IDENTIFICATION OF SMART STRUCTURES BY KAUTZ FILTER WITH MULTIPLE POLES

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Abstract. *Impulse response functions (IRFs) identification in mechanical systems and structural dynamics is important in many engineering applications, particularly in experimental modal analysis. Several approaches are available for identify the IRFs directly in the time domain by using input and output time series measured. The classical covariance method is one of the most used and it is based on the sum of convolutions of the input forces. In this sense, the main goal of this paper is to identify the IRF of mechanical structures lightly damped by considering the input force signals filtered by Kautz filter with multiple poles. If a Kautz filter is chosen adequately, the number of parameters needed to estimate the IRF is reduced drastically. The inclusion of different poles in the Kautz functions allows the use of a wide range of frequency without preprocessing the signals. In order to illustrate the applicability and the efficacy of the proposed approach, some experimental tests by using a smart beam with PZTs sensor/actuator coupled are performed. The results show the advantages and drawbacks of the methodology with some further directions research necessary to improve the procedure in order to implement in real-world engineering system.*

Keywords: *impulse response functions, orthogonal functions, Kautz filter.*

1. INTRODUCTION

In several engineering aplications is necessary to know the dynamic behaviour of the system in order to be able to predict events and let it more productive and safe. One method to obtain this is by using a white box modeling, that is based on the physicals events and theory. This method can be complex and very expensive both in relation of time and money. In order to overcome these drawbacks, an empirical procedure can be utilizated. This method is called black box method and it are based only in the input and output signals measured experimentally in a real-world plant. In that case, it is assumed no knowledge of the system theory (Aguirre, 2007).

In order to perform this, the aim of this paper is to use the covariance method (CM) basead on the sum of convolution of orthogonal functions to indentify the impulse response functions (IRF) of mechanical systems. The ortogonal function used in this paper is the Kautz filter, that is an orthogonal function in Hilbert space, and it is addressed to reduce the number of parameters that must be found to describe the IRF of the system and reduce the complexity of the problem. This approach is introduced to transform the input and output signals to an orthogonal Kautz basis functions. Once this transformation is made, standard identification techniques are applied to the signals. The CM methodology has been directed towards for the data reduction caused by the Kautz function. The CM approach estimates the terms of finite impulse response. Thus, the goal of this paper is to show clearly this method, based on the input and output signals to identify the impulse response function (IRF) of a low damped smart structure.

In other works, e. g. Pacheco and Junior (2004), different orthogonal functions were used to transform a set of differencial equations into a set of algebraic equations. This procedure have some drawbacks, one is the need of a huge amount of expansion terms and another limitation is the need of the knowledge of the mathematical model of the system (da Silva *et al.*, 2009). Moreover, the Kautz filter is described by complex conjugated poles that well describe oscillatory systems, ideal for vibration applications. Additionally, non-linear system identification of Volterra kernels can be well represented by Kautz filter. In this sense, da Silva (2011a) and da Silva *et al.* (2010) presented applications of these approach by using mechanical systems, as beam and portal frame, and discussed the advantageous and the practical enforceability. da Silva (2011b) presented a procedure involving the Kautz filter to be able to include different poles in a filters set, seeking applications for modal analysis and structural dynamics of mechanical systems. This allowed the use of a wide range of frequency without pre-processing the output vibration signal. However, the results showed were based only in numerical simulation. The goal of the present paper is to realize an application of IRF identification with Kautz filter with multiple poles by using experimental data provided by a smart beam in order to illustrate and validate experimentally the methodology shown in da Silva (2011b).

The paper is organized as follows: first a review of covariance method expanded in ortonormal basis functions is presented. Next, the Kautz filter with multiple poles and a briefly procedure to choose the poles in the Kautz filter based on optimization is described. In order to illustrate the procedure, a smart structure composed by an aluminium beam with PZT actuator/sensor coupled is used to identify the IRFs. The boundary conditions considered in the beam are free-free. The experimental setup is controlled by using a dSPACE® 1104 data acquisition board with ControlDesk® software. The numerical identification is performed by using the Matlab® and Simulink®. In order to show some features, it is provided a number of simulations to illustrate the applicability and drawbacks of the approach. Finally, the final remarks in this paper are presented with further direction researches.

2. COVARIANCE METHOD EXPANDED IN ORTHONORMAL BASIS

The classical Wiener-Hopf equation describes the relationship between the correlation function $R_{uu}(k)$ and cross-correlation $R_{uy}(k)$ trough the impulse response function (IRF) $h(k)$ (Godfrey, 1986):

$$R_{uy}(k) \approx \sum_{j=0}^{N} h(j) R_{uu}(k-j) \tag{1}$$

where the correlation function $R_{uu}(k)$ computed with the input signal $u(k)$ and cross-correlation $R_{uy}(k)$ computed with $u(k)$ and output signal $y(k)$ can be estimated by using different methods, for example, the Levinson-Durbin recursion method.

A least-square identification method can be implemented to estimate the expansion coefficients in the time-series $h(k)$ that describes the finite impulse response model (FIR model). However, this identification method often leads to conservative results because a vibrating system is hardly ever represented by a FIR model. Thus, the practical drawback is that a large number of parameters $h(k)$ must be considered in order to obtain a good approach in eq. (1).

The idea is to describe the IRF by using the Z transform and as a linear combination of the functions $\Psi_i(z)$:

$$H(z) = \alpha_0 \Psi_a(z) + \alpha_1 \Psi_1(z) + \cdots + \alpha_J \Psi_J(z) = \sum_{j=0}^{J} \alpha_j \Psi_j(z) \tag{2}$$

where α_j, $j = 0, 1, \ldots, J$ are the expansion coefficient values in the functions described by $\Psi_j(z)$. $\Psi_j(z)$ is a filter chosen in order to obtain the minimum possible number of elements and a good approximation of the IRF.

After substituting eq. (2) in eq. (1) yields:

$$\begin{aligned} R_{uy}(k) &\approx \sum_{i=0}^{N} h(i) R_{uu}(k-i) \equiv \sum_{i=0}^{N}\sum_{j=0}^{J} \alpha_j \psi_j(i) R_{uu}(k-i) \\ &= \sum_{j=0}^{J} \alpha_j \sum_{i=0}^{N} \psi_j(i) R_{uu}(k-i) = \sum_{j=0}^{J} \alpha_j v_j(k) \end{aligned} \tag{3}$$

where $v_j(k)$, $k = 0, \cdots, N$ is the input signal $R_{uu}(k)$ filtered by each element of the discrete-time function $\psi_j(k)$, $j = 0, 1, \ldots, J$ that is an approximation base functions:

$$v_j(k) = \sum_{i=0}^{N} \psi_j(i) R_{uu}(k-i) = \psi_j(k) * R_{uu}(k) \tag{4}$$

where the symbol $*$ represents the convolution operator.

The eq. (3) can be used to form the following matricial equation:

$$\left\{ \begin{array}{c} R_{uy}(0) \\ R_{uy}(1) \\ \vdots \\ R_{uy}(N) \end{array} \right\} = \left[\begin{array}{cccc} v_0(0) & v_1(0) & \cdots & v_J(0) \\ v_0(1) & v_1(1) & \cdots & v_J(1) \\ \vdots & \vdots & \ddots & \vdots \\ v_0(N) & v_1(N) & \cdots & v_J(N) \end{array} \right] \left\{ \begin{array}{c} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_J \end{array} \right\} \tag{5}$$

Finally, the IRF estimated $\hat{h}(k)$ can be defined by:

$$\hat{h}(k) = \sum_{j=0}^{J} \alpha_j \psi_j(k), \qquad k = 0, 1, \ldots, N \tag{6}$$

or in the matricial form:

$$\left\{ \begin{array}{c} \hat{h}(0) \\ \hat{h}(1) \\ \vdots \\ \hat{h}(N) \end{array} \right\} = \left[\begin{array}{cccc} \psi_0(0) & \psi_1(0) & \cdots & \psi_J(0) \\ \psi_0(1) & \psi_1(1) & \cdots & \psi_J(1) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_0(N) & \psi_1(N) & \cdots & \psi_J(N) \end{array} \right] \left\{ \begin{array}{c} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_J \end{array} \right\} \tag{7}$$

It worth noting that if the filter $\Psi_j(z)$ is properly chosen, the order $J << N$. Thus, it is easier to identify the coefficient expansion α_j than the IRF in physical base $h(k)$ once the convergence properties of $\Psi_j(z)$ subsets is related to the completeness properties of these subsets of functions.

3. KAUTZ FILTER WITH MULTIPLE POLES AND CHOICE OF POLES

An appropriate choice of poles in $\Psi_j(z)$ that reflects the dominant dynamics of the process is very important and which still preserving the orthogonality accelerates the convergence for certain classes of transfer functions. The Kautz function is a second-order filter described by pairs of complex conjugate poles in the z-domain, $\beta = \sigma + j\omega$ and $\beta^* = \sigma - j\omega$, which provides a good generalization of mechanical vibration systems (da Silva *et al.*, 2009).

da Silva *et al.* (2009) estimated the IRFs by using only one pole induced in the Kautz filter. This procedure required a pre-filtering of the signals involved in the range of modal contribution to be identified. A sequence of filters can be used with different poles in each section reflecting the modal behavior in the range of interest (da Silva, 2011b).

The elements in a set of Kautz filters can be also given by (den Hof and Bokors, 1995; Wahlberg, 1994; Heuberger *et al.*, 2005):

$$\Psi_{2n}(z) = \frac{\sqrt{(1-c^2)(1-b^2)z}}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{n-1} \tag{8}$$

$$\Psi_{2n-1}(z) = \frac{\sqrt{1-c^2}z(z-b)}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{n-1} \tag{9}$$

where the constants b and c are relative to the poles β_j and β_j^* in the j-th filter through the relations:

$$b = \frac{(\beta_j + \beta_j^*)}{(1+\beta_j\beta_j^*)}, \tag{10}$$

$$c = -\beta\beta_j^* \tag{11}$$

A procedure for estimating the poles and the IRF simultaneously in an iterative way can be implemented based on application of equation:

$$y(k) \approx \sum_{j=0}^{N} h(j)u(k-j) \tag{12}$$

and the output experimental signal $y_e(k)$. An error function can be described by:

$$e(k) = \hat{y}(k) - y_e(k) \tag{13}$$

where $\hat{y}(k)$ is the predicted output signal by the IRF $\hat{h}(k)$ estimated considering Kautz basis defined by the poles β_j and β_j^* in the z-domain:

$$\hat{y}(k) = \sum_{i=0}^{N} \hat{h}(i)u(k-i) \tag{14}$$

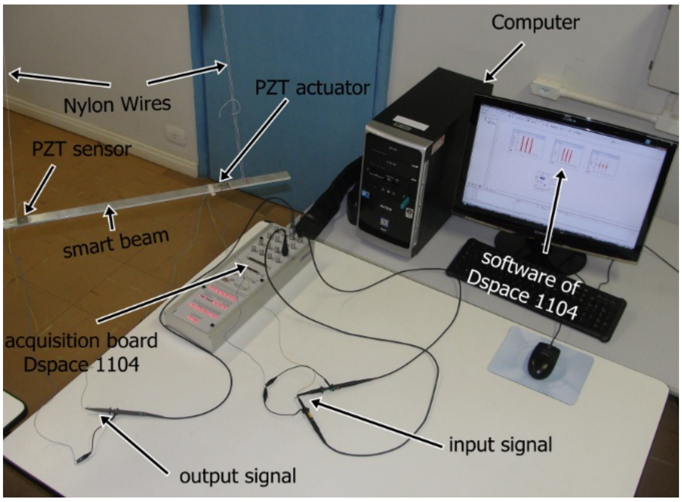
The optimization problem can be described by the following objective function that employs an Euclidean norm:

$$\min \quad ||e(k)|| = \sqrt{\sum_{k=1}^{N} |e(k)|^2} \tag{15}$$

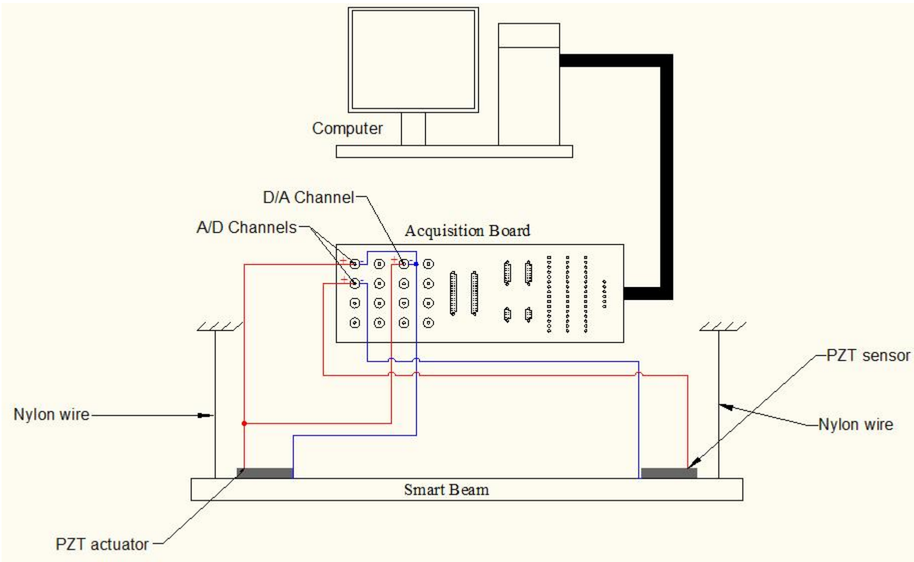
where the error $e(k)$ is function of the placement of the Kautz poles. The Kautz poles are functions of the frequencies and damping factors that are the optimization parameters. These parameters can be limited in a range searching. This optimization problem can be solved by several classical approaches, as for example, sequential quadratic program (SQP) (Goldbarg and Luna, 2000). However, in this paper was solved by using genetic algorithms (Goldberg, 1989; Michalewicz, 1996)

4. EXPERIMENTAL APPLICATION

In order to illustrate the approach, experimental tests are performed in a smart beam, see Fig. 1(a). The length, width and thickness of the beam are $600 \times 25 \times 4.73$ mm, respectively. In both ends of the smart structure are coupled two PZTs ceramics used as actuator and sensor. The boundary condition simulated is the free-free (Fig. 1(a)) . The signal input is produced by Matlab® and Simulink® and a Digital/Analogic board is used to applied the excitation signal in



(a) View of the smart beam with data acquisition system.



(b) Schematic diagram of the experimental setup.

Figure 1. Smart beam with experimental setup.

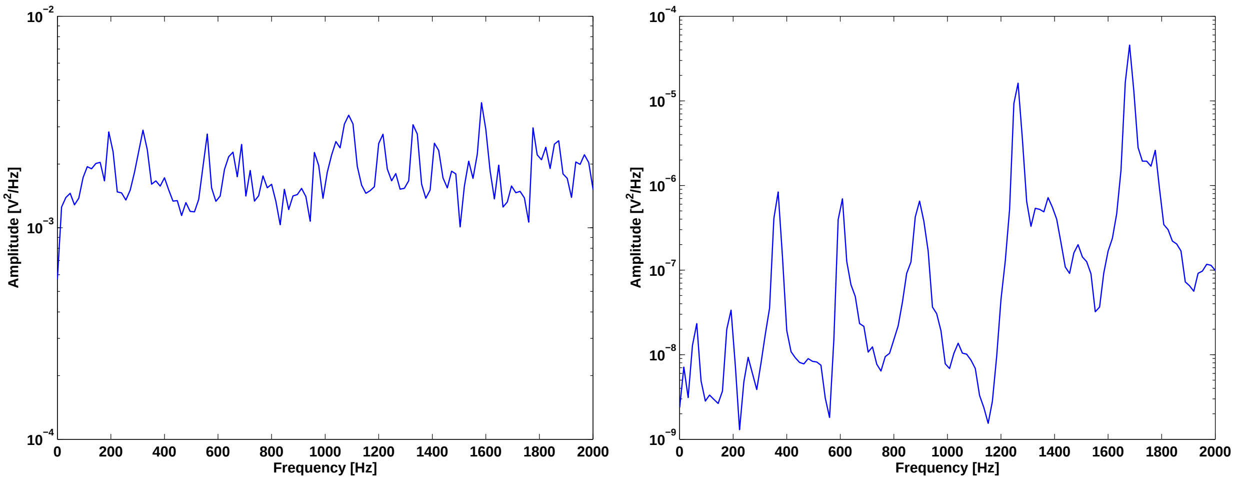
Table 1. Channels of the dSPACE® 1104 used in the tests.

Function	Description	Voltage range (V)	Resolution (bits)	Resolution (mV)
Signal generator	DAC 1	-10 to +10	16	0.3
Measurement	ADC 1 and ADC 2	-10 to +10	16	0.3

the PZT actuator. All data acquisition are realized by dSPACE® 1104 data acquisition board. Figure 1(b) presents the experimental procedure for the data acquisition. Information about the voltage range and resolution of the channels used in dSPACE® 1104 are presented in Tab. 1.

The sampling rate used in all tests is 8 kHz. The range of frequency to identify the modal contribution was from 0 to 2 kHz. For the boundary condition, material and geometric properties, the beam has several natural frequencies into this range. Two signal were used to excite the beam. The first one is a white noise with 0.4 seconds of duration filtered by a digital lowpass filter centered in 2 kHz. The second signal input used is a linear chip from 1 Hz to 2 kHz with the same time duration. All signals were recorded with a sampling rate of 8 kHz with 3200 samples. Thus, to identify the IRF by classical CM should be necessary to estimate 3200 parameters.

Figures 2(a) and 2(b) present the power spectral density (PSD) of the white noise input signal and the output measured by PZTs, respectively. Both PSDs were estimated by Welch method using 500 samples, 60 % of overlap and Hanning window.



(a) PSD of the input signal applied in the PZT actuator.

(b) PSD of the output signal measured by the PZT sensor.

Figure 2. Power spectral density (PSD) estimated by Welch method.

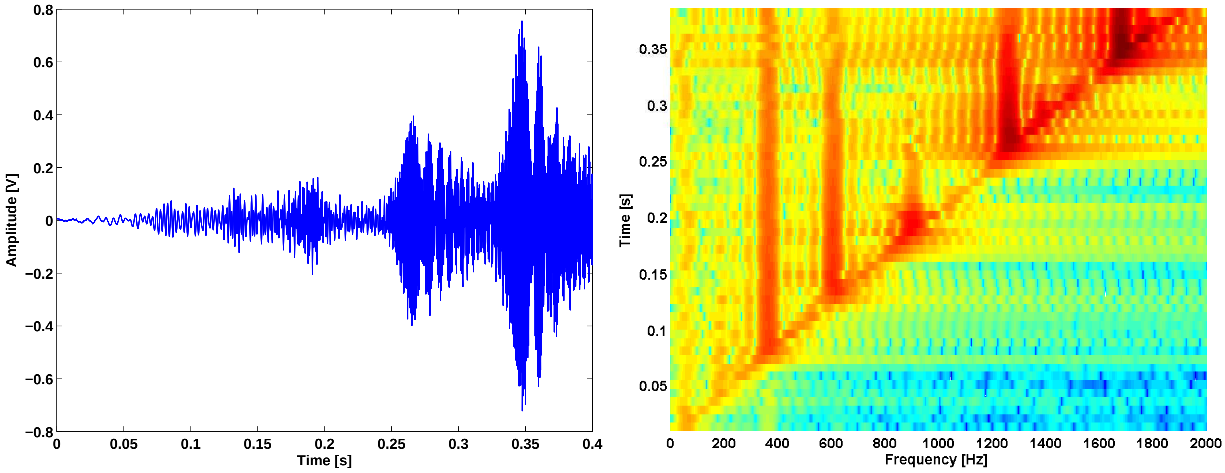
Figure 2(b) shows that the frequencies of 365 Hz, 605 Hz, 902 Hz, 1260 Hz and 1680 Hz can be chosen as candidates to natural frequencies, once that the PSD of the output signal has amplitudes much bigger in these frequencies. Now the modal damping factor are very small (less than 0.1). The candidate of natural frequencies and damping factors are used to chose the initial poles in the Kautz filter. These candidates can also be confirmed by analyzing Fig. 3 which shows the PZT response when a chirp signal is applied. It is worth noting that the amplitude of PSD in frequencies smaller than 300 Hz are not relevant and it will not be considered, see Fig. 2(b). In Tab. 2 is shown the analytical natural frequencies of the beam in the free-free boundary condition (Rao, 2011). Thus, the candidates values are adequately close to the analytical natural frequency.

Table 2. Analytical natural frequencies (Rao, 2011).

	$\omega_n[Hz]$
pole 1	68
pole 2	189
pole 3	371
pole 4	614
pole 5	917
pole 6	1281
pole 6	1706

In order to identify the IRF is necessary firstly to chose the poles by an optimization problem. The optimization procedure with genetic algorithm (GA) is done based on the initial values of the natural frequencies suggested before. The GA codified the optimization parameters in binary vectors representing the genetic code of the chromosome. A population of randomly generated chromosomes are created and submitted to crossover and mutation operators based on a quality measure of the chromosome called fitness, given by Eq. (15). This process is iteratively repeated until a pre-established stop criteria is reached or until a maximum number of generations. The best chromosome of the last generation is selected as the possible solution of the optimization problem. The flowchart in Fig. 4 shows this procedure. In order to ensure that the best chromosome in each generation has been selected until the GA reaches the stop criteria, an elitist selection was implemented. This procedure records the best chromosome in each generation and ensure that this elite chromosome remains to the next generation. The genetic algorithm applied to solve the optimization problem used the configurations showed in Tab. (3). More information about the algorithm parameters can be founded in the literature Goldberg (1989), Michalewicz (1996).

In the range of natural frequencies, one considered intervals into the frequencies centered in the candidates values presented and with a small range of variation of the damping factor ζ. These two set of parameters define the five Kautz poles in the continuos time (s domain). The range of searching is presented in Tab. 4. If one of the parameters shown in Tab. 4 remains in the limit after of the optimization, the process can be done again with new intervals proposed. All simulations were performed in a computer with Intel® core 2 Quad with 2.83 GHz and 4 GB(RAM) and with the Matlab®. Once the poles in s domain is chosen, it are converted to the z domain trough the bilinear transformation



(a) Time response of the PZT output.

(b) Time-frequency response of the PZT output computed by short-Fourier transform.

Figure 3. PZT output response when a chirp signal is applied in the PZT actuator.

Table 3. Genetic algorithm configurations.

Configuration	**Value**
Type of selection	Tournament
Type of crossover	One point
Population size	100
Number of participants of the tournament	5
Number of iterations (generations)	50
Crossover tax	0.8
Mutation tax	0.05

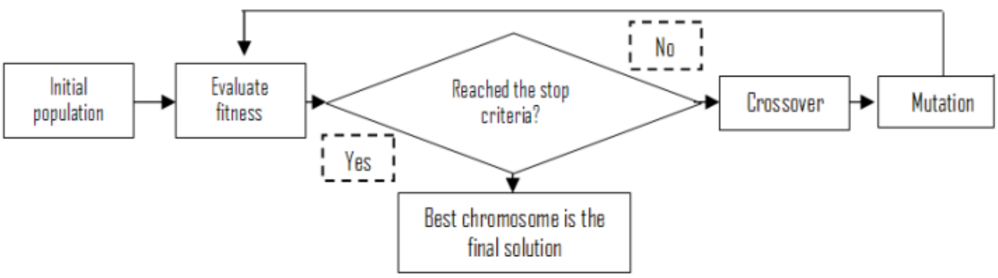


Figure 4. Flowchart of a basic genetic algorithm

because the Kautz filter is a discrete time filter.

Table 4. Range of searching of poles $s_i = -\xi_i \omega_{ni} \pm j\omega_{ni}\sqrt{1-\zeta_i^2}$ in the optimization procedure.

	$\omega_n[Hz]$	ζ
pole 1	$345 - 385$	$0.001 - 0.01$
pole 2	$585 - 625$	$0.001 - 0.01$
pole 3	$882 - 922$	$0.001 - 0.01$
pole 4	$1240 - 1280$	$0.001 - 0.01$
pole 5	$1660 - 1700$	$0.001 - 0.01$

Figure 5 presents the IRF estimated using the optimal parameters found, these results are presented in Tab 5. In this case were used the data set provided by the noise input signal. Interestingly, the number of parameters to estimate in the case of using the classic covariance method is given by $N = 3200$. On the other hand, the number of parameters α_j to be estimated using the expansion of Kautz filter is given by $J = 2 \times 5 = 10$. Clearly $J << N$. Figure 6 shows the comparison between the IRF estimated through the Kautz filter with multiple poles and by using the classical covariance method without expansion into orthogonal basis functions. The classical covariance method is implemented by using the Matlab® command *CRA* in the System Identification Toolbox. It is observed a good approach between the two responses, but the identification without Kautz filter required more parameters into the problem. The differences between the IRFs is due to use only five poles to described the dynamic behavior into Kautz filter. When the classical covariance method is performed all modal contributions is considered in the data. However, the computational complexity is higher ($J << N$). Figure 6 also presents the IRF estimated by applying the inverse discrete Fourier transform (DFT) in the frequency response function (FRF) computed by using the H_1 estimate through spectral analysis, see Fig. 7 .

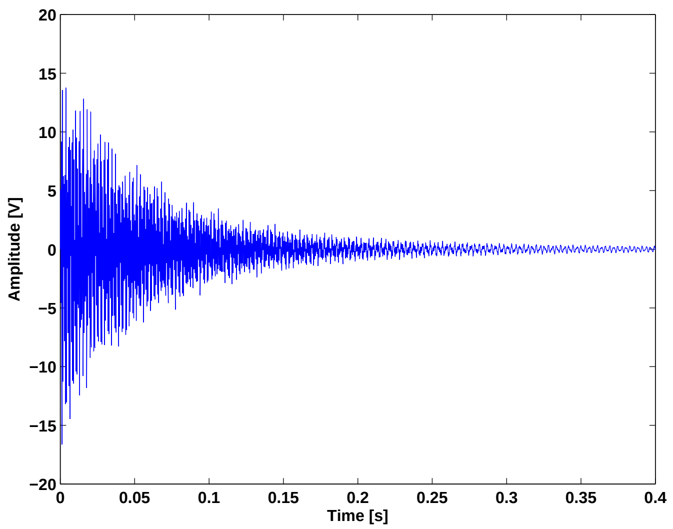


Figure 5. Impulse response function of the smart beam by considering the covariance method with Kautz filter with multiple poles. The poles are set based on results shown in Tab. 5.

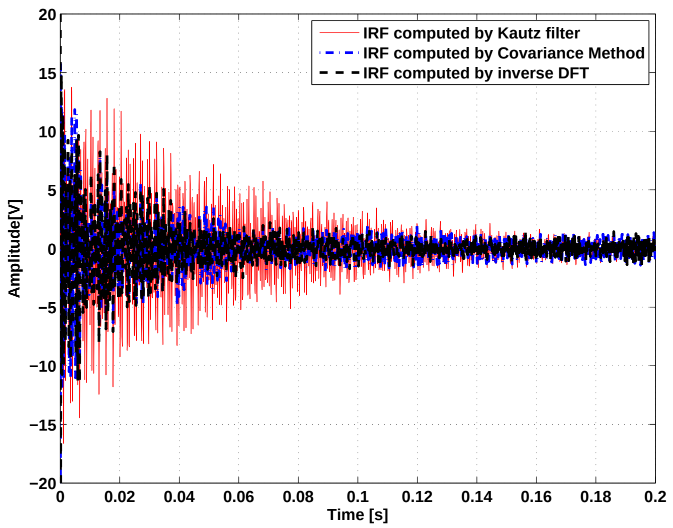


Figure 6. Comparison of the IRF identified by using Kautz filter with multiple poles, by using classical covariance method without orthogonal basis functions and through inverse discrete Fourier transform in the experimental FRF from Fig. 7.

Figure 8 shows the output estimated by Eq. (14) through the IRF estimated by Kautz filter. In order to better evaluate each of these estimates, the simulated response from eq. (14), with $\hat{h}(k)$ shown in Fig. 5, is compared with the experimental response due the chirp input signal. These results are presented in Fig. 9(a) and 9(b). The poles used in this validation tests are presented in Tab. 5.

Figure 9(b) ilustrates the response signal caused by a chirp input estimated by Kautz filter with non-optimized poles showed in Tab. 6. This ilustrates the sensitivity of the estimated IRF due the choice of the poles. As near of the real poles

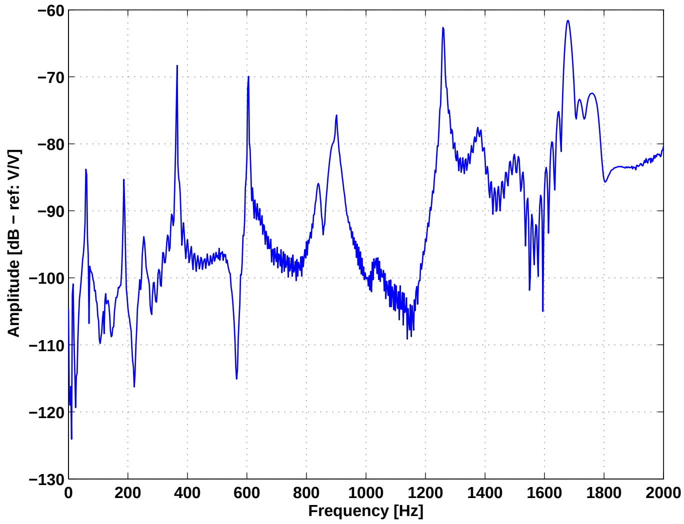
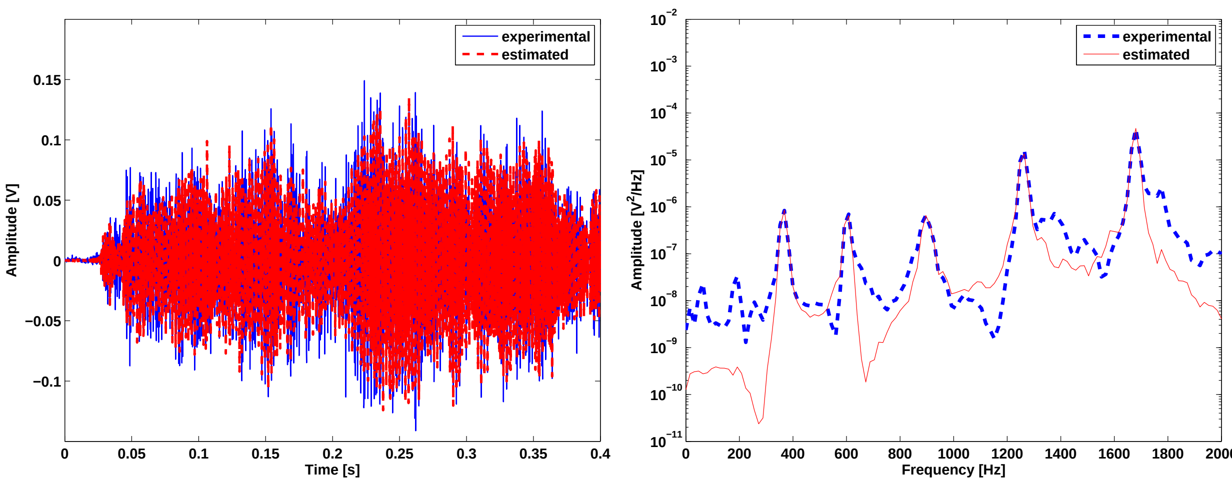


Figure 7. Experimental FRF computed by H_1 estimate by using the Welch method with 3000 samples, 50 % of overlap and rectangular window.

Table 5. Parameters of the optimum poles found by GA.

	$\omega_n[Hz]$	ζ
pole 1	365	0.00171
pole 2	604	0.00166
pole 3	900	0.00820
pole 4	1259	0.00262
pole 5	1679	0.00158



(a) Direct comparison - validation data.

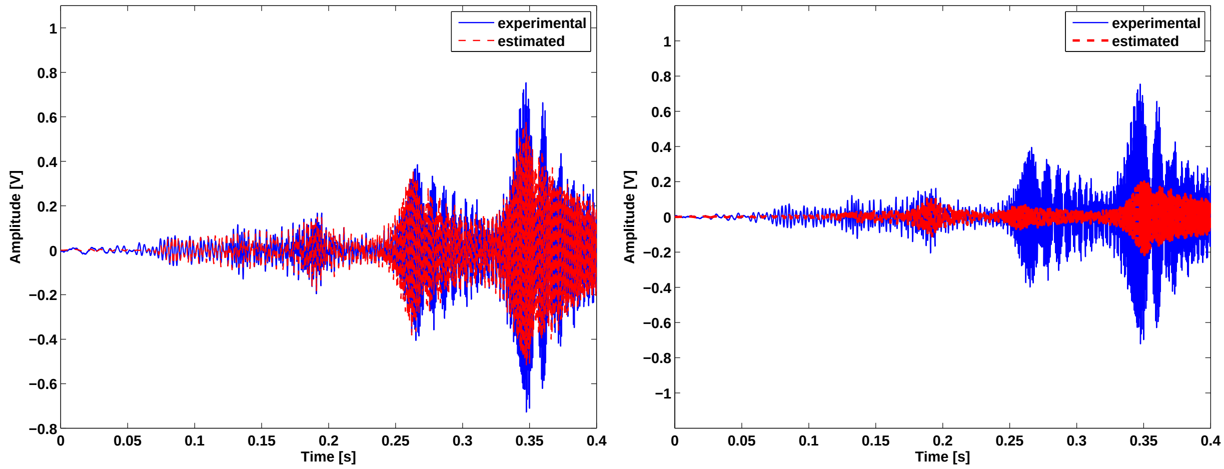
(b) PSD of the output.

Figure 8. Comparison between the experimental output signal PZT and the estimated by eq. (14) through the IRF identified with Kautz filter.

are the used in the Kautz filter, more the IRF will be close to the real system.

5. FINAL REMARKS

The present paper illustrated that the use of expansion into orthogonal basis functions of the excitation force signals can be effectively employed to describe the impulse response function (IRF). The Kautz filter allows to reduce the number of parameters needed in the estimation of IRFs considerably since the multiple Kautz poles are correctly chosen to esti-



(a) Experimental and estimated response of the beam due the chirp input signal trough Eq. (14) and the IRF shown in Fig. 5.

(b) Experimental and estimated response of the beam due the chirp input signal with the IRF computed with Kautz filter with non-optimized poles (Tab. 6).

Figure 9. Experimental and estimated response of the beam due to the chirp input signal.

Table 6. Non-optimized poles used in Fig. 9(b).

	$\omega_n[Hz]$	ζ
polo 1	380	0.0015
polo 2	610	0.0019
polo 3	910	0.008
polo 4	1240	0.0025
polo 5	1695	0.0013

mate a range of modal contributions of the IRFs. An optimization procedure based on genetic algorithms was proposed and adjusted seeking to find what are the best conditions in which there is a good agreement between the experimental measurements and the predicted responses using convolution PZT actuator signal with the impulse response identified. Experimental results were used to identify the IRFs from a smart beam. Based on the results shown, this procedure can be extremely useful in applications for the identification and modal analysis with the requirement of using IRFs in the time domain. Other applications can be performed with Kautz filter, for example, non-linear system identification of Volterra kernels or non-parametric damage detection. It is also worth while to note that the chosen Kautz pole is directly related to the ability to predict the output behavior of the system. Therefore, future research effort should be made to applications easier and faster in determining the poles of filters. Further methods for choosing the optimal poles need to be improved.

6. ACKNOWLEDGEMENTS

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