AN EXTENDED GLAUERT'S MODEL APPLIED TO DESIGN OF WIND TURBINES WITH DIFFUSERS

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Abstract. *This paper presents a mathematical model based on the Glauert's model to the design of wind turbines with diffusers, considering the extrapolation of the Betz's limit due to the influence of variation in the diffuser area. Such influence seems to be important, since the increase in wind turbine power is considerable according to Hansen* et al. *(2000), justifying the need for formulations which can predict this increase of efficiency in power generation. The mathematical model presented here is based in Glauert's model, with corrections in order to extend it to the case of turbines with diffusers. Comparision with experimental data shows that the present formulation can predict adequately the increase in power generation.*

Keywords: *Wind Turbines, Diffusers, BEM Method, Glauert's Model*

1. INTRODUCTION

The study of mathematical models applied to the design of wind turbines has become significant due to the use of energy generation technologies with low environmental impact (Vaz *et al.*, 2010b). Such models are based generally on the theory of Blade Element Momentum (BEM), which considers that the induction factor at the wake as twice the induction factor in the plane of the rotor (Eggleston and Stoddard, 1987), in addition to applying the empirical relation of Glauert (1926), on which the present work develops a correction for the turbines with diffusers.

In the present work, for the finite number of blades correction, the Prandtl's model is applied (Hibbs and Radkey, 1981). The Viterna and Corrigan model (1981), which modifies the airfoils data when operating in the post-stall region, in order to more accurately predict the behavior of an axial rotor turbine, is also considered here. Finally, we present the results obtained using the proposed model compared with the results obtained by Hansen *et al.* (2000). It is possible to evaluate a significant amount of extra power generation due to influence of the diffuser.

2. MATHEMATICAL MODEL

The Betz's limit can be exceeded when an axial turbine is placed inside a diffuser, since the flow inside this structure shows an increase in mass flow through the rotor plane due to suction caused by the diffuser (Rodrigues, 2007; Hansen *et al.*, 2000). Figure 1 illustrates the flow through a wind turbine with a diffuser.

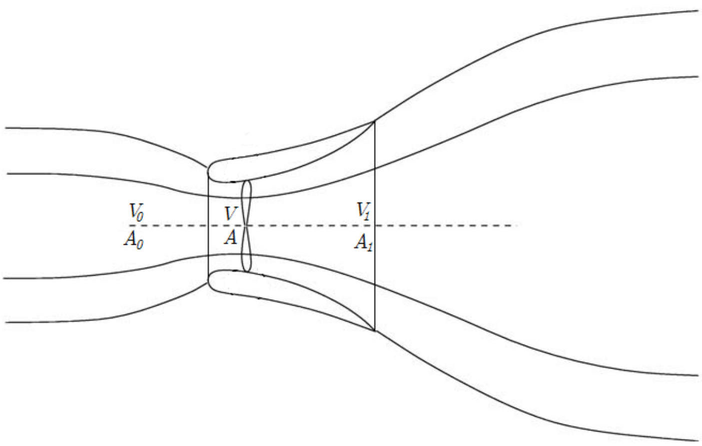

Figure 1. Simplified diagram of the velocities in the rotor plane and the diffuser

In Figure 1, V_0 is not induced axial velocity of flow, V is the velocity of flow in the axial plane of the rotor and V_1 is the axial velocity of flow at the exit of the diffuser. A_0, A_1 and A are the flow areas in turbine inlet, the rotor plane and the exit of the diffuser, respectively. In dimensional terms, the kinetic energy available to be converted into mechanical energy by the turbine is given by (Brasil-Junior *et al.*, 2006)

$$E_c = \frac{1}{2}\rho A V_0^3 \tag{1}$$

where ρ is the fluid density. The turbine efficiency is defined as the ratio between the energy absorbed by the rotor and the kinetic energy transported by the fluid Eq. (1). From an one-dimensional analysis for a rotor inside a diffuser is possible to obtain the following expression for the power coefficient

$$C_p = \frac{P}{\frac{1}{2}\rho A V_0^3} = \frac{TV}{\frac{1}{2}\rho A V_0^2 \frac{V_0}{V} V} = C_T \varepsilon \tag{2}$$

where P is the power developed by the rotor, T is the thrust on the rotor and C_T is the coefficient of thrust and ε is the rate of increase given by:

$$\varepsilon = \frac{V}{V_0} \tag{3}$$

To determine a expression for ε depending on the area at the exit of the diffuser A_1 and in the rotor plane A, it is necessary to apply the mass conservation equation to obtain

$$V = V_1 \frac{A_1}{A} \tag{4}$$

Setting $n = \frac{A_1}{A}$ and substituting in Eq. (4) na Eq. (3) one has

$$\varepsilon = \frac{V_1}{V_0} n \tag{5}$$

To the velocity at diffuser exit, it is proposed in the present work

$$V_1 = (1 - \lambda a)\, V_0 \tag{6}$$

This assumption is reasonable since the velocity in the free wake, according to Glauert's model is $V_{wake} = (1-2a)V_0$. Where λ takes the value 2. In a more general way, Vaz *et al.* (2010a) show that for slow speeds rotors the velocity in the free wake may take the form $V_{wake} = (1 - \varphi(a))V_0$, where $\varphi(a)$ is a function of induction factor a in the rotor plane. In this case, λ takes the value of 1. Rodrigues (2007) shows that for the exit velocity of an hydrokinetic turbine diffuser is $V_1 = (1 - a)V_0\eta$, where η is the ratio of the area in rotor plane and the area at diffuser exit, λ in this formulation also takes the value of 1. Figure 3 shows the sensitivity of turbine efficiency to the λ parameter considering a constant speed of $100rpm$. This figure shows the variation of the power coefficient as a function of the ratio between the blade tip and flow speed (Tip Speed Ratio - TSR). Note that when λ approaches 2, the efficiency of the turbine with diffuser approaches the classical Glauert's model for a free-flow turbine, showing that the model proposed here is a more general case than that presented by Glauert (1926). The fact that the curves coincide in the range of $TSR = 4.0$ to 5.0, is due to the fact that the rotation should be considered constant in the simulation. This discussion will be addressed in the section 4.

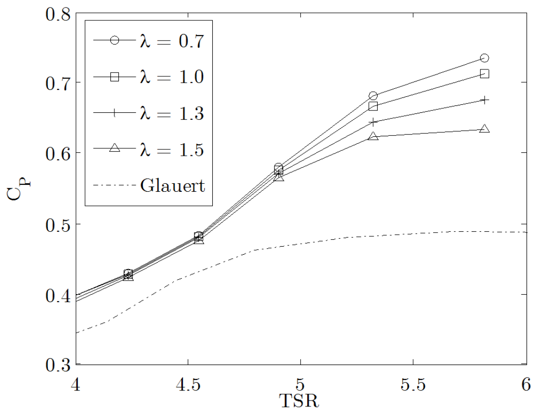

Figure 2. Sensitivity of the power coefficient to variations in λ parameter

Substituting Eq.(6) in Eq.(5), it is obtained a relation to the rate of increase ε as

$$\varepsilon = (1 - \lambda a)n \tag{7}$$

Once the formulation ε is established, a relation between C_P and C_T should be proposed. Hansen (2008) obtained relations for the coefficient of thrust C_T by fitting of experimental data of Glauert (1926) result as

$$C_T = \begin{cases} 4a(1-a)F & a \leq \frac{1}{3} \\ 4a\left(1 - \frac{a}{4}(5-3a)\right)F & a > \frac{1}{3} \end{cases} \tag{8}$$

where F is the Prandtl's correction factor, given by:

$$F = \frac{2}{\pi} cos^{-1}(e^{-f}), \text{ to } 0 \leq F \leq 1 \tag{9}$$

where $f = \frac{B}{2}\frac{R-r}{rsen\phi}$ in the outer edge of the blade and $f = \frac{B}{2}\frac{r-r_{cub.}}{r_{cub} sin\phi}$ in the inner edge of the blade. r_{cub} is the radius of the rotor hub (Mesquita and Alves, 2000).

Using eqs.(2) and (8)

$$C_P = C_T\varepsilon = \begin{cases} 4a(1-a)\varepsilon F & a \leq \frac{1}{3} \\ 4a\left[1 - \frac{a}{4}(5-3a)\right]\varepsilon F & a > \frac{1}{3} \end{cases} \tag{10}$$

In rotor plane C_P can also be given by (Hansen, 2008):

$$C_P = (1-a)^3 \frac{\sigma C_N}{\sin^2 \phi} \tag{11}$$

where

$$\sigma = \frac{Bc}{2\pi r} \tag{12}$$

$$C_N = C_L \cos\phi + C_D \sin\phi \tag{13}$$

C_L is the lift coefficient, C_D is the drag coefficient, B is the number of rotor blades, r is the radial position on the rotor and c is the airfoil chord. From eqs.(10) and (11), it is obtained the model proposed in this paper:

$$\begin{cases} a = \frac{2K+1-\sqrt{4K(1-\lambda)+1}}{2(K+\lambda)} & a \leq \frac{1}{3} \\ a^4 - \frac{(3+5\lambda+4K)}{3\lambda}a^3 + \frac{(5+4\lambda+12K)}{3\lambda}a^2 - \frac{(4+12K)}{3\lambda}a + \frac{4K}{3\lambda} = 0 & a > \frac{1}{3} \end{cases} \tag{14}$$

where

$$K = \frac{\sigma C_N}{4nF\sin^2\phi} \tag{15}$$

Note that Eq.(14) contains the parameter n, which carrys information about the area increase due to the presence of the diffuser. The solution of the equation of fourth degree for induction factor in the axial rotor plane, eq.(14), corresponds to an extension of the empirical relation of Glauert (1926), whose purpose is to correct the high values of a in the case of axial wind turbine with diffusers. This solution is obtained analytically through four roots, where one of the roots, Eq.(16), exhibits behavior consistent with the physical constraints of the problem.

$$a = Re\left(\frac{-\Gamma - \sqrt{\Gamma^2 - 4\Phi}}{4}\right) \tag{16}$$

where

$$\Gamma = -\frac{3+5\lambda+4K}{3\lambda} + \sqrt{\left(\frac{3+5\lambda+4K}{3\lambda}\right)^2 - \frac{4\,(5+4\lambda+12K)}{3\lambda} + 4\gamma} \tag{17}$$

$$\Phi = \gamma - \sqrt{\gamma^2 - \frac{16K}{3\lambda}} \tag{18}$$

$$\gamma = \sqrt[3]{R + \sqrt{Q^3 + R^2}} + \sqrt[3]{R - \sqrt{Q^3 + R^2}} + \frac{5+4\lambda+12K}{9\lambda} \tag{19}$$

$$Q = \frac{3\delta_2 - \delta_1^2}{9} \tag{20}$$

$$R = \frac{9\delta_1\delta_2 - 27\delta_3 - 2\delta_1^3}{54} \tag{21}$$

$$\delta_1 = -\frac{5 + 4\lambda + 12K}{3\lambda} \tag{22}$$

$$\delta_2 = \frac{(4 + 12K)(3 + 5\lambda + 4K)}{9\lambda^2} \tag{23}$$

$$\delta_3 = \frac{16K(5 + 4\lambda + 12K) - (4 + 12K)^2}{9\lambda^2} - \frac{4K(3 + 5\lambda + 4K)^2}{27\lambda^3} \tag{24}$$

The flow angle ϕ is given by (Hansen, 2008):

$$\phi = \tan^{-1}\left[\frac{V_0}{\Omega r}\frac{(1-a)}{(1+a')}\right] \tag{25}$$

where Ω is the rotor angular velocity. For the computation of a', Hansen (2008) proposed

$$C_t = C_L \sin\phi - C_D \cos\phi \tag{26}$$

$$a' = \left(\frac{4\sin\phi\cos\phi}{\sigma C_t} - 1\right)^{-1} \tag{27}$$

The iterative procedure for the calculation of induction factors given r, $c(r)$, $b(r)$, $C_L(a)$, $C_D(a)$ and V_0 is as follow:

(i) Assign initial values to a and a'. In this work, it was set $a = \frac{1}{3}$ and $a' = 0$;

(ii) Compute ϕ with Eq.(25);

(iii) Compute C_L and C_D from experimental (or other data for given profile) given $\alpha = \phi - \beta$;

(iv) Use extended Glauert's model to compute a and a' with Eqs.(14) and (27);

(v) Check convergence for a and a'. If the error tolerance was not achieved, return to (ii). In this work, the tolerance set as 10^{-3}.

The power coefficient Cp is given by (Vaz *et al.*, 2010c):

$$Cp = \frac{8}{X^2}\int_0^X (1 - aF)Fa'x^3 dx \tag{28}$$

The power output of the turbine is given by:

$$P = \frac{1}{2}\rho A Cp V_0^3 \tag{29}$$

3. CORRECTION MODEL FOR THE POST-STALL REGION

According to Alves (1997), the quality of results obtained from models based on the BEM method depends heavily on an accurate knowledge of lift and drag characteristics of the blade which, for small angles of attack before stall, are well established both theoretically and by experimental data. Lissaman (1994) shows that the area where the boundary layer remains attached is usually confined to attack angles of approximately 15^o. During the operation of a rotor, profiles can experiment much higher angles of attack, stall development (up to 30^o) or even completely separation of boundary layer, between 30^o and 90^o, where usually it is not know the lift and drag characteristics profiles.

The aspects mentioned above are important in predicting the maximum power developed by a rotor blade in the occurrence of high velocities, when much of the blade experiences high angles of attack. Negleting these effects leads to an underestimation of maximum power. So Viterna and Corrigan (1981) proposed an empirical model to modify profile data in all three regimes of operation in order to more accurately predict the behavior of a rotor axis. When the angle of attack is equal to or higher than that at which begins the separation ($\alpha \geq \alpha_s$), the Viterna and Corrigan (1981) model provides the following values for drag and lift coefficients:

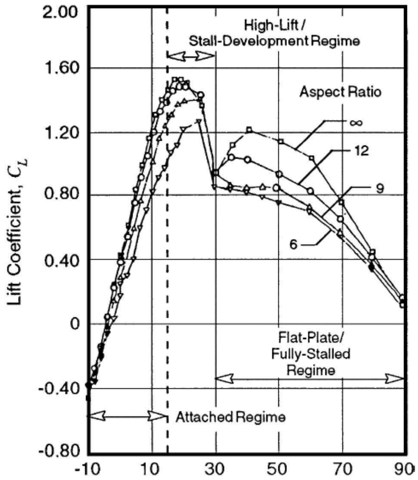

Figure 3. Typical Variation of lift coefficient with angle of attack and aspect ratio in the three regions of operation of a profile (Hansen, 2008)

$$C_l = \frac{C_{d,max}}{2}\sin^2\alpha + K_l\frac{\cos^2\alpha}{\sin\alpha} \tag{30}$$

$$C_d = C_{d,max}\sin^2\alpha + K_d\cos\alpha \tag{31}$$

$$K_l = (C_{l,s} - C_{d,max}\sin\alpha_s\cos\alpha_s)\frac{\sin\alpha_s}{\cos^2\alpha_s} \tag{32}$$

$$K_d = \frac{C_{d,s} - C_{d,max}}{\sin^2\alpha_s}\cos\alpha_s \tag{33}$$

$$C_{d,max} = 1.11 + 0.018\mu \qquad \mu \leq 50 \tag{34}$$

$$C_{d,max} = 2.01 \qquad \mu > 50 \tag{35}$$

$$\mu = \frac{R - r_{cub}}{c(r)} \tag{36}$$

where $C_{d,max}$ is the maximum drag coefficient in the completely separate regime.

As in the present work we use the profile NACA $66_4 - 421$ (Abbott and Von Doenhoff, 1959) the Viterna and Corrigan (1981) model is appropriate, since this model gives good results for the NACA profiles.

4. RESULTS AND DISCUSSION

The results are compared with data of Hansen *et al.* (2000), in which the flow through the rotor under the influence of the diffuser causes an increase in the kinetic energy extraction, compared with a turbine without diffuser. Figure (4) shows the result obtained for the power coefficient as a function of thrust coefficient, obtained by the combination of Eq.(2) with Eq.(7). All results were obtained for $\lambda = 1.3$ as this value was obtained from the simulated data by Hansen *et al.* (2000).

Just as the work of Hansen *et al.* (2000) the ratio of the power coefficient for the rotor with and without diffuser $\frac{C_{p,d}}{C_{p,b}}$ varies linearly with the ratio of the mass flows $\frac{m_d}{m_b}$ as shown in Fig.(5). As discussed in Hansen *et al.* (2000), this linear relationship can be given by

$$\frac{C_{p,d}}{C_{p,b}} = \frac{m_d}{m_b} = \frac{\varepsilon}{1-a} \tag{37}$$

Using the proposed model for ε, Eq.(7), in Eq (37)

$$\frac{C_{p,d}}{C_{p,b}} = \frac{m_d}{m_b} = \frac{1-\lambda a}{1-a}n \tag{38}$$

Note that Eq.(38) represents an explicit way to calculate the ratio between the mass flows in the wind rotor with and without diffuser.

Figure (6) shows the variation of ratio between the mass flows to a turbine with and without diffuser with the thrust coefficient. This fact occurs due to the decrease of movement created by the presence of the diffuser, confirming the

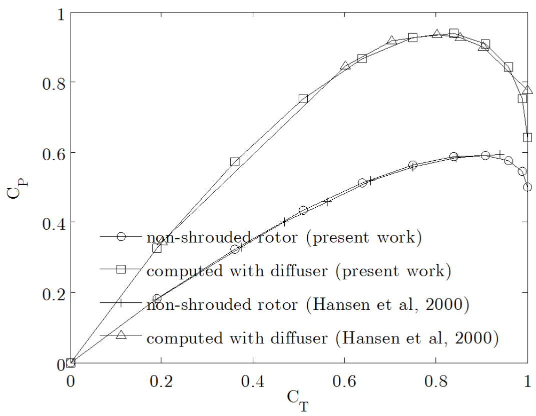

Figure 4. Power coefficient calculated based on the thrust coefficient.

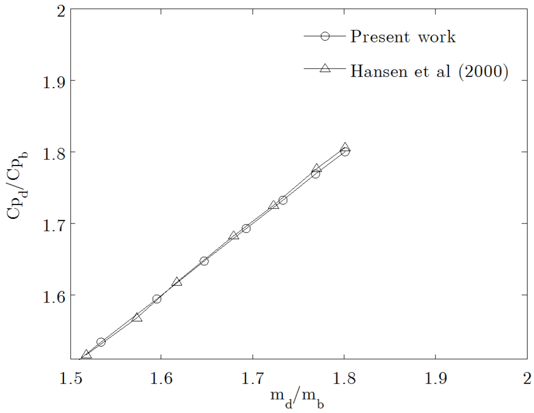

Figure 5. Power coefficients ratio as function of mass flows ratio for a turbine with and without diffuser.

assumption of Hansen *et al.* (2000) that the increase of mass flow in the rotor is influenced by the diffuser. Equation (41) shows the mathematical formulation obtained to compute the data shown in Fig.(6), where the ratio of the mass flow depends on the thrust coefficient. Therefore, for a rotor without diffuser Eq.(39) can be applied, which is obtained from Eq.(8), where $C_T = 4a(1-a)$, without the Prandtl correction factor F.

$$a = \frac{1}{2}\left(1 - \sqrt{1 - C_T}\right) \tag{39}$$

Substituting Eq.(39) in Eq.(37), we have:

$$\frac{m_d}{m_b} = \frac{2\varepsilon}{1 + \sqrt{1 - C_T}} \tag{40}$$

and using Eq.(7):

$$\frac{m_d}{m_b} = \frac{2n\left[1 - \frac{1}{2}\lambda\left(1 - \sqrt{1 - C_T}\right)\right]}{1 + \sqrt{1 - C_T}} \tag{41}$$

4.1 SOME RESULTS OBTAINED USING THE PROPOSED MODEL FOR A SMALL WIND TURBINE

The turbine used in the simulation done here has the following characteristics:

- Rotor diameter = $3.00m$
- Hub diameter = $0.30m$
- Diameter at diffuser exit = $4.0584m$
- Air density = $1223Kg/m^3$

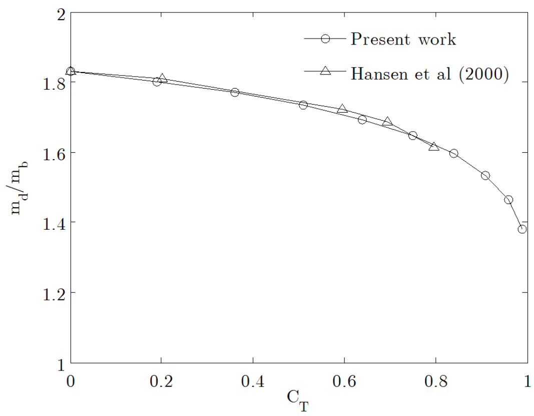

Figure 6. Mass flows ratio as function of thrust coefficient.

- Turbine rotation = $70, 80, 100$ and 120 rpm
- Airfoil NACA $66_4 - 421$
- Number of blades = 3
- Separation angle: $\alpha = 15^o$

The chord and mounting angle distributions along the rotor radius are shown in Figure 7 and 8.

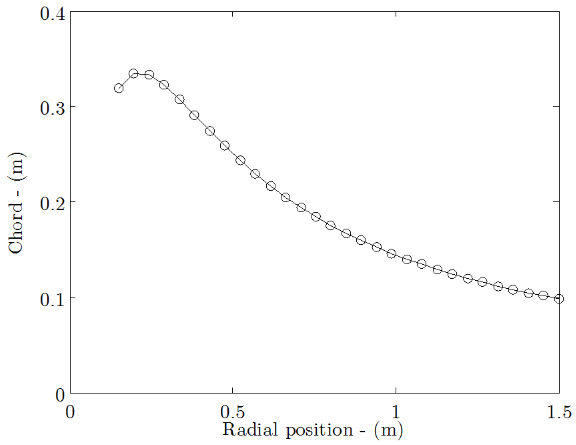

Figure 7. Chord distribution.

All the simulations were carried out only for $\lambda = 1.3$ since the best results, compared with Hansen *et al.* (2000), were obtained with this value.

Figure 9 shows the power curves of turbine under the influence of the diffuser. It is noticeable an improvement in power generation. This fact occurs because the Betz's limit is exceeded, promoting a significant increase in the rotor efficiency.

Figure 9 shows the variation of power coefficient as a function of axial velocity of the flow, at rotations 70, 80, 100 and 120 rpm. In this case, the highest efficiency obtained comprises the speed range of 2.5 to $3.5m/s$, for all rotations considered in the simulation. For an axial flow machine, the turbine rotation varies proportionally with the fluid velocity, resulting in a higher kinetic energy extraction as the rotation increases.

Figure 10 shows that all the curves of power coefficient in function of TSR coincide in a single trend, that depends on the turbine rotation.

5. CONCLUSIONS

The mathematical model proposed in this paper represents an alternative tool for the design of wind turbines with diffusers. However, it is necessary to consider some limitations of the model like not take into account the loss effects due to diffuser. It is possible that the rate of increase ε is a function of such losses, caused mainly by the geometry of the

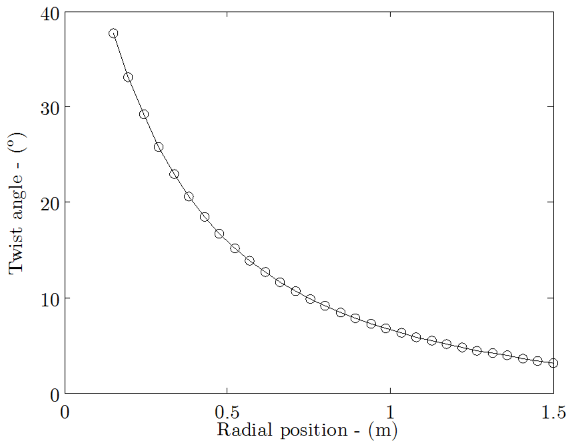

Figure 8. Mounting angle distribution.

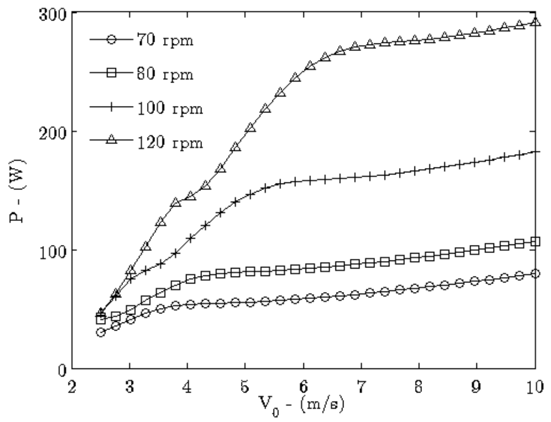

Figure 9. Power as a function of fluid velocity.

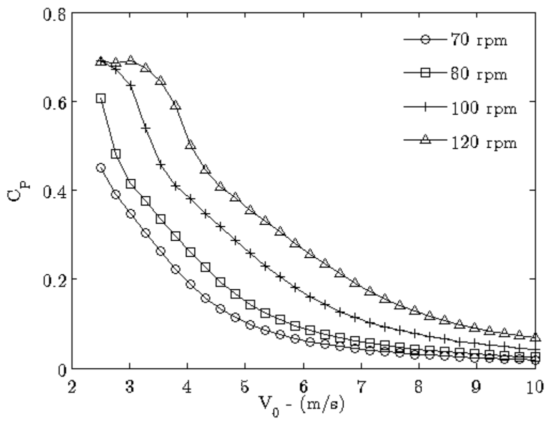

Figure 10. Power coefficient as function of fluid velocity.

diffuser. Experimental data are scarce in the literature and they are necessary to compute the λ parameter. However, when compared with data simulated using CFD of Hansen *et al.* (2000), the model showed good agreement. Remains as future work to implement the necessary fixes to make the model more efficient. As for mathematical stability, the proposed model showed no discrepancies in the results, showing good performance in comparison with the results obtained by Hansen *et al.* (2000). In the case of small turbine presented in this paper, the proposed model showed good performance for high and low turbine rotations.

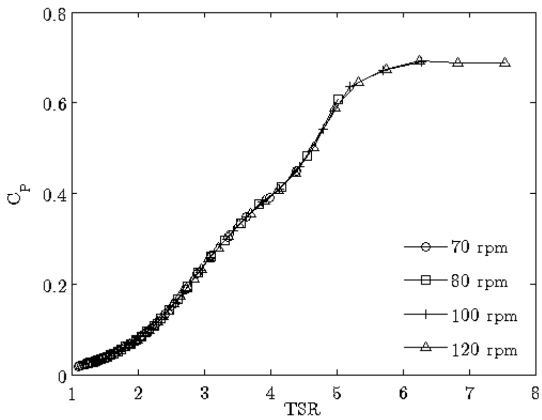

Figure 11. Power coefficient as a function of TSR.

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