

A VORTEX LATTICE PROGRAM FOR STEADY STATE AERODYNAMIC ANALYSIS OF WIND TURBINE BLADE LOADS

Da Silva, Cláudio Tavares, ctavares@ita.br
UTFPR - Universidade Tecnológica Federal de Paraná, Avenida Sete de Setembro, 3165, Centro, Curitiba - PR, CEP 80230-901.
Donadon, Maurício Vicente, donadon@ita.br
Menezes, João Carlos, menezes@ita.br
Silva, Roberto Gil Annes, gil@ita.br
ITA - Instituto Tecnológico de Aeronáutica, Praça. Marechal Eduardo Gomes, 50, Vila das Acácias, São José dos Campos - SP, CEP 12228-900.

Abstract. *This paper presents a revised MATLAB® program for estimating the subsonic aerodynamic characteristics of an optimal design of wind turbine blades using the Vortex Lattice Method (VLM). The aerodynamic characteristics of interest are principally pressure distribution and aerodynamic loads, but lift, rolling moments and pitching moments are also investigated. It is included the camber line of the airfoil which represents its characteristic parameters of lift and moments. The results of rolling and pitching moments, as well as flapping moments, are compared with the ones obtained from previous works based on the Blade Element Method. The program is developed and revised to evaluate the viability of using the VLM to carry out virtual tests in order to obtain consistent and confident data for the structural design of the wind turbine blades. Numerical results are presented, compared with classical results of the literature and discussed.*

Keywords*: Aerodynamics, Vortex Lattice Method, Wind turbine blade loads, Blade Element Method.*

1. INTRODUCTION

The demand for energy, more specifically electricity, has increased drastically over the last 100 years. It has now become important to consider the environmental impacts of energy production. Therefore, there is general agreement that to avoid energy crisis, the amount of energy needed to sustain society will have to be contained and, to the extent possible, renewable sources will have to be used. As a consequence, conservation and renewable energy technologies are going to increase in importance and reliability.

Within this context, this paper focuses on the development of a blade design tool for horizontal axis wind turbines with variable geometry. The design tool consists of an in-house MATLAB® program based on the Glauert Blade Element Theory (Burton, 2001), including tip and rotational wake losses as well as blade pitching and blade twisting effects, for the aerodynamic design. The program enables predictions of aerodynamic power, efficiency and forces acting on the wind turbine blades for a given operating condition.

One of the crucial and fundamental steps of the wind turbine blade design is the structural design. The determination of the aerodynamic loads associated with a given morphing blade configuration is essential. For this purpose, a Vortex Lattice based program has been also adapted and presented in this paper. The program enables the prediction of lift, pitching moments, rolling moment and pressure distribution for twisted plane shaped blade. One sample case for a 2 MW wind turbine is presented and discussed.

2. AERODYNAMICS OF WIND TURBINES AND OPTIMIZATION

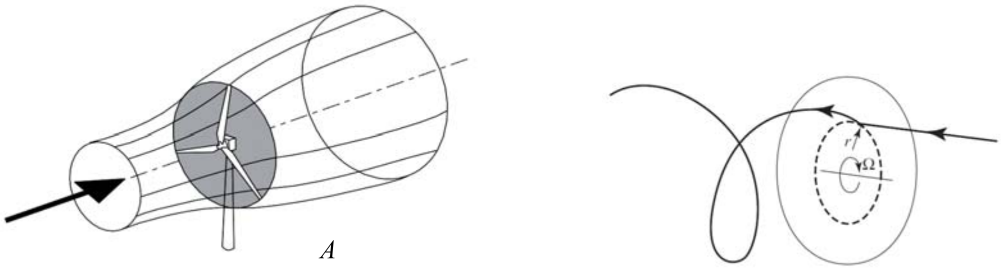
Blade Element Theory (BET) is a mathematical process originally developed to determine the behavior of propellers. Glauert adapted the theory to wind turbines. Blade Element Theory attempts to address information on rotor performance or blade design by considering the effects of blade design, i.e., shape, section, twist, etc. Blade element theory models the rotor as a set of isolated two-dimensional blade elements to which one is able to apply bi-dimensional aerodynamic theory individually and then perform an integration to find thrust and torque.

A model attributed to Betz (Burton, 2001), can be used to determine the power from an ideal turbine rotor. This model is based on a linear momentum theory. Assuming a decrease in wind velocity between the free stream and the rotor plane, an axial induction factor can be defined as $a=\left(U-U_d\right)/U$ where U is the free stream wind velocity and U_d is the wind velocity at the rotor plane disk.

The governing principle of conservation of flow momentum can be applied for both axial and circumferential directions. For the axial direction, the change in flow momentum along a stream-tube starting upstream, passing through the propeller disk area and then moving off into the downstream must be equal to the thrust produced by this element of the blade. To remove the unsteady effects due to the propeller's rotation, the stream-tube, according to Fig. 1a, is covering the complete area A of the propeller disk swept out by the blade element. In this model all variables are assumed to be time averaged values. The axial thrust on the rotor plane disk is given by

$$T=\frac{1}{2}\rho A U^2\left[4a\left(1-a\right)\right], \quad (1)$$

where ρ is the air density.



(a) (b)

Figure 1. (a) The energy extracting stream-tube of a wind turbine. (b) The trajectory of an air particle passing through the rotor disc.

The power output is equal to the thrust times the velocity at the disk.

$$P = \frac{1}{2}\rho A U^3 4a\left(1 - a\right)^2 . \tag{2}$$

Wind turbine rotor performance is usually characterized by its power coefficient which represents the fraction of the power in the wind that is extracted by the rotor.

$$C_p = \frac{Rotor\ power}{Power\ on\ the\ wind} = \frac{P}{1/2\,\rho A U^3} . \tag{3}$$

If one considers the wake rotation, according to Fig. 1b, where the angular velocity imparted to the flow stream is ω, while the angular velocity of the wind turbine rotor is Ω, then, across the flow disk, the angular velocity of the air relative to the blade increases to $(\omega+\Omega)$. One can prove that the tangential component of velocity is $\Omega r(1+a)$, according to Fig. 2. There, α is the angle of attack, ϕ is the angle of relative wind and β is section pitch angle.

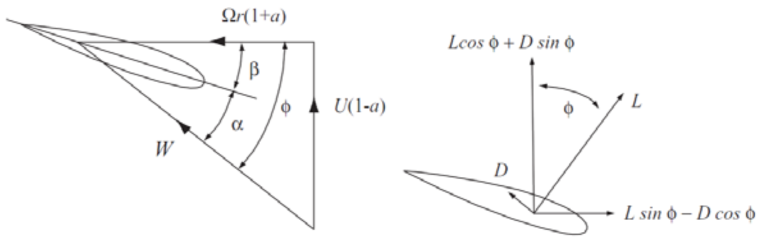


Figure 2. Blade element velocities, forces and angles.

Taking into account the lift force L and the drag force D, one can define the lift and the drag coefficients as

$$C_l = \frac{Lift\ force}{Dynamic\ force} = \frac{L}{1/2\,\rho U^2 c} \text{ and } C_d = \frac{Drag\ force}{Dynamic\ force} = \frac{D}{1/2\,\rho U^2 c} . \tag{4}$$

From Fig. 2, one can determine the following relationships:

$$W = \frac{U\left(1 - a\right)}{sin\phi} , \tag{5}$$

$$F_N = Lcos\phi + Dsin\phi \text{ and } F_T = Lsin\phi - Dcos\phi , \tag{6}$$

where W is the relative wind velocity, F_N is the normal force, and F_T is the tangential force to the disk.
If the rotor has B blades, the differential normal force on the section at the distance, r, from the center is

$$\mathrm{dF_N} = B\frac{1}{2}\rho W^2\left(C_l\cos\phi + C_d\sin\phi\right)cdr \tag{7}$$

and the differencial torque is

$$\mathrm{dQ} = B\frac{1}{2}\rho W^2\left(C_l\sin\phi - C_d\cos\phi\right)crdr\,. \tag{8}$$

The power coefficient can be calculated as a function of the tip speed ratio $\lambda = \Omega R/U$ and the local speed ratio $\lambda_r = \lambda\, r/R$ (Gash and Twele, 2002) by

$$C_p = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda}\sin^2\phi\left(\cos\phi - \lambda_r\sin\phi\right)\left(\sin\phi + \lambda_r\cos\phi\right)\left[1 - \frac{C_d}{C_l}\cot\phi\right]\lambda_r^{\,2}d\lambda_r\,, \tag{9}$$

One can perform the optimization of the blade shape for an ideal rotor by taking the partial derivative of C_P which is a ϕ function, and setting it equal to zero, to reveal that

$$\phi = \frac{2}{3}tan^{-1}\left(\frac{1}{\lambda_r}\right) \tag{10}$$

and

$$c = \frac{8\pi r}{BC_l}\left(1 - cos\,\phi\right). \tag{11}$$

Applying the Blade Element Theory (Donadon et al., 2008), the total wind turbine thrust and torque is obtained by summing the results of all the radial blade elements along the radial direction.

$$T = \sum_{i=1}^{N}\Delta T \text{ and } Q = \sum_{i=1}^{N}\Delta Q\,, \tag{12}$$

where N is the number of blade elements along the radial direction.

3. VORTEX LATTICE METHOD (VLM)

In order to determinate the aerodynamic loads, a computational program was developed based on the Vortex Lattice Method (VLM) (Donadon and Iannucci, 2006) and adapted to wind turbine blades in this work. This section presents a short review on the VLM and the fundamental equations used in the numerical implementation.

The VLM is the simplest of the methods to solve incompressible flows around wings of finite span (Bertin, 1989). The method represents the wing as a planar surface on which grids of horseshoe vortex are superimposed. The computation of the velocities induced by each horseshoe vortex at each specified control point is based on the Biot-Savart law (Bertin, 1989). A summation is performed for all control points on the wing to produce a set of linear system of equations for the horseshoe vortex strengths that satisfy the boundary conditions of no flow through the wing. The control points of each element (or lattice) are located at three-fourth of the element’s chord and the vortex strengths are related to the wing circulation and the pressure difference between the upper and lower surface of the wing. The pressure differentials are then integrated to yield the total forces and moments. In the approach used here to solve the governing equations, the continuous distribution of bound vorticity over the wing surface is approximated by a finite number of discrete horseshoe vortices, as shown in Fig. 3. The individual horseshoe vortices are placed in rectangular (or trapezoidal) panels also called finite elements or lattices. The bound vortex coincides with quarter-chord line of the element and is therefore, aligned with the local sweepback angle. In a rigorous theoretical analysis, the vortex lattice panels are located on the mean chamber of the wing and, when the trailing vortices leave the wing, they follow a curved path. However, for many engineering applications, suitable accuracy can be obtained with a linearized theory in which straight line vortices extend downstream to infinity.

The easy implementation, high speed running and consistent result are the most important reasons to adopt this method to calculate the aerodynamic load. This procedure will make possible an optimization routine that is the final objective of the entire project this work is inside on.

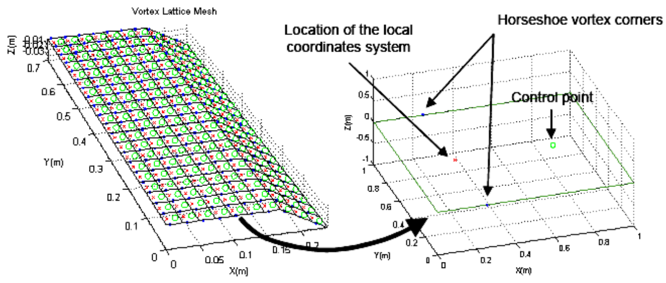


Figure 3. Vortex Lattice discretization.

3.1. Velocity induced by a general horseshoe vortex

Let's assume a typical three dimensional horseshoe vortex composed by a bound vortex segment and two trailing vortex segments as shown in Fig. 4.

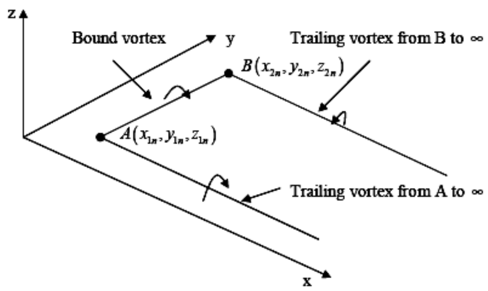
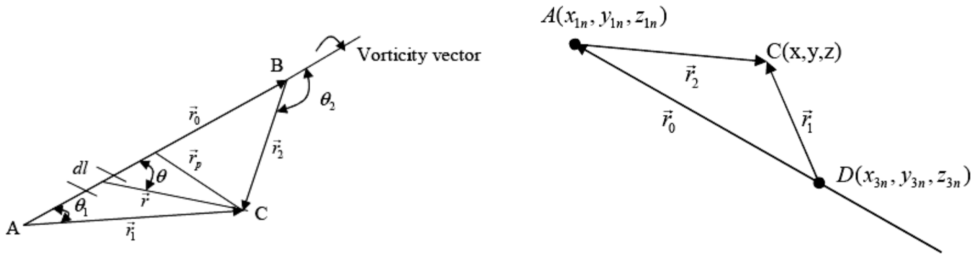


Figure 4. Typical horseshoe vortex.

The computation of the velocity induced at some point *C*(*x,y,z*) by a vortex segment *AB* for instance (See Fig.5a) is based on the Biot and Savart law as follows (Bertin, 1989),

$$\vec{V}_{AB} = \frac{\Gamma_n \cdot \vec{r}_1 \times \vec{r}_2}{4\pi \left|\vec{r}_1 \times \vec{r}_2\right|^2} \left[\vec{r}_0 \left(\frac{\vec{r}_1}{r_1} - \frac{\vec{r}_2}{r_2} \right) \right]. \tag{13}$$



(a) (b)

Figure 5. (a) Vortex segment *AB*. (b) Vortex segment *A*∞.

To calculate the velocity induced by the filament that extends from *A* to ∞, let us first calculate the velocity induced by the collinear, finite-length filament that extends from *A* to *D* according to Fig. 5b.

Applying again Biot and Savart law one can show that the velocity induced by the filament that extends from *A* to ∞ is given by,

$$\vec{V}_{A\infty} = \frac{\Gamma_n}{4\pi} \left\{ \frac{\left(z - z_{1n}\right) j + \left(y - y_{1n}\right) k}{\left(z - z_{1n}\right)^2 + \left(y_{1n} - y\right)^2} \right\} \left[1 + \frac{x - x_{1n}}{\sqrt{\left(x - x_{1n}\right)^2 + \left(y - y_{1n}\right)^2 + \left(z - z_{1n}\right)^2}} \right]. \tag{14}$$

Similarly, the velocity induced by the vortex filament that extends from *B* to ∞ is written.

The total velocity induced at some point (*x, y, z*) by the horseshoe vortex representing one of the surface elements with trailing vortices parallel to *x* axis is

$$\vec{V} = \vec{V}_{AB} + \vec{V}_{A\infty} + \vec{V}_{B\infty} . \tag{15}$$

Assuming the point *C* to be the Control Point (CP) of the *mth* panel, with coordinates (x_m, y_m, z_m) located at midspan of the element and three-fourth of the elemental chord, we can express the velocity induced at the *mth* control point by the vortex representing the *nth* panel as,

$$\vec{V}_{m,n} = \vec{C}_{m,n}\Gamma_n = \left(C^u_{m,n} i + C^v_{m,n} j + C^w_{m,n} k\right)\Gamma_n \tag{16}$$

where the influence coefficient *C* depends on the geometry of the *nth* panel and its distance from the control point of the *mth* panel. Since the governing equation is linear, the velocities induced by the *2N* vortices are added together to obtain an expression for the total induced velocity at the *mth* control point:

$$\vec{V}_m = \vec{C}_{m,n}\Gamma_n = \sum_{n-1}^{2N}\left(C^u_{m,n} i + C^v_{m,n} j + C^w_{m,n} k\right)\Gamma_n . \tag{17}$$

3.2 Computation of the vortex strengths

To compute the strengths of the vortices Γ_n which represent the lifting flow field of the wing, we use the boundary condition that the surface is a streamline. That is, the resultant flow is tangent to the wing at each and every control point. If the flow is tangent to the wing, the component of the induced velocity normal to the wing at the control point balances the normal component of the free-stream velocity. By doing that we can obtain the vortex strengths solving the following linear system of equations (Bertin, 1989),

$$\left\{\Gamma_n\right\} = \left[\overline{C}^w_{m,n} - \overline{C}^v_{m,n}\tan\left(\phi\right)\right]^{-1} 4\pi U_\infty \left\{\alpha_m\right\} . \tag{18}$$

where U_∞ , ϕ and α_m are the flow velocity, wing dihedral angle and elemental local angle of attack at the *mth* control point.

The derivation of the elemental local angle of attack is based on Fig.6 and is given by

$$\alpha_m = a\cos\left(\frac{\vec{U}\cdot\left(\vec{p}_1\times\vec{p}_2\right)}{\left|\vec{U}\right|\left|\vec{p}_1\times\vec{p}_2\right|}\right) - \frac{\pi}{2} \tag{19}$$

with $\vec{U} = U_\infty\cos\left(\alpha_i\right)\vec{i} + U_\infty\sin\left(\alpha_i\right)\vec{j}$ where α_i is the angle of attack associated with the incident flow in the plane *xz*.

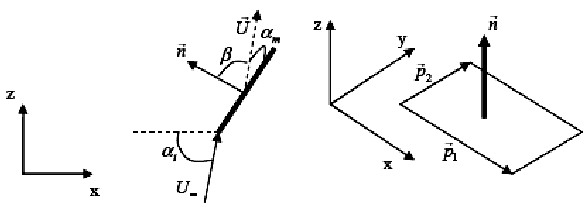


Figure 6. Elemental angle of attack.

3.3 Lift computation

Having determined the strength of each of the vortices, the lift of the wing may be calculated. The expression to compute the local lift acting on the *nth* panel of a wing with no dihedral is given by (Bertin, 1989),

$$\Delta l_n = \rho_\infty U_\infty \Gamma_n . \tag{20}$$

which is also the lift per unit of span. The total lift of a wing subjected to a symmetric flow is

$$L = 2\rho_\infty U_\infty \sum_{n=1}^{N}\Gamma_n . \tag{21}$$

where *N* is the number of panels of the blade, Δl_n is the elemental span length and ρ_∞ is the air density.

3.4 Incremental pressure coefficient calculation

The incremental pressure coefficient for the *nth* element is given by (Margason, 1971),

$$\Delta c_{p,n} = \frac{2\Gamma_n}{c_n U_\infty} . \tag{22}$$

where c_n is the elemental chord.

4 SOLUTION PROCEDURE FOR BLADE ELEMENT THEORY

The analysis described here assumes that all fluid particles undergo the same loss of momentum, i.e., there are a sufficient number of blades on the rotor for every fluid particle passing through the rotor disc to interact with a blade. With a small number of blades some fluid particles will interact with them but most will pass between the blades and, clearly, the loss of momentum by a particle will depend on its proximity to a blade as the particle passes through the rotor disc. If the axial flow induction factor a is large at the blade position then the inflow angle will be small and the lift force will be almost normal to the rotor plane. The component of the lift force in the tangential direction will be small and so will be its contribution to the torque. A reduced torque means reduced power and this reduction is known as tip loss because the effect occurs only at the outermost parts of the blades. In agreement with Burton (2001) this tip loss can be described with Prandtl's approximation by

$$f_T(\mu) = \frac{2}{\pi} \cos^{-1} e^{R_b E_b} , \tag{23}$$

where $\mu = r/R$, $R_b = \sqrt{1+(\lambda\mu)^2/(1-a)^2}$ and $E_b = (-B/2(1-\mu)/\mu)$.

At the root of a blade the circulation must fall to zero as it does at the blade tip and so it can be presumed that a similar process occurs. The blade root will be at some distance from the rotor axis and the airflow through the disc inside the blade root radius will be at the free-stream velocity. It is usual, therefore, to apply the Prandtl tip-loss function at the blade root as well as at the tip,

$$f_R(\mu) = \frac{2}{\pi} \cos^{-1} e^{E_R R_b} . \tag{24}$$

where $E_R = (-B/2(1-\mu_R)/\mu)$ and μ_R refers the position of the hub.

So, the tip/root loss factor is written as

$$f(\mu) = f_T(\mu) f_R(\mu) . \tag{25}$$

The tip/root loss factor leads to the new value of the induction factor.

$$a = \frac{1}{3} + \frac{1}{3} f - \frac{1}{3}\sqrt{1 - f + f^2} . \tag{26}$$

Once this induction factor is know, its derivative also is obtained,

$$a' = \frac{1}{(\lambda\mu)^2} a \left(1 - \frac{a}{f}\right) . \tag{27}$$

The derivative of Eq. 9 with respect to μ now can be written. It represents the span-wise variation of power extraction in the presence of losses.

$$R\frac{dC_p}{dr} = 8\left(1-a\right)a'\lambda^2\mu^3 . \tag{28}$$

The new inflow angle can be determined by

$$\phi = \tan^{-1}\frac{1-a/f}{\lambda\mu\left(\left(1+a\right)\left(1-a/f\right)/f\left(\lambda\mu\right)^2\right)} . \tag{29}$$

The power coefficient can be found now. Considering the Blade Element Theory already nominated and using equation 29, the C_P is obtained by a summation of all blade elements. Once C_P is already estimated, the diameter of the rotor can be found.

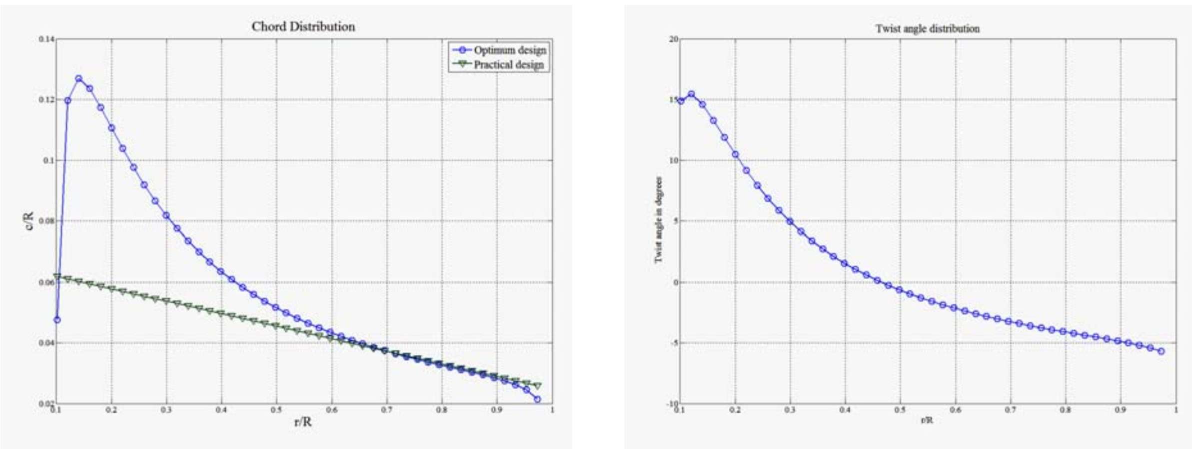
5 NUMERICAL SIMULATION

The theory presented was used to predict the aerodynamic performance of a 2 MW wind turbine.

It has three blades equally spaced along the circumferential direction. For the present work, the aerodynamic characteristic curves in terms of lift coefficient versus angle of attack and drag coefficient versus angle of attack of a NACA 4412 airfoil was taken into account. The adopted criterion for choosing the best airfoil for the wind turbine blade is based on how much aerodynamic power the airfoil can effectively generate and transfer to the wind turbine shaft for a given operating condition. This quantity is measured by the power coefficient C_P, as defined before. The turbine's aerodynamic performance was evaluated for each one of the three airfoils described previously using an in-house MATLAB® program based on the Glauert Blade Element Theory cited in section 2 (Menezes and Donadon, 2009). For this studied case the same airfoil is used in all sections of the blade along the radius direction.

The values of the input data is U=12.5 m/s (rated wind velocity) and N=45 (number of blade elements), adopted for the numerical simulation. The final value for radius found is R=35.8 m. For the simulated case, the air properties were considered for sea level at 20^{0}C.

Figure 8a shows the behavior of the blade chord along the blade radius direction. Figure 8b presents the variation of twist along the radius direction. The chord c is normalized according to the value of R.



(a) (b)

Figure 8. (a) Blade chord along the blade radius direction. (b) Variation of twist angle along the radius direction.

This data are used to construct the geometric model for the VLM.

In earlier works some simplification hypothesis were used in order to facilitate the acquisition of the results on a simple manner (Da Silva et alli, 2010) intending to produce the preliminary results. At this stage of the work those simplifications are not taken into account and a more consistent and realistic model is adopted to ensure the technique produces consistent results. A better accuracy is also expected.

Figure 9 shows the Vortex Lattice mesh produced using the optimal chord and twist distribution, as described before. For the blade span of 35.8 meters, the maximum root chord is about 4.55 meters and tip chord is about 0.77 meters. The twist angle also has an optimal distribution along the blade span. The maximum twist is about -21 degrees considering zero degrees at root chord.

The Vortex Lattice mesh with 704 panels was been obtained. They are 44 elements spanwise and 16 elements chordwise. It is possible to see the nodes and the control points all over de vortex lattice mesh. The camber line of the airfoil NACA4412 of the blade is also represented.

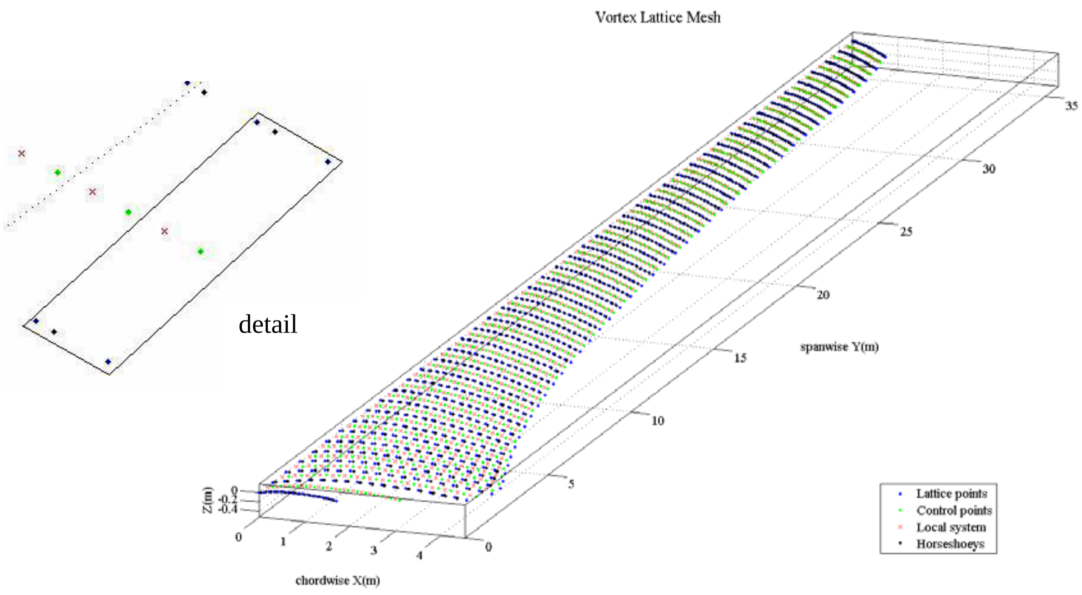


Figure 9. Vortex Lattice mesh.

The most important result obtained at this point of the work is the lift distribution all over the blade span. With the lift distribution, the structural loads can be obtained for the blade surface, as a whole. These results are shown on figures 10 and 11.

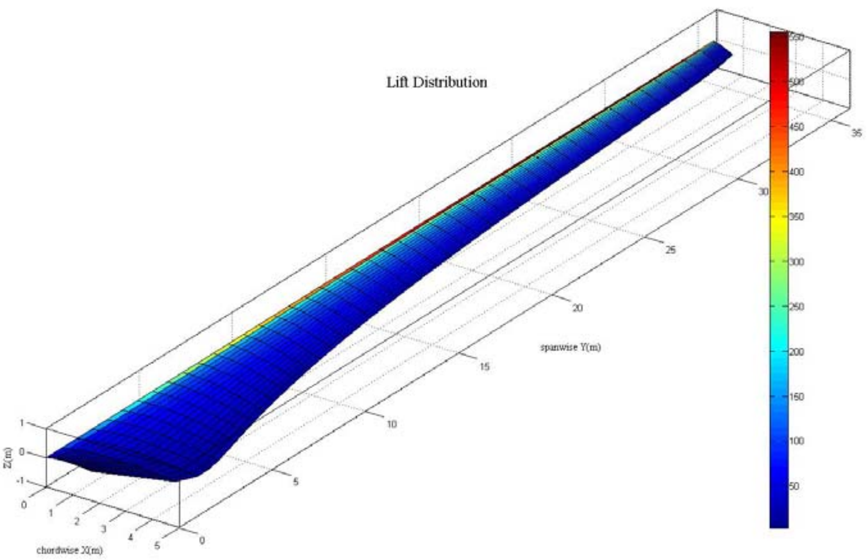


Figure 10. Lift distribution (fring).

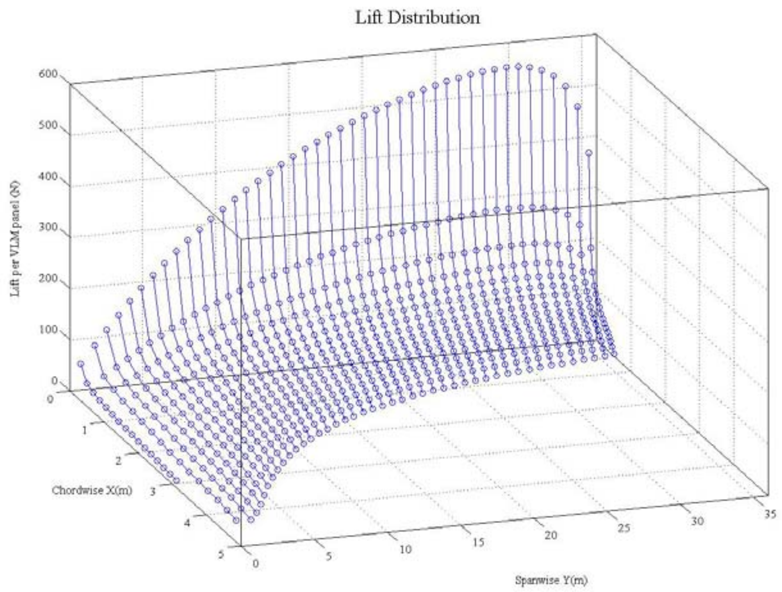


Figure 11. Lift distribution graph.

The VLM can not capture the information about drag at the present stages of the work. To perform some kind of comparison between the results due to BEM and VLM with the objective of adjustment of the models we can compare the total lift. Using BEM, the total lift per blade can be obtained by the following (Burton, 2001).

$$\delta L = \frac{1}{2}\rho W^2 c C_l \delta r . \tag{30}$$

where $W = \sqrt{U_\infty^2(1-a)^2 + \Omega^2 r^2 (1-a')^2}$.

Using VLM the total lift per blade is obtained simply by summation of all lift forces per panel. The comparison of this results in shown in table 1.

Table 1. Comparison of results for total lift per blade.

Composite Properties	BEM	VLM
Total Lift per Blade	106.2 kN	76.1 kN
C_p	0.54	0.39

6 CONCLUSIONS

An aerodynamic model for wind turbines with variable geometry was presented and discussed in this work. Details about the numerical implementation were also presented and discussed. The proposed formulation is based on the Glauert Blade Element Theory which accounts for tip and rotational wake losses described with Prandtl’s approximation, which enables the prediction of torque, thrust and power coefficient for wind turbines with different airfoils geometries and subjected to a wide range of operating conditions. A study case in terms of aerodynamic performance was presented for a 2.0 MW wind turbine, in which a nominal velocity of 12.5 m/s was considered. The numerical predictions indicated a final radius R and a C_p value coherent with expected values for a 2.0 MW wind turbine blade. The same results may be compared with wind turbine blades commercially adopted.

This paper also presented a detailed formulation of the Vortex Lattice Method (VLM) for plane shapes. The proposed formulation has been implemented into MATLAB software and a user interface with pre-processor, solver and post-processor capabilities was developed to help designers. A detailed description of the numerical implementation was also presented and discussed. Some modifications were implemented on the original MATLAB software for the use of wing turbine blades, especially related with the geometry of the blade and the $r\Omega$ dependency of the flow and inflow characteristics of the wind turbines. Numerical simulations were carried out for the same example studied before of a 2.0 MW wind turbine (Menezes and Donadon, 2009).

This paper is an extension of previous work (Da Silva, et alli, 2010). The final goal is to have a computational package able to perform an optimal design for wind turbine blades, from the point of view of aerodynamic and structural predictions. This optimization will include geometrical and structural parameters which will make possible the evaluation of aerodynamic loads and structural responses as well as aerodynamic and structural performances.

The results obtained up to the present stage are coherent and promising. The pressure, as well as the total lift, revealed the expected behavior. The evaluation of the present numerical results compared to previous works has shown an considerable improvement due to the VLM implementation, since smaller values of lift have been found as expected, and smaller values of C_P were obtained. Other results found at literature confirm the coherency of these assumptions. Some of these results can be seen at Belessis, 1998.

The distribution of the pressure along the blade spanwise can also be compared with the extraction of energy found with BEM theory (Donadon, at alli, 2008).

The present results allow one to conclude that the Vortex Lattice Method is a powerful numerical method for obtaining aerodynamic loads on wind turbine blades. It is fare to say the results obtained until here permit us to elect the Vortex Lattice Method as the principal numerical method for obtain the aerodynamic loads on the wind turbine blade.

7 FUTURE WORK

The next steps of the work are the implementation of the optimization procedure. Parameters like camber, chord and span of the blade will be taken as optimization parameter of a multi objective procedure. It is expected that this procedure can obtain the optimal design of the wind turbine blade for the best wind turbine efficiency.

Another step that also will be included is the structural design based on the loads obtained with the VLM method.

The evaluation of transient loads is an issue to be investigated in future and it is expected to become the next step in the implementation of the VLM method.

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9 RESPONSIBILITY NOTICE

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