.254644	0.254645	0.254646	0.254646	0.254648	0.254654
0.7	0.210893	0.210894	0.210896	0.210896	0.210897	0.210903
0.8	0.174659	0.174660	0.174662	0.174662	0.174663	0.17467
0.9	0.144651	0.144653	0.144654	0.144654	0.144655	0.144661
1.0	0.119799	0.119801	0.119802	0.119802	0.119803	0.119808

Figure 3 shows the comparison of the behavior of temperature along the centerline for the case theoretical and practical case presented by Li, W. (1990). In the Fig. 3 was used a number Biot = 10^5, Re = 430, Pr = 0.7 and thermal parabolic entry profile.

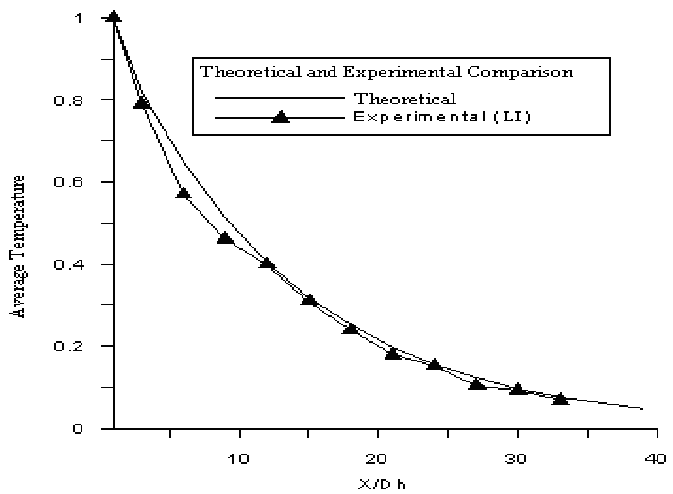

Figure 3. Comparison of temperature along the centerline of the channel between the present work and experimental work presented by Li, Bi = 10^5, Ω = 0.01 and $\Delta\theta$ (y) = 1-y^2, Re = 430 and Pr = 0.7

8. CONCLUSIONS

The Generalized Integral Transform Technique has proven to be an excellent alternative to solve problems purely convective, because beyond the safety of proven results, it still offers the main advantage of a low computational cost when you want the result of engineering.

This method was the application of GITT in transverse and longitudinal directions to the fluid flow and allowing accurate analysis of the results.

Another significant help is the use of the property of orthogonality of the eigenfunctions in the elimination of directional dependency, allowing greater freedom in the choice of auxiliary problems.

.The reliability of the results compared to traditional numerical methods is mainly due to greater analytical treatment of the equations representing the phenomenon. Thus it is possible to simplify the solution without the need to change the simplifying assumptions of mathematical modeling. The GITT may be more conservative than the traditional numerical methods, since it can have an automatic control of precision.

The development of GITT, some years ago, allowed a huge field of work both in the scientific and technological fields. It is past the stage of proof of the advantages. The point is to exploit this technique in all its possibilities.

Given the above mentioned, it appears that the technique used in this work is efficient in solving the heat flow in developing, obtaining also an excellent agreement with results obtained by Cheroto (2002), where the results are valid for long times.

9. REFERENCES

CAVALCANTE, F.F., "Convecção Forçada Transiente Interna com Entrada Periódica: Solução Via Transformada Integral", Dissertação de Mestrado, UFPB, Brasil, 2000.

CHEROTO, S., "Theoretical Investigation of Unteady Forced Convection in Parallel-Plate Channels for Thermally Developing Flow", Master Thesis, University of Maiami, Coral Gables, FL, USA, December 1995.

COTTA, R.M. and OZISIK, M.N., "Laminar Forced Convection Inside Ducts with Periodic Variation of Inlet Temperature", International Journal of Heat and Mass Transfer, vol.29, pp. 1495-1501, 1986.

COTTA, R.M., MIKHAILOV, M.D. and OZISIK, M.N., "Transient Conjugated Forced Convection in Ducts with Periodically Varying Inlet Temperature", International Journal of Heat and Mass Transfer, vol. 30, pp. 2073-2082, 1987.

COTTA, R.M. and GERK, J.E.V., "Mixed Finite-Difference/Integral Transform Approach for Parabolic-hyperbolic Problems in Transient Forced Convection", Num. Heat Transfer, part B. Vol. 25, pp.433-448, 1994.

GONDIM, R.R., "Convecção Forçada Transiente Interna com Difusão Axial Solução Via Transformação Integral", Tese, UFRJ/COPPE, Rio de Janeiro, Brasil, 1997.

LI, W., and KAKAÇ, S., Experimental and Theoretical Investigation of Unsteady Forced Convection in Ducts, Ph.D. Dissertation, University of Miami, Coral Gables, FL, USA, 1990.

MIKHAILOV, M. D and OZISIK, M N, "Unified Analysis and Solutions of Heat and Mass Diffusion"; Johhn Wiley, NewYork, 1984.

SANTOS, S.J., "Analise Teórico de convecção Forçada Laminar Transiente com entrada Periódica", UFPB, João Pessoa, 2002.

SANTOS, J.C., "Análise Teórica de Convecção Forçada Laminar Transiente de Fluido Não-Newtoniano Com Entrada Periódica", Annals of 10th Brazilian Congresso of Thermal Sciences and Engineering, ENCIT 2004, Rio de Janeiro, 2004.

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