# THERMODYNAMICS ANALYSIS OF STIRLING ENGINE

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***Abstract.*** *The present research seeks to study the thermodynamics of the Stirling engine with the goal of using solar energy as a source. Analysis is performed for a numerical simulation for the alpha configuration. The development of the Stirling engine meets the appropriate use of renewable energy, as discussed today. The Stirling engine is theoretically the thermal engine with improved thermal efficiency compared to other cycles of internal combustion engines, such as Otto and Diesel. Your cycle is closed and consists of four thermodynamic processes, two isothermal and two isometric. We present results of numerical simulation of ideal models, isometric and adiabatic. The equations presented here were used to create a mathematical model that resembles the real model. All simulations took into account two types of mechanism of the Ross-Yoke and Schmidt's. According to the findings of the ideal model stand out from the yield of 50.85% and a power output of 4.423 kW. In the isothermal models considering that the pressure is constant inside the engine and using the Ross-yoke mechanism, found a power of 1.418 kW using the Hydrogen on the other hand the results with helium were similar to that makes your viable use due to helium is an inert gas and not harm the environment. The present results show feasibility of using this cycle for energy generation and sustainability for the future.*

***Keywords****:Stirling Engine, Thermodynamics, Simulation.*

## 1. INTRODUCTION

In 1816, Robert Stirling (1816) developed an engine and a regenerative closed cycle regenerative heat exchanger. This closed thermodynamic cycle engine operates according to Figure 1.

The cycle consists of four processes, they were, isothermal compression and expansion and addition and heat rejection isometric as shown in Figure 1.

Considering that an initial assessment to have an engine composed of two opposing cylinders and a regenerator in between. The regenerator in this example acts as a sponge that alternates between absorbing and releasing heat from the gas circulating inside the engine. There are two large volumes, one of them is the volume expansion and the other is volume compression.

The heat will be added to the system by the side of the piston and the expansion side of the piston compression will be maintained at a lower temperature. This temperature gradient directly influences the performance of Stirling cycle engine.

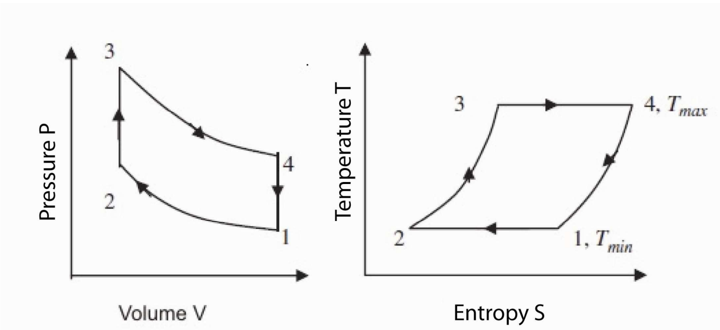


Figure 1. P-V and T-S diagram of a Stirling Engine

## 2. STIRLING CYCLE ENGINE

### 2.1. Processes contained in the Stirling cycle

Process 1-2 (Isothermal compression) - Assumed that the compression piston is at its maximum expansion and the piston is at the minimum. The work done is equal to the magnitude of the heat rejected in the process.

There is no change in the amount of internal energy, but there is a drop in the value of entropy. Using the ideal gas equation where *P* is pressure in kPa, *V* is the volume in m³, *m* is mass in kg, *R* is the gas constant in kJ / kg K, *T* is temperature in K and $r_v$ is the compression ratio we have:

$$PV = mRT \tag{1}$$

$$P_1V_1 = P_2V_2 \tag{2}$$

$$r_v = V_1 / V_2 \tag{3}$$

Heat transferred *Q* = Work done in the compression chamber *Wc*

$$W_c = P_1V_1\ln(1/r_v) = mRT_1\ln(1/r_v) \tag{4}$$

Entropy rate = $s_2$-$s_1$

$$s_2 - s_1 = R\ln(1/r_v) \tag{5}$$

Process 2-3 (Regeneration at Constant Volume) - The fluid is transferred from the amount of compression to expansion through the regenerator. Occurs a gradual increase in temperature $T_{min}$ to $T_{max}$ as the fluid passes through the regenerator and the way the pressure is increased.
No work is carried out during this process, there is an increase in entropy and internal energy of the fluid. In the equations τ is the ratio of temperatures $T_{min} / T_{max}$ and $C_v$ is the specific heat at constant volume in kJ / kg K.

$$P_2 / T_2 = P_3 / T_3 \tag{6}$$

$$\tau = T_2 / T_3 \tag{7}$$

Heat Transferred *Q*:

$$Q = c_v(T_3 - T_2) \tag{8}$$

Work done *W*=0
Entropy rate = $s_3$-$s_2$

$$s_3 - s_2 = c_v \ln(1/\tau) \tag{9}$$

Process 3-4 (Isothermal Expansion) - The piston continues to expand away from the regenerator to the maximum extent while the compression piston remains stationary at its minimum.

The temperature remains constant with the addition of external heat. The work done is equal to the magnitude of the heat supplied. There is variation in the amount of internal energy, but there is an increase in entropy of the fluid.

$$P_3V_3 = P_4V_4 \tag{10}$$

Heat Transferred *Q* = Work done in the expansion chamber $W_e$

$$W_e = mRT_3\ln(rv) \tag{11}$$

Entropy rate = $s_4$-$s_3$

$$s_4 - s_3 = R\ln(r_v) \tag{12}$$

Process 4-1 (Regeneration at Constant Volume) - The pistons move simultaneously transferring the fluid volume expansion to compression through the regenerator. During the fluid flow through the regenerative heat is transferred to the regenerator of the fluid thereby reducing its temperature to *Tmin*. No work is done, there is a drop in the value of internal energy and entropy of the fluid.

$$P_4 \,/\, T_4 \;=\; P_1 \,/\, T_1 \tag{13}$$

Heat Transferred $Q$:

$$Q \,=\, c_v\,(T_1 - T_4) \tag{14}$$

Work done $W$=0
Entropy rate = $s_1$-$s_4$

$$s_1 - s_4 \,=\, c_v\,\ln(\tau) \tag{15}$$

Thus after the analysis concludes that the heat used is equal to $R.T_3.\ ln(r_v)$ and the heat rejected is equal to $R.T_1.ln(r_v)$ and the cycle efficient $\eta_t$ is given by the equation below:

$$\eta_t \,=\, \frac{mRT_3\,\ln(r_v) - mRT_1\,\ln(r_v)}{mRT_3\,\ln(r_v)} \tag{16}$$

As $m$, $R$ e $ln(r_v)$ are constants they can be removed from the equation, such as $T_1 = T_{min}$ and $T_3 = T_{max}$ we have:

$$\eta_t \,= 1 - \frac{T_{\min}}{T_{\max}} \tag{17}$$

Noted that Eq. (17) is equal to the yield equation Carnot cycle. It is known that this model is ideal and that these results are achieved, there must be heat transfer between the walls and the working fluid, in addition to all the processes involved in the cycle and the exchange of heat exchangers made by should be ideal .

## 2.2. Isothermal Analysis

For a better analysis of the Stirling Engine, Uriel Berchowitz (1984) divided the motor into 5 parts which are: C-Space Compression, K-Cooler, R-Regenerator, H-and E-Space Heater Expansion. The volumes that these spaces are among the volumes are already included for simplification. The analysis is always made in relation to the rate of heat transferred to the gas, in other words the area enclosed within the PV diagram.

Considerations:
1 - $P$ is considered constant pressure on the engine;
2 - The temperature in the cooler and compression is constant and in the expansion area and heater is constant and equal respectively to the $T_c = T_k$, $T_h = T_e$;
3 - The total gas mass of the system is constant and equal to the sum of the masses contained in each part of the system.
So using the Ideal gas equation we have:

$$M \;=\; p \,/\, R\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e}\right) \tag{18}$$

The effective temperature of the regenerator can be written as:

$$T_r \,=\, (T_h - T_k) \,/\, \ln(T_h \,/\, T_k) \tag{19}$$

Isolating the value of the instantaneous pressure $p$ in the cycle are:

$$p \,=\, MR\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r\,\ln(T_h \,/\, T_k)}{(T_h - T_k)} + \frac{V_h}{T_h} + \frac{V_e}{T_e}\right)^{-1} \tag{20}$$

The equation given by Senft (1985) for the fluid pressure of work is due to the variation of volume $V_c$ and $V_e$.
Thus all the work produced by the engine in the full cycle is the sum of work done by compression and expansion spaces.

$$W = \oint p\left(\frac{dV_c}{d\phi} + \frac{dV_e}{d\phi}\right)d\phi \tag{21}$$

**2.3. Heat Transfer in the Isothermal Model Engine**

For a better analysis of heat transfer between the heater and cooler is necessary to use the energy equation for gas work. Rallis (1977) created a model of control volume shown in Fig. 2 that can be used in the control volume of workload or volume of heat exchange. The enthalpy is transferred to the control volume is given in terms of mass input $m_i$ and entry temperature $T_i$ and exit mass $m_o$ and the outlet temperature $T_o$. The operator $D$ is used as secondary and $Dm$ refers to the derivative of mass *(dm / dt)*

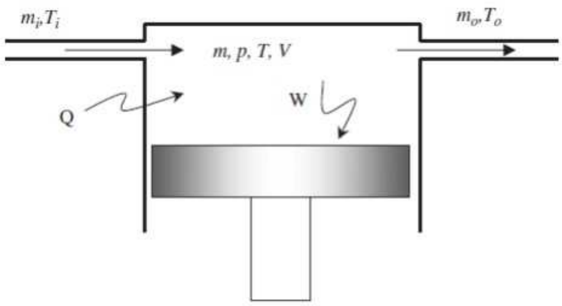


Figure 2 – Control Volume used as a model for C.J.Rallis.

Looking at Fig (2) we have the following equation:

$$DQ + (c_p T_i m_i - c_p T_o m_o) = DW_d + c_v D(mT) \tag{22}$$

Where the values of $c_p$ and $c_v$ are the specific heats at constant pressure and volume respectively.
For a control volume that is in the volumes of compression and expansion, in an isothermal model we have $T_i = T_o = T$, thus:

$$DQ + (c_p T(m_i - m_o) = DW_d + c_v TDm \tag{23}$$

Taking into account the conservation of mass and mass variation between the input and output can be expressed as $DM = m_i\text{-}m_o$ and remembering that the definition of $R=c_p\text{-}c_v$ and integrating we have the equation that defines the heat transfer fluid during the cycle can be defined as:

$$\oint DQ = \oint DW_d + RT\oint Dm \tag{24}$$

Knowing that the *DM* variation that represents the mass variation inside the chamber is zero we have that the variation of heat in the system is transformed into work. Thus the output power will depend solely on the performance of the system limited to the amount of income of Carnot. Creating a volume control on the camera that we have all the heat input less the heat that is lost in the system represents the work output *Wd*.

**2.4. Heat Transfer Model for the Adiabatic Engine.**

In an engine the Isothermal compression and expansion volumes are maintained at constant temperatures. Rankine (1859) proved that this is not possible in practice. This leads to a very important issue where volumes are warmer or cooler are redundant. All the necessary heat transfer occurs across the boundaries of the workload isothermal. So in actual engines, the volume of work tends to be adiabatic rather than isothermal. This implies that the heat exchange fluid through the cycle must be supplied by heat exchangers, as shown in Fig. 3.

We will be considering the model for the analysis of Uriel Ideal, which is composed of 5 parts and considers whether the case exchangers in the refrigerator, heater and regenerator are ideal, so the gas cooler and the heater is kept in isothermal conditions and these temperatures $T_k$ and $T_h$ The workloads are assumed to be adiabatic so the temperatures are $T_c$ and $T_e$ and vary according to the adiabatic nature of the spaces, we note that variation in Fig. 3.

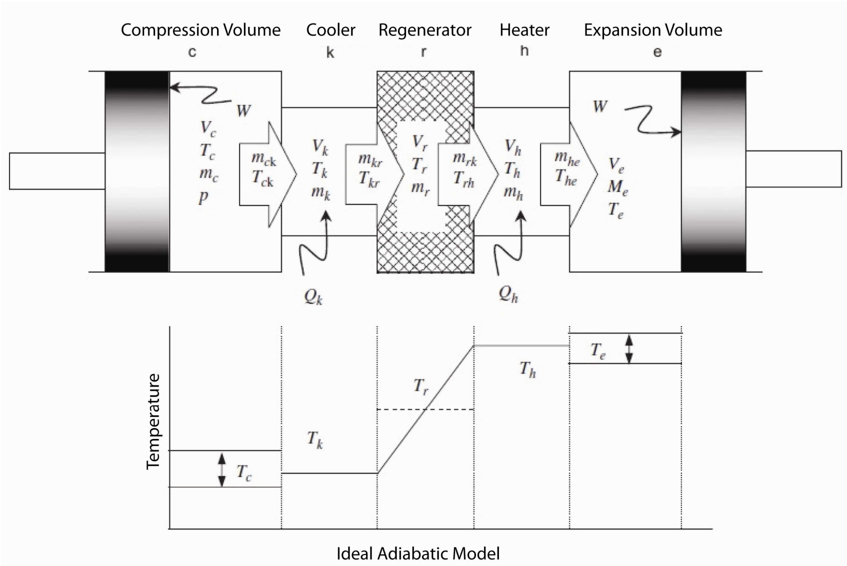


Figure 3 – Ideal model of Adiabatic Stirling engine.

The energy equation for the volume can be written as:

$$DQ + (c_p T_i M_i - c_p T_o m_o) = dW_d + c_v D(mT) \tag{25}$$

Making logarithm on both sides and differentiating is obtained by the differential form of the equation of state:

$$\frac{Dp}{p} + \frac{DV}{V} = \frac{Dm}{m} + \frac{DV}{V} \tag{26}$$

Recalling that the mass of the system is constant and equal to the sum of the masses contained in each of the five volumes. As we have *DV / V* on both sides of the equation can be reduced by applying the three volumes to the cooler, heater and regenerator are:

$$Dm_c + Dm_e + (Dp / R)\left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) = 0 \tag{27}$$

Applying the energy equation for the volume compression are:

$$DQ_c + (c_p T_{ck} m_{ck}) = DW_c + c_v D(m_c T_c) \tag{28}$$

As the amount of compression is adiabatic so *DQc=0*, favoring the work done *DWc=pDVc*, considering the rate of gas accumulation *Dmc* is equal to the mass that enters the gas which is given by $m_{ck,}$ we have:

$$c_p T_{ck} m_{ck} = pDV_c + c_v D(m_c T_c) \tag{29}$$

Substituting the equation of state and ideal gas in the volumes of compression and expansion and simplifying we have:

$$Dp = \frac{-\gamma\, p((DV_c / T_{ck}) + (D_{ve} / T_{he}))}{[V_c / T_{ck} + \gamma(V_k / R_k + V_r / T_r + V_h / T_h) + V_e / T_{he}]} \tag{30}$$

Where:

$$\gamma = \frac{c_p}{c_v} \tag{31}$$

We observe that Eq. (30) is due to or *pe mc me*. The missing variables can be found with mass balance and the equation of state. Temperatures interface $T_{ck}$ e $T_{he}$ they are conditional depend on the direction of mass flow. In order to evaluate the mass flow and thus the direction of flow, we will be using the continuity equation.

## 3. RESULTS AND NUMERICAL ANALYSIS OF STIRLING ENGINE

To perform the simulation took as basis a Stirling engine of the article: Analysis and design consideration of mean temperature differential Stirling engine for solar application, authoring Iskander Tlili, Youssef Timoumi, Sassi Ben Nasrallah (2008). By having the same application of this work, power generation using solar energy. Below is the data from the article:

Mean pressure of work: 8,7 bar(870kPa)
Swept Volume:75cm³
Volume of heat exchangers:
Heater:165cm³
Cooler:165cm³
Freqüency :75hz
Fluid :Hydrogen

Power Output: 256,77W
Thermal Efficiency : 48,11%
Working Temperature:
Tc: 390K
Th: 590K
Phase Angle: 90º

According to the collected data and equations related to the mechanism used to model the Ross-Yoke. Using the equations presented above, the following situations were simulated: Ideal Model, Model Isothermal, Adiabatic Model. The simulations used two fluids, helium and hydrogen which are more commonly used in high performance Stirling engines.

**Table 1 – Characteristics of heat transfer fluid used.**

| Fluid | Heat Transfer Coefficient | Capacity Factor | Molar Mass (Mkg/kmol) | Gas constant R (kJ/kg.K) | Cp (kJ/kg.K) | Cv (kJ/kg.K) |
|---|---|---|---|---|---|---|
| Ar | 1,00 | 1,00 | 29,00 | 0,29 | 1,01 | 0,72 |
| H2 | 3,42 | 0,68 | 2,00 | 4,12 | 14,20 | 10,08 |
| He | 1,42 | 0,83 | 4,00 | 2,08 | 5,19 | 3,11 |

Helium is a monatomic gas, inert, stable, high specific heat and high thermal conductivity, as a composite of the two hydrogen atoms has a thermal conductivity higher and have a higher specific heat, has less than the mass of helium as a disadvantage but it is flammable.

### 3.1 Results found with the simulation of the Ideal Model.

From the data found as compression ratio of 1.123 and equations of the volumes to the angle of the crankshaft can start the simulation model using ideal fluid chosen. According to the results, to find both a fluid yield of 50.85%. In the ideal model there are many considerations which are independent of the type of fluid and make the income directly related to the temperatures used. The yield in the case of the ideal model is given by equation yields Carnot. It was noted that regardless of the fluid used is the power output of 4.423 kW for a frequency of 75 Hz

### 3.2 Results found with the simulation of the Isothermal Model.

According to the considerations taken in topic 2.2 and equations designed to simulate this condition, it was simulated model of a Stirling engine isothermal. The results are shown in Fig. (4) to (12). According to the results we highlight the average pressure in the cycle of 673.3 kPa for the analysis of Schmidt's and 702.6 kPa for the Ross Yoke mechanism, regardless of the used fluid output powers are 1.1 kW Schmidt's and Ross

Yoke 1.418 kW for a frequency of 75Hz. In conclusion, both fluids can be used and has satisfactory results, using a choice for security issues and operations.

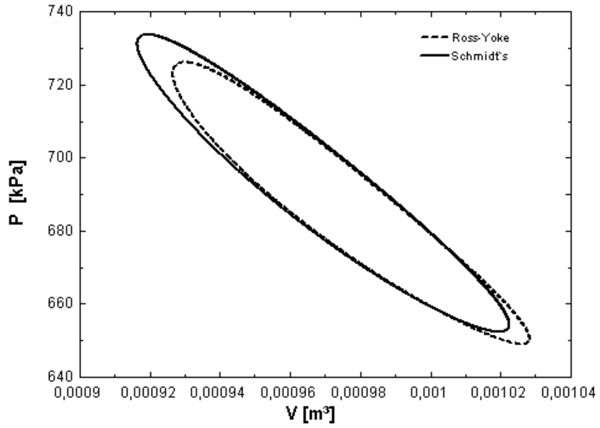


Figure 4 – *PV* diagram of the Stirling engine by comparing the analysis of Schmidt's and Ross Yoke mechanism, using helium and hydrogen.

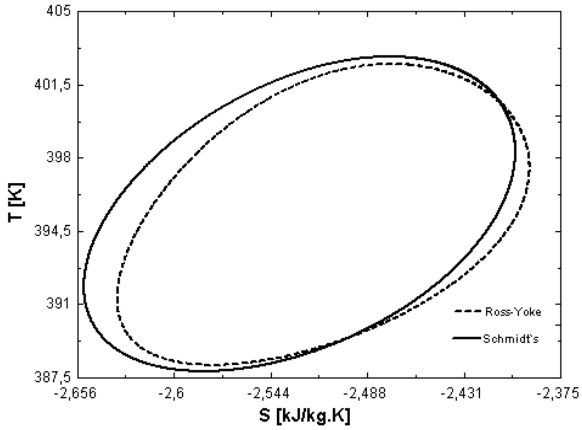


Figure 5 – *T-S* diagram using Helium.

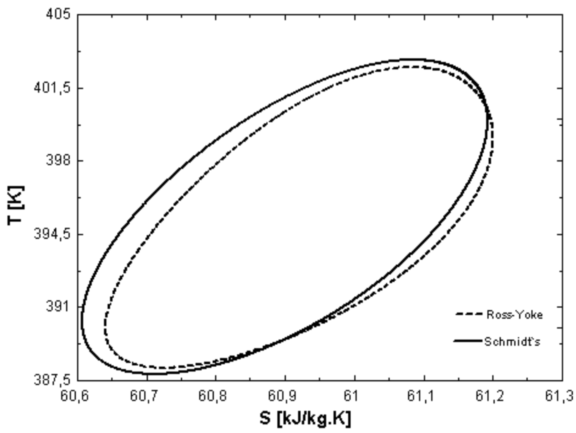


Figure 6 – *T-S* diagram using hydrogen H2.

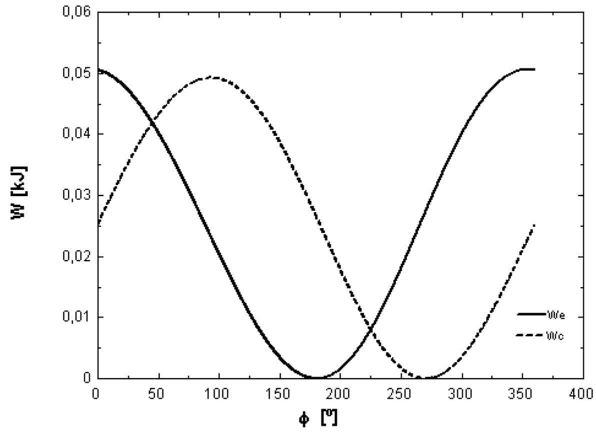


Figure 7 –*W*- $\phi$ diagram representing the values of entries in each chamber.

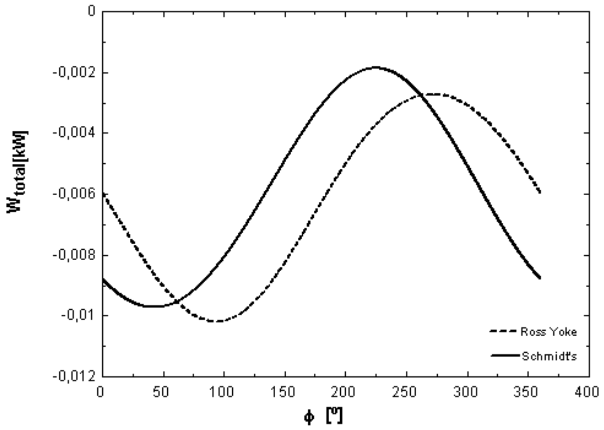


Figure 8 –*Wtotal* – $\phi$ diagram representing the values of the two mechanisms used.

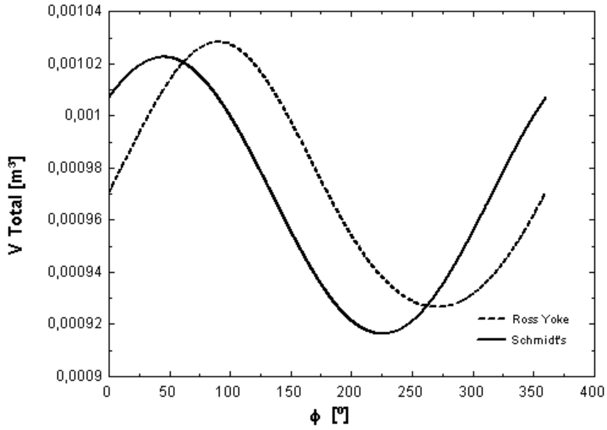


Figure 9 – *V total* – $\phi$ diagram representing the values obtained in the two types of mechanisms.

 

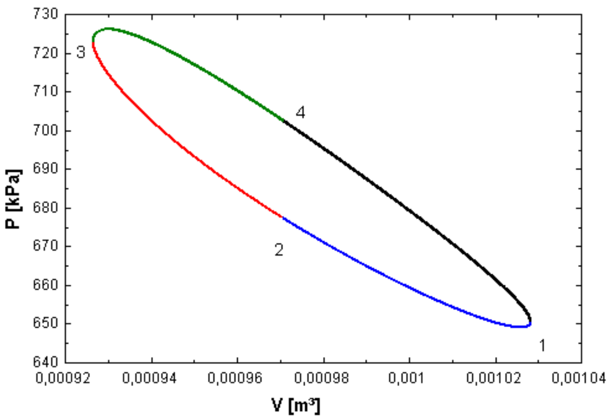


Figure 10 – *P-V* diagram showing the main points of the cycle.

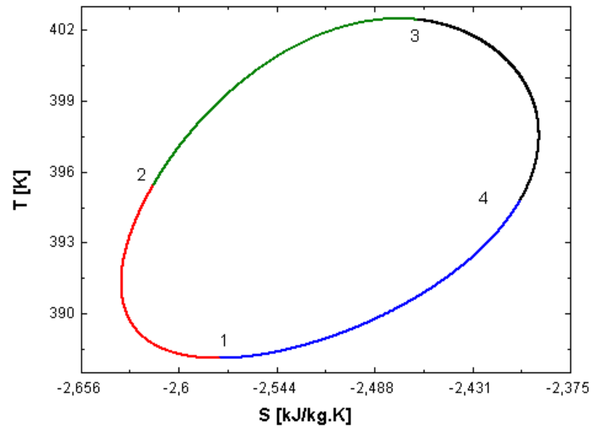


Figure 11 –*T-S* diagram using helium representing the main points of the cycle.

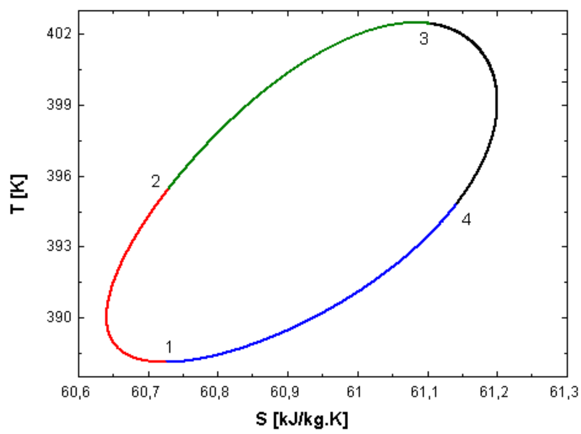


Figure 12 –*T-S* diagram using Hydrogen representing the main points of the cycle.

**3.3 Results of heat transfer in the Isothermal Model.**

According to the topic 2.3, one can make some considerations regarding the change of mass within the control volume CJRallis. Whereas the mass entering is equal to the mass coming out, we have all the heat transferred to the system is equal to output power more heat loss. So for a motor with 50.85% efficiency means that for every 50.85 W 49.15 W output have been lost. Although high values of the Stirling engine has lost income higher than that found in other heat engine.

**3.4 Results of the simulation of heat transfer in Adiabatic Model.**

Note that the equations relating the output power in isothermal and adiabatic model are equal and directly proportional to pressure for change in volume. Adiabatic process in the Stirling engine is divided into five parts thus making the analysis more complete. With this the process is also assessed in relation to the direction of gas flow inside the engine. According to the equations and the simulation results as we have the following diagrams.

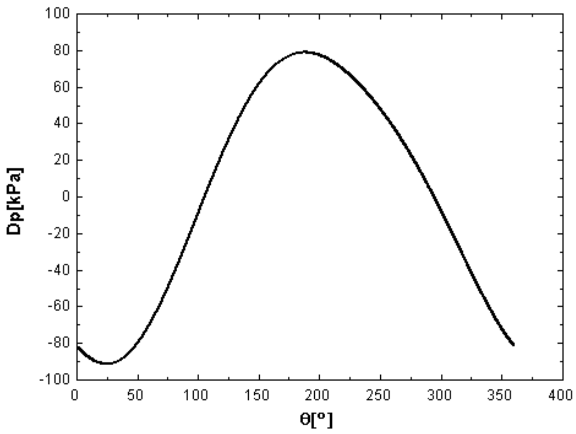


Figure 13 – *Dp* - $\theta$ diagram using helium and hydrogen as fluids.

As we can see there is a pressure variation in the cycle that is directly linked to the variation of the total volume of the engine.

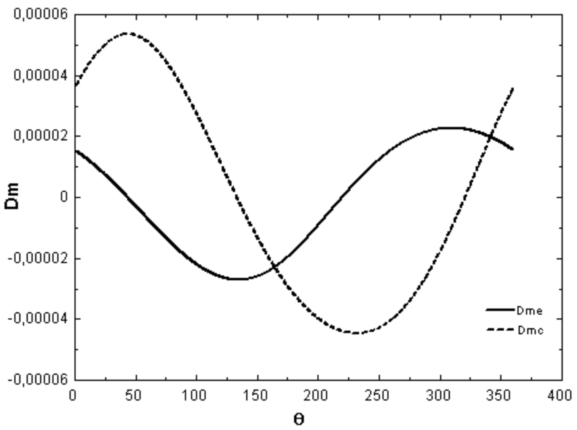


Figure 14– *Dm* - $\theta$ diagram fluid used helium and hydrogen

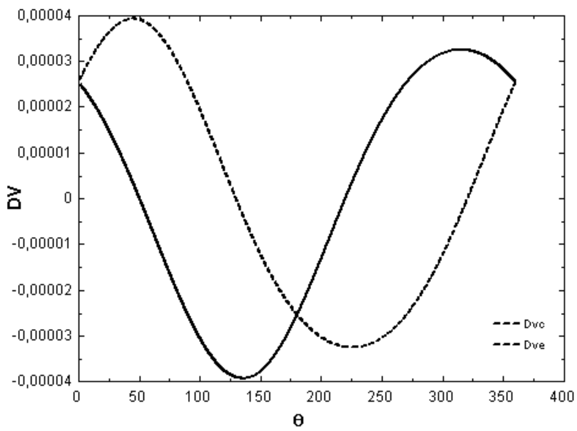


Figure 15 – *DV* - $\theta$ diagram fluid used helium and hydrogen.

## 4. CONCLUSIONS

This work aimed to study the Stirling engine, its main characteristics, thermodynamic processes involved, processes and simulation models covered. Among the findings in the simulations we can highlight the importance of knowledge of each of the variables and their influence on the functioning of the Stirling engine as a whole, the models adopted for the simulation were satisfactory getting the results as expected. Because the simulation does not rely on actual data for its validity it is necessary to make a more rigorous analysis, especially in the design and simulation of the regenerator. The regenerator is the part mainly responsible for obtaining an efficient engine, and a detailed study on the distribution of temperature and pressure throughout the engine, it is necessary. Thus making the simulation closer to a real model, however, this work is to be a useful tool for future work and research related to Stirling engines and their possible applications.

According to the results of the ideal model in the simulations we can see that the output powers are equal regardless of the working fluid used, either helium or hydrogen. According to these results the use of helium as the working fluid is more advisable because it is an inert gas that will not cause damage to equipment or whoever is operating. In this simulation we find the same value of 50.85% yields a high value compared to income from internal combustion engines that are between 20 and 35%, that if we stick to the values of ideal simulation. As we know the values in an ideal simulation differ from those found in practice. In the simulation model we can see the two isothermal fluids continue to exhibit the same behavior. The Ross-yoke mechanism provides greater power output at around 28.8% over According to Schmidt's models for the simulation and their special considerations, we can see any major difference between the ideal and the real model. Based on the results in the transfer of heat in the isothermal model we note that much of the energy supplied to the system is dissipated to the environment, yet it is known that the maximum income may not exceed the value of Carnot and that this waste is inevitable to increase further should focus on project and create new mathematical models closer to reality. In modeling an adiabatic system can notice a change in system pressure which could not be noted in the isometric system as in the isometric system we consider a constant pressure throughout the system. The values of mass change in each of the rooms are large and have these values due to the temperature of each of the cameras that constantly changes according to the angle of the crankshaft of the engine, therefore we can see the same situation in relation to the volume of each one of the chambers.

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