A FINITE ELEMENT FOR COMPOSITE LAMINATED BEAMS WITH A SHEAR CORRECTION FACTOR MODEL

Murilo Sartorato, murilosart@gmail.com
Volnei Tita, voltita@sc.usp.br
Rômulo Vinícius Vera, romulo.vera@hotmail.com
USP – Universidade de São Paulo, EESC – Escola de Engenharia de São Carlos – Av. Trabalhador São Carlense, 400 – 13 São Carlos /SP, Brasil

Abstract. *In the last decades, the use of composite material in several industries that need a high strength to weight ratio has greatly increased. Thus, several studies about the behavior of such structures, especially composite laminated structures have been performed and provided an abundant number of theoretical and semi-empiric models that are used in the commercial finite element packages nowadays. The majority of models included in these packages are based on the first order shear deformation theories which require shear correction factors to account for the shear stiffness and transverse shear effects. As the most common use of this kind of material consists on thin laminated plates where the transversal shear effects are small, the classical isotropic rectangular solution of k equal 5/6 is commonly used. However, there are some particular structures and analysis cases such as wing-box fixing frames in the aeronautical industry, deep-water oil drilling and draining tubes in the petroleum industry or modal analysis of high frequency modes in which the effects of transverse shear stresses are predominant, and as such need a better mathematical model of the shear corrections factors. Studies are being made on new models, but most of them are time consuming with low computational efficient, invalidating their use. Therefore, a one-dimensional simple calculated shear correction factor expression was used with basic finite elements based on the first order shear deformation theory for composite laminated beams that were implemented as a UEL (User Element Subroutine in FORTRAN) and linked to software ABAQUS. Analysis of the influence of the material properties, number of layers and fiber orientation in the shear corrections factors were done. Finally, comparisons of the simulated results with experimental ones found in the literature are presented.*

Keywords*: finite elements; composite materials; shear correction factor*

1. INTRODUCTION

In the last decades, the use of composite material in several industries has been steadily increasing, especially in areas that focus on highly optimized structural projects that aim for a weight per resistance ratio as low as possible, such as the aeronautical, aerospace and petroleum industries. This fact is due to the advantages of these materials over traditional engineering materials like its high density to strength ratio and the great number of possible structure configurations that can be produced and assembled. In special, the laminate composites are greatly used and studied as its inherent anisotropy can be used in a positively away in a structural project by aligning the principal directions of the material to the greatest loads through the direction of the fibers in each layer, enabling to choose a configuration specifically designed for each application. However, one of the downsides of such materials is its complex mechanical behavior that leads to costly and sometimes inaccurate models. Consequently, several studies about the behavior of such structures providing an abundant number of theoretical and semi-empiric models that are used for simulating their mechanical behavior in the commercial finite element packages nowadays.

The majority of models included in these packages are based on the first order shear deformation theories, which require shear correction factors to account for the shear stiffness and transverse shear effects. As the most common use of this kind of material consists on thin laminated plates where the transversal shear effects are small, classical solutions of the problem where the correction factors are constant, including the classical isotropic plate solution of Reissner/Mindlin (Bathe, 1996) of k equal 5/6, are commonly used. However, some authors such as Whiney (1973), Reddy *et al*. (1992) and Dong *et al*. (2010) say that there are some particular structures and analysis cases such as wing-box fixing frames in the aeronautical industry, deep-water oil drilling and draining tubes in the petroleum industry in which the effects of transverse shear stresses are predominant, and as such need a better mathematical model of the shear correction factor. Also, as Reddy and Ochoa (1993) pointed out, in dynamic analysis, some of the higher modes, as well as the torsion modes can be heavily influenced by the transversal shear effects. This has led to several studies and the creation of models and corrections that better calculate the effects of transverse shear stresses, and although there are several modern models that can predict accurately this phenomena (Dong *et al.,* 2010; Wooran *et al.* 2010; Singh, 2010) most of them use higher order shear deformation theories or hybrid stress-strain formulations that are not computational efficient making impossible for mass use in commercial finite element packages in an industrial environment.

Therefore, a simple, one-dimensional shear correction factor expression, based on the models proposed by Raman and Davalos (1996) was used to analyze the influence and sensibility of different laminate configurations over its value such as: material properties; fiber orientation angles, disposition of layers (symmetric or anti-symmetric), number of layer and aspect ratio. Then both classical one-dimensional beam element and two-dimensional plate elements based on the first order shear deformation theory for composite laminated beams, (Bathe, 1996) including the model, were implemented as an UEL (User Element Subroutine in FORTRAN) and linked to software ABAQUS.

The advantage of the implementation of the models into pre-processing software like ABAQUS is the ability to model complex structures as, like is shown in Figure 1, the inclusion of an UEL affects only its main core processing, specifically the residue and stiffness and mass matrixes calculations. As such, other phases of finite element analysis that are time consuming and difficult to program like geometry, node, element and indexing matrixes creation are covered by ABAQUS' functions and user interface.

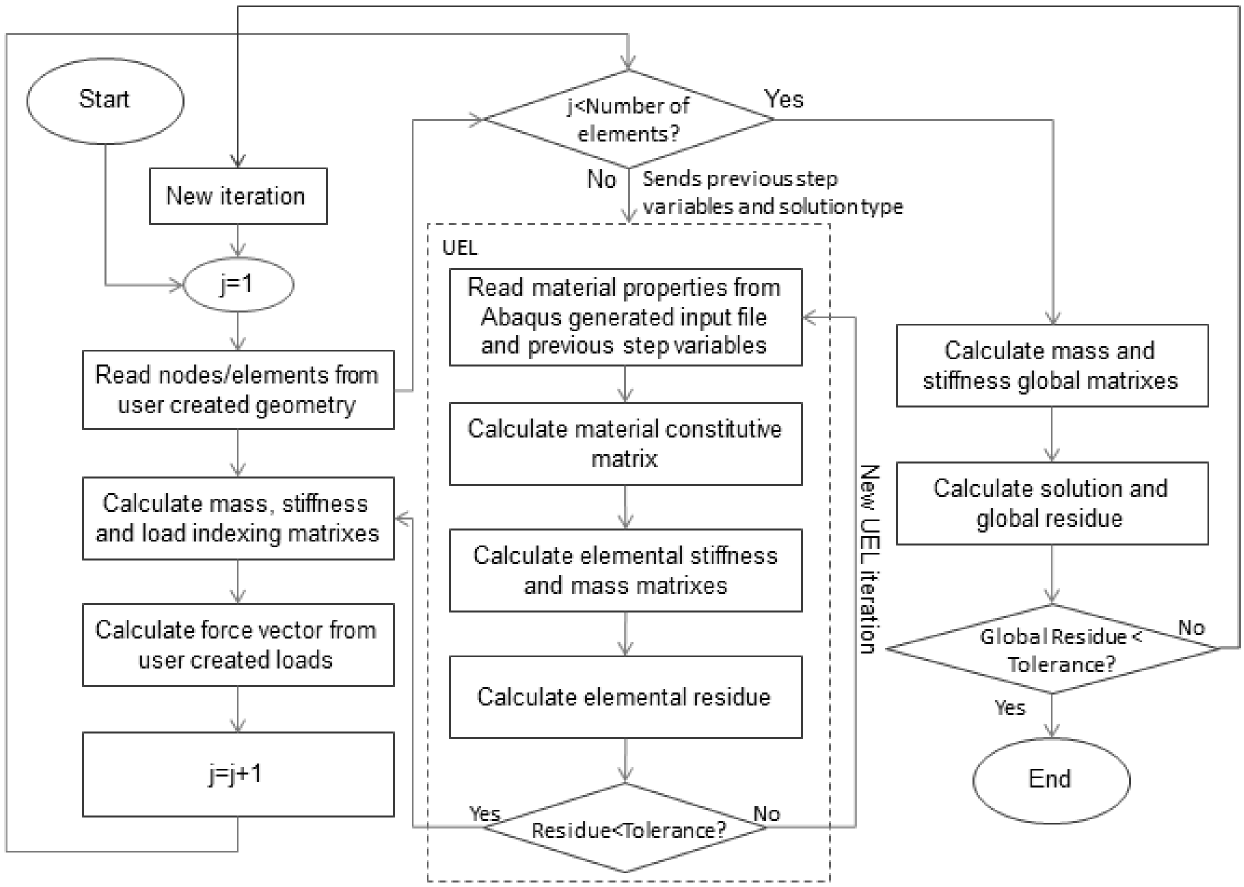

Figure 1. Summary of the Abaqus and UEL subroutine interactions

Finally, these elements were used in comparisons of simulated analysis with existing models and experimental results found in the literature.

2. SHEAR CORRECTION FACTOR MODEL

In this section the formulation of the shear correction factor is presented. The transversal shear coefficients of a laminated beam are derived by utilizing the method proposed by Raman and Davalos (1996): first the constitutive equations for a composite laminated plate are degenerated into beam equations by applying the classic beam theory contour and constitutive relations and other simplifying assumptions on the expressions from the classical laminated plate theory; the transversal shear strain energy are then calculated by integrating these relations over the thickness of the beam. Then, using Bert (1973) definition of the strain energy with the shear correction factors, these two expressions can be manipulated into a final expression for the coefficients.

2.1. Laminated Plate Constitutive Relations

The constitutive relations for a composite laminated plate based on the classical laminated plate theory, as presented by Reddy and Ochoa (1993) and, are obtained from the integration of the constitutive equations of each layer, found on Eq. (1), over the plate thickness. The final relations of the laminated can be separated into the axial and bending relations and the transverse shear relations. Thus, we have the relations for axial and bending stresses in Eq. (2) and the relations for the transverse shear stresses in Eq. (5).

$$\{\sigma^k\} = [\bar{Q}^k]\{\varepsilon^k\} \tag{1}$$

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{bmatrix} \begin{Bmatrix} \{\varepsilon_0\} \\ \{\kappa\} \end{Bmatrix} \tag{2}$$

Where $\{\sigma^k\} = \{\sigma_x \quad \sigma_y \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz} \quad \sigma_z\}^T$, $\{\varepsilon^k\} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz} \quad \varepsilon_z\}^T$ are the stresses and deformations of each layer; $\{N\} = \{N_x \quad N_y \quad N_{xy}\}^T$ are the axial loads and on-plane moment, $\{M\} = \{M_x \quad M_y \quad M_{xy}\}^T$ are the bending and torsional moments, $\{\varepsilon_0\} = \{\varepsilon_{x_0} \quad \varepsilon_{y_0} \quad \gamma_{xy_0}\}^T$ are the in-plane deformation, $\{\kappa\} = \{\kappa_x \quad \kappa_y \quad \kappa_{xy}\}^T$ are curvatures; $\bar{Q}^k$ are the stiffness matrix of each k^th layer and $\underline{A}$, $\underline{B}$ and $\underline{D}$ are the stiffness matrix integrated through the thickness of the laminated. This expression can be rewritten as:

$$\begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix} = \begin{bmatrix} \underline{\alpha} & \underline{\beta} \\ \underline{\underline{\beta}} & \underline{\delta} \end{bmatrix} \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} \tag{3}$$

Where:

$$\begin{bmatrix} \underline{\alpha} & \underline{\beta} \\ \underline{\underline{\beta}} & \underline{\delta} \end{bmatrix} = \left(\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{bmatrix} \right)^{-1} \tag{4}$$

Also, using k_1 and k_2 as the plane shear correction factors, the transverse shear loads Q_x and Q_y can be related to the transversal shear deformations γ_{xz} and γ_{yz} by the relation (Raman and Davalos, 1996) bellow. It is important to note that in this formulation, according to Bert, the transversal strains γ_{xz} and γ_{yz} are constant over the thickness of the plate and their values are corrected by k_1 and k_2.

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} k_1^2 A_{44} & k_1 k_2 A_{45} \\ k_1 k_2 A_{45} & k_2^2 A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{5}$$

2.2. Laminated Beam Contour and Constitutive Relations

The first assumption made for degenerating the plate relations into beam relations is that, on beams, the shear load on the xy plane Q_y is negligible, thus the shear deformation γ_{xy} can be computed from Eq. (5) as:

$$\gamma_{xy} = \left[k_1^2 \left(A_{44} - \frac{A_{45}^2}{A_{55}} \right) \right]^{-1} Q_x \tag{6}$$

Also, from this assumption the equilibrium equation for the stresses on a beam on the xz plane, in the absence of external body forces, can be written as in the Eq. (7) (Bert, 1973).

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \tag{7}$$

Integrating Eq. (7) through the thickness and using the relations presented in Eq. (2) an expression for σ_{xz} is obtained:

$$\sigma_{xz} = -\int_{-\frac{h}{2}}^{z} \frac{\partial}{\partial x} \left[\left(\bar{Q}_{11}\varepsilon_{x_0} + \bar{Q}_{12}\varepsilon_{y_0} + \bar{Q}_{13}\gamma_{xy_0} \right) + z\left(\bar{Q}_{11}\kappa_x + \bar{Q}_{12}\kappa_y + \bar{Q}_{13}\kappa_{xy} \right) \right] dz \tag{8}$$

Substituting the deformations and curvatures for the external loads using Eq. (3) we get a final expression for σ_{xz}

$$\sigma_{xz} = -\int_{-\frac{h}{2}}^{z} \frac{\partial}{\partial x}\left[\left(N_x\bar{Q}_{1i}\alpha_{j1} + N_y\bar{Q}_{2i}\alpha_{i2} + N_{xy}\bar{Q}_{6i}\alpha_{i3} + M_x\bar{Q}_{1i}\beta_{j1} + M_y\bar{Q}_{2i}\beta_{i2} + M_{xy}\bar{Q}_{6i}\beta_{i3}\right)\right.$$
$$\left. + z\left(N_x\bar{Q}_{1i}\beta_{j1} + N_y\bar{Q}_{2i}\beta_{i2} + N_{xy}\bar{Q}_{6i}\beta_{i3} + M_x\bar{Q}_{1i}\delta_{j1} + M_y\bar{Q}_{2i}\delta_{i2} + M_{xy}\bar{Q}_{6i}\delta_{i3}\right)\right]dz\,, i = 1,2,3 \qquad (9)$$

Using the second assumption that, for the classic beam theory,$\partial\{N\}/\partial x = 0$, $\partial M_x/\partial x = Q_x$ and that $M_y = M_{xy} = 0$ we can simplify Eq. (9) into

$$\sigma_{xz} = -\int_{-\frac{h}{2}}^{z} Q_x(\bar{Q}_{1i}\beta_{i1} + z\bar{Q}_{1i}\delta_{i1})dz\,, i = 1,2,3 \qquad (10)$$

2.3. Derivation of the Shear Correction Factor

The transversal shear strain energy per unit of length can be calculated using Eq. (9) and (12) by integrating the transversal shear stress σ_{xz} over the thickness of the plate. Also, from Eq. (8) and Bert (1973) assumption that by using the shear correction factors γ_{yz} is approximated constant over the thickness of the plate we can have two distinct expressions for the energy.

$$\bar{U} = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\sigma_{xz}^2}{\left(\bar{Q}_{44} - \frac{\bar{Q}_{45}^2}{\bar{Q}_{55}}\right)} dz = \frac{1}{2}\frac{Q_x^2}{k\left(A_{44} - \frac{A_{45}^2}{A_{55}}\right)} \qquad (11)$$

Equation 11 can be manipulated into one final expression for the shear correction factor:

$$k = k_1^2 = \left[\left(A_{44} - \frac{A_{45}^2}{A_{55}}\right)\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\left(\int_{-\frac{h}{2}}^{z} Q_x(\bar{Q}_{1i}\beta_{i1} + z\bar{Q}_{1i}\delta_{i1})dz\right)^2}{\left(\bar{Q}_{44} - \frac{\bar{Q}_{45}^2}{\bar{Q}_{55}}\right)} dz\right]^{-1}\,,\ i = 1,2,3 \qquad (12)$$

It is interesting to note that, by using an isometric material and a rectangular beam with b and h dimensions, the expression in Eq. (12) is simplified to the expression in Eq. 19 below. By doing so the shear correction factor returns to the classical value of 5/6 and retrieves Reissner/Mindlin plate theory (Raman and Davalos, 1996). Also, the proposed expression is different from the classic plate transversal shear distribution presented in Eq. (14) in that, if normalized and transformed in a warping function, it is discretized along each layer of the laminated, and it depends on the material properties.

$$k = \left[G\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\left(\int_{-\frac{h}{2}}^{z} z\right)^2}{G} dz\right]^{-1} = \frac{5}{6} \qquad (13)$$

$$f(z) = \frac{5}{4}\left(1 - \left(\frac{z}{h/2}\right)^2\right) \qquad (14)$$

3. FINITE ELEMENT FORMULATIONS

With the shear correction factor model, both a 2-node Timoshenko beam finite element, including the torsion rotation degrees of freedom and a 4-node Reissner/Mindlin plate were implemented into Abaqus FORTRAN subroutines (UEL). The formulation for the beam element can be found at the (Bathe) reference, and is summarized by the expressions in Eq. 15 to Eq. 23.

$$\boldsymbol{H_w} = \left\{\frac{1}{2}(1+\xi) \quad \frac{1}{2}(1-\xi) \quad 0 \quad 0\right\}, \boldsymbol{H_\beta} = \left\{0 \quad 0 \quad \frac{1}{2}(1+\xi) \quad \frac{1}{2}(1-\xi)\right\}, \boldsymbol{B_w} = \frac{\partial \boldsymbol{H_w}}{\partial \xi}, \boldsymbol{B_\beta} = \frac{\partial \boldsymbol{H_\beta}}{\partial \xi} \qquad (15)$$

$$\boldsymbol{K^e} = \frac{1}{h}\left(A_{11} - \frac{A_{12}^2}{A_{22}}\right) I \int_{-1}^{1} \boldsymbol{B'_\beta B_\beta}\,|det\boldsymbol{J}|d\xi + \frac{k}{h}\left(A_{44} - \frac{A_{45}^2}{A_{55}}\right) A \int_{-1}^{1} (\boldsymbol{B_w} - \boldsymbol{H_\beta})'(\boldsymbol{B_w} - \boldsymbol{H_\beta})\,|det\boldsymbol{J}|d\xi \qquad (16)$$

$$\boldsymbol{M^e} = \int_{-1}^{1}\{\boldsymbol{H_w} \quad \boldsymbol{H_\beta}\}\begin{bmatrix}\rho bh & 0\\ 0 & \frac{\rho bh^3}{12}\end{bmatrix}\begin{Bmatrix}\boldsymbol{H_w}\\ \boldsymbol{H_\beta}\end{Bmatrix}|det\boldsymbol{J}|d\xi \tag{17}$$

In the same way, complete formulations for the plate element can be found in the Reddy reference and can be summarized by the expressions found in Eq. 1 a 3.

$$\boldsymbol{H} = \left\{\frac{1}{4}(1-\xi)(1-\eta) \quad \frac{1}{4}(1+\xi)(1-\eta) \quad \frac{1}{4}(1+\xi)(1+\eta) \quad \frac{1}{4}(1-\xi)(1+\eta)\right\} \tag{18}$$

$$\boldsymbol{B_m} = \begin{bmatrix}\partial h_i/\partial x & 0 & 0 & 0 & 0\\ 0 & \partial h_i/\partial y & 0 & 0 & 0\\ \partial h_i/\partial y & \partial h_i/\partial x & 0 & 0 & 0\end{bmatrix} \tag{19}$$

$$\boldsymbol{B_b} = \begin{bmatrix}0 & 0 & 0 & 0 & \partial h_i/\partial x\\ 0 & 0 & 0 & -\partial h_i/\partial y & 0\\ 0 & 0 & 0 & -\partial h_i/\partial y & \partial h_i/\partial x\end{bmatrix} \tag{20}$$

$$\boldsymbol{B_s} = \begin{bmatrix}0 & 0 & \partial h_i/\partial x & 0 & h_i\\ 0 & 0 & \partial h_i/\partial y & -h_i & 0\end{bmatrix} \tag{21}$$

$$\boldsymbol{K^e} = h\int_{-1}^{1}\int_{-1}^{1}(\boldsymbol{B'_m A B_m})|det\,\boldsymbol{J}|d\xi d\eta + \frac{h^2}{2}\int_{-1}^{1}\int_{-1}^{1}\boldsymbol{B'_m B B_b}|det\boldsymbol{J}|d\xi d\eta + \frac{h^3}{12}\int_{-1}^{1}\int_{-1}^{1}\boldsymbol{B'_b D B_b}|det\boldsymbol{J}|d\xi d + kh\int_{-1}^{1}\int_{-1}^{1}\boldsymbol{B_s G B_s}|det\boldsymbol{J}|d\xi d\eta \tag{22}$$

$$\boldsymbol{M^e} = h\int_{-1}^{1}\int_{-1}^{1}\rho\boldsymbol{H'H}|det\,\boldsymbol{J}|d\xi d\eta \tag{23}$$

Where $\boldsymbol{J}$ is the jacobian matrix for coordinate transformation from the global system to the element isoparametric local system.

4. RESULTS

4.1. Comparison with literature results

The first analysis was a comparison between the transverse shear model discussed in the present work with other models found in the literature, contained on Tab. 1. It can be seen that the maximum percentage difference between the values is 12.9%, which is reasonable considering the simplicity of the formulation. Also, it can be seen that the greatest differences were for thin laminates with angles other than 0°, 45° and 90° which have coupling effects, this may be due to the assumptions in the formulation neglecting the effects outside the xz plane.

Table 1. Comparison between the calculated shear correction factor and examples found in the literature

Study Case	Shear Correction Factor (k)		Difference [%]
	Present Theory	Reference Value	
[+45° -45°] (Bert, 1973)	0.8209	0.8212	0
[+45° -45°]$_S$ (Whitney, 1973)	0.5952	0.5953	0
[+30° -30°] (Whitney, 1972)	0.7605	0.8592	12.9
[+30° -30°]$_S$ (Whitney, 1972)	0.6221	0.6730	7.50
[0° 90° 0°]$_S$, E_{11}/E_{22}=13 (Whitney, 1972)	0.8343	0.8212	1.57
[0° 90° 0°]$_S$, h=2 mm (Pai, 1995}	0.8343	0.8538	2.28
[0° 90° 0°]$_S$, h=2 mm (Pai, 1995)	0.8343	0.8676	3.83
[0° 90° 0° 90°]$_S$, h=1mm (Raman and Davalos, 1995)	0.8963	0.8969	0.664

4.2. Parameters sensibility

The expression on Eq. (12) was then used to measure the sensibility of the shear correction factor to various parameters of a composite beam: material properties, number of layers and aspect ratio, and ply angle orientation.

Symmetric and anti-symmetric composite laminates in the form $[(0°\ \theta°)_N]_S$ and $[(0°\ \theta°)_N]_{AS}$ with N a number of layers ranging from 3 to 9, θ ply angles ranging from 0° to 90° and with material properties as follow: E_{11}/E_{22}=10 or 50; E_{33}/E_{22}=1; G_{12}/E_{22}=G_{13}/E_{22}=0.5; G_{23}/E_{22}=0.2; ν_{12} =0.25 had the corrections factors evaluated.

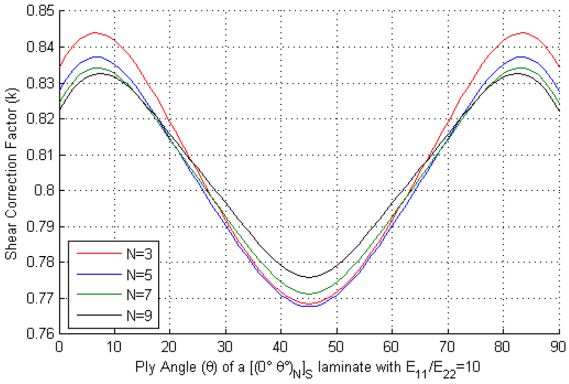

Figure 2. Variation of the shear correction factor for a symmetric composite laminate with E_{11}/E_{22}=10 for both number of layers and ply orientation angles

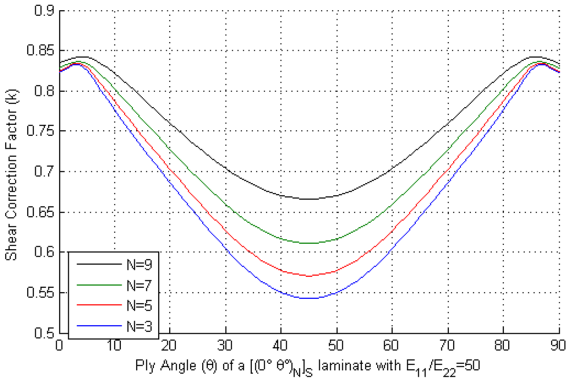

Figure 3. Variation of the shear correction factor for a symmetric composite laminate with E_{11}/E_{22}=50 for both number of layers and ply orientation angles

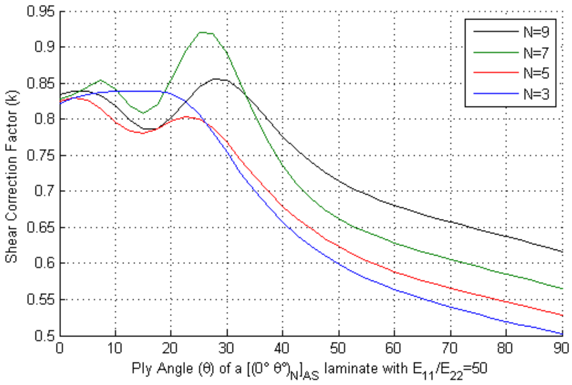

Figure 4. Variation of the shear correction factor for an anti-symmetric composite laminate with E_{11}/E_{22}=50 for both number of layers and ply orientation angles

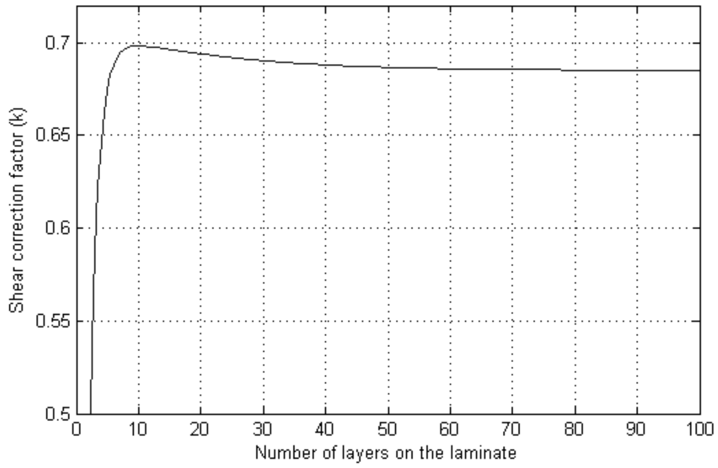

Figure 5. Evolution of the shear correction factor with the thickness of the plate

The average of the factors obtained was below the classical value of 5/6, but there were cases where higher factors were found. It is interesting to note that while for the symmetric laminates, the minimal values occurs for 45°, this angle is 90° for the anti-symmetric laminates which probably occurred because the transversal shear effects diminishes as the difference between the ply angles increases. Another interesting fact is that the number of layers, and consequently the thickness and aspect ratio of the laminate are more influent on the transversal shear as the difference between the strength of the fiber and transverse directions increases, which reduces the magnitude of the normal and consequently the in-plane stresses. From Fig. 5 can be observed that, as the laminate gets thinner, the shear correction factor gets smaller is explained by the reduced effects of the transversal shear stresses; at the same time, as the plate starts to grow thick the correction factor stabilizes at k=0.678.

4.3. Comparison between existing commercial finite element packages

These finite elements containing the shear correction factor model were used in two different tests. The first was a static analysis of a single clamped laminated plate subjected to a constant concentrated bending-torsional load F=40N, modeled using the plate element. The material properties used were: E_{11}=132GPa, E_{22}=10.3GPa, ν_{12}=0.357, ν_{23}=0.306, ρ=1470kg/m3, G_{12}=G_{13}=6.5GPa, G_{23}=4.3GPa, h=2mm with a composite laminate with the ply angles orientations equal to $[0° \; 90° \; +45° \; -45°]_S$. The vertical displacement (u_z) on the B point was recorded as the aspect ratio of the plate were changed by running simulations with increasing values of L.

This is a classical benchmark example for testing effects of shear-locking (Simulia Dassault, 2008) as the plate is over a strong bending-torsional load and the predominant effects are out-of-the plane shear stresses. Thus, it was used to compare the results of the existing elements within the Abaqus software and the proposed element, as well as studding the influence of the shear correction factor on the shear-locking problem and its influence in a case where it is one of the most influential effects. For that the analysis was made using three different elements: Abaqus' S8R, an 8-node plate element with reduced and selective integration and hour-glass control as the reference, as this kind of element would hardly fell any kind of shear-locking effect; Abaqus' S4, an 4-node full-integrated plate element that is highly susceptive to this problem, and the created element containing the shear correction factor model.

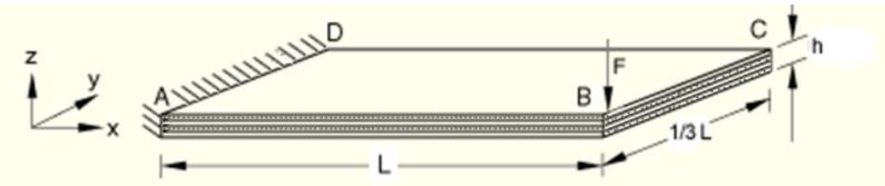

Figure 5. A schematic view of the first numerical problem studied

Remembering that the only difference between the created element and Abaqus' S4 element is the shear correction factor model, it can be observed from Figure 5 that the shear locking, although still having a strong effect over the created element, was lessened by the shear correction factor model.

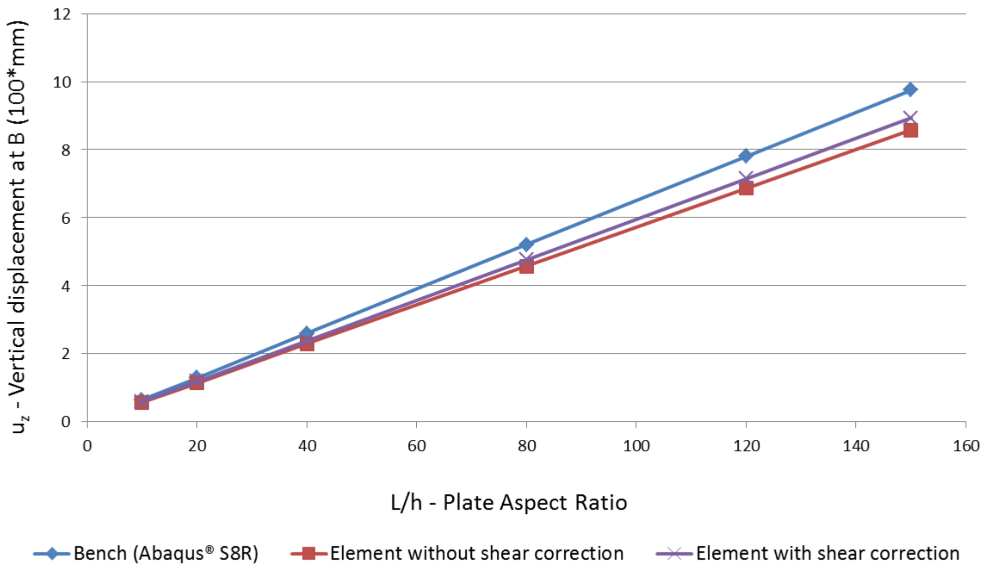

Figure 5. Results from the shear-locking Abaqus benchmark test

4.4. Comparison with experiments found in the literature

Another test was made using the beam element: a modal analysis of a composite laminated beam was done and compared to experimental results obtained by Tita (1999). The beam has 425 mm x 25 mm x 1.6 mm. The material properties are E_{11}=48.6GPa; E_{22}=11.27GPa; ν_{12}=0.28; G_{12}=G_{13}=4.85GPa; G_{23}=4.45Gpa e ρ=1780kg/m^3. Two different pile-ups were prepared: a [(+45° -45°)$_3$ (0° 90°)]$_S$ and a [(0° 90°)$_4$]$_S$. The beam was created on Abaqus and the modal results were obtained for both Abaqus' solution, the created element and the experimental results.

Table 2. Vibration Modes for the [(+45° -45°)$_3$ (0° 90°)]$_S$ case

Mode	Abaqus (undamped) [Hz]	Created Element (undamped) [Hz]	Experimental (damped) [Hz]
1	4.734	4.369	4.0
2	28.90	27.33	25.5
3	80.12	67.70	-
4	82.83	76.52	74.0
5	163.5	141.3	139.0
6	182.8	150.0	-
7	252.6	248.5	-
8	386.3	372.1	-

Table 3. Vibration Modes for the [(0° 90°)$_4$]$_S$ case

Mode	Abaqus' B31 (undamped) [Hz]	Created Element (undamped) [Hz]	Experimental (damped) [Hz]
1	4.802	5.333	4.5
2	29.96	33.36	28.0
3	83.01	82.91	-
4	94.84	93.27	84.0
5	164.5	93.59	111.5
6	192.8	182.4	155.5
7	272.6	281.5	-
8	315.1	301.1	-

As it was expected, both MEF simulations got higher natural frequencies than the experimental ones as the formers are undamped analysis. However, for most of the values the model with the shear correction factor obtained a closer result to the experimental ones, especially on the first case. That makes sense as in the cross-ply case there are a greater shear stresses, as in the (0° 90°) the internal loads are basically normal which can great influence the results. Also, the simplification applied in Eq. 5 can also have had an effect as in the (45° -45°) case the Q_y loads have a lower effect as the transversal shear effect is more homogenized. Another interesting fact is that, in the first case, the model with the shear correction factor got much better results than the common element in higher modes, where there's an increased influence of the transversal shear effects as the smaller distance between nodes in these modes has an effect of virtually increase the aspect ratio of the beam.

5. CONCLUSIONS

A simple, one-dimensional model of the shear corrections factors for composite laminated plates was formulated based on the works of Pai (1995) and Ramon and Davalos (1996). The final expression was used to evaluate the sensibility of the shear correction factor to various properties of a composite material such as number of layers, thickness, ply angles orientation, configuration (symmetric or anti-symmetric). The model was then used in two classic beam and plate finite elements which were used to test the characteristics and limitations of the formulation using comparisons between experimental results found in the literature and the existing finite-element models. Those tests showed that the formulation used for the transversal shear to be more precise than the classical theories in specific cases where the transversal shear loads have a prominent effect at the structural system behavior and the shear loads are close to a one-dimensional load case. Also, because of its simple implementation that causes a great computational efficiency, this model can be further developed into a more complex, bi-dimensional model in future works as, even if existing modern theories are more precise, they are time consuming solutions.

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7. REFERENCES

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