INTEGRATION OF ROTOR-HYDRODYNAMIC BEARINGS-SUPPORT STRUCTURE SYSTEMS FOR NUMERICAL RESOLUTION

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Abstract. *Rotordynamics occupies a prominent position in the design of machines and structures, due to the wide application of rotating machinery and the importance of the knowledgment of its operation for purposes of optimization and design for cost reduction, quality improvement and increasing of the components life cycle. These machines and their components have peculiar characteristics in its operation, which will reflect in the dynamic behavior and, consequently, in the mechanical problems associated with them. The behavior of rotating machinery involves considerable complexity, as they are usually composed by a system of equipments which includes, besides the rotor, shafts and bearings, foundation and other auxiliary equipment, for example, liquid seals. Therefore, to study the dynamic of these systems, it is necessary to determine the interaction of all components to understand all the phenomena involved. The rotating system is mathematically modeled by Finite Element Method of the shaft, through a set of disks and beam elements with elasticity and mass distribution. The hydrodynamic bearings are represented through the equivalent coefficients of stiffness and damping. The foundation has also its influence covered by Concentrated Parameters, Mechanical Impedance or Mixed Coordinates. Therefore, a model of a turbo-group is developed with three rotating shafts, seven bearings and the supporting structure, with the purpose of modeling the rotating system as close as possible to a practical application and to predict its dynamic behavior.*

Keywords*: Rotordynamics, Systems integration, Finite element method*

1. INTRODUCTION

Rotor machines are designed to transmit power through its movement of rotation. These equipments are often part of production plants or power plants, so that a sudden stop can cause great financial loss.

The operation of these machines involves a large amount of rotational energy, and a small amount of vibration energy. The goal of the rotor dynamics as applied to rotating machines is to analyze how the vibration levels can be kept as small as possible (KRAMER, 1993; CHILDS, 1993; WEBER, 1992; STEFFEN, 1981). Therefore, as its design and optimization is focused on reducing costs, improving quality and preventing catastrophic failures of most machines, it becomes essential to study the behavior of the system.

The behavior of rotating machinery involves considerable complexity, as these machines are made, usually by a system of equipments which includes, in addition to the rotor, shafts and bearings, foundation and other auxiliary equipments, for example, liquid seals and couplings. Therefore, to study the dynamic behavior of this system, it is necessary to determine the interaction of all components involved. Several mathematical models have been developed to simulate the working conditions of rotating systems and assess their actual behavior, in other words, methods are developed to better model real machines, as in the case of large rotating machinery. Early in the rotor dynamics, Rankine (1869) reported the existence of critical velocities. In 1883, Laval built the turbine, which later received its name. Jeffcott (1919) explains the rotordynamics science in a graphical form, still used today.

However, since 1970, many researchers in the field of rotor dynamics have studied the use of the finite element method for modeling and numerical solution of rotating systems, as introduced by Archer (1963).

Concurrently, studies related to the influence of the dynamic behavior of lubricated bearings on rotating machines have been accomplished dating from 1883 and 1885. The studies were performed by Tower reporting the fact that a rotor when properly placed in rotation would be supported by the oil film. In 1884, with some simplifications in the Navier-Stokes equations, Reynolds established the differential equation for the profile of pressure that acts between two surfaces in relative motion, due to the variation of internal pressure in the fluid film.

The idea of representing the dynamic characteristics of a journal bearing by means of stiffness and damping coefficients was introduced by Stodola (1925) and Hummel (1926). The goal was to improve the calculation of the critical speed including the flexibility of the oil film. Lund (1964) published a method for calculating the linearized dynamic coefficients of stiffness and damping, to be introduced into the equation of motion of the rotating system. Numerical models for cylindrical bearings, elliptical and tri-lobe were proposed by Pinkus (1956, 1958, 1959) and recently implemented by Machado (2009).

The foundation is the structure that supports the rotor, and the bearings are the connection points of the support structure with the rotor. In other words, the rotor is connected with the foundation through the bearings (LEPORE, 1988). Through these connections, the forces caused by movement of the unbalanced rotor are transmitted to the foundation, which reacts in the bearings, interfering with the response of the rotor. This reaction of the foundation depends on its impedance. As the foundation can provide high rigidity, therefore, is often regarded as infinitely rigid.

The rotor-bearing-foundation system model is divided into two subsystems (rotor-bearings and foundation), which are analyzed separately. The response of the complete system is obtained by integrating the models of the two sub-systems. The supporting structure can be modeled either by the method of Mechanical Impedance or by the method of Mixed Coordinates (CAVALCA, 1993).

Therefore, the aim of this work is to study the interaction of the several components. The rotating system was mathematically modeled using the finite element method, through a set of hard disks and segments of shaft with elasticity and distributed mass, and hydrodynamic bearings, represented by the equivalent coefficients of stiffness and damping.

2. MATHEMATICAL MODELING

2.1. Coordinate System

A typical configuration of a rotating system, which includes disks, shaft elements and bearings, is illustrated below, along with the two coordinate systems that are used to describe the motion of the system (NELSON and McVAUGH, 1976).

The reference system *XYZ* is the inertial reference frame. *X* is the axial axis, while *Y* and *Z* are the vertical and horizontal axes, respectively. The reference system *xyz* is the rotational reference and is defined in relation to the inertial coordinate system by the rotation W*t* around the axis *X*, with w denoting the precession speed, since the axes *X* and *x* are collinear and coincident with the centerline of the non-deformed rotor.

The degrees of freedom *V* and *W* are the displacements in the horizontal direction *Y* and vertical direction *Z*, respectively, and B and G and are the angular displacement around *Y* and *Z* axes, respectively. The angle of rotation f, for a constant angular velocity of the system and neglecting the torsional deformation, is W*t* where W denotes the spin rotation of the rotor.

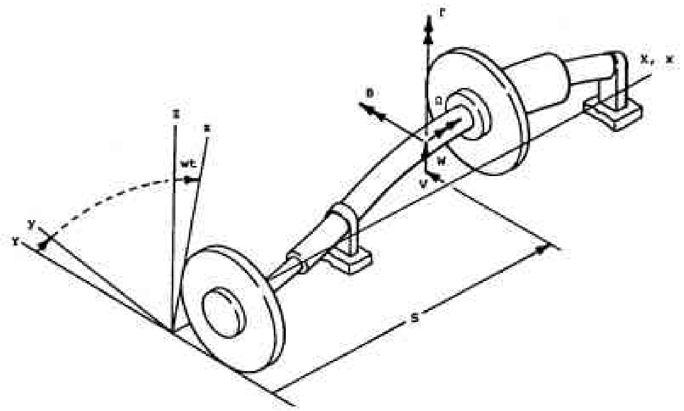

Figure 1. Typical configuration of a rotating system (NELSON and McVAUGH, 1976)

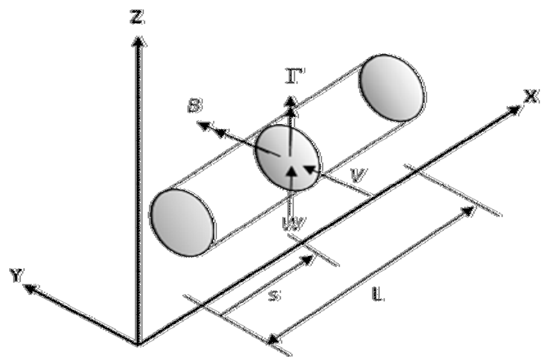

Figure 2. Beam Element and its Cartesian coordinate system

2.2. Cylindrical Beam Element

A typical beam element with uniform circular cross section (located between nodes i and j), based on the work of Nelson and McVaugh (1976) and Nelson (1980), is developed in the inertial coordinate system. The Timoshenko beam element has four degrees of freedom per node, two translational and two rotational. This element includes the effects of inertia for translation and rotation, gyroscopic moments, elastic bending energy, shear deformation and structural damping. Figure 2 presents a schematic representation of a beam element with its coordinates and degrees of freedom.

It is important to note that the displacements (*V*,*W*,B,G) are function of time and of axial position *s* along the *X* axis of the element. The displacement vector in the fixed system for the beam element is:

$$\{q\}^T = \{V_i \ W_i \ B_i \ \Gamma_i \ V_j \ W_j \ B_j \ \Gamma_j\} \tag{1}$$

The Hamilton Extended Principle, with energy and work, produces the following equation for the finite element, using the restriction of constant speed, $\dot{\phi} = \Omega$:

$$([M_T]+[M_R])\{\ddot{q}\}+(\Omega[G]+[C])\{\dot{q}\}+([K_B])\{q\}=\{F\} \tag{2}$$

Where: $[M_T]$ is the translation inertia matrix; $[M_R]$ is the rotational inertia matrix; [G] is the gyroscopic matrix; [C] is the structural damping matrix; $[K_B]$ is the bending stiffness matrix; and {F} is the external force vector.

This equation relates the movement of the element in the inertial coordinate system and all matrices are symmetric, except for the gyroscopic matrix [G], which is anti-symmetric. The matrices and force vector include the effect of shear deformation. If the shear effect is ignored, the model becomes identical to the Euler-Bernoulli beam.

The matrices of the cylindrical beam element are given in Tückmantel (2010). It is important to note that the structural damping is considered proportional to the mass and stiffness matrices of the element (SANTANA, 2009).

2.3. Bearings

The basic equation that describes the behavior of the lubricating fluid in the radial hydrodynamic bearing is the Reynolds equation. The equation of Reynolds (1886) is the basis of modern theory of hydrodynamic lubrication. When applied in the study of the oil film bearing, the solution of this equation gives the pressure distribution in the oil, ie, determines $p(x,\theta)$ according to the geometry of the bearing. This pressure field is the information needed to solve most of the basic problems in the analysis of hydrodynamic bearings.

The basic differential equation that governs the oil film pressure distribution, in the gap within a radial bearing, is the Reynolds equation.

$$\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)+\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)=6\mu U\frac{\partial h}{\partial z}+12\mu\frac{\partial h}{\partial t} \tag{3}$$

Where: t denotes time, x and z the Cartesian coordinates, μ the viscosity of the lubricant, U the tangential velocity in the rotor surface and h the thickness of oil film.

The lubricant is discretized through a spring-damper model (Fig. 3) and it is characterized by equivalent coefficients of stiffness and damping, K and C, respectively.

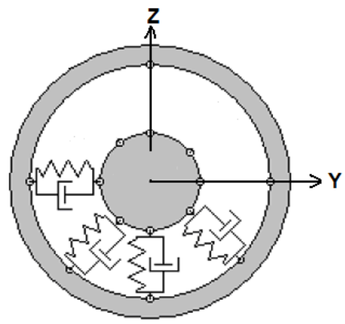

Figure 3. Model for fluid film bearings (MACHADO and CAVALCA, 2009)

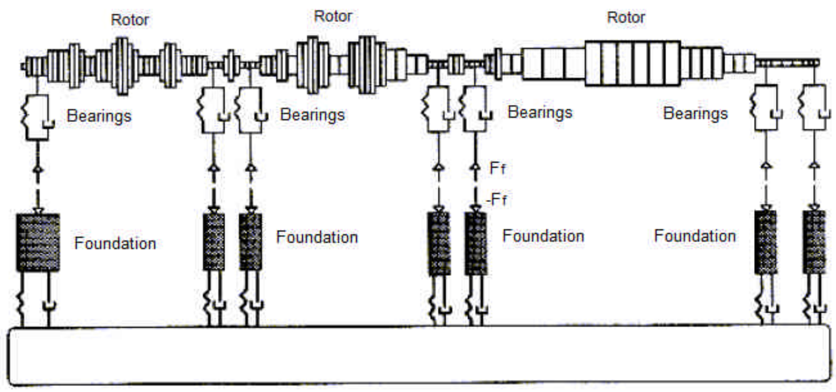

Figure 4. Representation of a rotor-bearing-foundation system (CAVALCA, 1993)

As can be seen, the reaction forces are function of y and z coordinates, and the instantaneous velocity of the shaft center, $\dot{y}$ and $\dot{z}$. As for small displacement amplitudes, Δy and Δz, measured from the static equilibrium position (y_0 and z_0), the Taylor series first order expansion leads to:

$$\begin{aligned} F_y &= F_{y0} + K_{yy}\Delta y + K_{yz}\Delta z + R_{yy}\Delta\dot{y} + R_{yz}\Delta\dot{z} \\ F_z &= F_{z0} + K_{zy}\Delta y + K_{zz}\Delta z + R_{zy}\Delta\dot{y} + R_{zz}\Delta\dot{z} \end{aligned} \tag{4}$$

The terms (K_{yy} and K_{zz}) and (R_{yy} and R_{zz}) are called direct coefficients of stiffness and damping, and the terms (K_{zy} and K_{yz}) and (R_{yz} and R_{zy}) are the cross-coupling coefficients. Since y_0 and z_0 is the equilibrium position, then $F_{y_0} = 0$, and F_{z_0} must equal the static load W.

In possession of all coefficients, the equation of motion for bearings elements is

$$[C]\{\dot{q}\}+[K]\{q\}=\{F\} \tag{5}$$

Being *[C]* and *[K]* the equivalent damping and stiffness matrices of the oil film, respectively, depending on the spin speed of the shaft, $\{q\}$ the displacement vector in the fixed reference system and $\{F\}$ the vector of reaction forces of the bearing in the same coordinate system.

2.4. Unbalance Excitation

The unbalance is known as one of the most common causes of vibration in machinery and it is present in greater or lesser degree, in virtually all rotating machinery. It is considered, yet, the most significant source of excitation of rotors. The unbalance creates a centripetal force, which will act on the bearings and support structure of the machine. This force is periodic (measured from a stationary point) when the machine is in operation, and rotates with the same spin speed of the shaft, so its frequency of vibration is, therefore, synchronous (1x Ω).

This condition will happen only at synchronous speed for the first harmonic, ie, when the precession of the rotor is synchronized with the spin speed, and therefore, if the system response vibrates at higher harmonic frequencies, this behavior will not be due to unbalance (WOWK, 2000; RAO, 2001).

According to Lalanne and Ferraris (1998), to obtain this force it is necessary to determine the position and velocity of the residual mass in relation to the inertial frame:

$$\{F\}=\{F_c\}\cos\Omega t+\{F_s\}\sin\Omega t=\Omega^2 \mathrm{hx}\begin{Bmatrix}\cos(\Omega t+\mathrm{f})\\ \sin(\Omega t+\mathrm{f})\\ 0\\ 0\end{Bmatrix} \tag{6}$$

Where: F_c is the force component in cosine and F_s is the force component in sine, being the unbalance applied to translational degrees of freedom of the node in (y,z) directions. Besides, x is the eccentricity, W is the spin speed of the shaft and F is the concentrated unbalance force, with its components in the horizontal and vertical directions.

2.5. System Equation of Motion

With the model of each component of the rotating system, it is possible to obtain the global system of equations. The matrices of each element are grouped into a global matrix and their positions in global matrices are related to the degrees of freedom. It is obtained, therefore, the dynamic equation of the system, which considers all degrees of freedom of the rotating system with N nodes.

$$[M]\{\ddot{q}(t)\}+(\Omega[G]+[C])\{\dot{q}(t)\}+[K]\{q(t)\}=\{F(t)\} \tag{7}$$

$$\{q(t)\}=\{V_1\ W_1\ B_1\ \Gamma_1\ V_1\ W_2\ B_2\ \Gamma_2\ \ldots\ V_N\ W_N\ B_N\ \Gamma_N\}^T \tag{8}$$

Equation 7 is time dependent. *[M]*, *[C]*, *[G]* and *[K]* are the matrices of inertia, damping, gyroscopic and stiffness; *{F}* is the vector of all external forces acting on the system. The assembly of the matrices of the elements in the global array is made by superposition. During this process, the terms of the elementary matrices are added to the terms of the same degree of freedom of another elementary matrix. The external forces are positioned in the corresponding GDL where the force is applied. The system of equations is solved, in this case, only for the first harmonic excitation.

$$\{q(t)\}=\{q_o\}e^{j(\Omega t+\mathrm{j})}=\{q_o\}e^{j\Omega t}e^{j\mathrm{j}} \qquad \{F(t)\}=\{F_o\}e^{j(\Omega t+\mathrm{f})}=\{F_o\}e^{j\Omega t}e^{j\mathrm{f}} \tag{9}$$

Where: *{q}* and φ are the amplitude and phase responses of the system and *{F}* and f are the amplitude and phase of the excitation forces, respectively.

After replacing the first and second derivatives of *q(t)* with respect to time, in Eq. 7, the equation of motion in the frequency domain is obtained:

$$\left(-\Omega^2[M]+j\Omega([C]+\Omega[G])+[K]\right)\{q_o\}=\{F_o\} \tag{10}$$

2.6. Foundation

The mathematical modeling consists in analyzing the complete system rotor-supports-foundation (Fig. 4) as two subsystems separately: rotor-supports and foundation (CAVALCA, 1993). Thus, each subsystem is analyzed and the response of the complete system is obtained by unifying the dynamic response of the subsystems.

Based on the method of mechanical impedance used to simulate the supporting structure, it was developed a new mathematical method, which consists in modifying the method of mechanical impedance, avoiding, thus, numerical problems when the inversion of the matrix of flexibility. The method consists in describing the displacement vector of the supporting structure as independent variables through a modal approach (DIANA et al., 1988 and CAVALCA, 1993). In this transformation, a mixed coordinate vector is used, physical coordinates for the rotor and generalized coordinates for the foundation, which describes the behavior of the complete system.

The equation of motion for the complete system, in mixed coordinates is:

$$\begin{bmatrix}[M_{rr}] & [0]\\ [0] & [m_f]\end{bmatrix}\begin{Bmatrix}\{\ddot{q}_r(t)\}\\ \{\ddot{p}(t)\}\end{Bmatrix}+\begin{bmatrix}[C_{rr}] & [C_{rf}][\Phi]\\ [\Phi]^T[C_{fr}] & [c_f]+[\Phi]^T[C_{ff}][\Phi]\end{bmatrix}\begin{Bmatrix}\{\dot{q}_r(t)\}\\ \{\dot{p}(t)\}\end{Bmatrix}+\begin{bmatrix}[K_{rr}] & [K_{rf}][\Phi]\\ [\Phi]^T[K_{fr}] & [k_f]+[\Phi]^T[K_{ff}][\Phi]\end{bmatrix}\begin{Bmatrix}\{q_r(t)\}\\ \{p(t)\}\end{Bmatrix}=\begin{Bmatrix}\{F_{ro}\}\\ \{0\}\end{Bmatrix} \tag{11}$$

Where: $\ddot{q}_r, \dot{q}_r, q_r$ are the vectors of acceleration, velocity and displacement of the rotor; $\ddot{q}_f, \dot{q}_f, q_f$ are the vectors of acceleration, velocity and displacement of the nodes of connection between the rotor and foundation; M_{rr}, C_{rr}, K_{rr} are the matrices of mass, damping (with gyroscopic matrix) and stiffness of the rotor; C_{ff}, K_{ff} are the damping and stiffness matrices of the oil film bearings; F_r is the external force due to the unbalance of the rotor; F_f is the force transmitted from the foundation to the rotor, through the connecting nodes positioned in the bearings.

3. COMPUTACIONAL MODELING

The computacional package *Rotortest* aims to support the dynamic analysis of rotating systems, modeling, thus, its components and their interactions. The logistics of information is therefore complex, as different components and models can be used. The main goal of this first version is to build a base from which other mechanical components, failure modes etc, can be implemented and integrated into the software in an organized and standardized way. Hence, in this work, the computational package configuration, the standardization of input and output files, the implementation of beam elements and disks, the unbalance excitation, the calculation of static loads on supports and, finally, the dynamics of the system taking account of the support structure influence were developed.

Seeking for robustness, flexibility to incorporate new models, reliability of results, besides usability and user-friendly interface, the software was structured into two distinct blocks:

• Interface: during pre-processing, comprehends defining the problem and discretization of the system, and during post-processing, visualization of results.

• Analysis: includes data processing and determination of results.

Rotortest flowchart is shown in Fig. 5. The processing phase is subdivided into four stages. The first phase includes to determine the static load on the bearings; loads that are of fundamental importance in the next stage of determining the equivalent coefficients of stiffness and damping of the supports. The third phase deals with the interpolation of the coefficients of the supports for the rotor frequency range of analysis, while the fourth and final phase accomplishes with the determination of the dynamic behavior of the system. The sequence followed for the processing is shown in the flowchart of the program by the color markings.

Thus, it was considered interesting structuring the software in a way that allows involving distinct contributions, directed to the same common goal, through this work of integrating the rotor-bearing-foundation systems. The software presents modular configuration, separated but mutually dependent, being structured to receive new contributions and new components to the system.

4. APPLICATIONS

A three shafts and seven bearings turbo-group IS developed (CAVALCA, 1993), whose purpose is to present the modeling of a rotational system close to a practical application (Fig. 6), discretized by the finite element method. The bearings are represented by triangles while the excitations are represented by vertical arrows. The geometric data of bearings and the characteristics of excitations by unbalanced mass are presented in Tables 1 and 2, respectively. The couplings between the shafts are considered rigid.

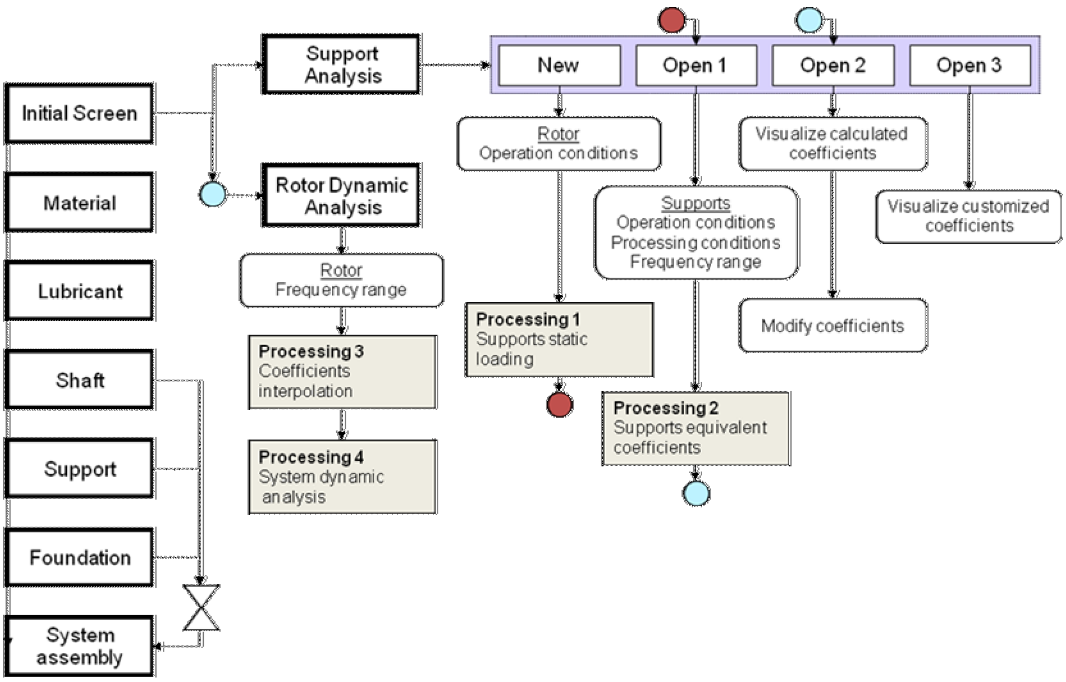

Figure 5. Flowchart of computational package *Rotortest*

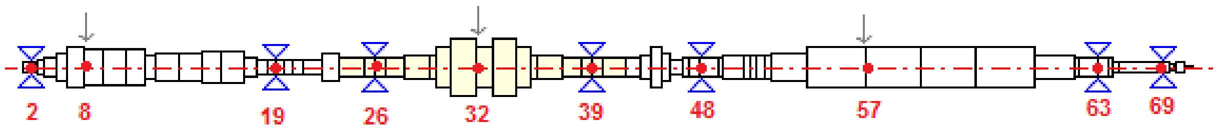

Figure 6. Turbo-group FEM.

Table 1. Geometrical characteristics of hydrodynamic bearings of the turbo-group.

Bearing	Type	Position [node]	Preload	L [mm]	D [mm]	Cr [m]	Reaction F_z [kN]
1	Tri-lobe	2	0,20	216	304	337	126,0
2	Tri-lobe	19	0,20	150	355,3	337	113,0
6	Elliptic	26	0,25	370	431,1	435	266,9
7	Elliptic	39	0,25	300	430,9	435	235,2
3	Elliptic	48	0,25	286	456	300	255,5
4	Elliptic	63	0,25	286	456	300	443,2
5	Cylindrical	69	0	160	220	400	-153,7

Table 2. Data for the external excitation by residual mass unbalance.

Excitation	Position [node]	Residual mass [kg]	Excentricity [m]	Angle [º]
1	8	2,5x10-2	4x10-1	90
2	32	5x10-2	4x10-1	180
3	57	1x10-1	4x10-1	0

Table 1 also shows the reactions forces used to calculate the pressure distribution, shaft locus and equivalent coefficients for each of the bearings. The greater force is at bearing 6.

The hydrodynamic bearings, operating at 30 ºC with oil ISO VG32 (60.45 mPa.s viscosity), have their coefficients determined for the frequency range from 15 up to 335 Hz, and the finite volume method was used for the resolution of the pressure distribution within the bearing.

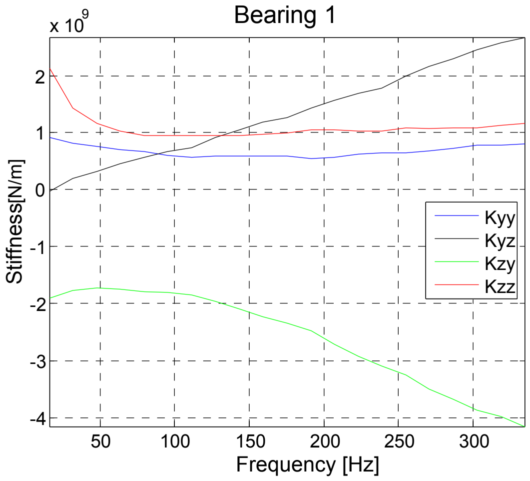

Figure 7a. Stiffness coefficients for bearing 1

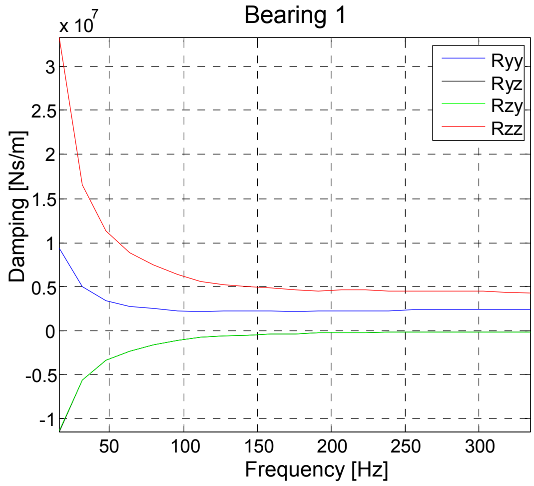

Figure 7b. Damping coefficients for bearing 1

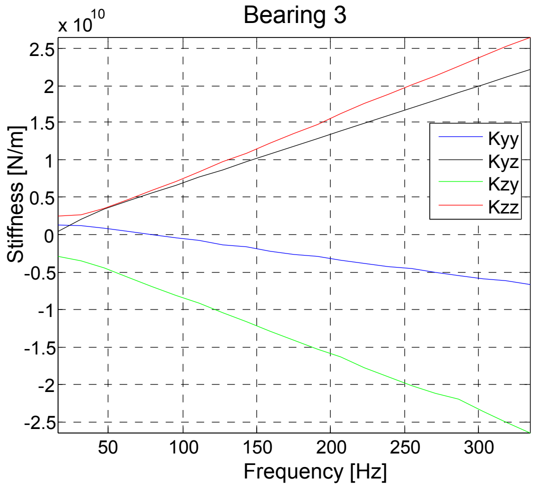

Figure 8a. Stiffness coefficients for bearing 3

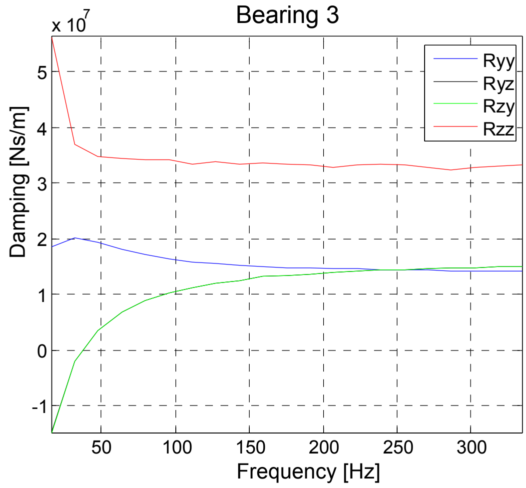

Figure 8b. Damping coefficients for bearing 3

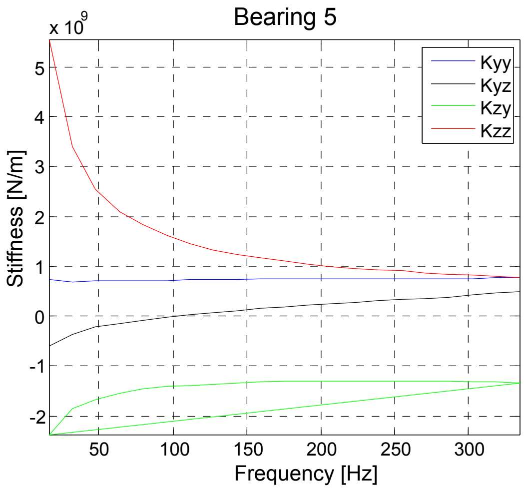

Figure 9a. Stiffness coefficients for bearing 5

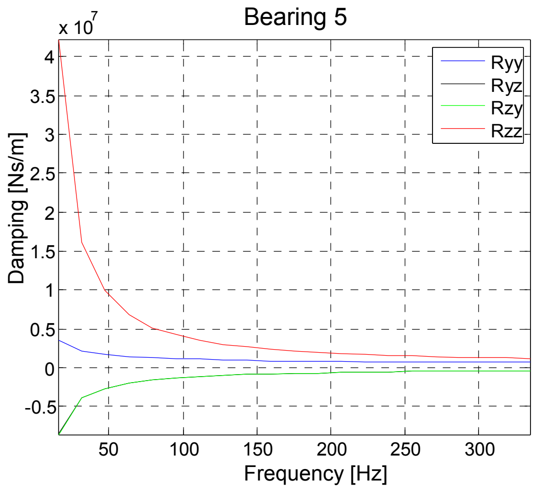

Figure 9b. Damping coefficients for bearing 5

Both the stiffness and damping coefficients behavior are similar between the pairs of bearings 1 and 2 (Fig. 7), 3 and 4, 6 and 7 (Fig. 8). The behavior of the bearing 5 (Fig. 9) differs enough from that obtained for the others, with the stiffness coefficients decreasing as the spin speed of the shaft increases, due mainly to its geometric configuration (journal bearing).

In Fig. 10, the first 16 natural frequencies are shown in the Campbell diagram. However, the frequencies at which the high amplitudes occur in operating condition should be investigated as many of the vibrating modes are highly damped, so some of these are not even detected in the frequency domain.

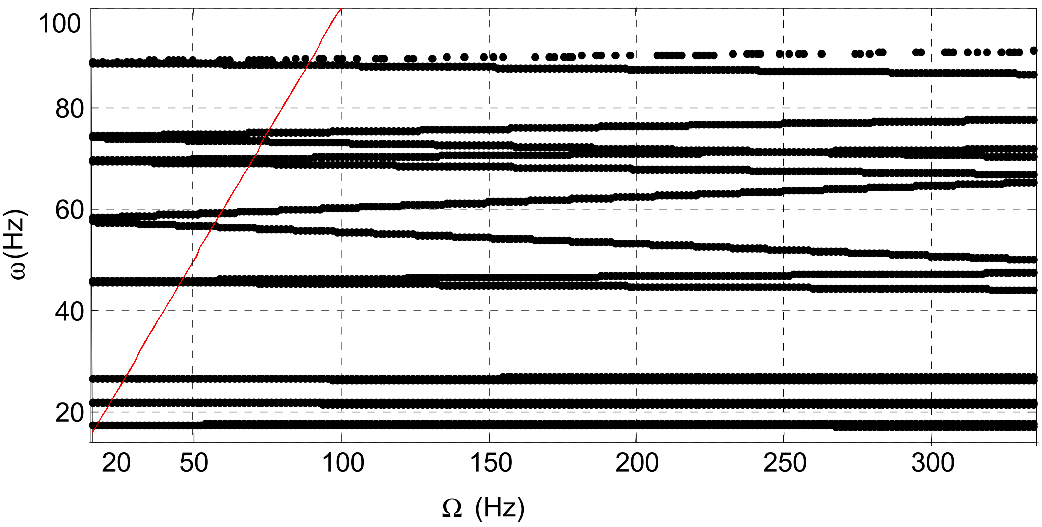

Figure 10. Campbell diagram

As an example of the natural frequencies occurring at the bearing nodes (48, 63, 69) and the excitation node (57), the unbalance response is evaluated for the frequency range from 15 to 335 Hz, respectively.

The vibration amplitude of the response in the horizontal direction is greater than in the vertical direction at the bearings (Figs. 11a, 11c, 11d). For the excitation node, the amplitudes in vertical and horizontal directions are very close, mainly at higher frequencies (Fig.11b). The journal bearing has the highest vibration amplitudes between the three types used (Fig. 11d). The effect of anisotropy for the excitation nodes is small, especially for high speeds, while the effect of anisotropy in the response of the bearings is significant because of the cross-coupled equivalent dynamic coefficients of the oil film.

The occurrence of a first natural frequency close to 17.5 Hz (Fig.10) corresponds to the operational mode strongly associated with the first mode of the alternator (shaft 3) of the Fig. 12a. Other vibration modes, indicated in Fig. 10, are shown in Figs. 12b (46.2 Hz), 12c (76.4 Hz) and 12d (87.5 Hz).

For the frequency of 46.2 Hz (Fig. 12b), the turbo-group operational vibrating mode is composed of the second mode of the low pressure turbine (shaft 1), the first mode of high pressure turbine (shaft 2) and the first mode of shaft 3 (alternator).

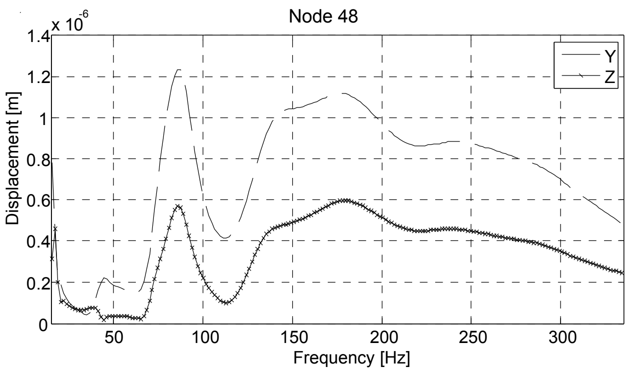

Figure 11a. Horizontal and vertical vibration amplitudes at node 48 (bearing 3)

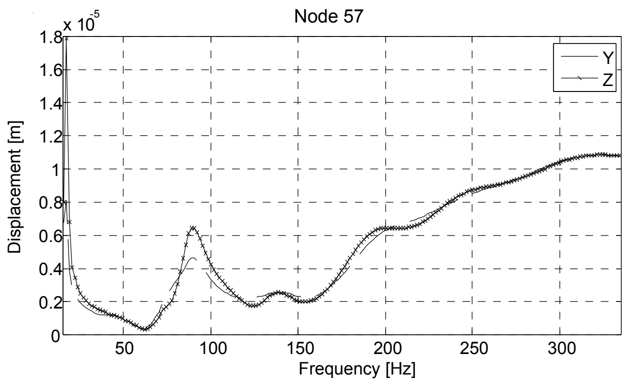

Figure 11b. Horizontal and vertical vibration amplitudes at node 57 (excitation 3)

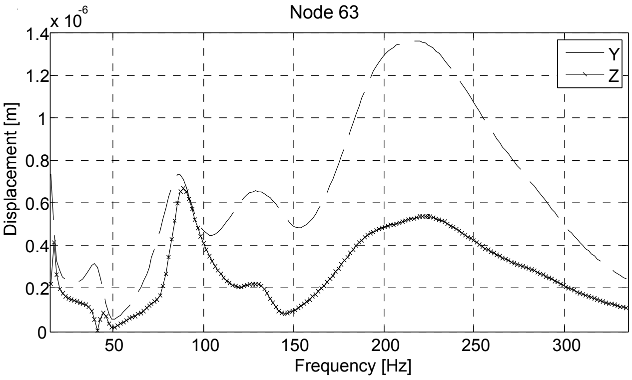

Figure 11c. Horizontal and vertical vibration amplitudes at node 63 (bearing 4)

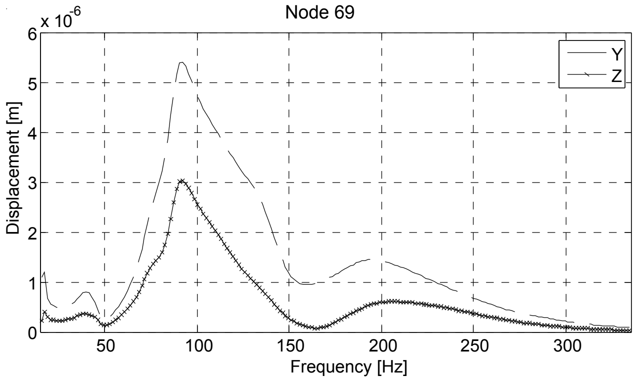

Figure 11d. Horizontal and vertical vibration amplitudes at node 69 (bearing 5)

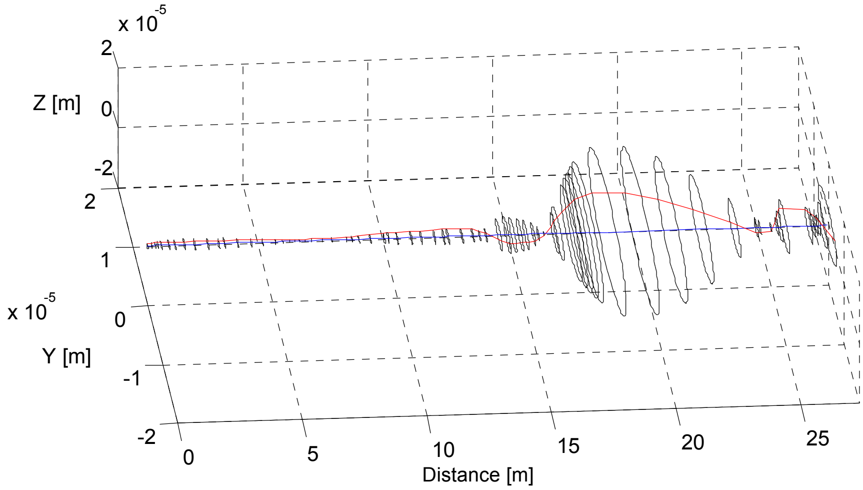

Figure 12a. Operational mode at 17.5 Hz

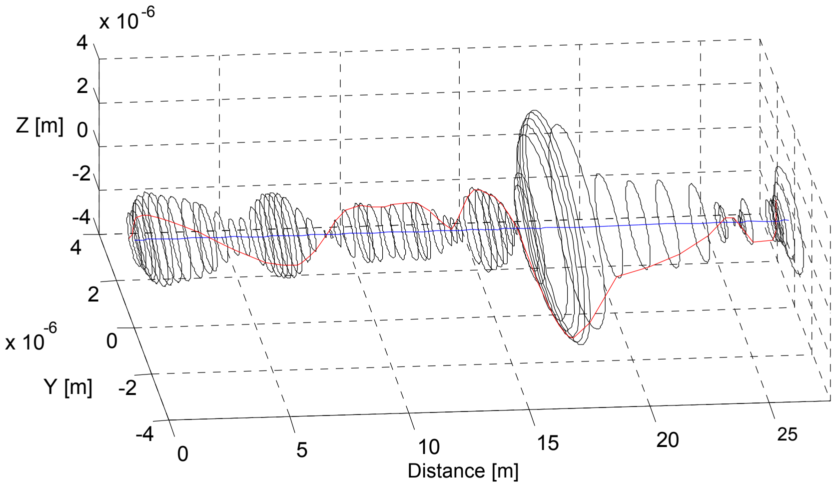

Figure 12b. Operational mode at 46.2 Hz

The operational mode for 76.5 Hz (Fig. 12c) presents the second vibrating mode of the alternator, whereas, the second mode component of high and low pressure turbines (shafts 1 and 2) starts in the system operational vibrating mode.

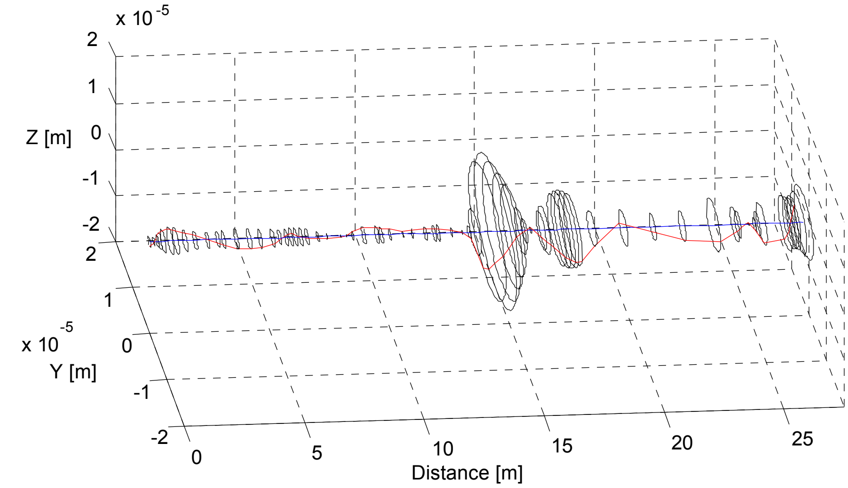

Figure 12c. Operational mode at 76.4 Hz

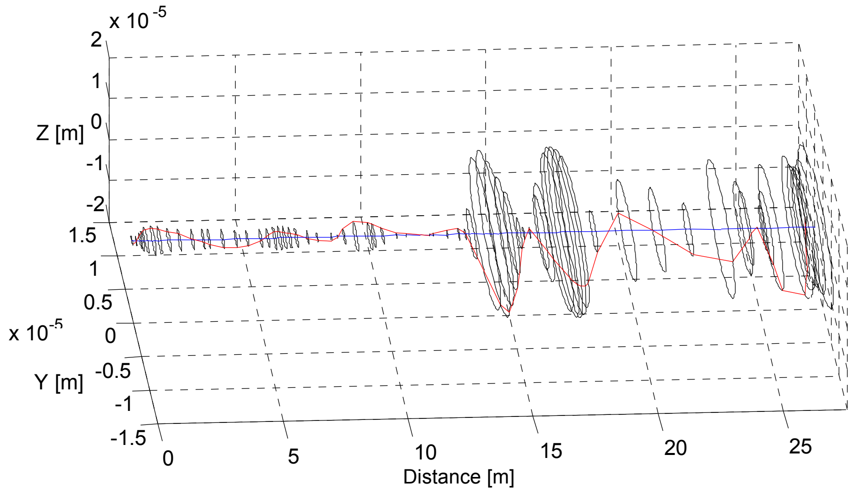

Figure 12d. Operational mode at 87.5 Hz

Figure 12d shows the second mode of the high pressure turbine (shaft 2) and of the alternator (shaft 3) simultaneously.

5. CONCLUSIONS

The package *Rotortest* shows that in the case of the turbo-group analyzed here, several combinations among the vibration modes of the three rotors generate a wide range of possibilities in terms of operational modes of the rotating system. Indeed, the rotation system, when in practical operation, behaves rather complex, and for modeling purposes components are incorporated in order to approach its behavior. Such contributions are now feasible within *Rotortest* in its current version.

Besides the analysis of each shaft, the analysis of the complete set is shown as an important design requirement for this group of machines. Therefore, a software that allows to study the dynamic of rotating system is promising for applications involving vibration control and model based fault diagnosis.

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