CHAOS IN THE DAISYWORLD USED FOR GLOBAL WARMING DESCRIPTION

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Abstract. *Global warming is the observed increase in the average temperature of the Earths´ atmosphere and oceans. The primary cause of this phenomenon is the greenhouse gases released by burning of fossil fuels, land cleaning, agriculture, among others, leading to the increase of the so-called greenhouse effect. The consequences of this warming is unpredictable, however, one could mention climate sensitivity and other changes related to the frequency and intensity of extreme weather events. The mathematical modeling of ecological phenomena has an increasing importance in recent years. These models may describe time evolution and spatial distribution and may explain some important characteristics of these systems. Although there are many difficulties related to the system description, their modeling may define at least a system caricature, which may be useful for different goals. This contribution deals with the modeling of global warming in a dynamical point of view. Mathematical formulation is based on the daisyworld model that is able to describe the global regulation that can emerge from the interaction between life and environment. This idea became famous as the Gaia theory of the Earth that establishes self-regulation of the planetary system. In brief, daisyworld represents life by daisy populations while the environment is represented by temperature. Here, two daisy populations are of concern, black and white daisies, and an extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Moreover, temperature evolution transient is of concern. The sinusoidal variation of the luminosity is incorporated in the model allowing the description of climate variability. Numerical simulations are investigated in order to present a qualitative description of the phenomenon. Special attention is dedicated to chaotic behavior.*

Keywords*: Global warming, daisyworld, nonlinear dynamics, chaos, ecology.*

1. INTRODUCTION

Global warming is a specific case of the more general term climate change. Although climate change can be related to either natural or anthropogenic causes, it is usually associated with human activities. In general, it is important to establish a difference between climate change and climate variability. Climate change is usually related to permanent changes while climate variability denotes deviations of climate conditions over a period of time due to natural phenomena (WMO, 2010).

There are numerous modeling efforts trying to analyze either climate change or climate variability and their effects on Earth. In general, one could establish the following classification (Alexiadis, 2007): general circulation models (GCMs); model-based methods (MBMs) or empirical models; planet's dynamics models (PDMs). Moreover, we can highlight the existence of models built upon time series analysis (Viola *et al.*, 2010). GCMs consider physical aspects of system dynamics including conservation of physical variables. MBMs use some empirical observations and/or statistical tools from experimental time series and therefore, do not deal with system's physics directly. PDM are based on a simplified description of the system dynamics and falls between the previous two categories. Time series analysis tries to build a model from experimental data.

The mathematical modeling of ecological phenomena has an increasing importance in recent years (Jorgensen, 1999, Savi, 2005, 2006). These models may describe time evolution and spatial distribution and may explain some important characteristics of these systems. Daisyworld was originally proposed by Watson & Lovelock (1983) representing a prototype of PDM approach. In brief, daisyworld establishes the self-regulation of the planetary system representing life by daisy populations while the environment is represented by temperature. The daisyworld is an archetypal of the Earth being able to describe the global regulation that can emerge from the interaction between life and environment (Lovelock, 1992, Lenton and Lovelock, 2000, 2001). The general aspects of the nonlinear dynamics of the daisyworld have not been explored yet. Chaotic behavior of the daisyworld was addressed in some references: Zeng *et al.* (1990) used logistic map to represent the continuous system but actually, could not reproduce continuous system results. On the other hand, Flynn (1993) used a delayed excitation. This work introduces the effect of climate variability in the daisyworld assuming that solar luminosity has a sinusoidal variation. Chaotic behavior is of concern, showing the main aspects of the nonlinear dynamics of the daisyworld.

2. DAISYWORLD MODEL

Climate system has an inherent complexity due to different kinds of phenomena involved. The equilibrium of this system is a consequence of different aspects related to the atmosphere, oceans, biosphere and many others, and the sun activity provides the driving force for this system. The Earth's heating mechanism may be understood as the balance between the radiation energy from the sun and the thermal radiation from the Earth and the atmosphere that is radiated out to space. The presence of greenhouse gases tends to break this balance since they are transparent to the sun short wave radiation, however, they absorb some of the longer infrared radiation emitted from the Earth. Therefore, the increase amounts of these gases makes the Earth cool more difficult increasing the Earth's surface temperature.

Watson & Lovelock (1983) proposed a model to demonstrate that global regulation can emerge from the interaction between life and environment. This behavior was represented by an archetypal model called daisyworld that represents an imaginary planet populated by organisms in coexistence. The daisyworld is basically composed by the environment, represented by the temperature, and by populations of daisies representing life. In brief, it is assumed that daisyworld is like the Earth but with less oceans and with the whole surface being fertile. The original daisyworld includes only two populations of daisies but further investigations include herbivores and carnivores as well as daisies.

The first step of the daisyworld modeling is the definition of life, represented by daisies, which evolution is described by the following general equation where $\alpha_i (i = 1,2,..N)$ represents the area coverage by daisy populations:

$$\dot{\alpha}_i = \alpha_i \left[\alpha_g \beta(T_i) - \gamma\right] \tag{1}$$

where dot represents time derivative, β is variable growth rate that is temperature dependent and γ is the death rate. Daisy colors define the amount of energy absorption and the balance between daisy populations can control the planet temperature. A first approach to this archetypal model is to consider only two daisy populations: black, α_b, and white, α_w. Black daisies absorb more energy while white daisies absorb less energy.

In order to incorporate the greenhouse gases in the daisyworld, a new population is included into the model. The idea is to represent the albedo increase due to the effect of the greenhouse gases. In this regard, the inclusion of the greenhouse variable G has an effect similar to black daisies. Therefore, the model is written as in the classical way, but now there is a function that establishes the greenhouse gases time history:

$$G = G(t) \tag{2}$$

The variable α_g is the fractional area coverage of the planet represented by:

$$\alpha_g = p - \sum_{i=1}^{N} \alpha_i - G \tag{3}$$

Here, p represents the proportion of land suitable for the growth of daisies and N represents the biodiversity related to the number of populations involved in the system.

The mean planetary albedo of the daisyworld, A, can be estimated from the individual albedo of each population (a_i for daisies, a_g for the bare ground and a_G due to greenhouse gases):

$$A = \alpha_g a_g + \sum_{i=1}^{N} \alpha_i a_i + G a_G \tag{4}$$

Afterwards, the local temperature of each population is defined as follows:

$$T_i^4 = q\left(A - a_i\right) + T^4 \tag{5}$$

$$T_g^4 = q\left(A - a_g\right) + T^4 \tag{6}$$

$$T_G^4 = q\left(A - a_G\right) + T^4 \tag{7}$$

where T is the globally-averaged temperature of daisyworld, and q is a constant used to calculate local temperature as a function of albedo (Watson & Lovelock, 1983). Finally, it is important to establish the thermal balance of the daisyworld (Foong, 2006), and therefore, the absorbed energy is given by (Nevison *et al.*, 1999):

$$\dot{T} = (1/c)[SL_v\left(1 - A\right) - \sigma T^4] \tag{8}$$

L_v is the solar luminosity and S is the solar constant that establishes the average solar energy, SL_v; σ is the Stefan-Boltzmann constant; c is a measure of the average heat capacity or thermal inertia of the planet. A key difference between climate change and climate variability is the persistence of anomalous conditions. In order to investigate the effect of climate variability in the daisyworld, it is assumed a solar luminosity with a sinusoidal variation represented as follows:

$$L_v = L + \left(L_0 \sin(\omega t)\right) \tag{9}$$

Note, that L may describe a linear increase and term $L_0 \sin(\omega t)$ represents a perturbation that can be associated with climate variability. The functional form for β_i is usually assumed to be a symmetric single-peaked function as follows:

$$\beta_i(T) = \begin{cases} B\left[1 - \left(\dfrac{T_{opt} - T_i}{k}\right)^2\right] & \left|T_{opt} - T_i\right| < k \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where T_{opt} is the optimal temperature usually assumed to be T_{opt} = 295 K = 22.5°C. The parabolic width k is chosen in order to establish proper life conditions as for example, between 5°C and 40°C (De Gregorio *et al.*, 1992), which is related to $k = 17.5$. In the same way, B alters these values in order to represent different environmental characteristics.
The daisyworld model can be simulated using classical procedures for numerical integration. Here, the fourth order Runge-Kutta method is employed. In general, the following parameters are assumed: q=2.06×10^9 K^4, σ = 1.789×10^3 W/m^2 K^4, S = 2.89×10^{13} W/m^2. These parameters are related to time scale of thousands of years. Other parameters are varied in order to analyze different situations. Moreover, it is important to highlight that only black and white daisy populations are considered.

3. DAISYWORLD AND CLIMATE VARIABILITY

The climate variability is now of concern by assuming a linear increase of luminosity ($0.75 \le L \le 1.7$) and a sinusoidal variability with L_0=0.1 and ω=0.01, as presented in Figure 1. Besides, it is assumed that c=3.0×10^{13} J/m^2K s. The situation with sinusoidal variability is compared to a situation with just linear increase (L_0=ω=0). Figure 2 presents the temperature evolution of the daisyworld showing a comparison between the situation without (represented by light line) and with sinusoidal variability (represented by dark line). The effect of greenhouse gases is also of concern. The left panel presents results without greenhouse gases while the right panel considers a situation with greenhouse gases. Figure 3 presents the correspondent evolution of the daisy populations. It should be observed that the system has an irregular behavior when sinusoidal luminosity is considered.

The daisyworld has self-regulation due to the interaction between life and environment, represented respectively by daisy populations and the planet temperature. Therefore, the planetary system tends to maintain a constant temperature due to the interaction between black and white daisy populations. The increase of the black daisies tends to increase the planet temperature since they absorb more energy, and the opposite occurs concerning white daisies. This occurs when the solar luminosity has small values. The increase in solar luminosity causes the decrease of the black daisies population and the increase of the white daises.

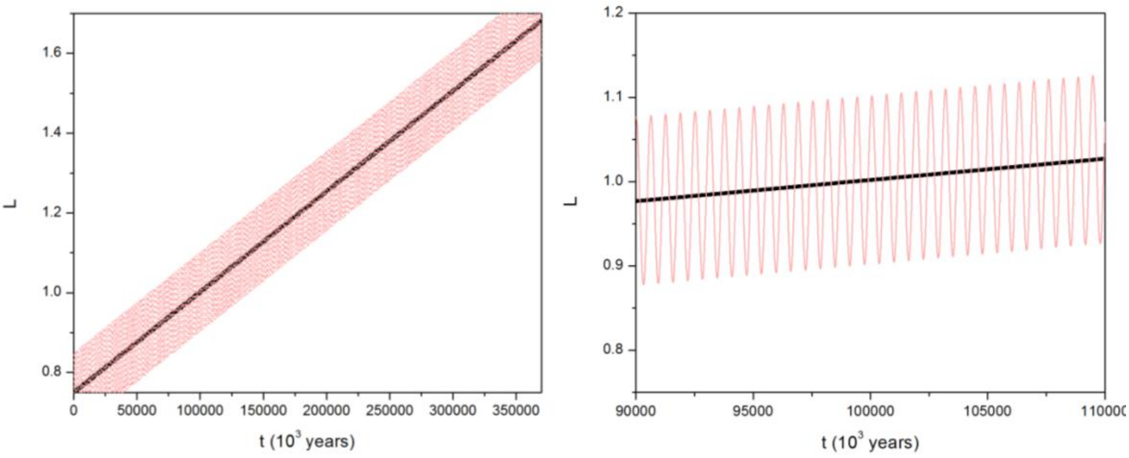


Figure 1. Linear luminosity (red line) and sinusoidal variation (black line).

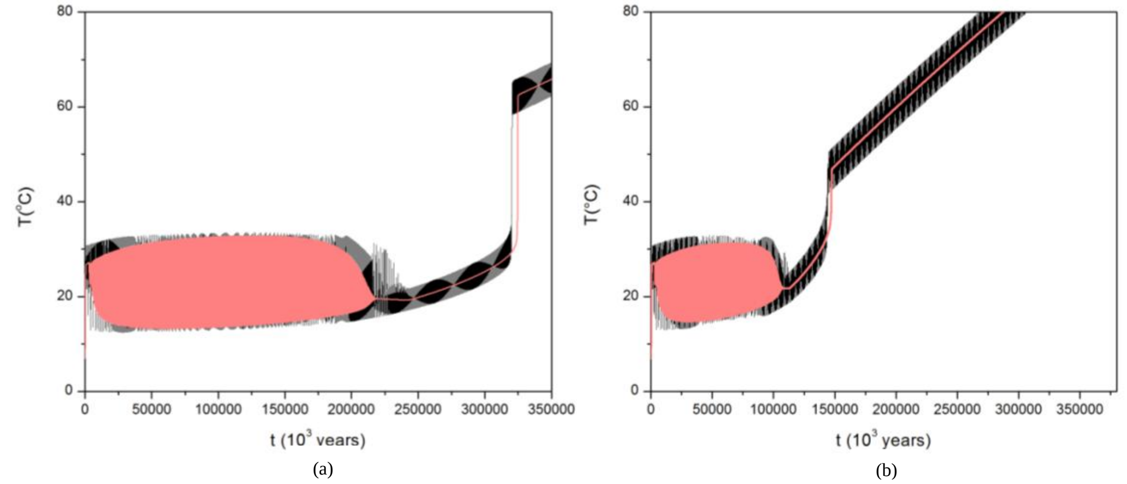


Figure 2. Temperature evolution due to solar luminosity linear increase. L_0=0.1 and ω=0.01 (dark line), L_0=0.0 and ω=0.0 (light line) without (a) and with (b) greenhouse gases ($0.0 \leq G \leq 0.8$).

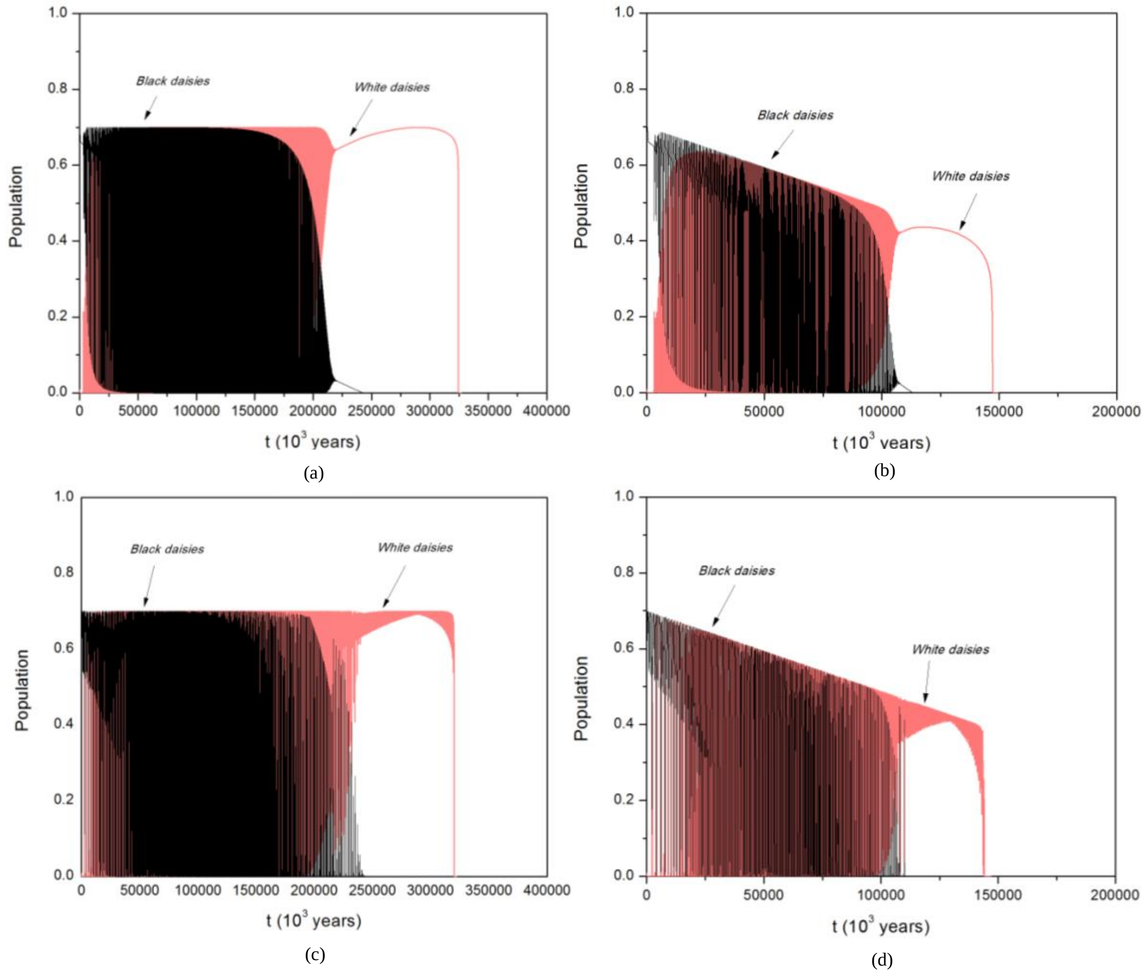


Figure 3. Daisy populations evolution with solar luminosity linear increase. (a) L_0=0.0, ω=0.0 and G=0; (b) L_0=0.0, ω=0.0 and ($0.0 \leq G \leq 0.8$); (c) L_0=0.1 and ω=0.01, G=0; (d) L_0=0.1, ω=0.01and ($0.0 \leq G \leq 0.8$).

The irregular behavior of the previous result motivates a deeper dynamical investigation of the daisyworld. Therefore, it is carried out the influence of the luminosity parameter in the system dynamics. The analysis starts with the bifurcation diagram that presents stroboscopically sampled temperature values under the slow quasi-static increase of the luminosity. This diagram demonstrates the influence of the parameter on system dynamics, showing the global behavior of the system. Initially, it is assumed that G = 0.21052, L_0=0.1, ω=0.01 and luminosity L is varying between 0.75 and 1.2. Figure 4 presents this bifurcation diagram that shows regions related to single points as well as regions associated with cloud of points. The influence of frequency parameter ω may be evaluated by considering other bifurcation diagrams presented in Figure 5 for ω=0.05 and ω=0.1. Note that this change can dramatically alter the general system dynamics. The influence of amplitude parameter L_0 is showed in Figure 6 by assuming L_0=0.1 and L_0=2.0. Once again, it is clear that the change of this parameter can dramatically alter the system dynamics. Greenhouse gases also influence the system dynamics. Figure 7 evaluates their influence by assuming different values of G (G=0.31 and G=0.46). Under these conditions, the system tends to be more regular, accelerating the end of the balance between life and environment.

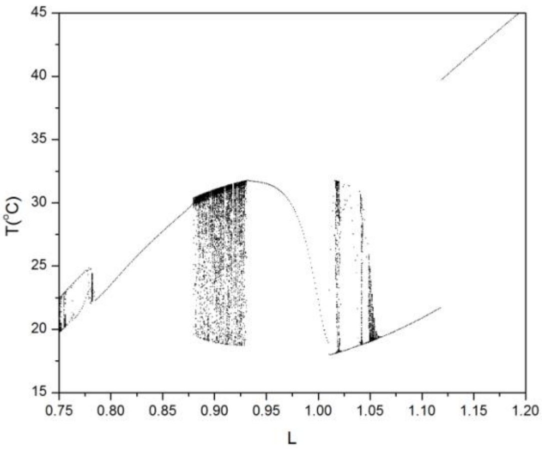
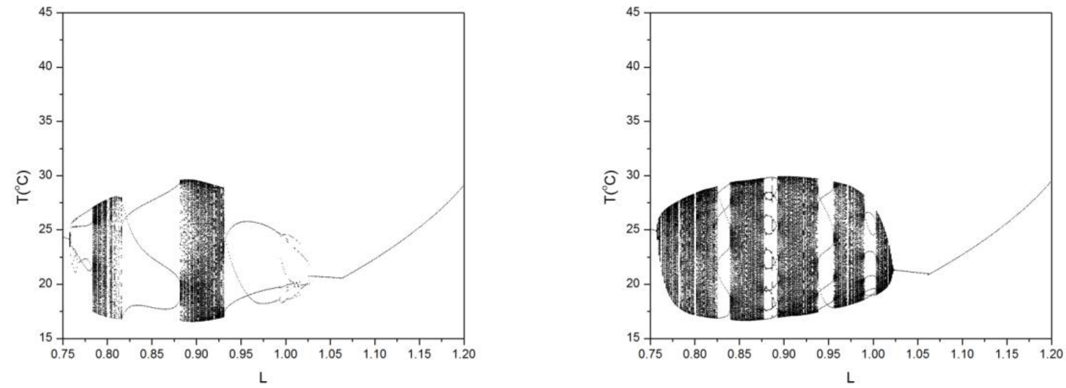


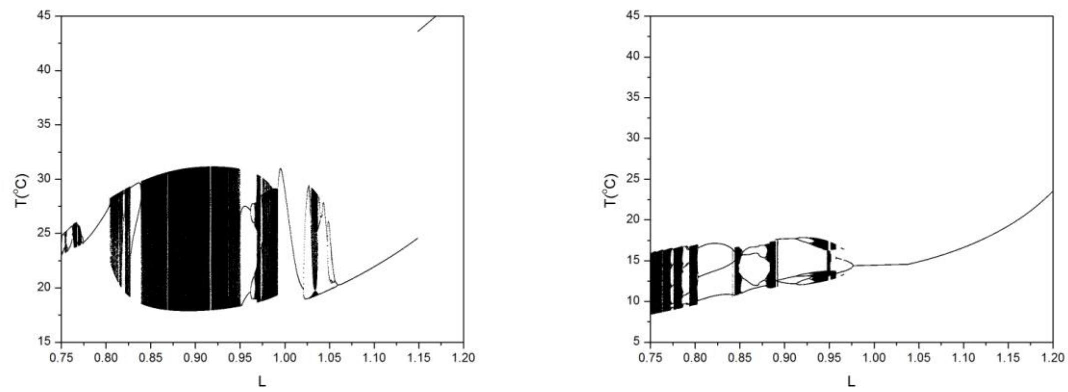
Figure 4. Bifurcation diagram varying L ($0.75 \le L \le 1.20$) with ω = 0.01.



(a)

(b)

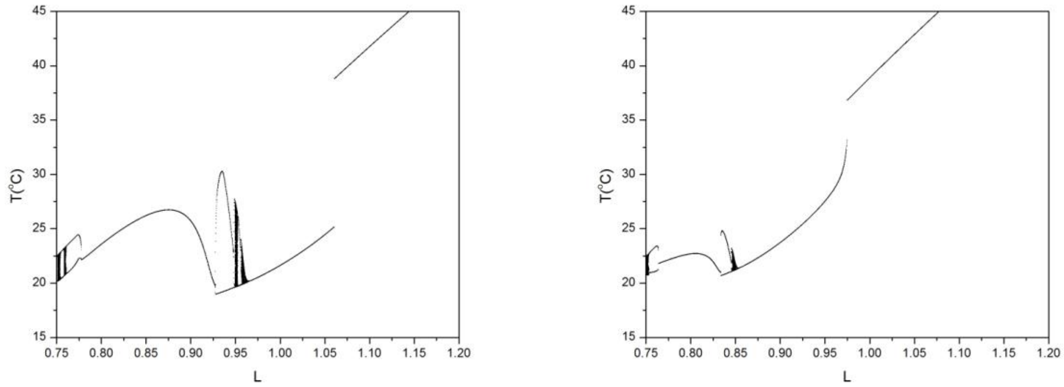
Figure 5. Bifurcation diagram varying L: (a) ω=0.05 e (b) ω=0.1.



(a)

(b)

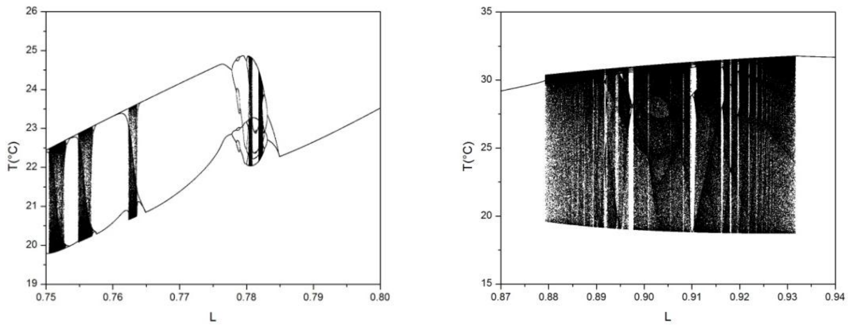
Figure 6. Bifurcation diagram varying L: (a) L_0=0.05, e (b) L_0=2.0.



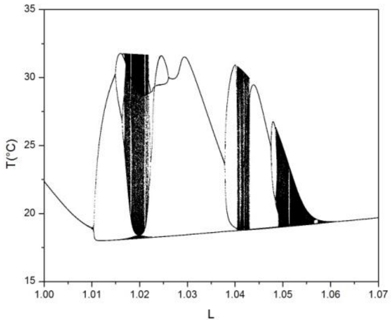
(a) (b)

Figure 7. Bifurcation diagram varying *L*: (a) *G*=0.31 e (b) *G*=0.46.

The details of the system dynamics is now in focus by revisiting the results presented in Figure 4. Figure 8 presents enlargements of the bifurcation diagram for different ranges of luminosity. Note that bifurcation and chaos are presumable present in the daisyworld dynamics. Figure 9 show some details of the daisyworld behavior. Figure 9a shows a period-2 response for L = 0.76 that is followed by a chaotic-like behavior when L = 0.7629 (Figure 9b). When L = 0.7787 (Figure 9c), the system presents a period-8 response that is followed by a period-4 when L = 0.78 (Figure 9d). The increase of the luminosity to L = 0.782 induces a new chaotic-like response (Figure 9e). A sequence of bifurcations occurs until L = 0.785 when a period-1 response appears. In the range between 0.88 and 0.93, quasi-periodic behavior emerges as when L = 0.88 (Figures 9f) and L = 0.90 (Figure 9g). Periodic windows are also present in this range as can be observed for the period-3, L = 0.9082 (Figure 9h) and period-7, L = 0.9097 (Figure 9i). Afterwards, chaotic behavior occurs again as can be observed for L = 0.919 (Figure 9j) and L = 0.9238 (Figure 9k). In the range between 0.932 and 1.01 the system presents new bifurcations. For L = 1.01 a period-1 response occurs (Figure 9l). Some bifurcations make the system increases periodicity reaching a chaotic regime as for L = 1.02 (Figure 9m). For L = 1.04 a period-3 response occurs (Figure 9n) and chaotic-like response occurs again for L = 1.05 (Figure 9o). For values greater than 1.06 the system presents period-1 response. Figure 10 presents the Poincaré section related to L = 0.9238 (Figure 9k) and L = 1.05 (Figure 9o) showing the general characteristics of these behaviors. It should be highlighted that daisyworld presents a chaotic-like behavior and that the system response is highly dependent on parameters.



(a) (b)



(c)

Figure 8. Bifurcation diagram varying L. (a) $0.75 \leq L \leq 0.80$; (b) $0.87 \leq L \leq 0.94$ and (c) $1.00 \leq L \leq 1.07$.

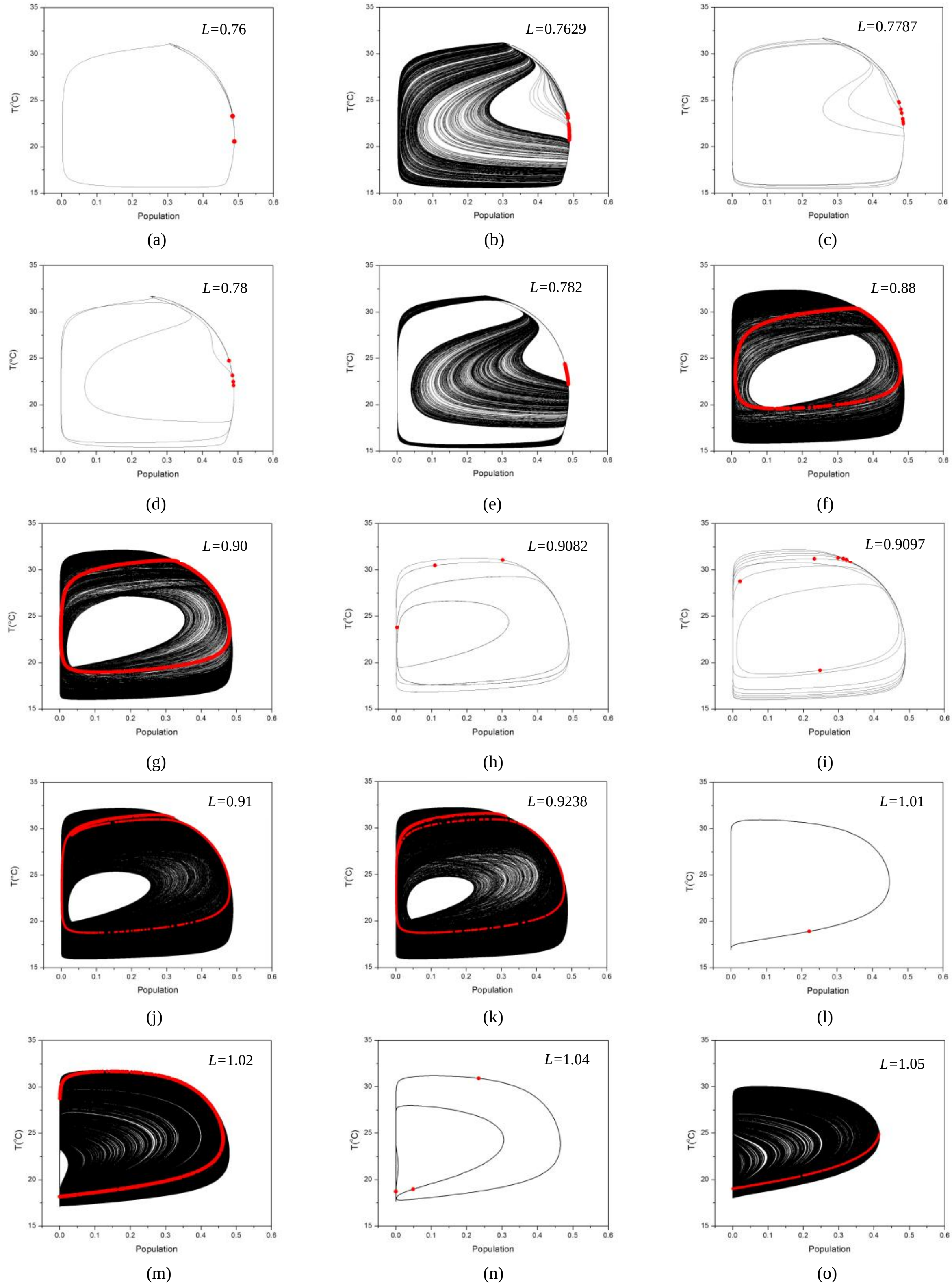


Figure 9. State space and Poincaré sections for different luminosity values.

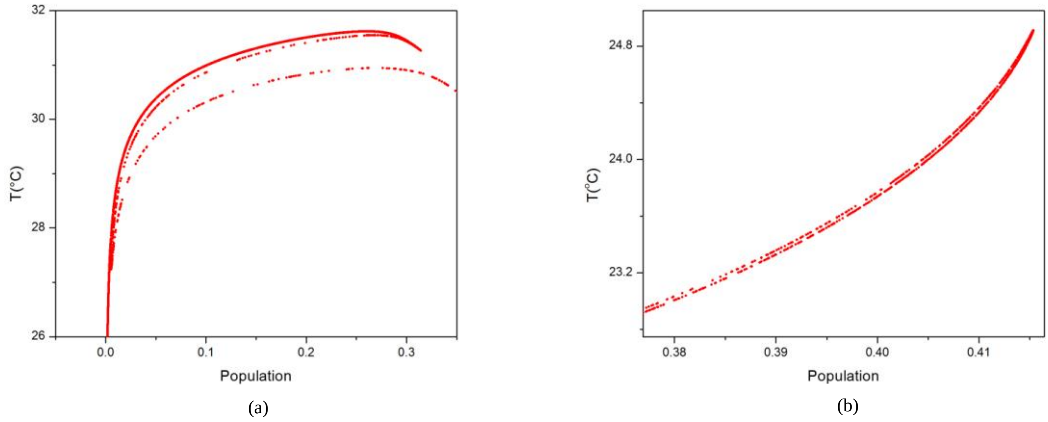


Figure 10. Poincaré sections details of figures 9k and 9o.

4. CONCLUSIONS

The daisyworld is an archetypal of the Earth and is able to describe the global regulation that can emerge from the interaction between life and environment. In brief, daisyworld represents life by black and white daisy populations while the environment is represented by temperature. An extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Besides, energy equation is considered in order to investigate transients phenomena related to temperature variation. The climate variability is described by considering a sinusoidal variation of the solar luminosity. A general analysis of the daisyworld is carried out analyzing different aspects of the solar luminosity. In general, greenhouse gases tend to increase the planet temperature, accelerating the death of populations and decreasing the capacity of global regulation. Thermal inertia of the planet is also of concern showing its influence in the system response. Results related to climate variability show irregular pattern that can be associated to rich responses that include periodic, quasi-periodic and chaotic-like behaviors. The authors believe that the proposed model can be used for a qualitative description of the global warming phenomenon that is an essential problem of this century. Besides, it should be highlighted the highly parameter dependence of the responses.

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